Building new states of matter with polar molecules: a route toward topological order

Gavin K. Brennen
Andrea Micheli
Peter Zoller

Stephen S. Bullock

Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences

Institute for Theoretical Physics, University of Innsbruck

Center for Computational Sciences, Bowie, Maryland

Motivation: Let's use ideas from QI to probe models of the natural world

AMO systems as a toolbox

designer traps precision measurement engineered interactions
coherent control reservoir engineering

Many body physics

Hubbard models
Lattice spin simulators

Quantum Information

Qubit registers
Quantum gates Memory

More speculative

Topologically phases

Gapped error protection
Topological QC

Outline

- **Protected quantum memory in surface codes as topological order**
  - Models with abelian quasi-particle excitations
- **Proposed implementation with polar molecules**
  - Structure of polar molecules
  - Engineering spin lattice models
  - Constructing a model due to Kitaev on a honeycomb lattice having topologically protected ground states
  - Verification: Measuring anyonic particle statistics
- **Extensions to spin one models**
- **Conclusions**

Topological order

- **Systems with topologically order have some emergent symmetry in the ground states that is not present in the microscopic eqs. of motion**
  - **All physical correlation functions are topological invariants**
  - **Ground state degeneracy that depends on the topology of the underlying space. Robust to perturbations (even those that break the symmetry of H)**
  - **Long range order exists despite absence of long range correlations of local operators**
  - **Energy gap between ground and excited states that is independent of the number of particles.**

- **Impact**
  - **Fundamental physics. Can be used as a model of emergent gauge fields and particles with quantum statistics**
  - **Quantum computation. Topologically protected quantum memory, fault tolerant computation**

I. Topologically protected q. memory

- **Isomorphism between spins and 1-chains (pieces of string) on a surface cellulation** $\Gamma = \Gamma(\mathcal{V}, \mathcal{E}, \mathcal{F})$

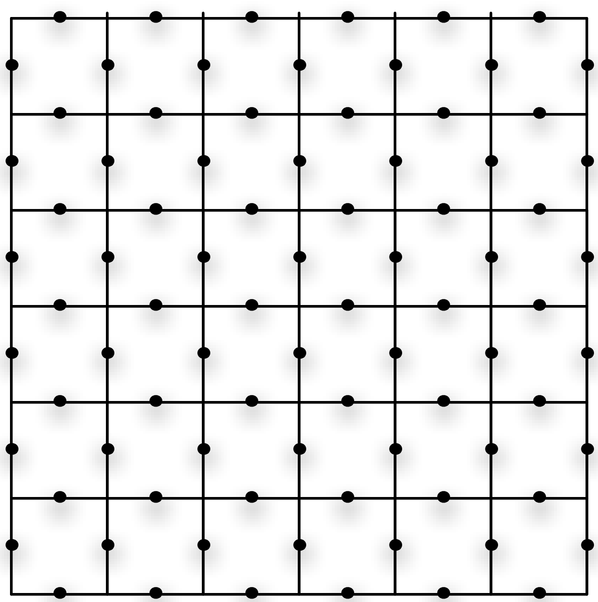
  - **e.g. n qubits on a square lattice**

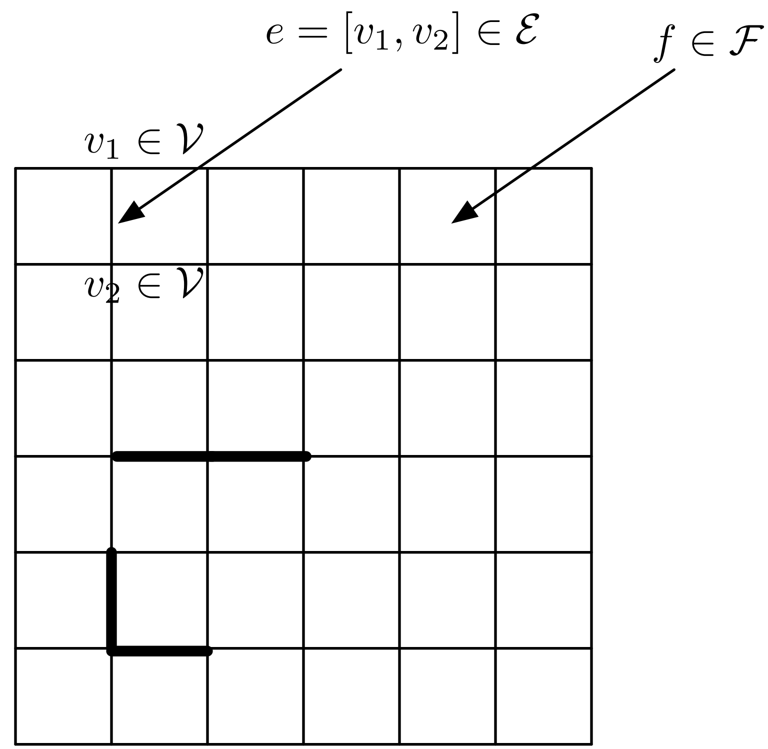
$$\mathcal{H} \cong (\mathbb{C}^2)^{\otimes n} \cong \mathbb{C}^{C_1(\Gamma, \mathbb{Z}_2)}$$

$$C_0(\Gamma, \mathbb{Z}_2) = \mathrm{span}_{\mathbb{Z}_2}(\mathcal{V})$$
$$C_1(\Gamma, \mathbb{Z}_2) = \mathrm{span}_{\mathbb{Z}_2}(\mathcal{E})$$
$$C_2(\Gamma, \mathbb{Z}_2) = \mathrm{span}_{\mathbb{Z}_2}(\mathcal{F})$$

no string (vacuum) $= 0$

string $= 1$





$$|\psi\rangle = \mathbf{1} \otimes \cdots \mathbf{1} \otimes X \otimes X \otimes \mathbf{1} \otimes X \otimes X \otimes \mathbf{1} \otimes \cdots \mathbf{1} |vac\rangle$$

The stabilizer formulation

- **Want a Hamiltonian with vertex and face operators that commute**

$$H = -U\Big(\sum_{v\in\mathcal{V}} g_v + \sum_{f\in\mathcal{F}} g_f\Big)$$

$$g_v = \prod_{e\in\{[*,v],[v,*]\}} Z_e \qquad g_f = \prod_{e\in\partial f} X_e \qquad [g_v, g_{v'}] = [g_f, g_{f'}] = [g_v, g_f] = 0$$

  - **Generators of the stabilizer group G** $\quad G = \langle\{g_v, g_f\}\rangle$

- **Ground states of H are ``stabilized" by G, i.e. they are eigenstates with eigenvalue +1**
  - **dimension of this eigenspace** $\quad \dim\,\mathcal{H}^G \;=\; \mathrm{Trace}\Big[\frac{1}{\#G}\sum_{g\in G} g\Big]$
  - **Examples**
    - For two qubits, define $\quad G\langle\{X_1X_2, Z_1Z_2\}\rangle = \{\mathbf{1}_4, X_1X_2, Z_1Z_2, (iY_1)(iY_2)\}$

$$\mathrm{Trace}\Big[\frac{1}{\#G}\sum_{g\in G} g\Big] = \frac{4}{4} = 1 \qquad \mathcal{H}^G = |\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

    - One qubit $\quad G = \langle\{X, Z\}\rangle = \{\pm\mathbf{1}_2, \pm X, \pm(iY), \pm Z\}$

$$\mathrm{Trace}\Big[\frac{1}{\#G}\sum_{g\in G} g\Big] = 0 \qquad \mathcal{H}^G = \emptyset$$

Qubits on a plane

- **Ground state degeneracy for qubits on a p x q plane***

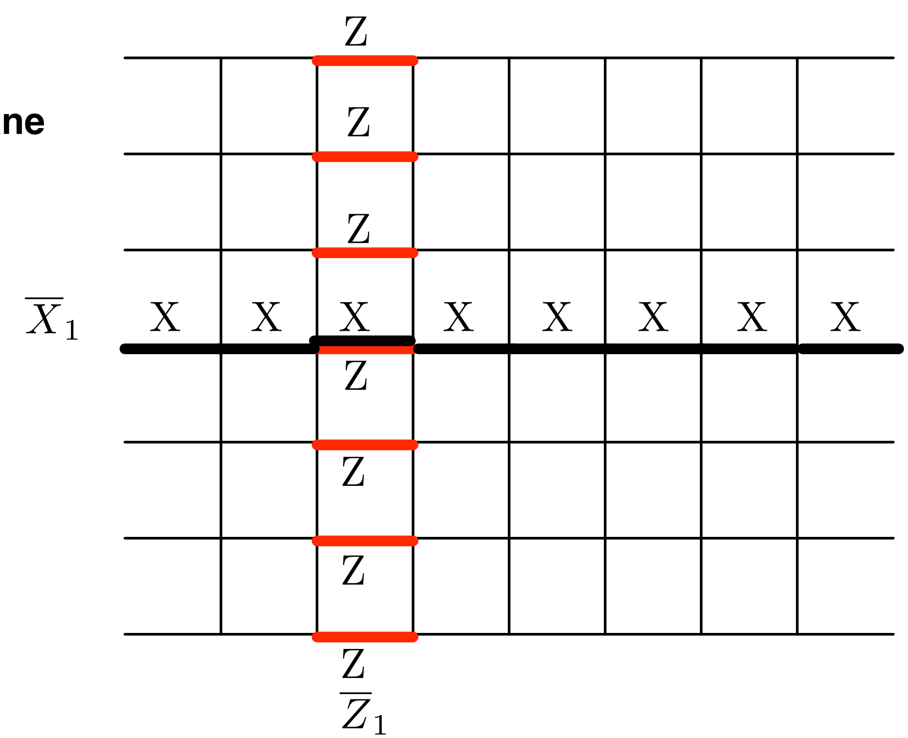
$$H = -U\Big(\sum_{+} Z_{e_1}Z_{e_2}Z_{e_3}Z_{e_4} + \sum_{\text{top,bottom}} Z_{e_1}Z_{e_2}Z_{e_3} + \sum_{\square} X_{e_1}X_{e_2}X_{e_3}X_{e_4} + \sum_{\text{right,left}} X_{e_1}X_{e_2}X_{e_3}\Big)$$

$$\#\mathcal{V} = (p+1)q \qquad \#\mathcal{F} = p(q+1) \qquad \#\mathcal{E} = n = pq + (p+1)(q+1) = \#\mathcal{V} + \#\mathcal{F} + 1$$

$$\dim\mathcal{H}_{\text{gr}} = \text{Trace}\Big[\frac{1}{\#G}\sum_{g\in G} g\Big] = \text{Trace}\Big[\frac{1}{2^{n-1}}\sum_{g\in G} g\Big] = 2$$

