Spin interferometry and entanglement generation with Rashba spin splitting

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and

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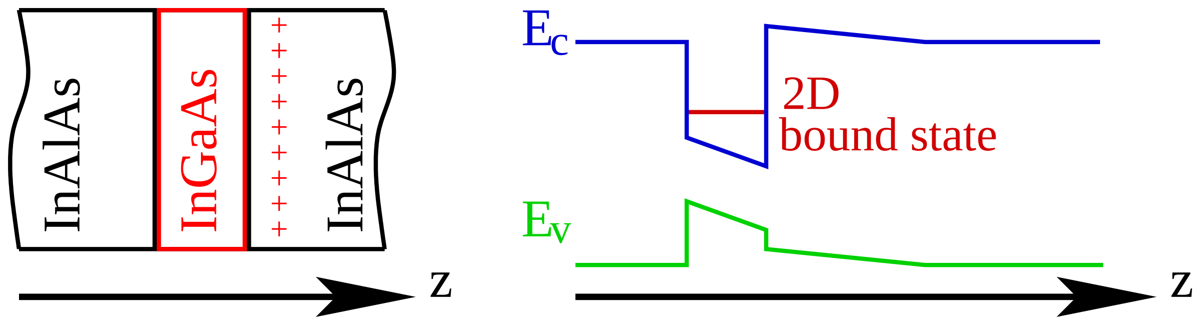
Outline

- Introduction
  - Rashba spin splitting: basics
  - electron–wave interference
- Single–electron interference at a Mach–Zehnder interferometer with Rashba spin splitting
  - theory: spin–resolved scattering matrix
  - results & applications
- Two–particle interference and entanglement generation
  - spin–dependent two–particle scattering matrix
  - entanglement from projective charge measurement
- Conclusions

Rashba spin splitting: Basics

- band bending in heterostructures: 2D electron system



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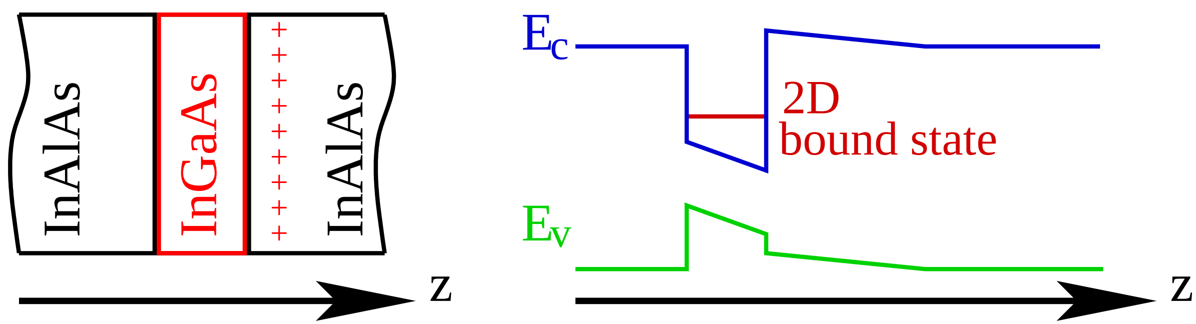

- structural inversion asymmetry $\Rightarrow$ spin–orbit coupling

$$H_{\mathrm{so}} = \frac{1}{2m_*}\left[\vec{p}\times\hbar\vec{\nabla}\left(\frac{V_{\mathrm{ext}}}{2E_{\mathrm{g}}}\right)\right]\cdot\vec{\sigma}$$

(for small band gap E_{g})

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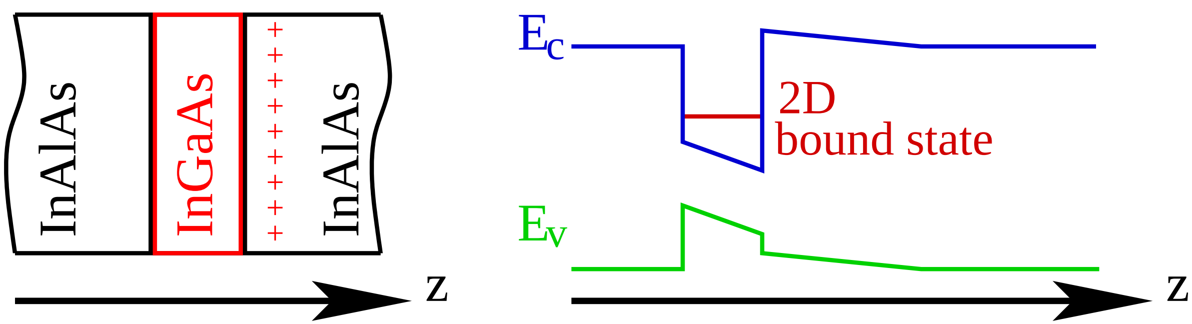

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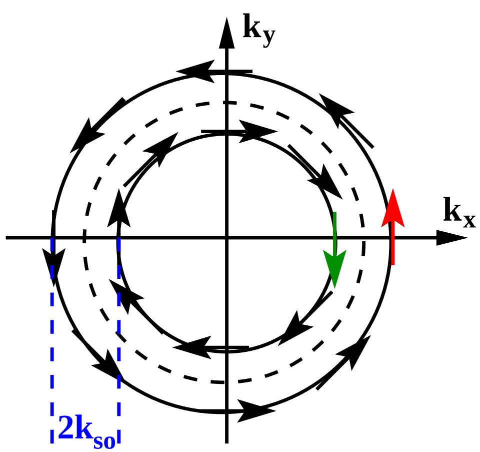
$H_\mathrm{R} = \frac{\hbar}{m} k_\mathrm{so}\, \hat{z} \cdot [\vec{\sigma} \times \vec{p}]$, w/ $k_\mathrm{so} = \frac{\pi}{L_\mathrm{so}}$ tunable by gate voltage



Fermi surface splits into two; electron spin ⊥ momentum $\hbar\vec{k}$; spin precesses

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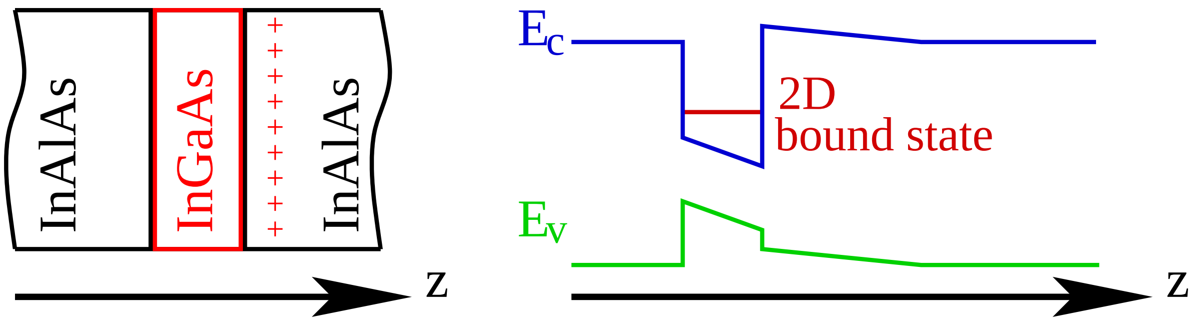

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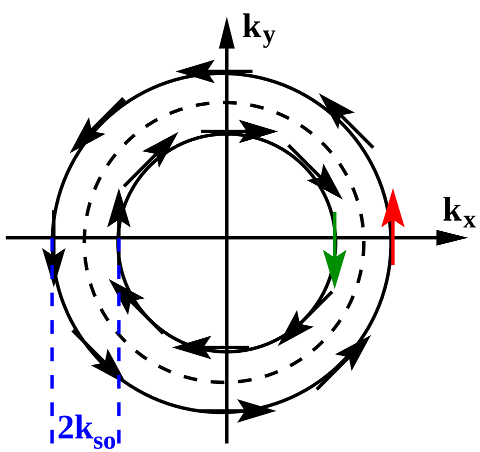

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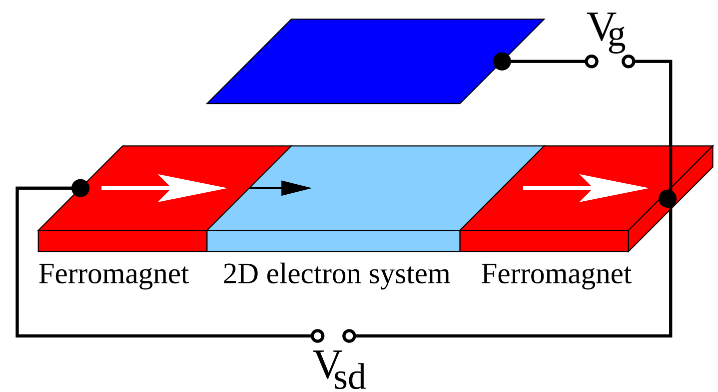

(Datta & Das, Appl. Phys. Lett. 1990)

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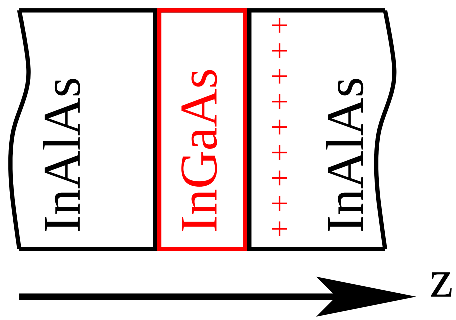

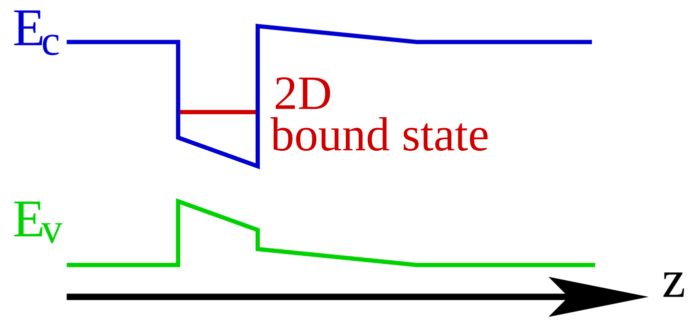

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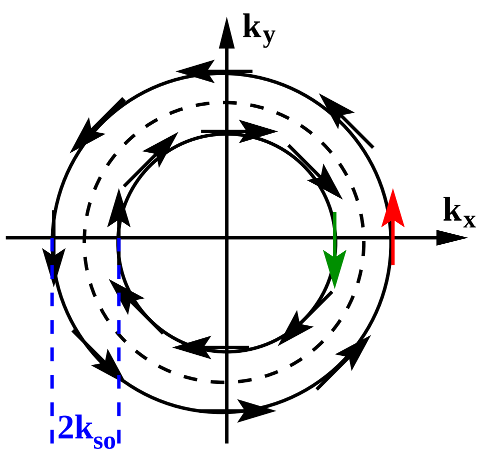

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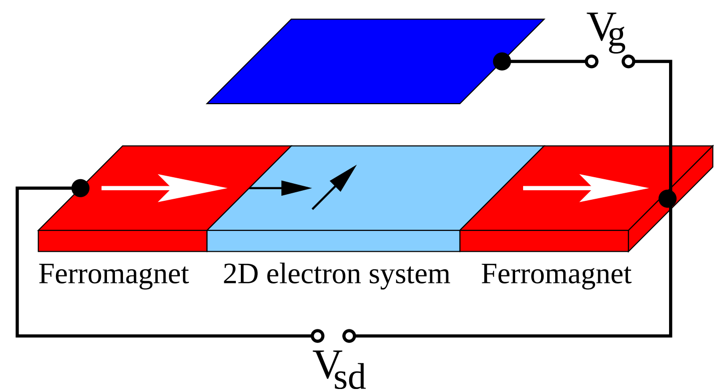