Can encode one qubit

6×7 **plane**



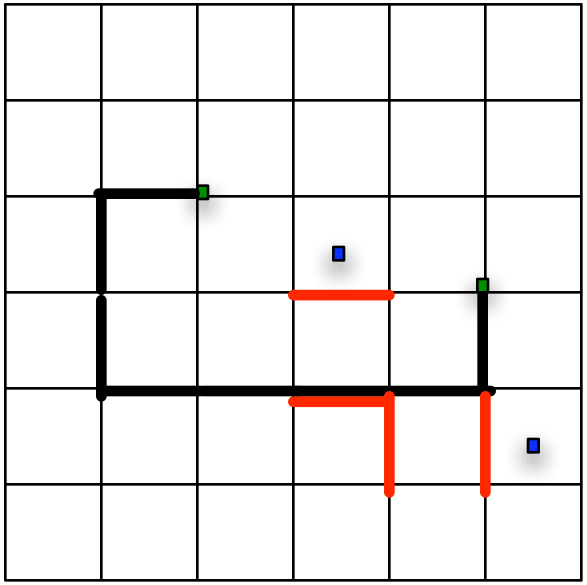
$$\{\overline{X}_1, \overline{Z}_1\} = 0$$

$$[H, \overline{X}_1] = [H, \overline{Z}_1] = 0$$

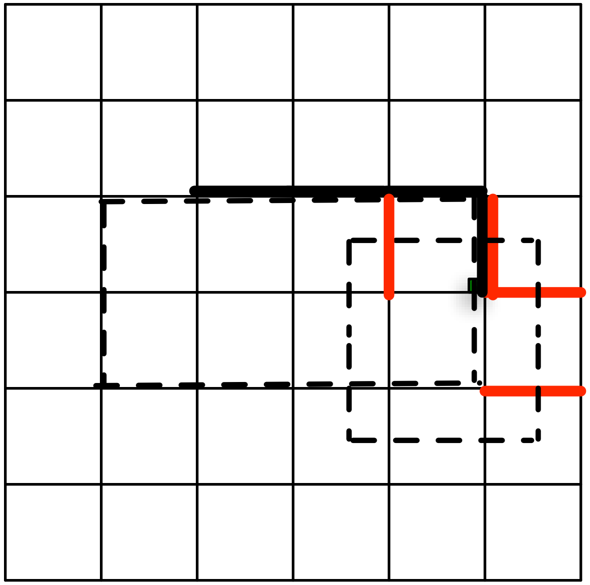
*A.Yu. Kitaev, Annals of Physics, **303**, 2 (2003); quant-ph/9707021

Error Correction

- **Single qubit (edge) flips create two boundaries (energy cost $4J_{\mathrm{eff}}$)**
  - **Z (X) errors create boundaries on the lattice (dual) lattice**
  - **Errors can be corrected by fusing boundaries because trivial cycles are in the stabilizer group**
  - **Boundaries are quasi-particles with anyonic statistics (more on this latter)**



Strings of X and Z errors



Strings of X and Z error correction

- **Logical error can occur on a k x k lattice due to non trivial cycle hence code can correct $\left\lfloor \frac{k-1}{2} \right\rfloor$ errors**

Slightly more generic construction using qudits*

- **Place a spin on each edge of lattice** $\Gamma(\mathcal{V},\mathcal{E},\mathcal{V})$. **Represent state space of each spin on a lattice by a qudit**

$$\mathcal{H}(1,d) = \mathbb{C}|0\rangle \oplus \cdots \oplus \mathbb{C}|d-1\rangle$$
$$\mathcal{H}(n,d) = \mathcal{H}(1,d)^{\otimes n} \qquad n = \#\mathcal{E}$$

Chain **Computational basis state**

$$\omega = \sum_{e\in\mathcal{E}} n_e e \quad \leftrightarrow |\omega\rangle$$

- **Operator basis**

$$X\,|j\rangle = |j+1 \bmod d\rangle$$
$$Z\,|j\rangle = \xi^j\,|j\rangle\,, \qquad \text{for } \xi = \exp(2\pi i/d)$$
$$X^a Z^b = \xi^{a\cdot b} Z^b X^a$$

Pauli-group

$$\mathcal{P}(n,d) = \{\xi^c X^{\otimes \mathbf{a}} Z^{\otimes \mathbf{b}}, \mathbf{a},\mathbf{b} \in (\mathbb{Z}_d)^n\}$$

- **Vertex constraints**

$$g_v = \prod\nolimits_{e=[*,v]} Z_e \prod\nolimits_{e=[v,*]} Z_e^{-1}$$
$$H_v = -(g_v + g_v^\dagger)$$

For a lattice of valence k, this is of the form

$$Z^{\otimes k} + (Z^{-1})^{\otimes k}$$

- **Potential term** $H_\partial = U\sum_{v\in\mathcal{V}} H_v, \quad U>0$

  – **Claim.** $|\omega\rangle$ **is a ground state iff** $\partial\omega = 0$

  – **Check:** $g_v\,|\omega\rangle = \xi^c\,|\omega\rangle$ where $\partial\omega = cv + \sum_{w\neq v} c_w w$

  – **hence,** $|\omega\rangle$ **is in the stabilizer** $\langle\{g_v\}\rangle \subseteq \mathcal{P}(n,d)$ **iff** $|\omega\rangle$ **is an eigenstate of each** H_v

  – **with minimal eigenvalue iff** $|\omega\rangle$ **is a ground state of** H_∂

***S.S. Bullock, GKB, in preparation**

Hamiltonian with TO cont.

- **We're not there yet**
  - The ground states of H_∂ are superpositions of cycles, but they are not topologically ordered because the cycle space is not yet a topological invariant. Changing the cellulation changes the degeneracy.
- **Face constraints**

$$\begin{aligned} g_f &= X_{e_1}^{o_1} X_{e_2}^{o_2} X_{e_3}^{o_3} \dots X_{e_p}^{o_p} \\ H_f &= -(g_f + g_f^\dagger) \end{aligned} \qquad \partial f = \sum_{k=1}^{p} o_k e_k \qquad o_k \in \{1, d-1\}$$

orientation (+/-)

- **Kinetic term**

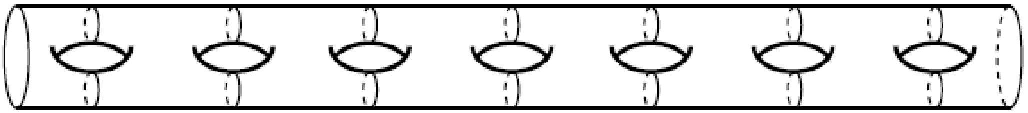
$$H_{\mathrm{KE}} = g \sum_{f \in \mathcal{F}} H_f$$

- **Total Hamiltonian** $H = H_\partial + H_{\mathrm{KE}}$
  - can show that

$$\dim_{\mathbb{C}}(H_{\mathrm{gr}}) = \# H_1(\Gamma, \mathbb{Z}_d)$$

  - for a compact, connected, orientable surface of genus g,

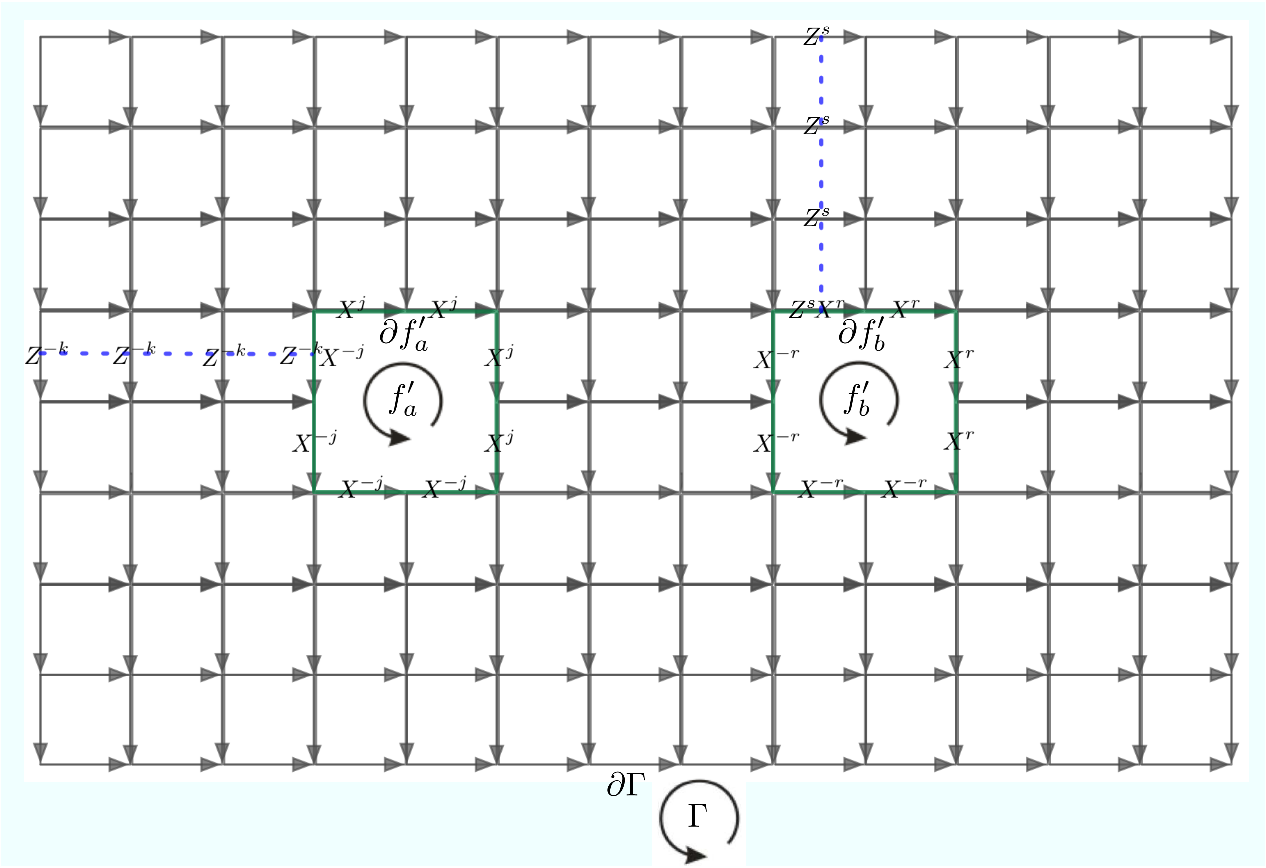
$$H_1(\Gamma, \mathbb{Z}_d) = (\mathbb{Z}_d)^{2g}$$