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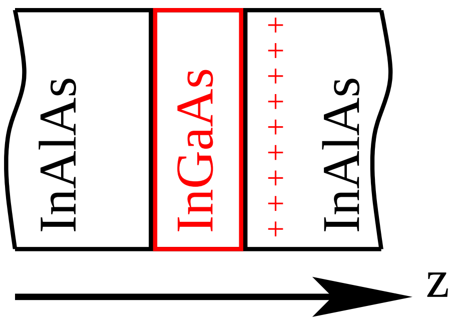

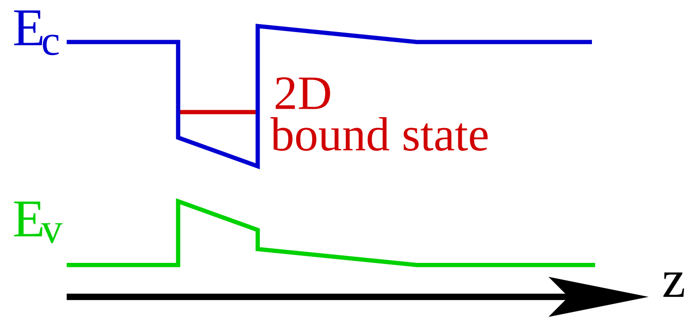

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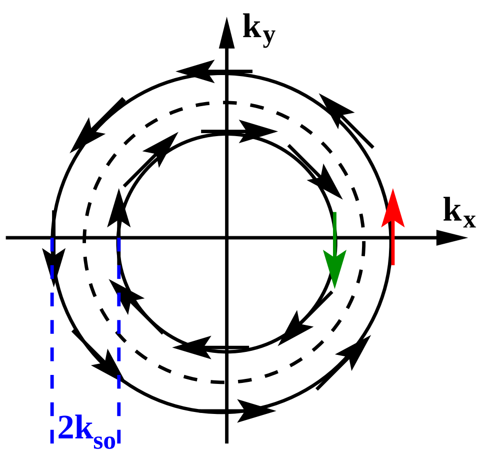

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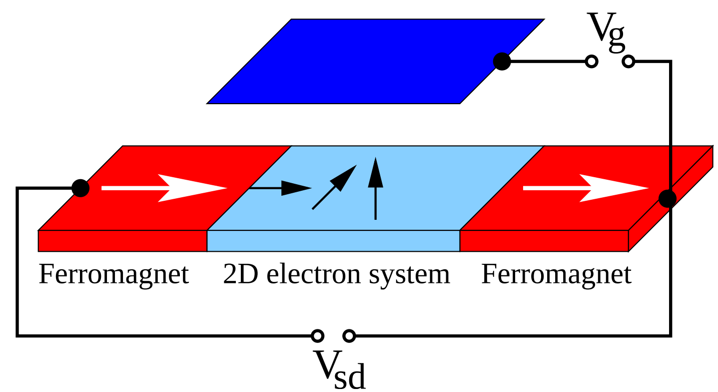

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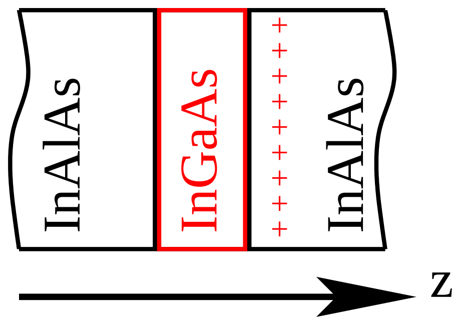

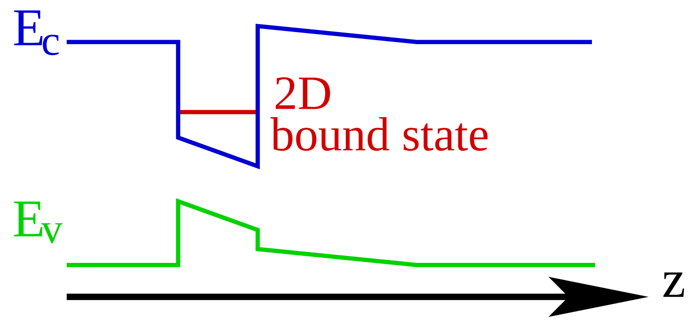

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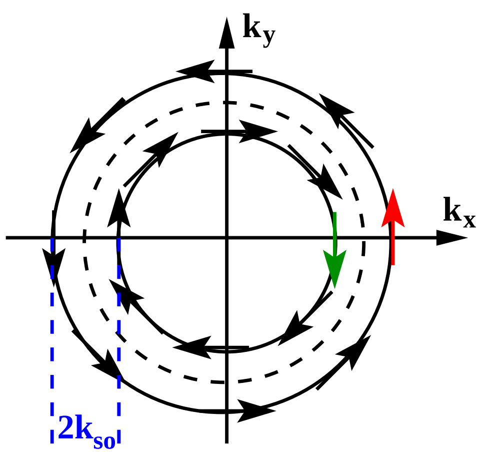

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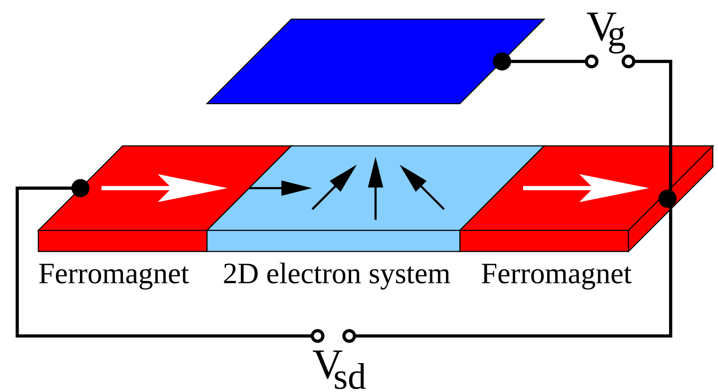

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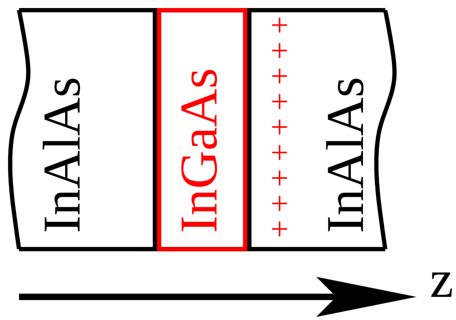

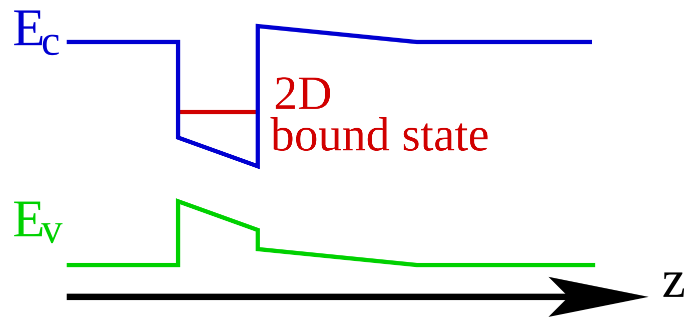

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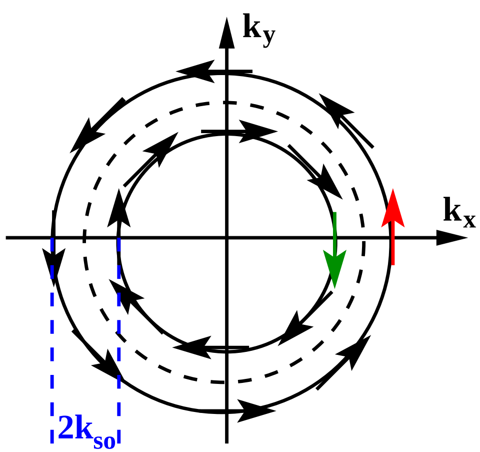

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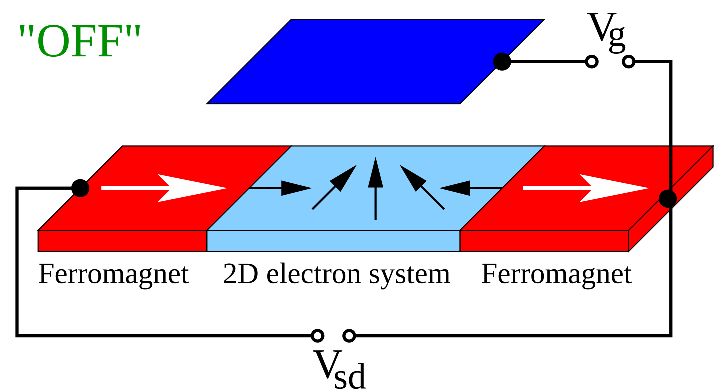

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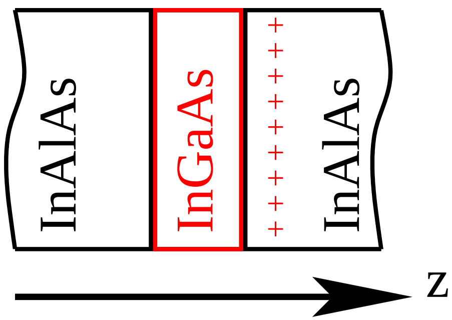

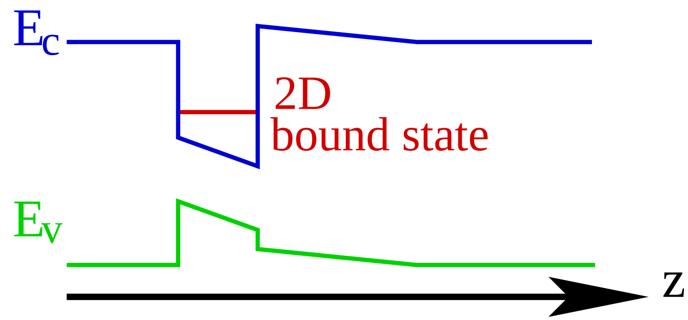

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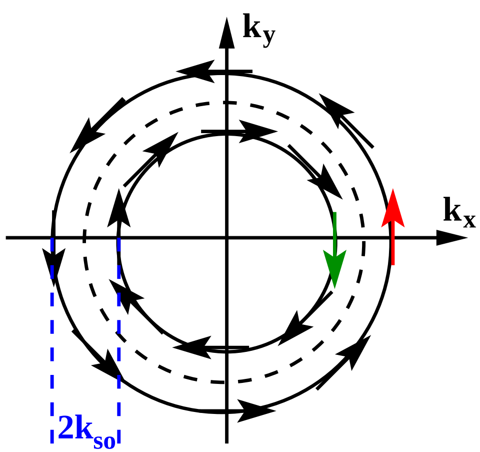

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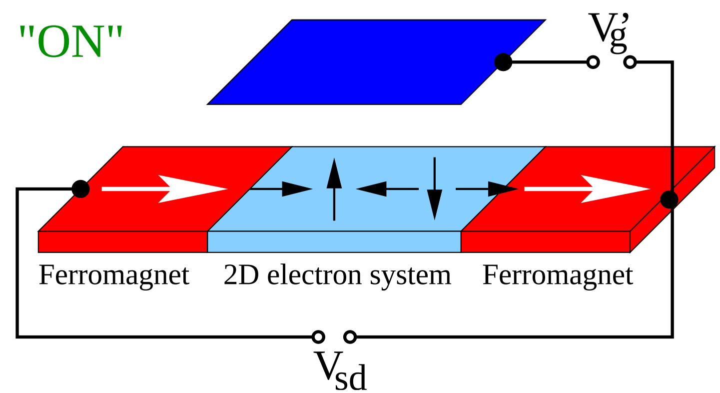

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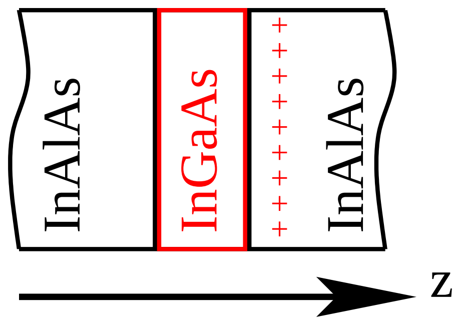

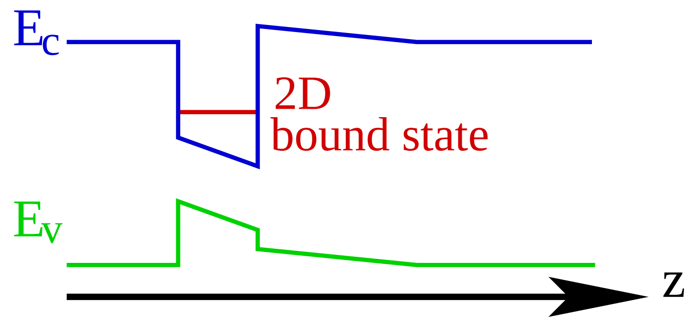

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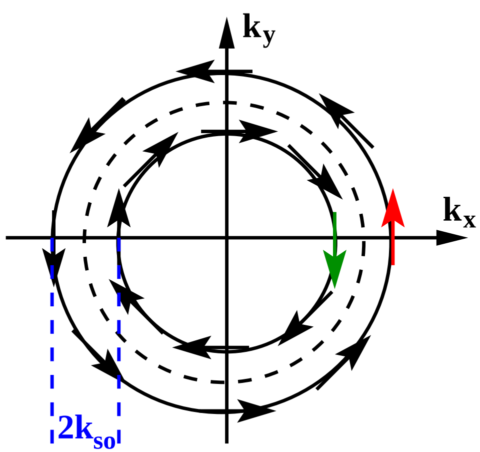

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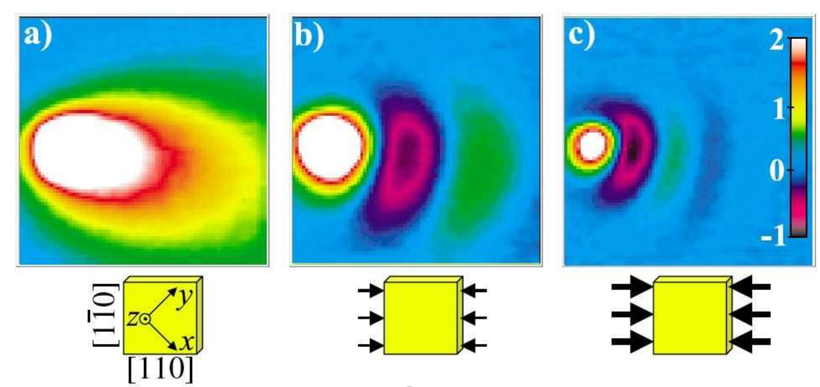

(Crooker & Smith, Phys. Rev. Lett. 2005)

Interference of electron waves

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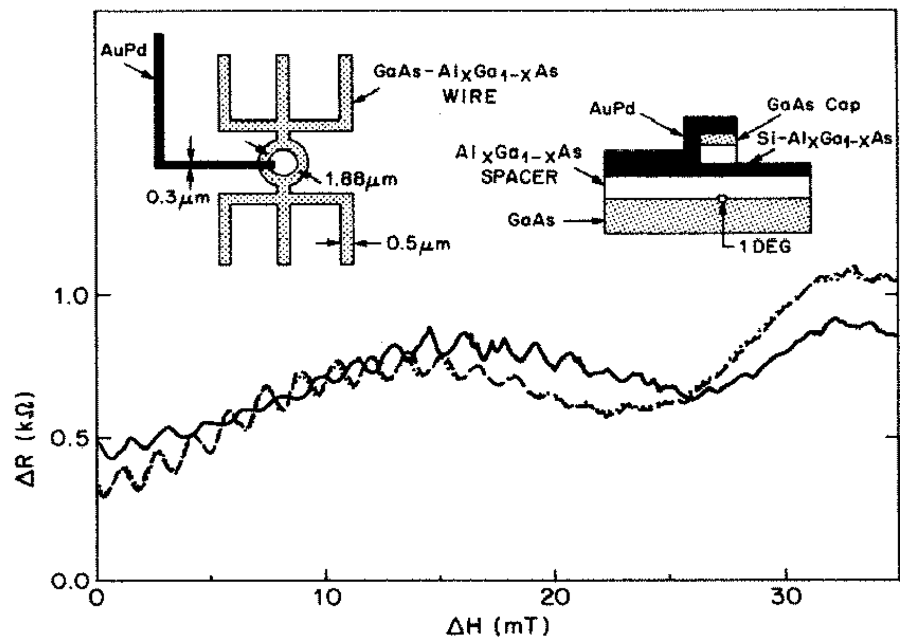
- celebrated examples: Aharonov–Bohm oscillations



(de Vegvar et al., PRB 1989)

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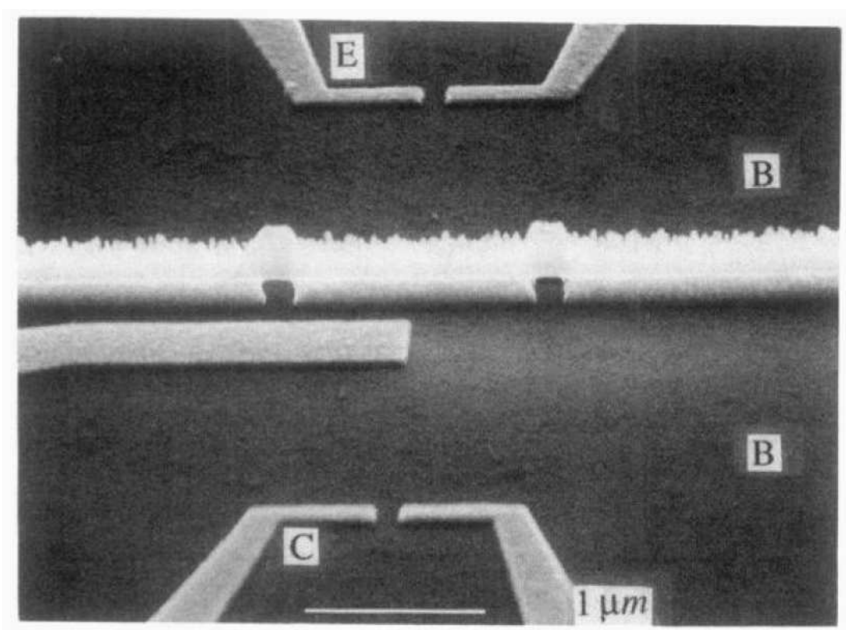
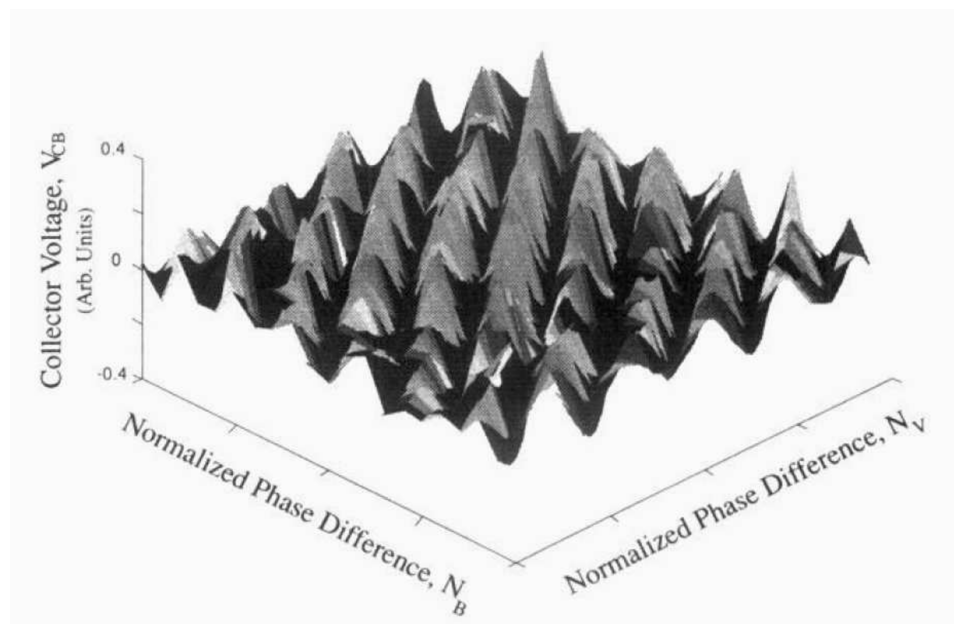
- celebrated examples: Young's Double–Slit experiment





(Yacoby et al., PRL 1994)

Interference of electron waves

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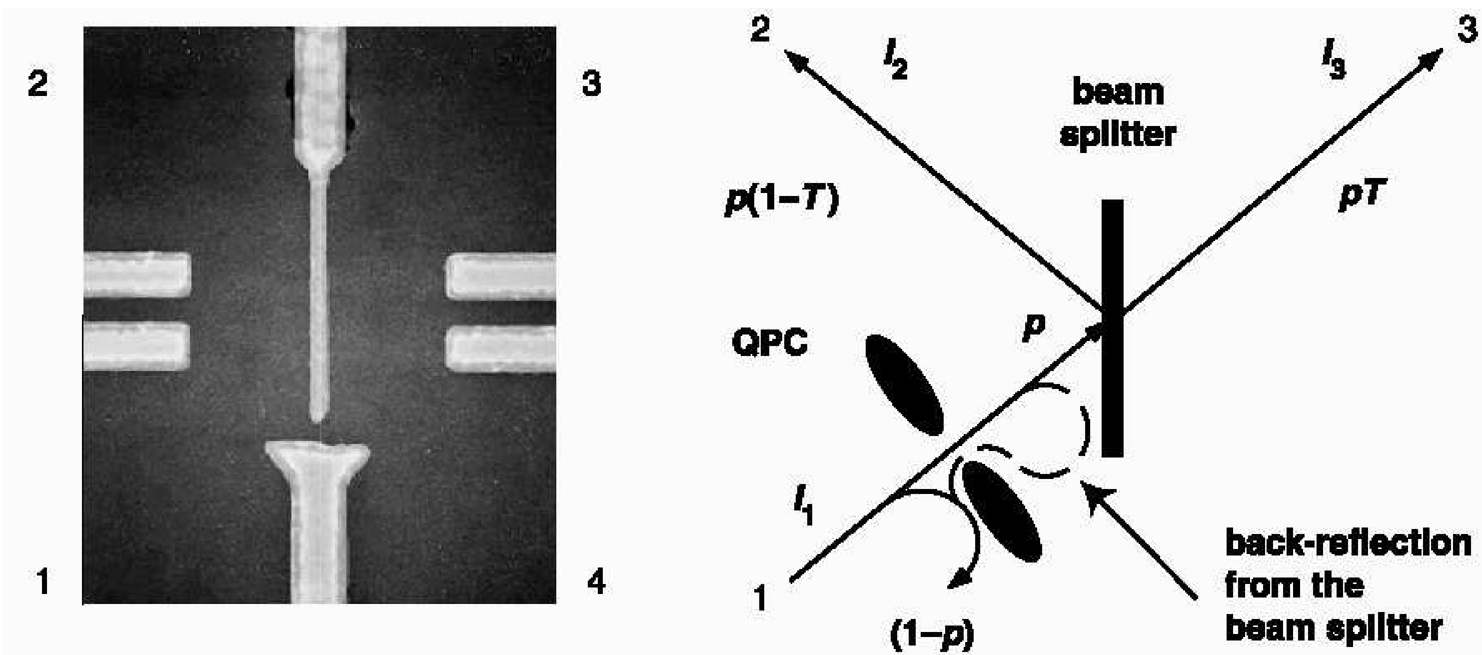
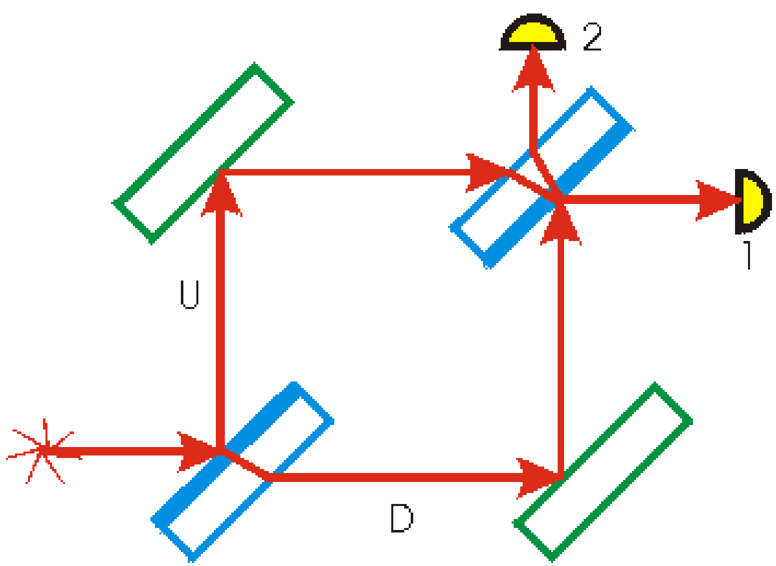
- phase–coherent electronics: conductance modulation (e.g., as function of gate voltage or magnetic field)
- celebrated examples: Hanbury-Brown and Twiss expt.