  - ground subspace (code space)

$$\mathcal{H}_{\mathrm{gr}} \cong (\mathbb{C}^d)^{2g}$$

Example: 2 punctured plane encoding 2 qudits



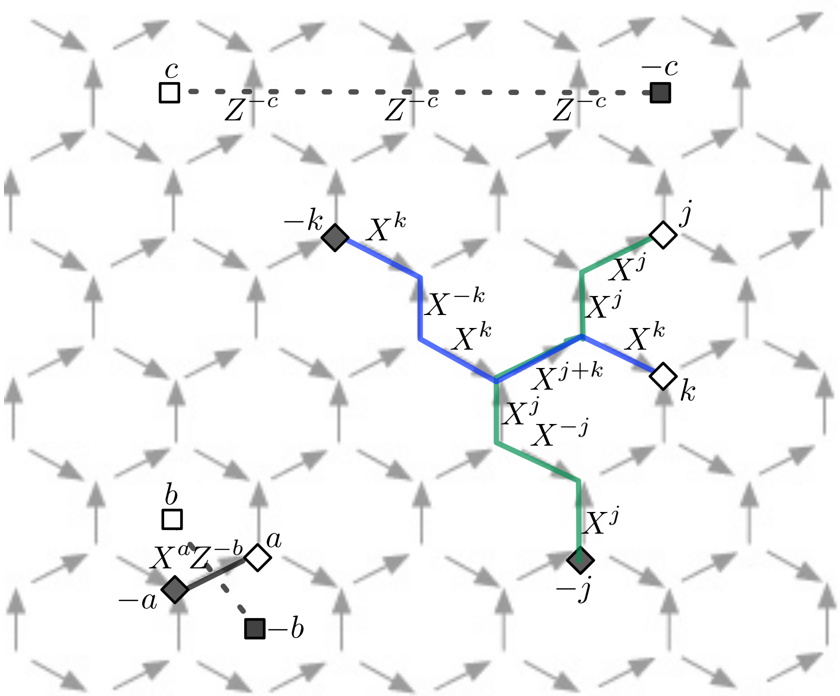
Excitations behave according to a $\mathbb{Z}_d$ gauge theory

Excitations come in particle anti particle pairs

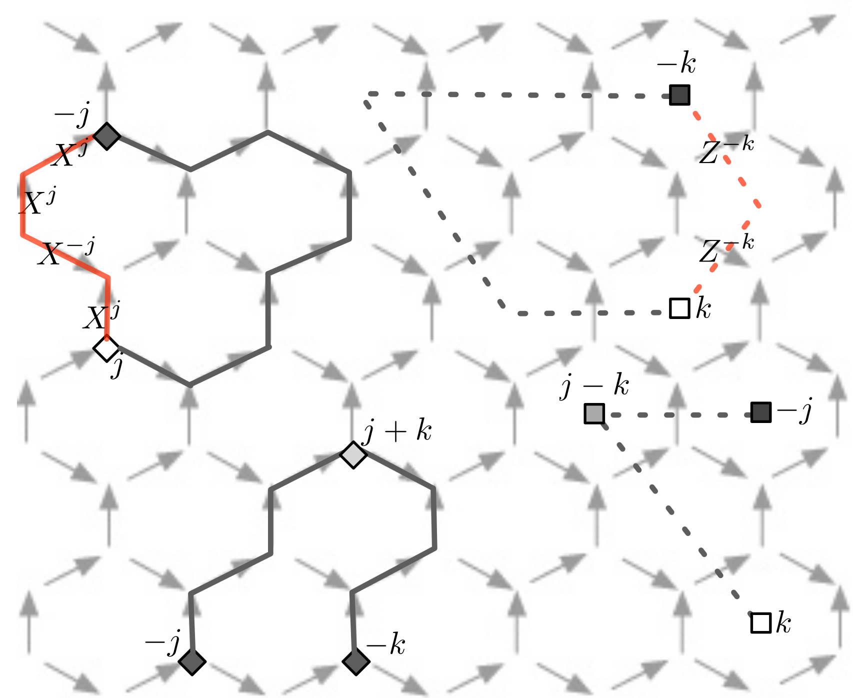
Each particle a charge-flux dyonic combination $(a,b)\in\mathbb{Z}_d\times\mathbb{Z}_d$

Particle mass: $2U(1-\mathrm{Re}[\xi^a])+2h(1-\mathrm{Re}[\xi^b])$ $\quad \xi=e^{i2\pi/d}$

Particle creation



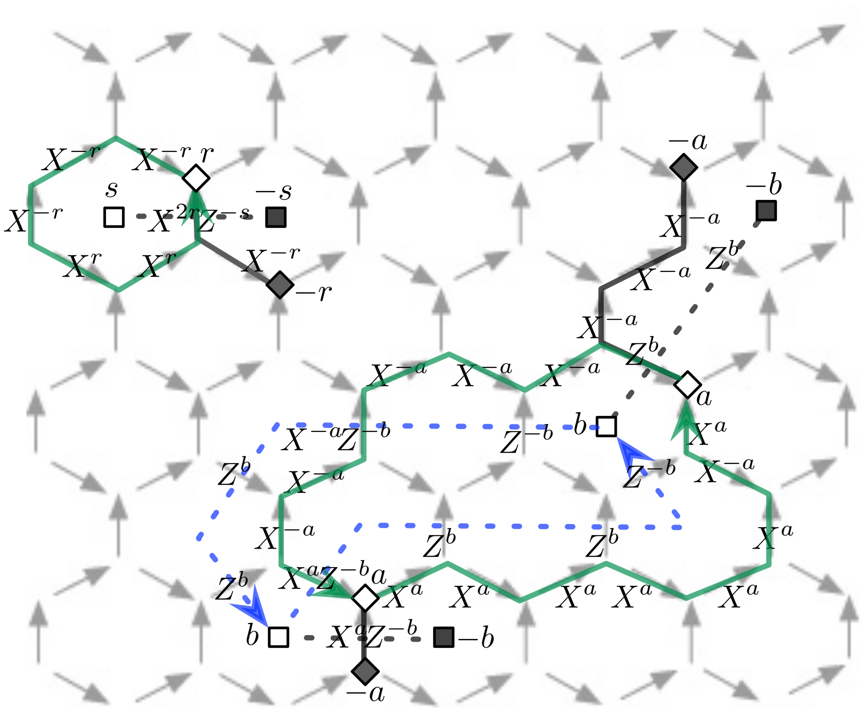
Fusion



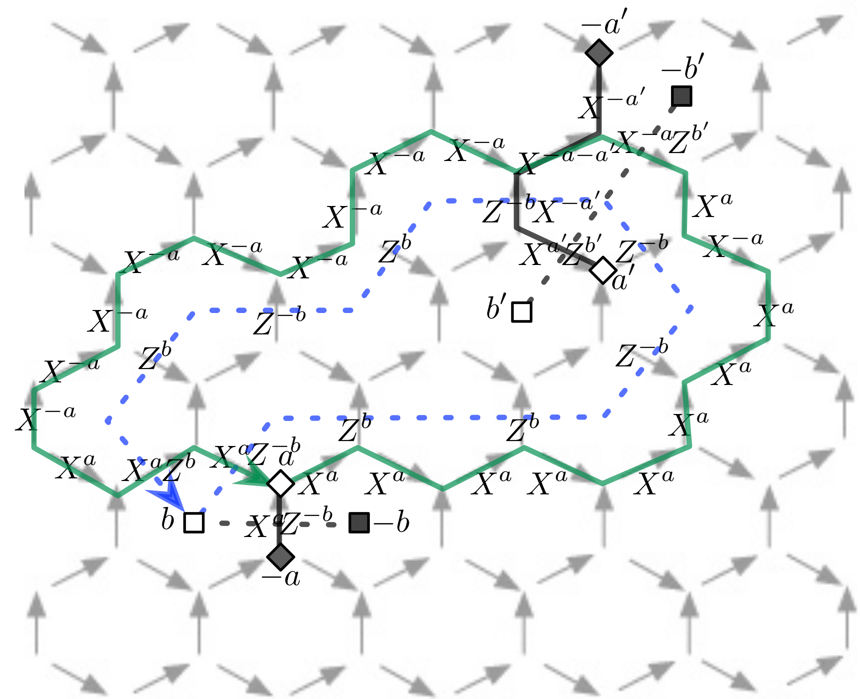
$$|(a,b;(v,f))\rangle\times|(a',b';(v,f))\rangle=|(a+a',b+b';(v,f))\rangle$$

Excitations cont.

Particle exchange



Braiding

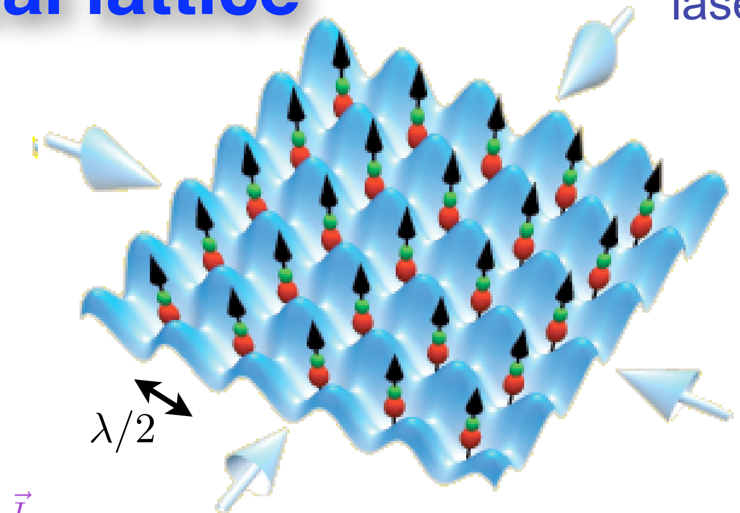


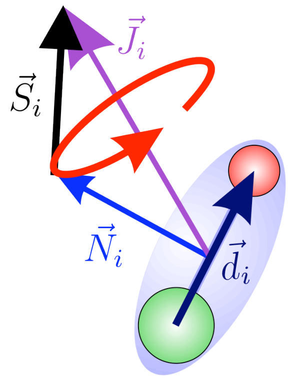
$$\mathcal{R}^2\,|(a,b)\rangle\,|(a',b')\rangle \;=\; \xi^{(a'b+b'a)}\,|(a,b)\rangle\,|(a',b')\rangle$$