(Oliver et al., Science 1999)

Mach–Zehnder interferometer

- invented in optics for measuring 1^{st}–order coherence



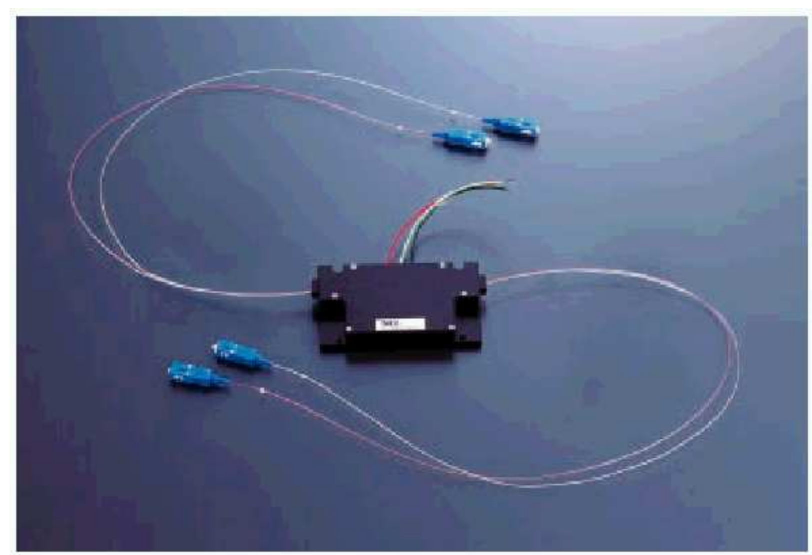
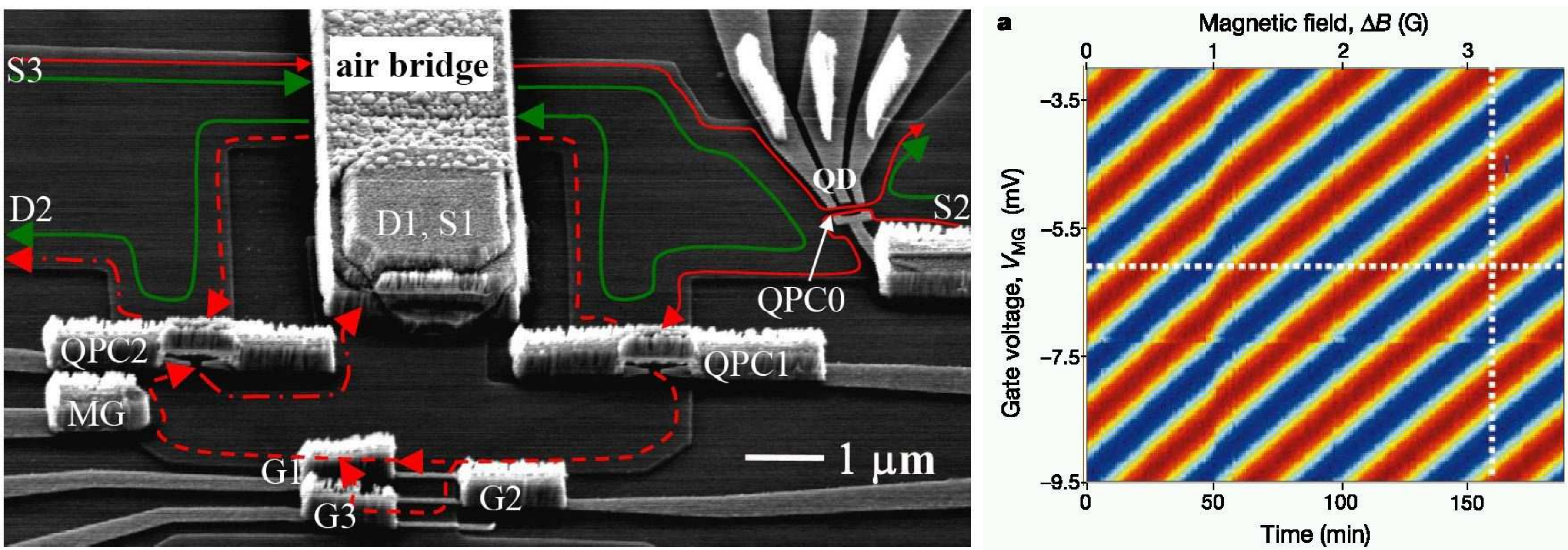


Illustration: D. M. Harrison, U Toronto

Photo: Website of NTT Electronics Corp.

Mach–Zehnder interferometer

- invented in optics for measuring 1^{st}–order coherence
- an electronic version was only recently realized using quantum–Hall edge states (Ji et al., Nature 2003)



(see also Neder et al., Phys. Rev. Lett. 2006)

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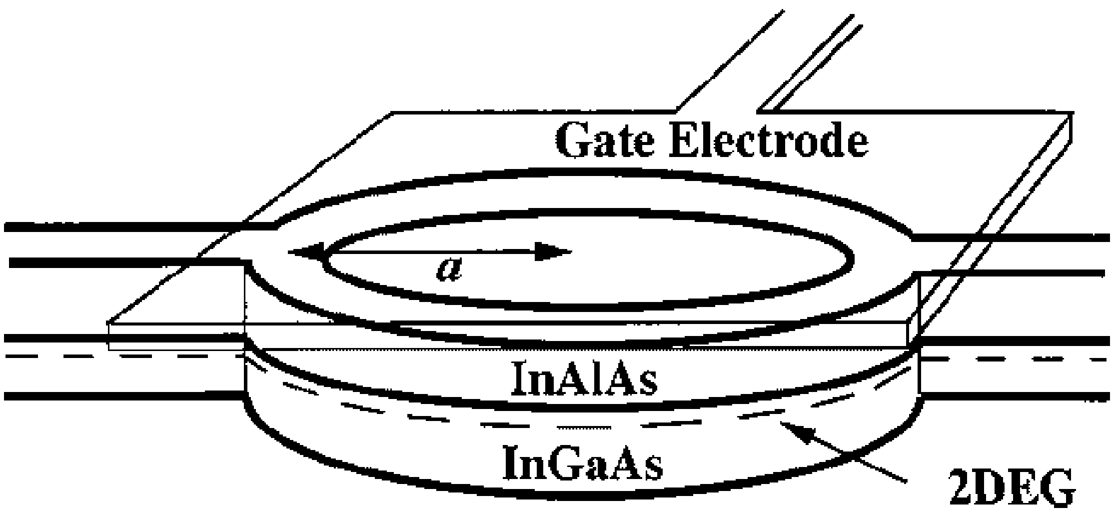
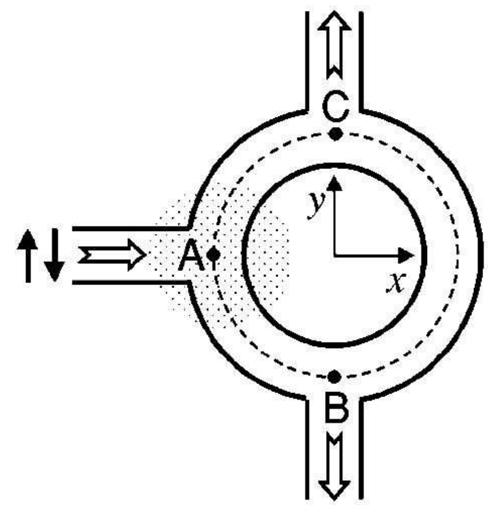


(Nitta et al., Appl. Phys. Lett. 1999)



(Kiselev & Kim, J. Appl. Phys. 2003)

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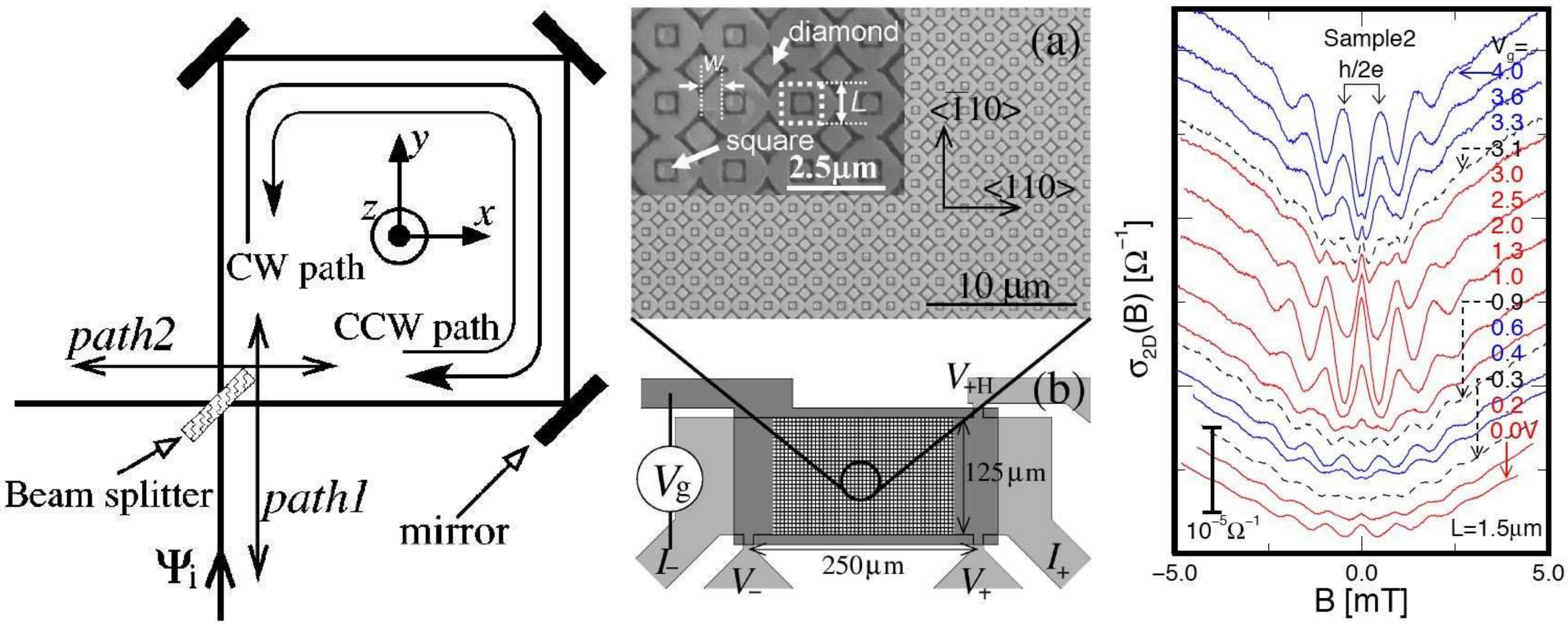
- recent experiments: Altshuler-Aronov-Spivak oscillations



(Koga et al., Phys. Rev. B 2004, see also cond-mat/0504743)

Spin–dependent electron interferometry based on Rashba spin–orbit coupling

Interferometer + spin splitting

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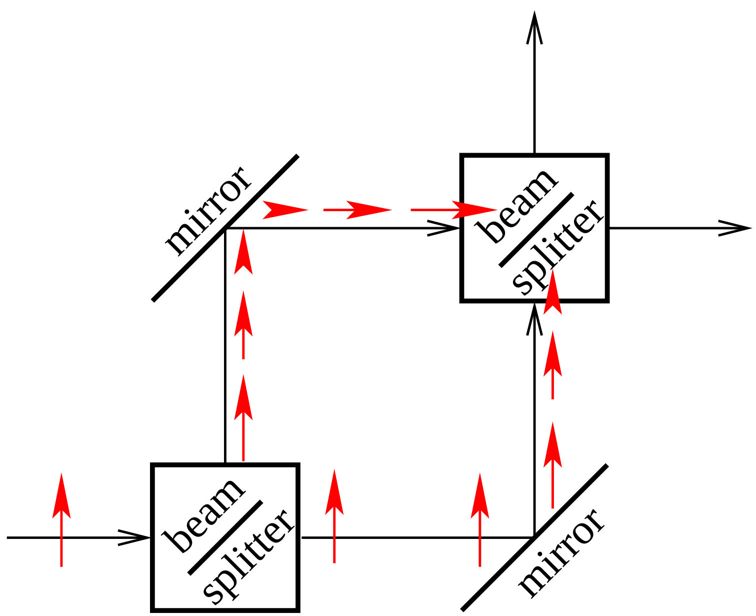
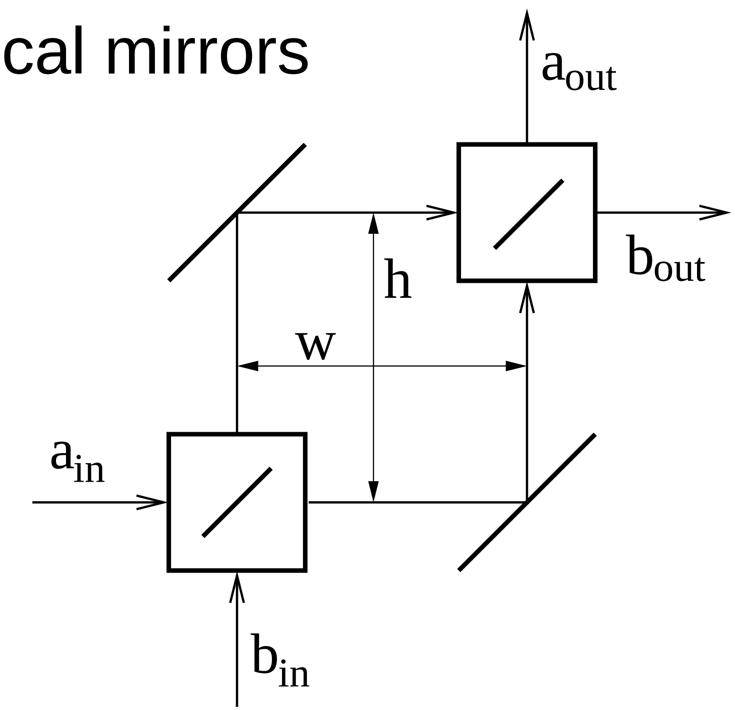
- spin-dependent feature: tuneable Rashba spin splitting
  (Nitta et al., Phys. Rev. Lett. 1997; Engels et al., Phys. Rev. B 1997)
- our model: single 1D subband, symmetric and identical beam splitters, perfect and identical mirrors





Theoretical description

- elements characterized by their 4×4 spin–resolved scattering matrix, use basis of spin–split eigenstates

$$\begin{pmatrix} a_{\text{out}+} \\ a_{\text{out}-} \\ b_{\text{out}+} \\ b_{\text{out}-} \end{pmatrix} = \underbrace{\begin{pmatrix} r_{1++} & r_{1+-} & t_{2++} & t_{2+-} \\ r_{1-+} & r_{1--} & t_{2-+} & t_{2--} \\ t_{1++} & t_{1+-} & r_{2++} & r_{2+-} \\ t_{1-+} & t_{1--} & r_{2-+} & r_{2--} \end{pmatrix}}_{\mathcal{S}} \begin{pmatrix} a_{\text{in}+} \\ a_{\text{in}-} \\ b_{\text{in}+} \\ b_{\text{in}-} \end{pmatrix}$$

Theoretical description

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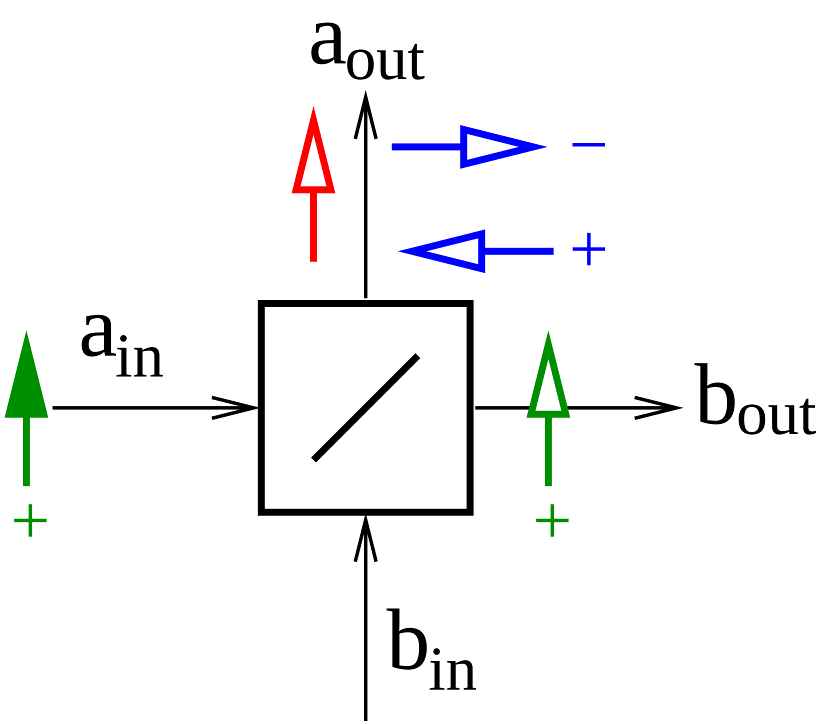
- spin conservation at beam splitter / mirror: incident basis state reflected into mixture of basis states in outgoing arm



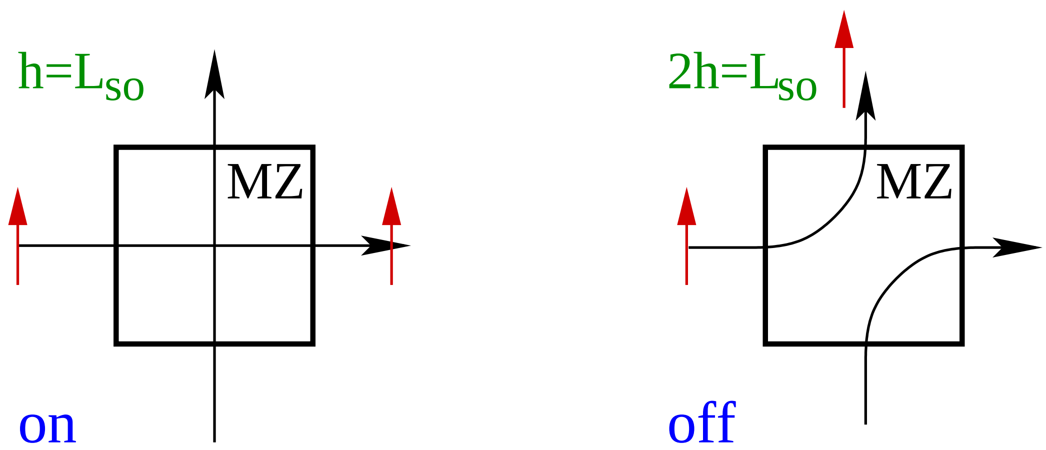
$$\mathcal{S}_{\mathrm{bs}} = \begin{pmatrix} \frac{i}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{i}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{2} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{i}{2} \end{pmatrix}$$

(Feve et al., Phys. Rev. B 2002)

Applications: Single–input manipulation

UZ, Appl. Phys. Lett. **85**, 2616 (2004)

- spin transistor (switch) without magnets: when $w = h$



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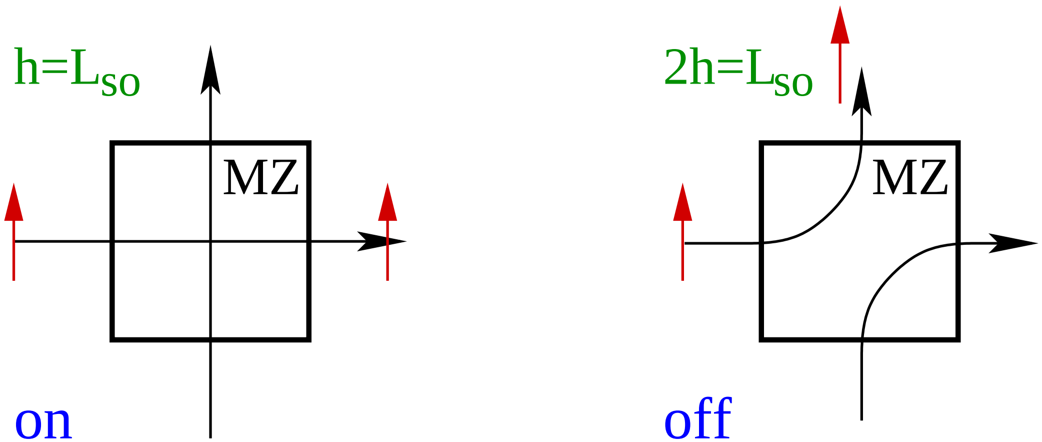
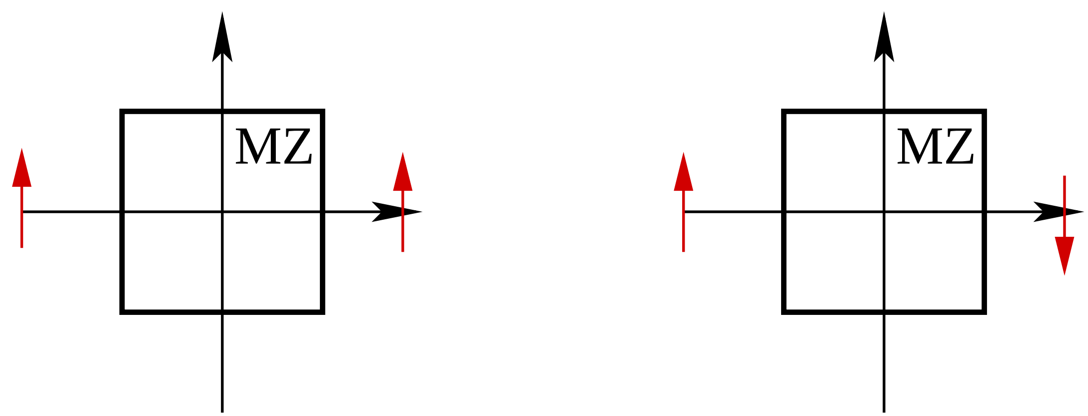