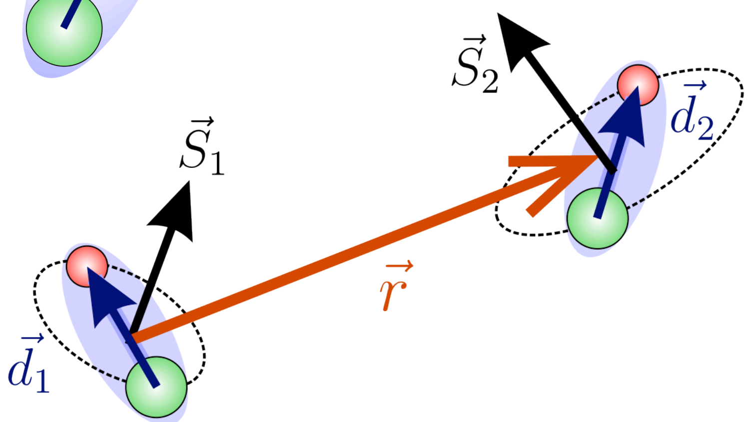
$$\mathcal{R}\,|(a,b;(v,f))\rangle\,|(a,b;(v',f'))\rangle = \xi^{ab}\,|(a,b;(v,f))\rangle\,|(a,b;(v',f'))\rangle$$

II. Implementation of qubit code with polar molecules in an optical lattice

- lattices:
  - prepare exactly one molecule per site in optical lattice, e.g. starting from a BEC.
  - cf. AMO-Hubbard models





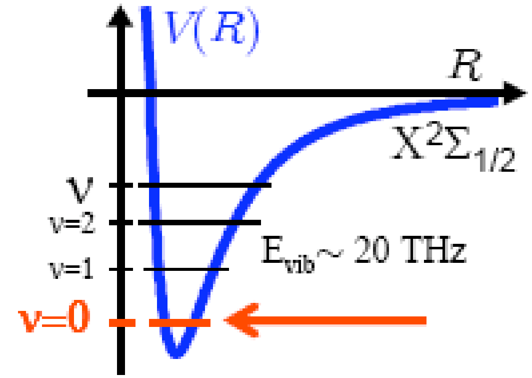


Energy scales:

$\gamma/\hbar \sim 100$ MHz	**Spin-rotational coupling**
$B/\hbar \sim 10$ GHz	**Rotational constant**
$\omega_{osc} \sim 100$ kHz -1MHz	**Lattice trap spacing**
$\Gamma/\hbar \sim 10^{-3}$ Hz	**Black-body scattering rate**
$\Gamma_{\rm scat}/\hbar \sim 10^{-1}$Hz	**Spontaneous emission**

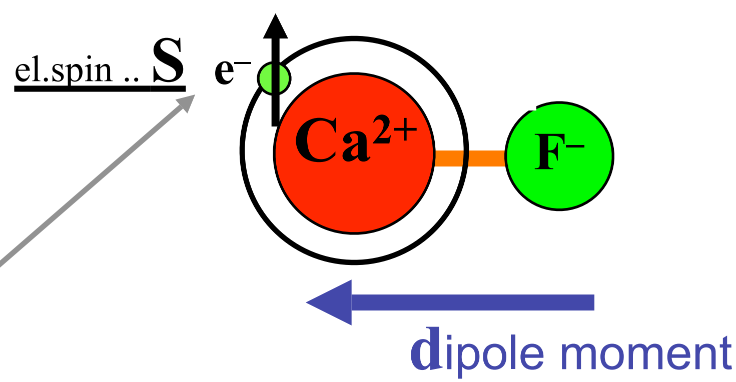
Primer to polar molecules

- System: $^2\Sigma_{1/2}$ hetero-nuclear molecules in electronic-vibrational ground-states
  - Alkaline-earth monohalides (CaF,CaCl,MgCl...)
  - single electron in outer shell
- Electric dipole moment in superposition
- of rotational states



* exp: Demille, Doyle, Mejer, Rempe, …

here e.g. CaF



optical excitation
Alkali-like

talks to optical radiation,
like an alkali atom

talks to microwave radation
... as rotations on ~20 GHz

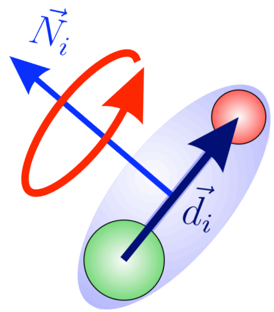
Rotational spectra of a single molecule

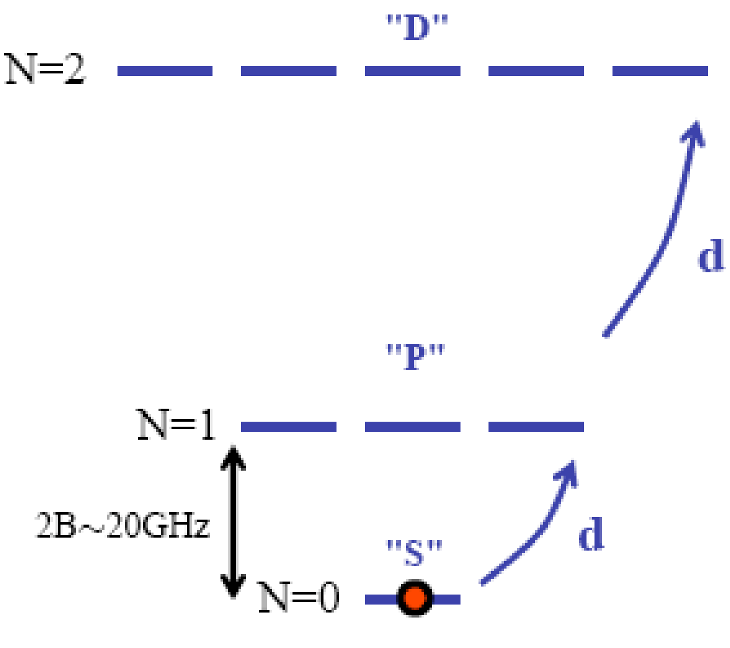
- rigid rotor

$$H = B\,\mathbf{N}^2$$

$$|N, M_N\rangle$$

$$E_N = B\,N(N+1)$$





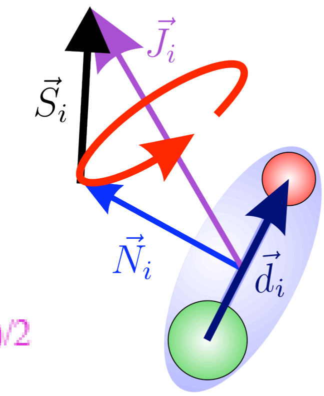
rotational ground state ...

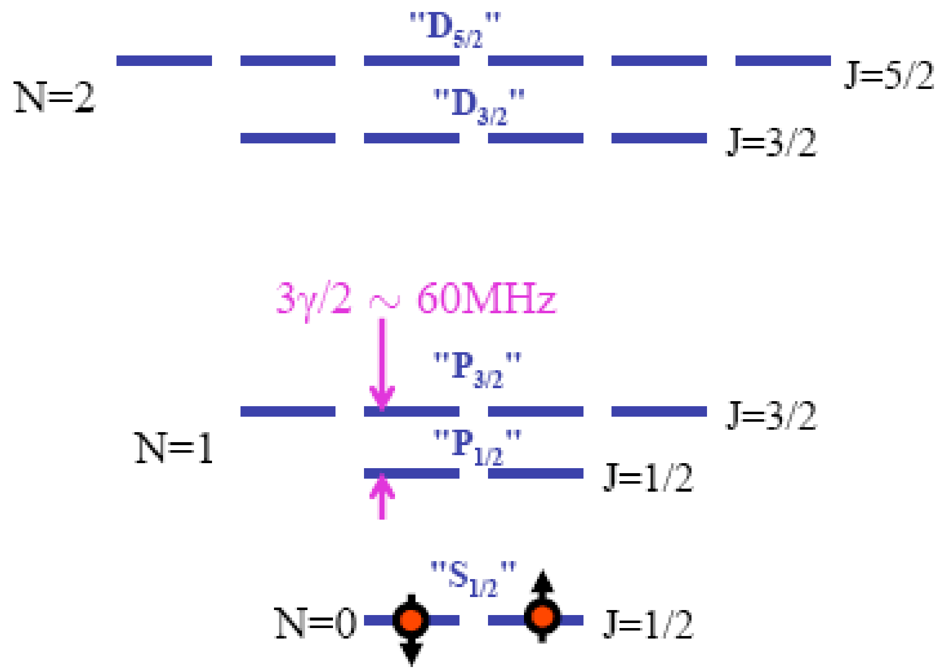
- add spin-rotation coupling

$$H = B\,\mathbf{N}^2 + \gamma\,\mathbf{N}\cdot\mathbf{S}$$

$$|N, J, M_J\rangle \quad (J=|N\pm1/2|)$$

$$E_{N,J=N\pm\frac{1}{2}} = B\,N(N+1) + \begin{cases} +\gamma N/2 \\ -\gamma(N+1)/2 \end{cases}$$

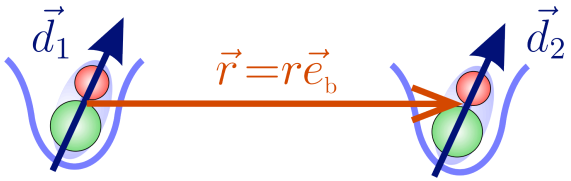




... as spin-1/2-system

Two polar molecules: dipole-dipole interactions

- interactions of two polar molecules

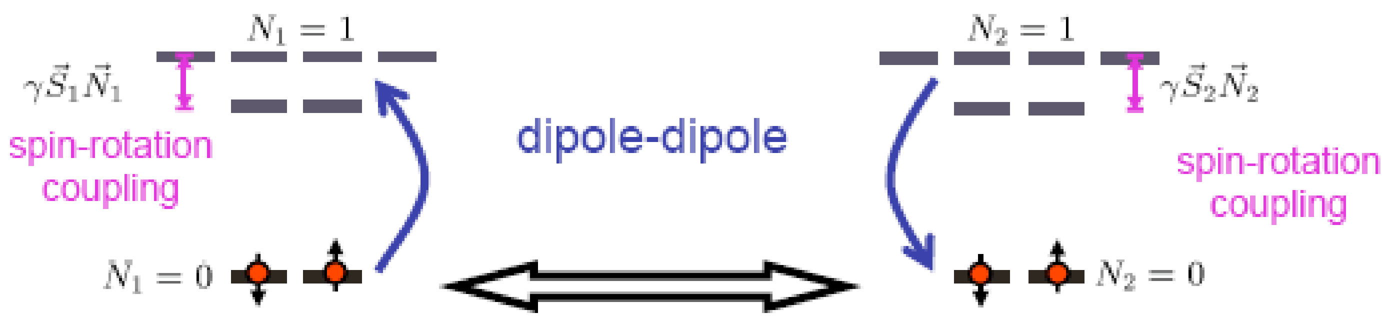


$$V_{dd} = \frac{\vec{d}_1\cdot\vec{d}_2 - 3\left(\vec{d}_1\cdot\vec{e}_b\right)\left(\vec{e}_b\cdot\vec{d}_2\right)}{r^3}$$

features of dipole-dipole interaction:

- long range ~$1/r^3$
- angular dependence (anisotropic)

- include spin-rotation coupling in adiabatic potentials for molecular dimers



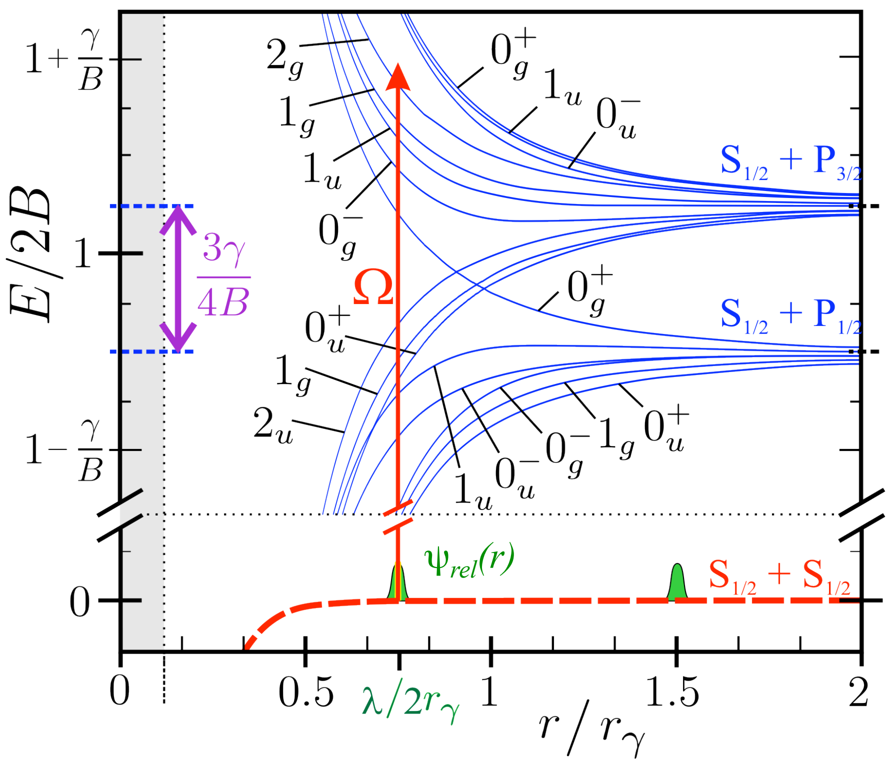
- At typical optical lattice spacing : $\lambda/2 \sim r_\gamma = (2d^2/\gamma)^{1/3}$
  - rotation of dimers strongly coupled to spins
  - Hunds case (c) excited states, $\{|Y|_{g,u}^{\pm}(r)\}$ $(Y=\Sigma_{i=1,2} M_{N,i}+M_{S,i})$
  - solvable in closed form due to symmetries

Microwave coupling with tunable spin patterns

$$H_{\mathsf{mf}} = -\sum_{j=1}^{2} \vec{d}_j \cdot \vec{E}(\vec{x}_j, t) = -\hbar\Omega \sum_j \vec{d}_j \cdot \vec{e}_F e^{-i(\vec{k}_F \cdot \vec{x}_j - \omega_F t)}/d + h.c.$$

$$H_{\text{eff}}(r) = \sum_{i,f} \sum_{\lambda(r)} \frac{\langle g_f | H_{\text{mf}} | \lambda(r) \rangle \langle \lambda(r) | H_{\text{mf}} | g_i \rangle}{\hbar\omega_F - E(\lambda(r))} |g_f\rangle\langle g_i|$$

$$H_{\text{spin}} = \langle H_{\text{eff}}(r) \rangle_{\text{rel}} = \sum_{\alpha,\beta} A_{\alpha,\beta} \sigma^\alpha \sigma^\beta$$



- Feature 1: Tuning close to a resonance one select a *specific* spin pattern, e.g.

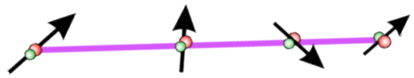
Polarization	Resonance	Spin pattern
$\hat{x}$	2_g	$\sigma^z\sigma^z$
$\hat{z}$	0_u^+	$\vec{\sigma}\cdot\vec{\sigma}$
$\hat{z}$	0_g^-	$\sigma^x\sigma^x + \sigma^y\sigma^y - \sigma^z\sigma^z$
$\hat{y}$	0_g^-	$\sigma^x\sigma^x - \sigma^y\sigma^y + \sigma^z\sigma^z$
$\hat{y}$	0_g^+	$-\sigma^x\sigma^x + \sigma^y\sigma^y + \sigma^z\sigma^z$
$(\hat{y}-\hat{x})/\sqrt{2}$	0_g^+	$-\sigma^x\sigma^y - \sigma^y\sigma^x + \sigma^z\sigma^z$

polarization rel. to body axis, here set $\vec{e}_b = \hat{z}$

Lattice Spin Models using multiple fields

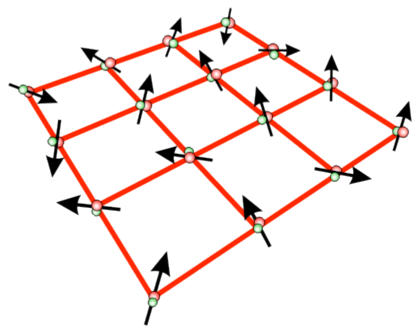
- Feature 2: for a *multifrequency* field spin textures are *additive* => toolbox

- 1D XYZ model



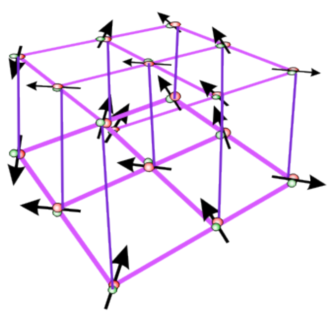
$$H = \sum_{<i,j>} J_x \sigma_i^x \sigma_j^x + J_y \sigma_i^y \sigma_j^y + J_z \sigma_i^z \sigma_j^z$$

- 2D Ising model



$$H = \sum_{<i,j>} J \sigma_i^z \sigma_j^z$$

- 3D Heisenberg model



$$H = \sum_{<i,j>} J \vec{\sigma_i} \cdot \vec{\sigma_j}$$

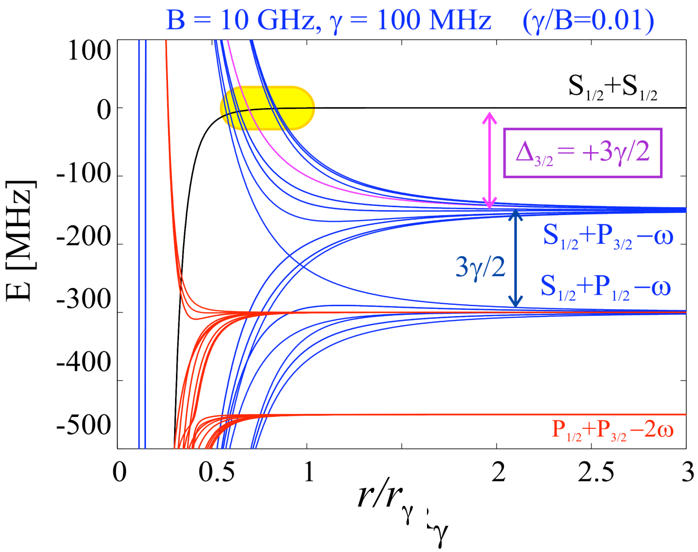
- Typical coupling strengths: $|J| \sim 10$ - $100\mathrm{kHz}$

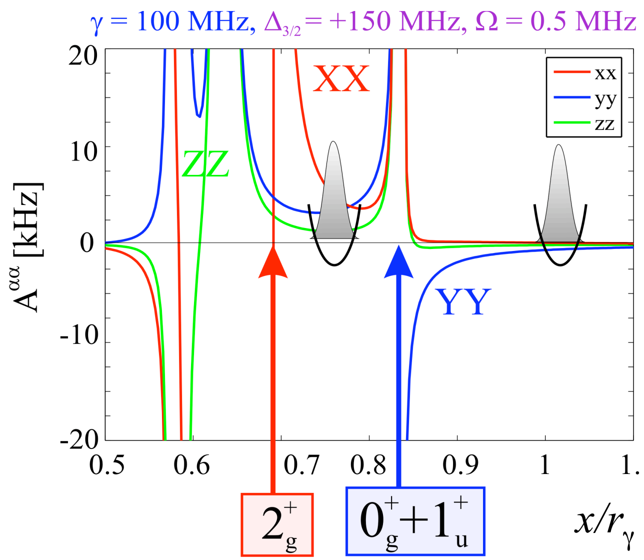
Polarization	Resonance
$\hat{z}$	0_u^+
$\hat{y}$	0_g^-
$\hat{y}$	0_g^+
$\hat{x}$	2_g
$\hat{x}$	0_u^+
$\hat{z}$	0_g^-
$\hat{z}$	0_u^+
$\hat{x}$	1_u