- single–qubit rotations (quantum negator when $h = w/2$)



Spin–dependent linear electron optics and quantum information processing

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- quantum mechanics: electrons are waves, too! $\Rightarrow$ can we realise flying qubits in electronic spin degree of freedom?
  - no efficient quantum computation possible with linear fermion optics only (Terhal & DiVincenzo, Phys. Rev. A 2002)

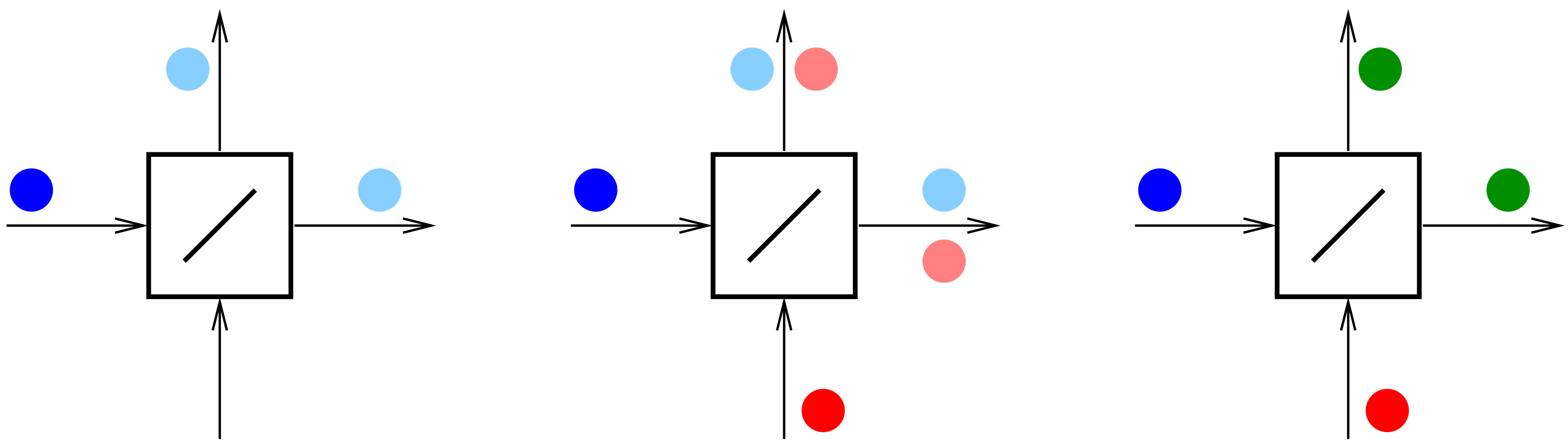
Linear optics & QIP

- efficient quantum computing can be achieved using linear photon optics (Knill, Laflamme & Milburn, Nature 2001)
  - photon = flying qubit in its polarisation degree of freedom (Milburn, Phys. Rev. Lett. 1988)
  - need only single–photon source + photodetectors, plus linear devices (beam splitters & phase shifters)
- quantum mechanics: electrons are waves, too! $\Rightarrow$ can we realise flying qubits in electronic spin degree of freedom?
  - no efficient quantum computation possible with linear fermion optics only (Terhal & DiVincenzo, Phys. Rev. A 2002)
  - charge detection enables quantum computation with linear fermion optics (Beenakker et al., Phys. Rev. Lett. 2004)

Single vs. two–electron input

- Fermi statistics affects scattering, e.g., at beam splitters

(Loudon, Phys. Rev. A 1998)



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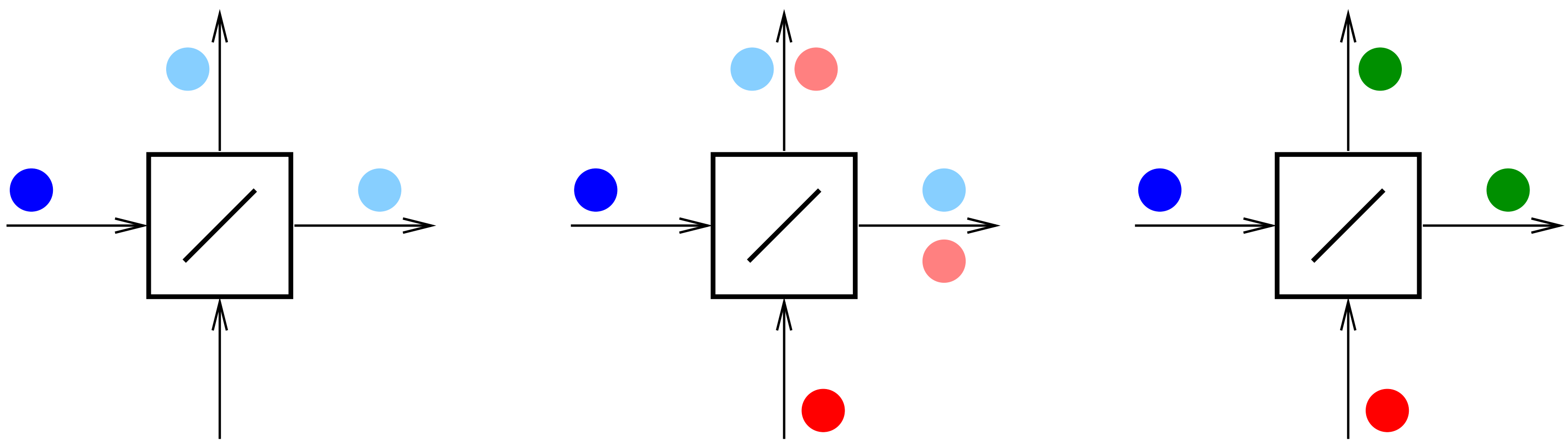
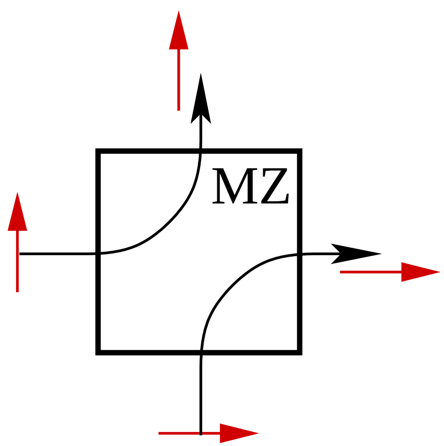


- at Mach–Zehnder interferometer: several scenarios



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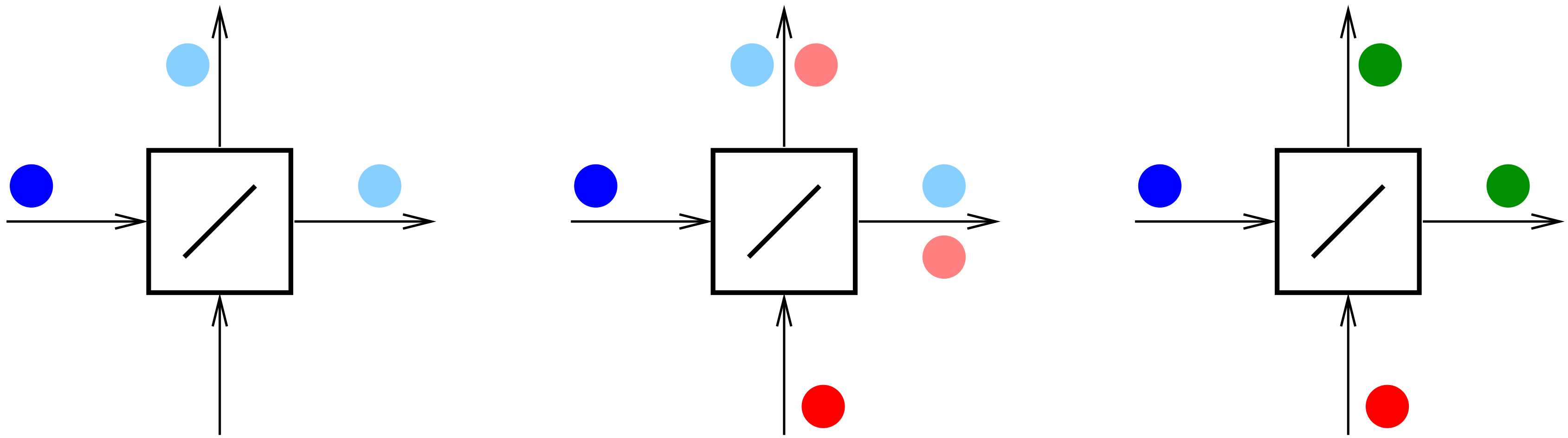

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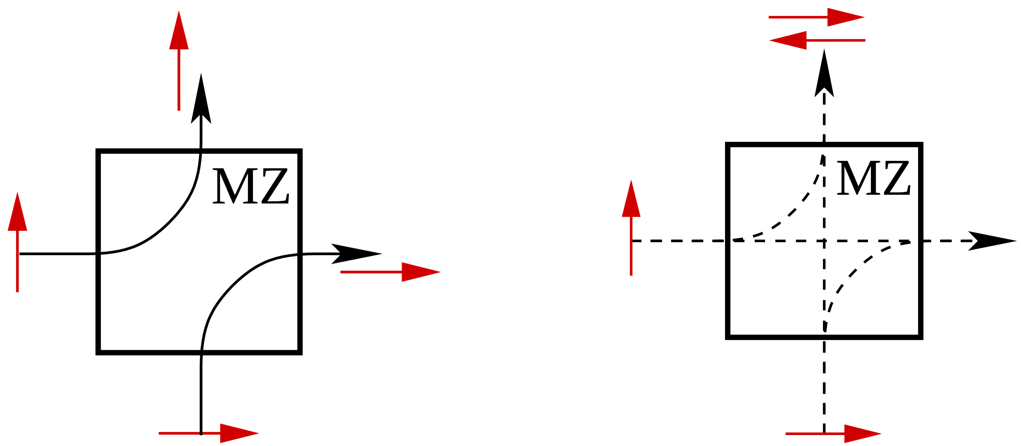

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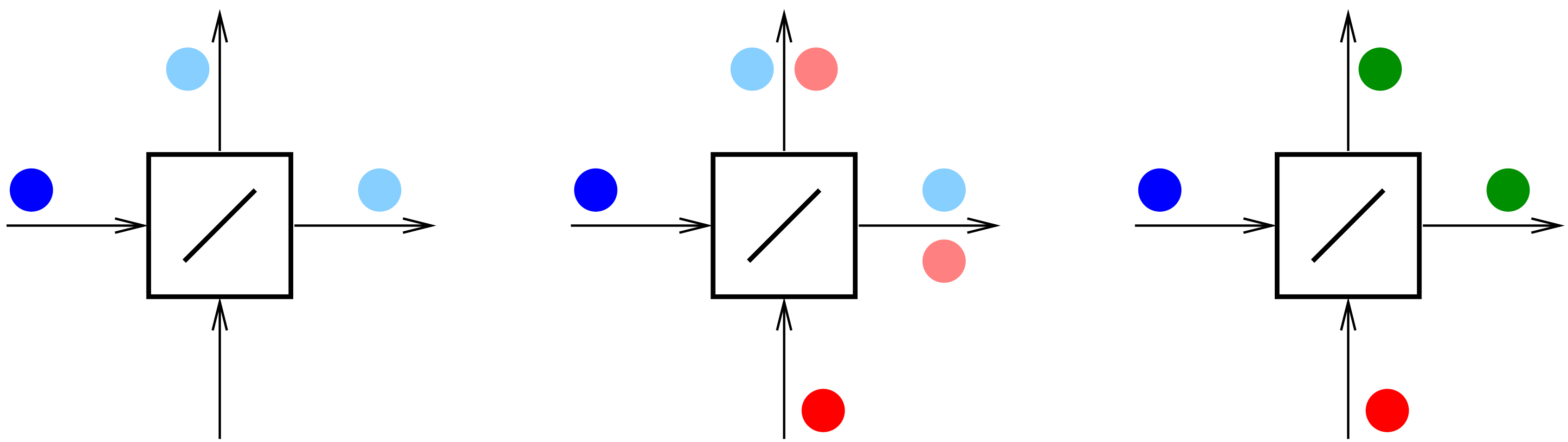