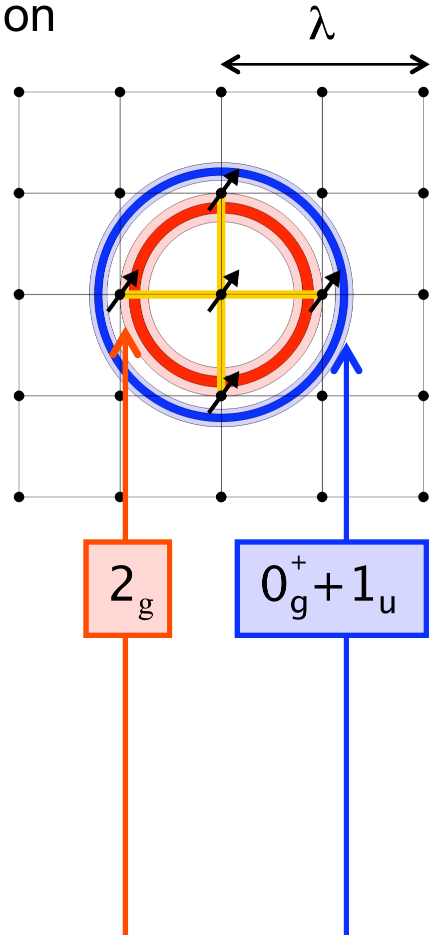
sign adjustable by tuning above or below given resonance

Spatial orientation dependent interactions

- Example: Ising interaction
- Realization by tuning MW far blue from bare $S_{1/2} \leftrightarrow P_{3/2}$ transition





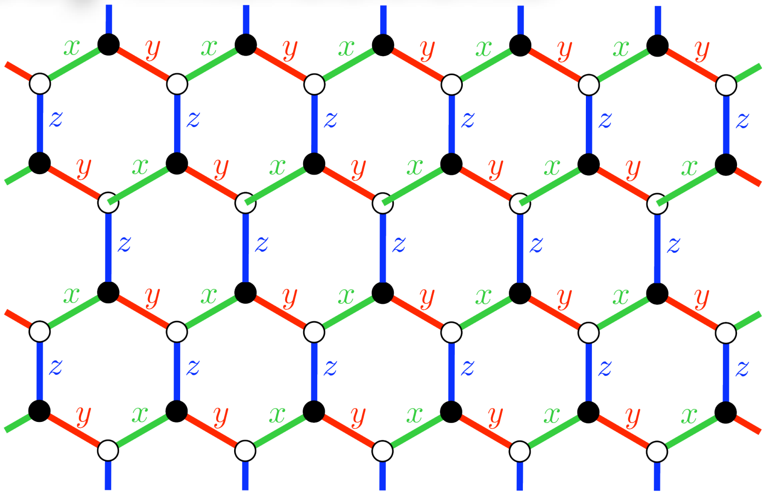


- interaction given effectively by interplay of 3 resonances
  - outer two yield single effective interaction
  - optimal regime near 2_g as spin-texture $\begin{cases} \sigma^z\sigma^z \text{ in direction } z \\ \sigma^x\sigma^x \text{ in direction } x \end{cases}$

- Feature 3: Can choose the *range* of the interaction for a given spin texture

Realization with 2-body interactions

- **Spin-1/2 particles on a honeycomb lattice***



*A.Yu. Kitaev, Annals of Physics, 321,2 (2006)

$$H = J_{\perp} \sum_{x-\text{links}} \sigma_j^x \sigma_k^x + J_{\perp} \sum_{y-\text{links}} \sigma_j^y \sigma_k^y + J_z \sum_{z-\text{links}} \sigma_j^z \sigma_k^z .$$

  - **Exactly solvable**

- **In the limit, $|J_z| \gg |J_{\perp}|$, pairs of spins along z-links are mapped to a qubit**
  - **New spin operators on each z-link:**

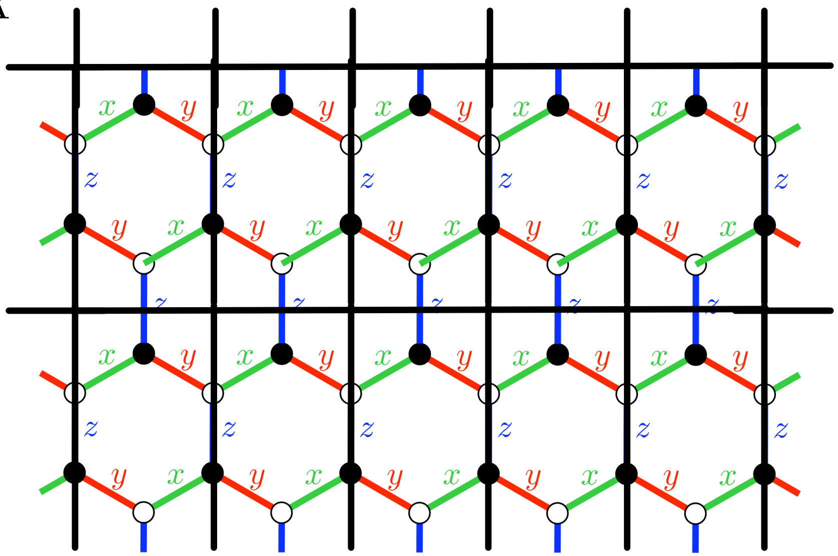
$$\mathbf{1}_{2(1)} \otimes \sigma_2^z \rightarrow Z \qquad \sigma_1^y \otimes \sigma_2^x \rightarrow Y \qquad \sigma_1^x \otimes \sigma_2^x \rightarrow X$$

$$H_{\text{eff}} = -J_{\text{eff}} \sum_{\diamond} Y_{\text{left}} Z_{\text{up}} Y_{\text{right}} Z_{\text{down}}$$

Unitary transformation: $\prod_{j \ni \text{white}} e^{iX_j \pi/4}$

$$H_{\text{eff}} = -J_{\text{eff}} \left(\sum_{+} Z_{e_1} Z_{e_2} Z_{e_3} Z_{e_4} + \sum_{\square} X_{e_1} X_{e_2} X_{e_3} X_{e_4} \right)$$

  - **Protected q. memory**



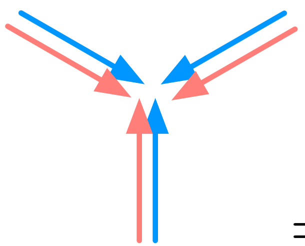
$$J_{\text{eff}} = \frac{J_{\perp}^4 |J_z|}{16 J_z^4}$$

Construction in an optical lattice

Bichromatic trapping beams in 2D, with relative phase shift

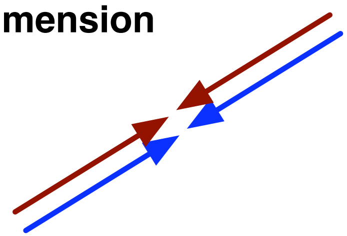
One triangular lattice staggered on top of another

Q*bert lattice with nearest neighbor honeycomb graph. Edges connecting nearest neighbors form orthogonal triads

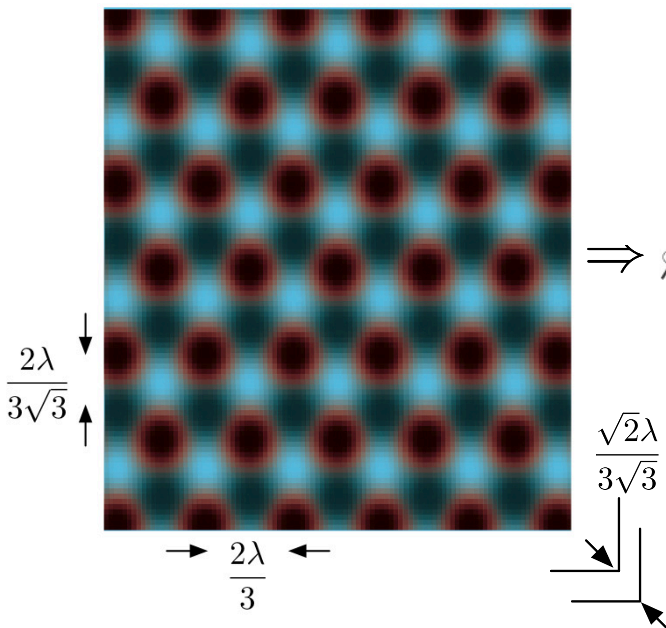


$+$

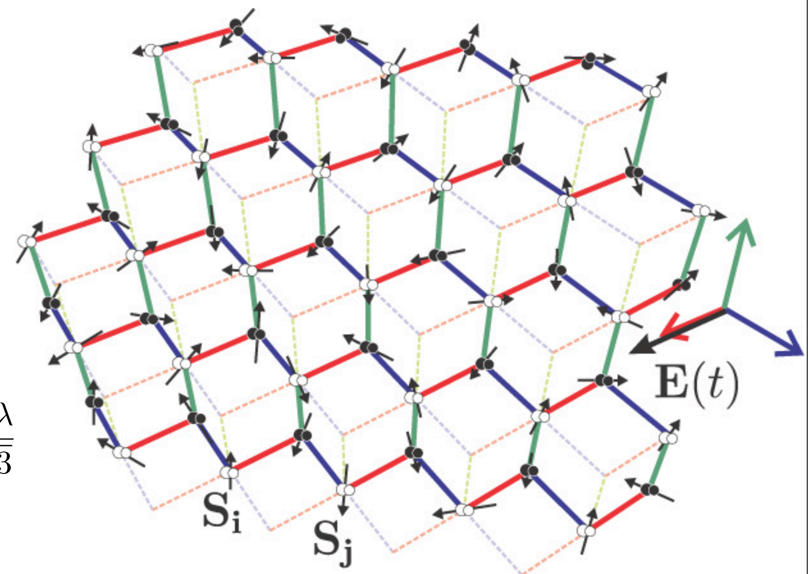
Biperiodic trap in third dimension



$\Rightarrow$

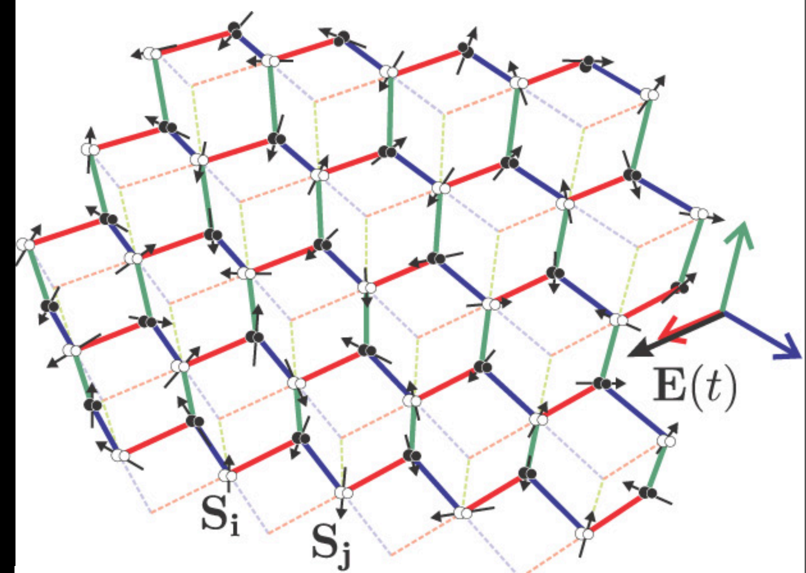


$\Rightarrow$



Construction in an optical lattice

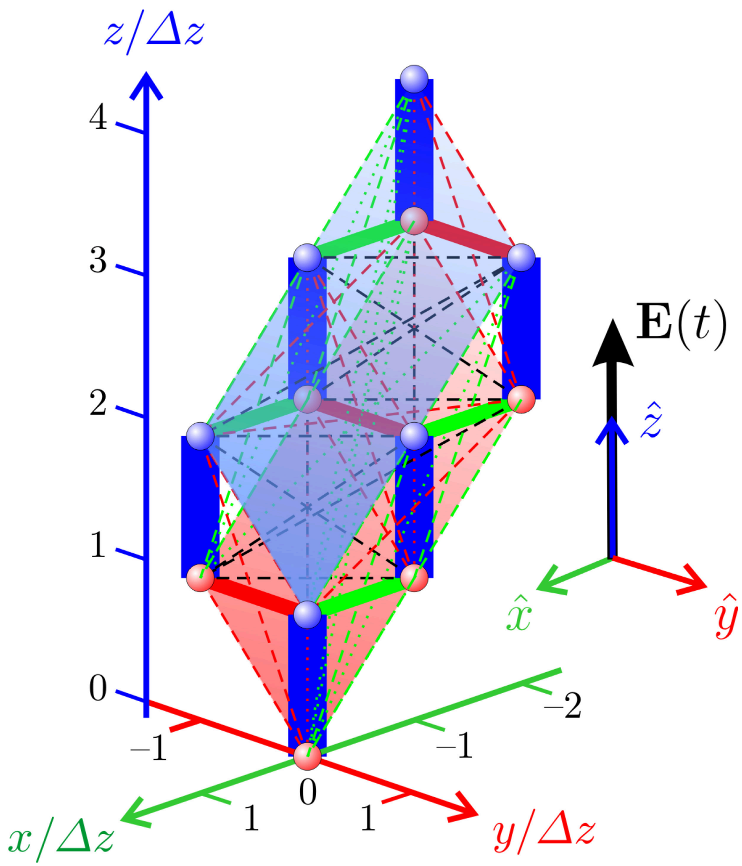
Q*bert lattice with nearest neighbor honeycomb graph. Edges connecting nearest neighbors form orthogonal triads



Results for system of 12 spins

- **Realization with 3 fields. Several field choices possible, e.g. all polarized along $\hat{z}$ tuned to $1_g, 0_g^-, 2_g$**

Coupling Graph



Spin pattern

$\sigma^z\sigma^z$

$\sigma^x\sigma^x$

$\sigma^y\sigma^y$

Other

$|J_\perp| = 0.4|J_z|$

Residual long range coupling strengths $|J_{lr}|$

$< 10^{-2}\ |J_z|$

$< 10^{-3}\ |J_z|$

Operator fidelity (on a 4 spin configuration)

$$\sup\left[||H_{\text{spin}} - H_{\text{spin}}^{(\text{II})}|\psi\rangle||_2;\ \langle\psi|\psi\rangle = 1\right] = 10^{-4}\ |J_z|$$

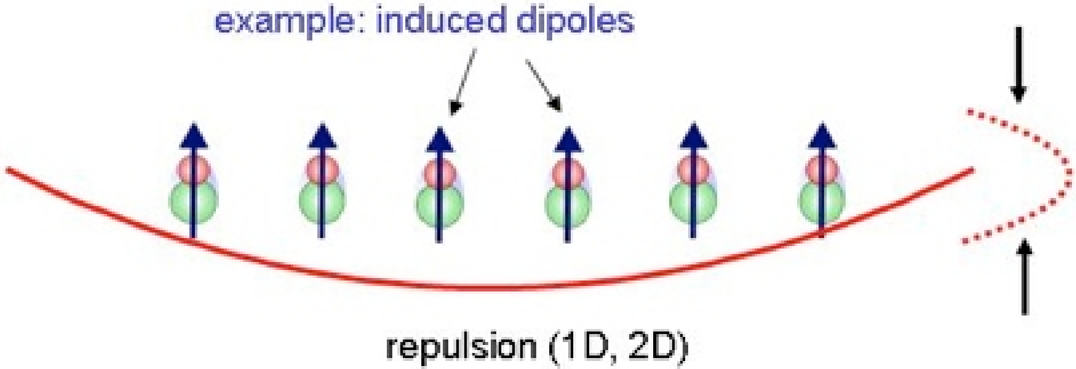
For realistic parameters

$$|J_z| = 100\ \text{kHz} \Rightarrow\ \ J_{eff} \sim 167\ \text{Hz}$$

A. Micheli, GKB, P. Zoller, Nature Physics, **2**, 341 (2006)

For larger gap need smaller lattice spacings: self assembled crystals

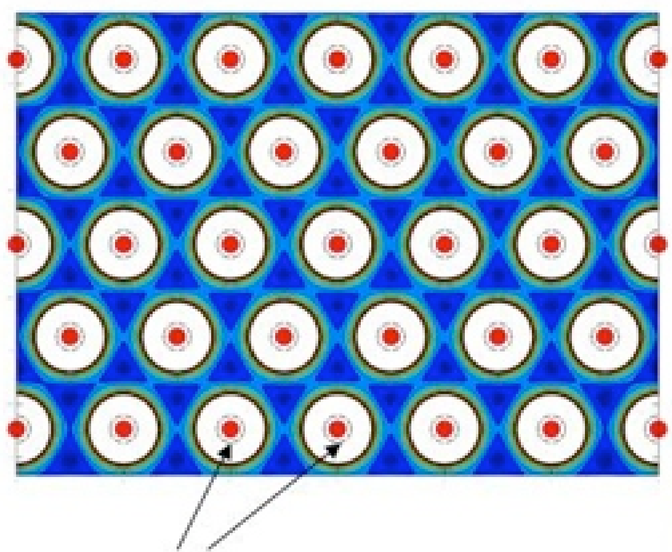
- **Engineer repulsive interactions***



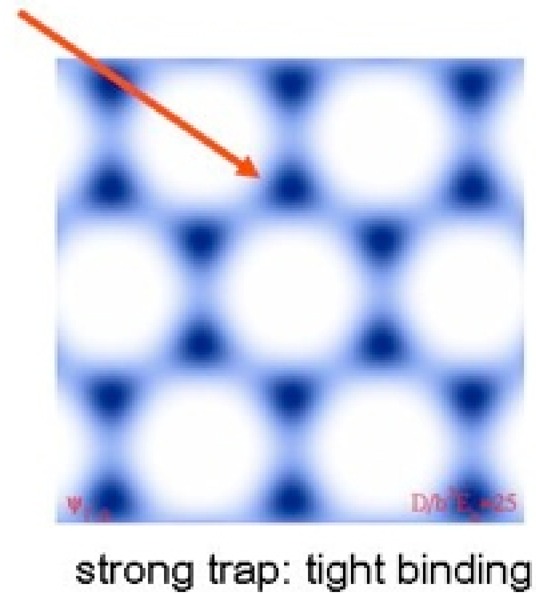
$$\hbar\omega_z \gg E_{\rm int} \sim \frac{d^2}{a^3}$$

$$a \sim 100\text{nm}$$

other particles see honeycomb lattice



dipoles generate 2D triangular lattice



strong trap: tight binding

H.P. Büchler, et al., *cond-mat/0607294*

Quasiparticle statistics

- **Excitations induced by single spin flips (along a z-link) represented by particle pairs**

  - **Consider translationally invariant 4-local interaction along diamonds with vertices on z-links**

$$H_{\rm eff} = -J_{\rm eff} \sum_{\diamond} Y_{\rm left} Z_{\rm up} Y_{\rm right} Z_{\rm down}$$

  - **Four superselection sectors: vacuum (no particles), Z particles ($\square$) on the left and right of a Z flipped spin, Y particles ($\diamond$) above and below a Y flipped spin, bound state of a Z particle and an Y particle ($\square\diamond$) flanking an X flipped spin.**
    - Fusion rules (as obtained from the action of the Pauli operators):

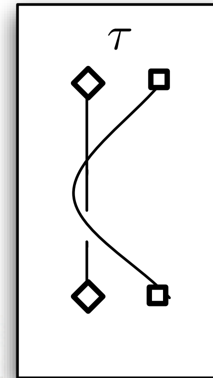
$$\square \times \square = 1 \qquad \diamond \times \diamond = 1 \qquad \square\diamond \times \square\diamond = 1$$

$$\square \times \diamond = \square\diamond \qquad \square \times \square\diamond = \diamond \qquad \diamond \times \square\diamond = \square$$

    - Relative statistics under braiding:

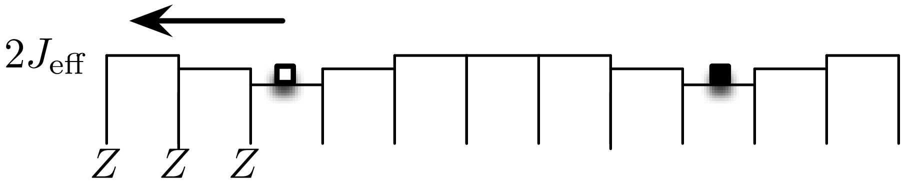
Particles	Statistical phase
$\square \ \square$	0
$\diamond \ \diamond$	0
$\square \ \diamond$	π
$\square\diamond \ \square\diamond$	0

Braiding



- $|\Psi(1)\rangle = S_A^Y S_B^Z |\lambda_g\rangle$
- $|\Psi(2)\rangle = e^{-iS_I^Z \pi/4}|\Psi(1)\rangle = \frac{1}{\sqrt{2}}(|\Psi(1)\rangle - iS_I^Z|\Psi(1)\rangle)$
- **Adiabatically drag □ left**

$$H'(t) = H + \sum_{e \in Path} \delta J_e(t)(\sigma_1^z \sigma_2^z)_e + \kappa(t) Z_e(t)$$



$$|\Psi(3)\rangle = \frac{1}{\sqrt{2}}(|\Psi(1)\rangle - iS_{B'\cup I}^Z|\Psi(1)\rangle)$$