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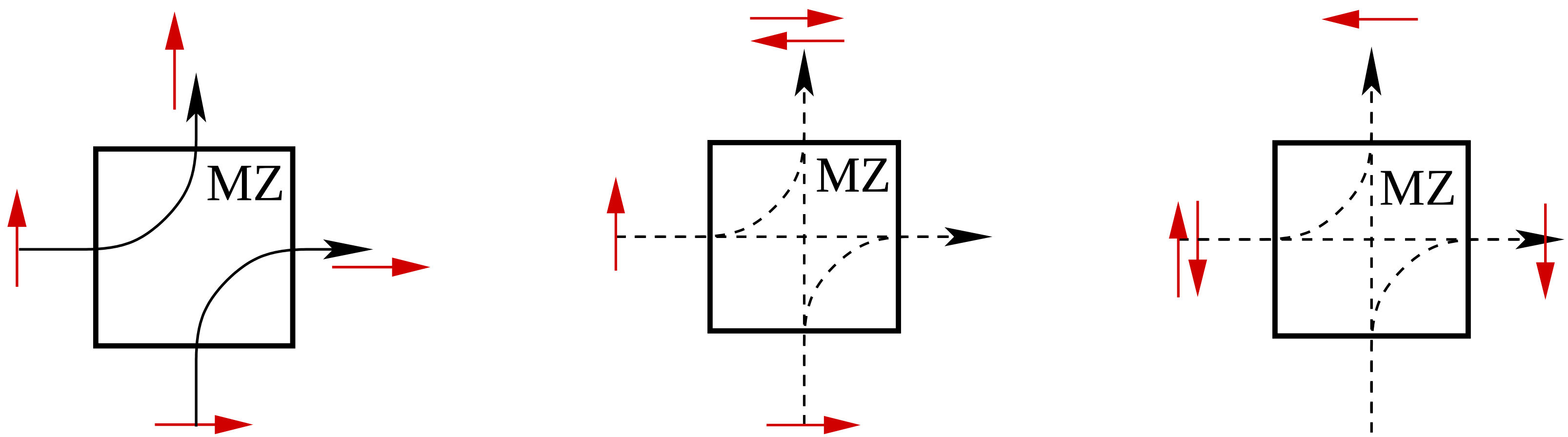

Theoretical description

- 4D single–electron space with basis $|\sigma = +,-;\alpha = \mathrm{a},\mathrm{b}\rangle$
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  with antisymmetric 4×4 matrix $\Psi_{\alpha\sigma}^{\alpha'\sigma'} \Rightarrow$ 6D space
- use magic basis of Bell states (Eckert et al., Ann. Phys. 2002)

$$\chi_1 = \frac{1}{\sqrt{2}}\left(c_{\mathrm{a}+}^{\dagger} c_{\mathrm{a}-}^{\dagger} + c_{\mathrm{b}+}^{\dagger} c_{\mathrm{b}-}^{\dagger}\right) \qquad \chi_2 = \frac{1}{\sqrt{2}}\left(c_{\mathrm{a}+}^{\dagger} c_{\mathrm{b}+}^{\dagger} - c_{\mathrm{a}-}^{\dagger} c_{\mathrm{b}-}^{\dagger}\right)$$

$$\chi_3 = \frac{1}{\sqrt{2}}\left(c_{\mathrm{a}+}^{\dagger} c_{\mathrm{b}-}^{\dagger} + c_{\mathrm{a}-}^{\dagger} c_{\mathrm{b}+}^{\dagger}\right) \qquad \chi_4 = \frac{i}{\sqrt{2}}\left(c_{\mathrm{a}+}^{\dagger} c_{\mathrm{a}-}^{\dagger} - c_{\mathrm{b}+}^{\dagger} c_{\mathrm{b}-}^{\dagger}\right)$$

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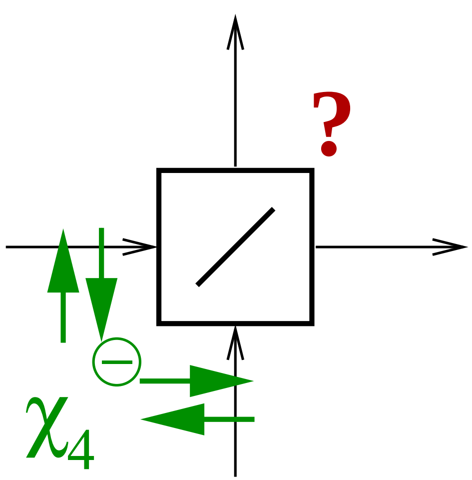
Theoretical description cont'd

- need the two–electron–state scattering matrix for each linear electron–optics element (e.g., beam splitter)



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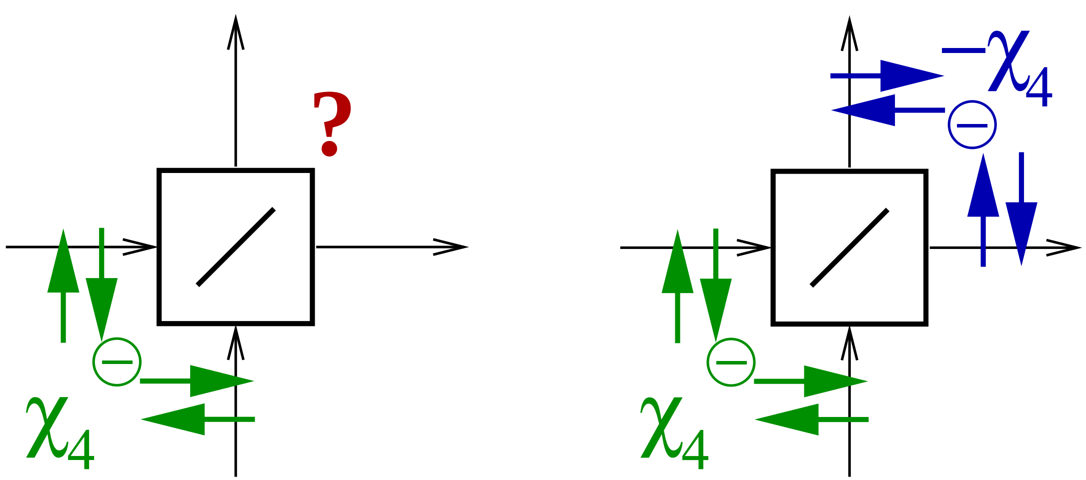
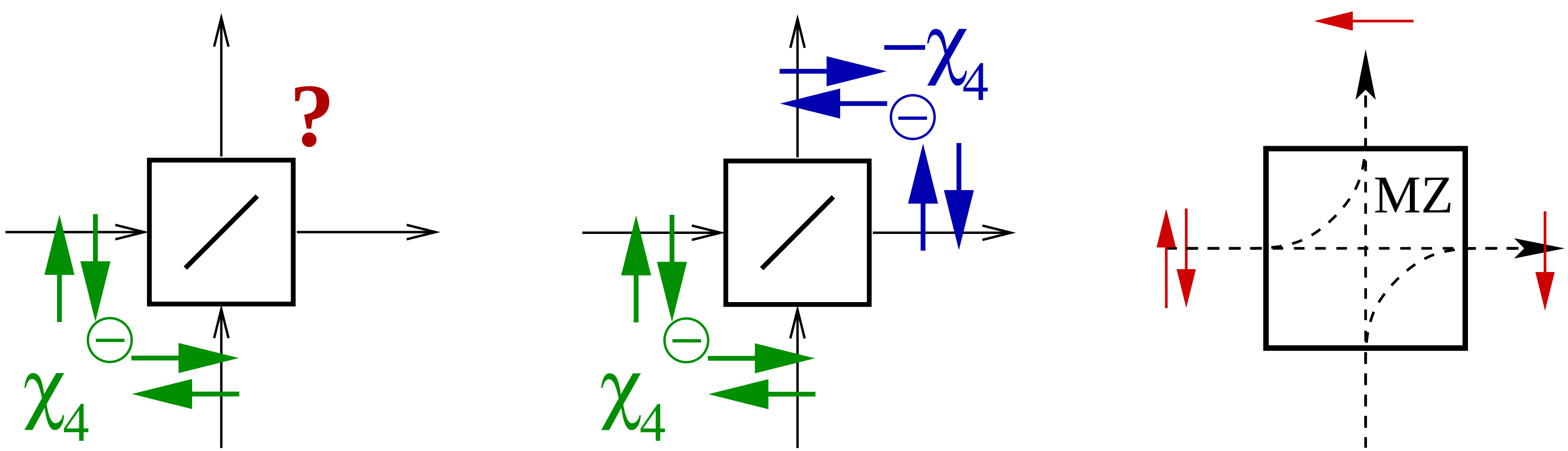


Theoretical description cont'd

- need the two–electron–state scattering matrix for each linear electron–optics element (e.g., beam splitter)
- can derive it from single–particle scattering matrix obtained before (UZ, Appl. Phys. Lett. 2004)
- find two–particle scattering matrix analytically for entire interferometer: oscillatory in size / spin–precession length



Entanglement generation at a spin MZI

Signal & UZ, Appl. Phys. Lett. **87**, 102102 (2005)

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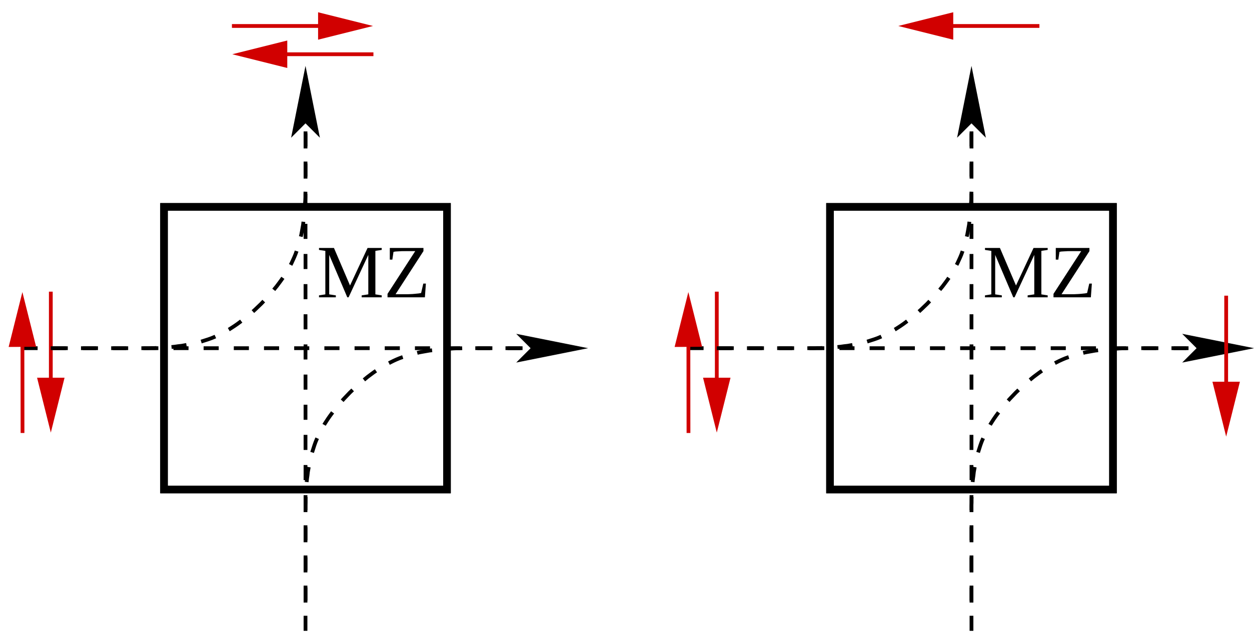

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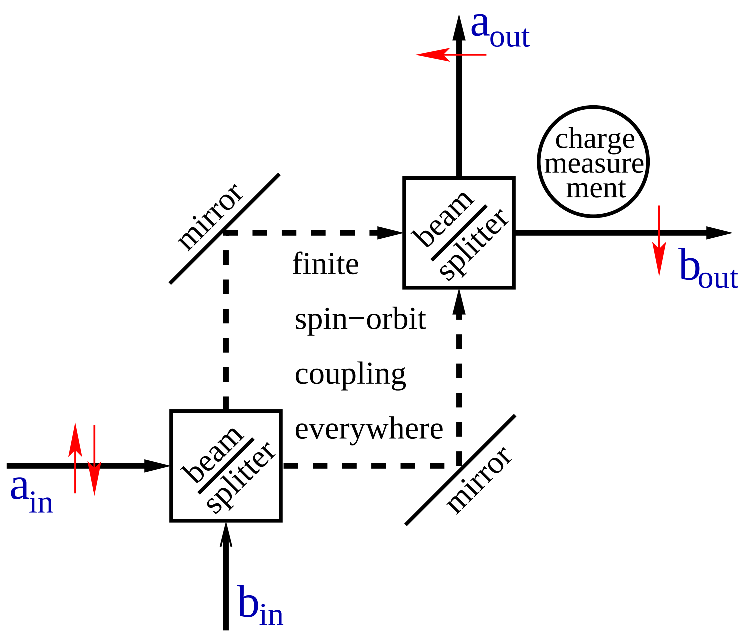

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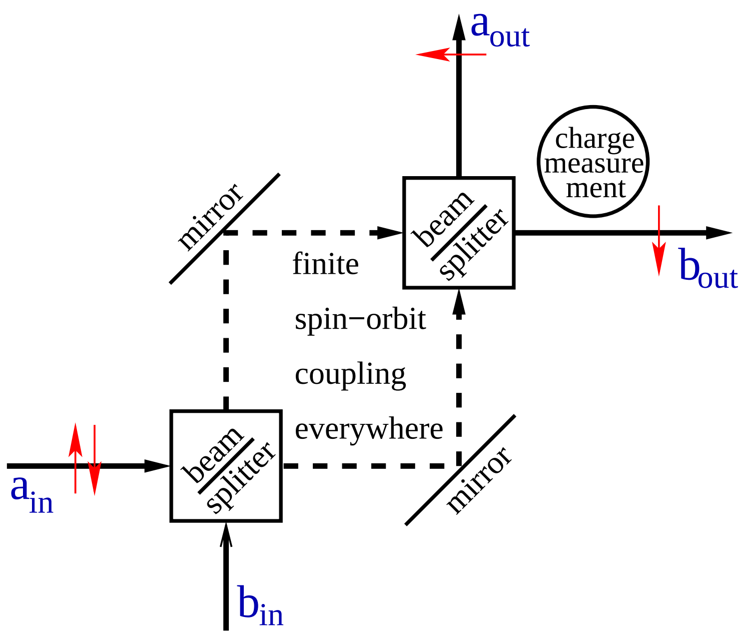

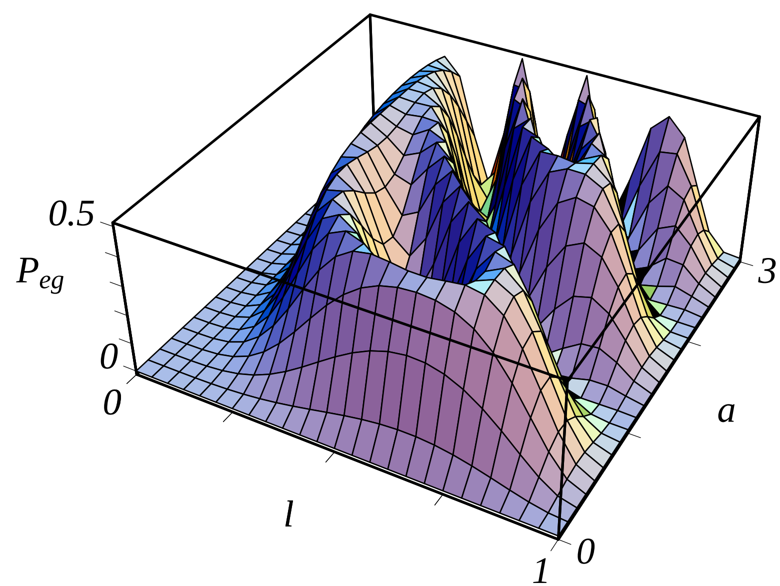

Tuneable entanglement generation

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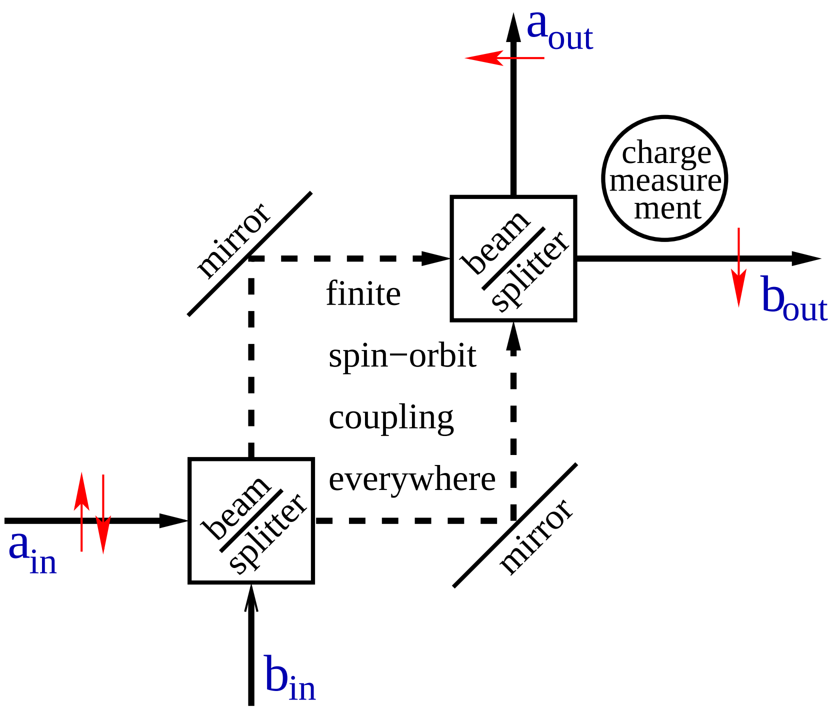
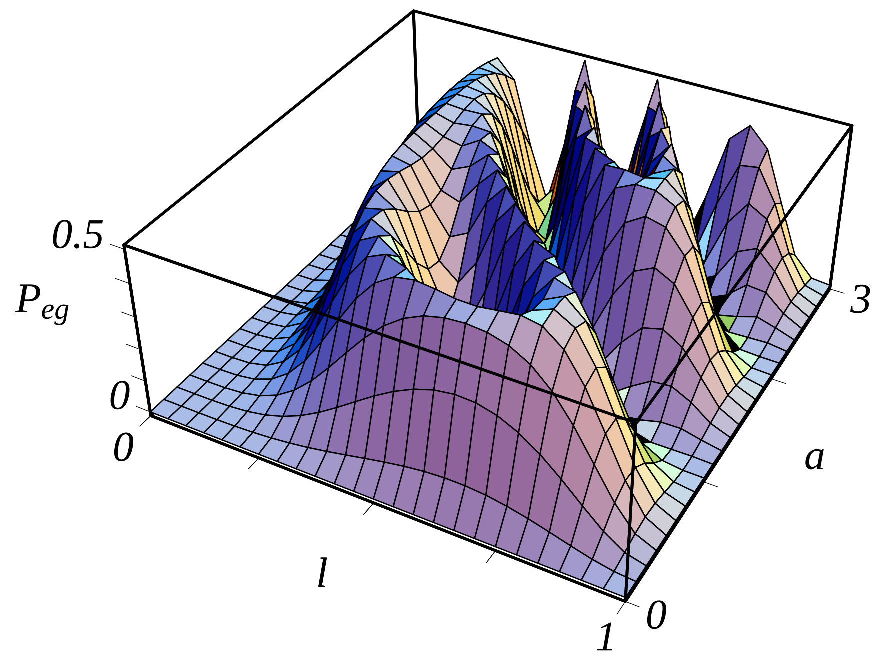
- efficiency P_{eg} of entanglement generation can be adjusted via gate voltage: switchable entangler





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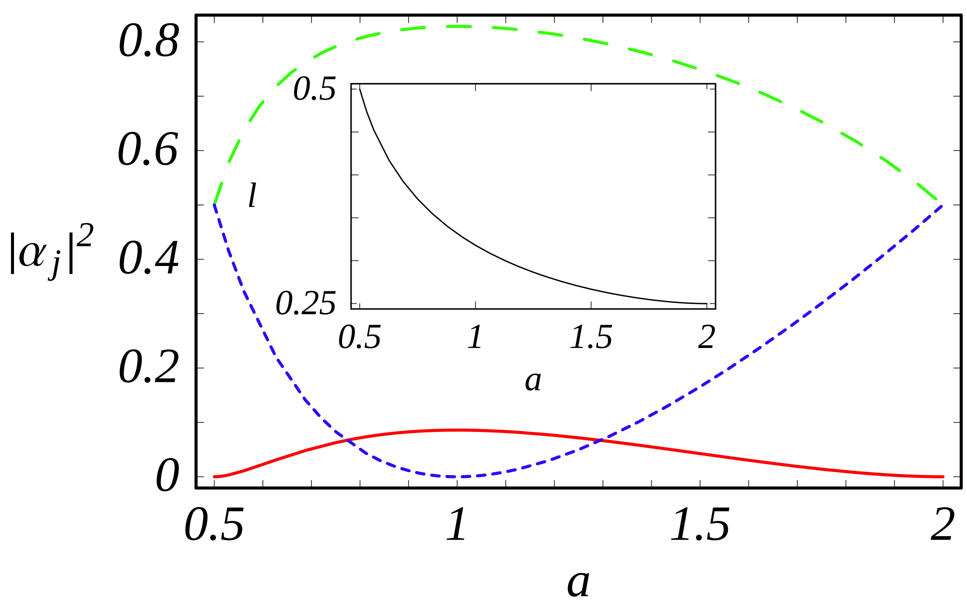
- efficiency $P_{\rm eg}$ of entanglement generation can be adjusted via gate voltage: switchable entangler
- detailed form of maximally entangled output state also tuneable: $|\Psi\rangle_{\rm out} = \alpha_2|\chi_2\rangle + \alpha_3|\chi_3\rangle + \alpha_5|\chi_5\rangle + \alpha_6|\chi_6\rangle$



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Conclusions, Outlook & Acknowledgment

- calculated single and two–electron interference at a spin–dependent Mach–Zehnder interferometer
- single–electron–input applications: magnet–less spin switch and quantum gates for mobile–electron spin qubit
- interferometer can act as entangler: 50% max. efficiency + electric-field control of entanglement generation!!
- in progress: classify possible two-qubit gates realised by spin–MZI (Bremner et al., Phys. Rev. Lett. ’02; Makhlin, Quant. Inf. Proc. ’03)

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	J. König	(Ruhr–U Bochum)
	Y. Tokura	(NTT Labs)
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