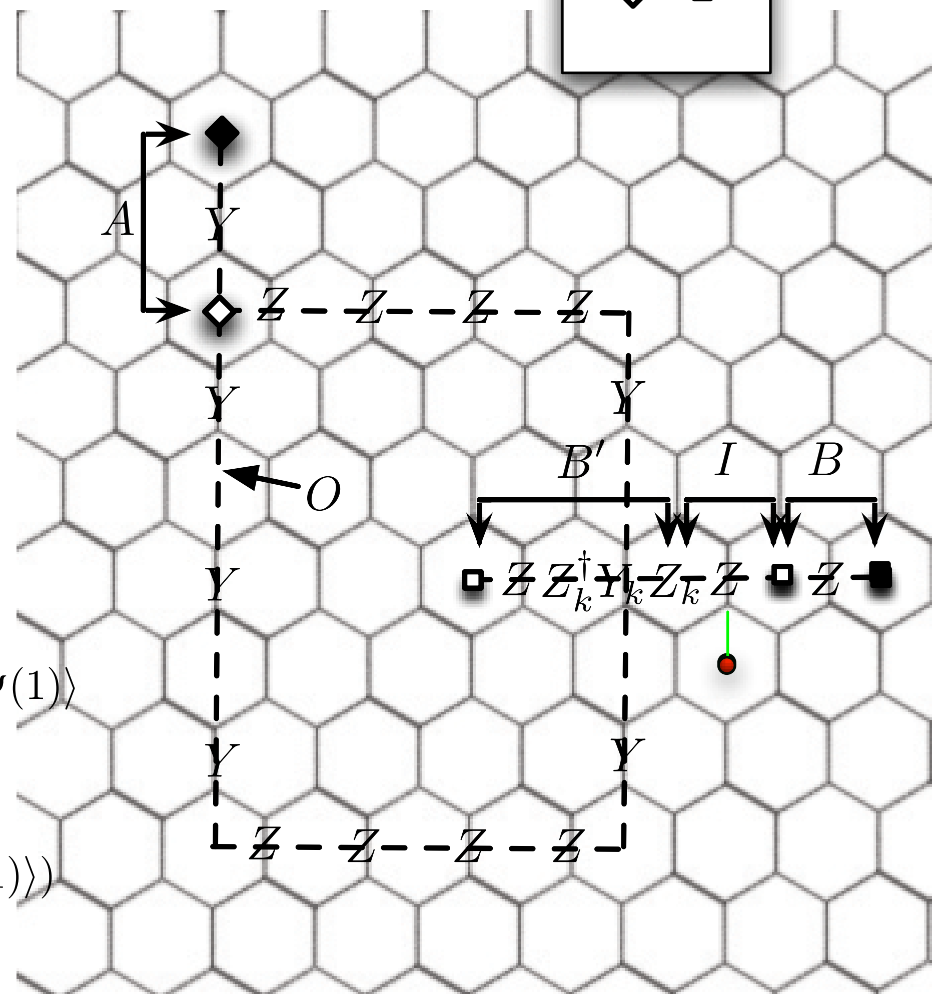
- **Adiabatically drag ◇ CCW around □**

$$|\Psi(4)\rangle = \frac{1}{\sqrt{2}}(O|\Psi(1)\rangle - iOS_{B'\cup I}^Z|\Psi(1)\rangle)$$

- **Adiabatically drag □ right**

$$\begin{aligned}|\Psi(5)\rangle &= \tfrac{1}{\sqrt{2}}(O|\Psi(1)\rangle - ie^{i\beta}(Z_k Y_k Z_k)Y_k O S_I^Z \Psi(1)\rangle) \\ &= \tfrac{1}{\sqrt{2}}(|\Psi(1)\rangle + ie^{i\beta}S_I^Z|\Psi(1)\rangle)\end{aligned}$$

- $$\begin{aligned}|\Psi(6)\rangle &= e^{iS_I^Z\pi/4}|\Psi(5)\rangle \\ &= \tfrac{1}{2}((1+e^{i\beta})iS_I^Z|\Psi(1)\rangle + (1-e^{i\beta})|\Psi(1)\rangle)\end{aligned}$$



- **Measure location of □**

$$\langle S_I^Z\rangle = \sin(\beta + \pi)$$

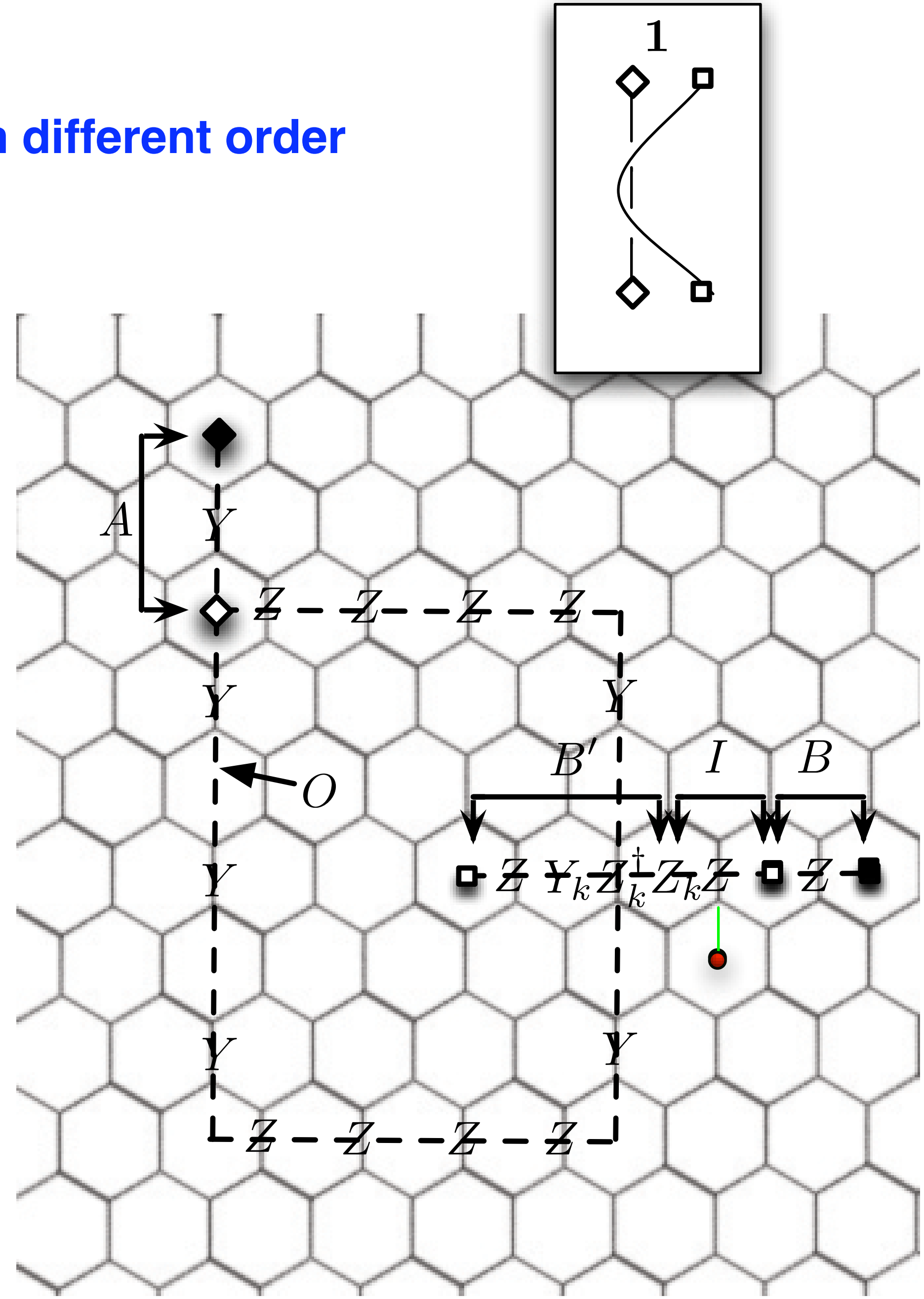
Dynamical+Berry phases

Statistical phase

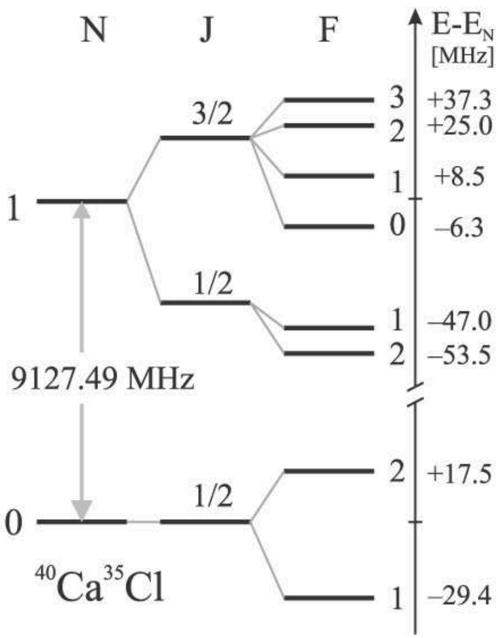
For trivial braid use same steps but in different order

- Adiabatically drag ◇ CCW around □
- Adiabatically drag □ left
- Adiabatically drag □ right
- Measure location of □

$$\langle S_I^Z \rangle = \sin(\beta)$$



III. Higher spin models

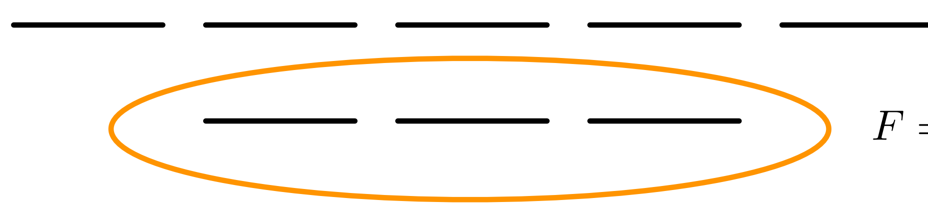


$I = 3/2$

$$H_{\mathrm{m}} = B\mathbf{N}^2 + \gamma \mathbf{N}\cdot\mathbf{S} + b\mathbf{I}\cdot\mathbf{S} + cI^z S^z + eQq\frac{3I^{z2} - I(I+1)}{4I(2I-1)}$$

Fermi contact **Dipolar** **Electric Quadrupole**

$N = 0$

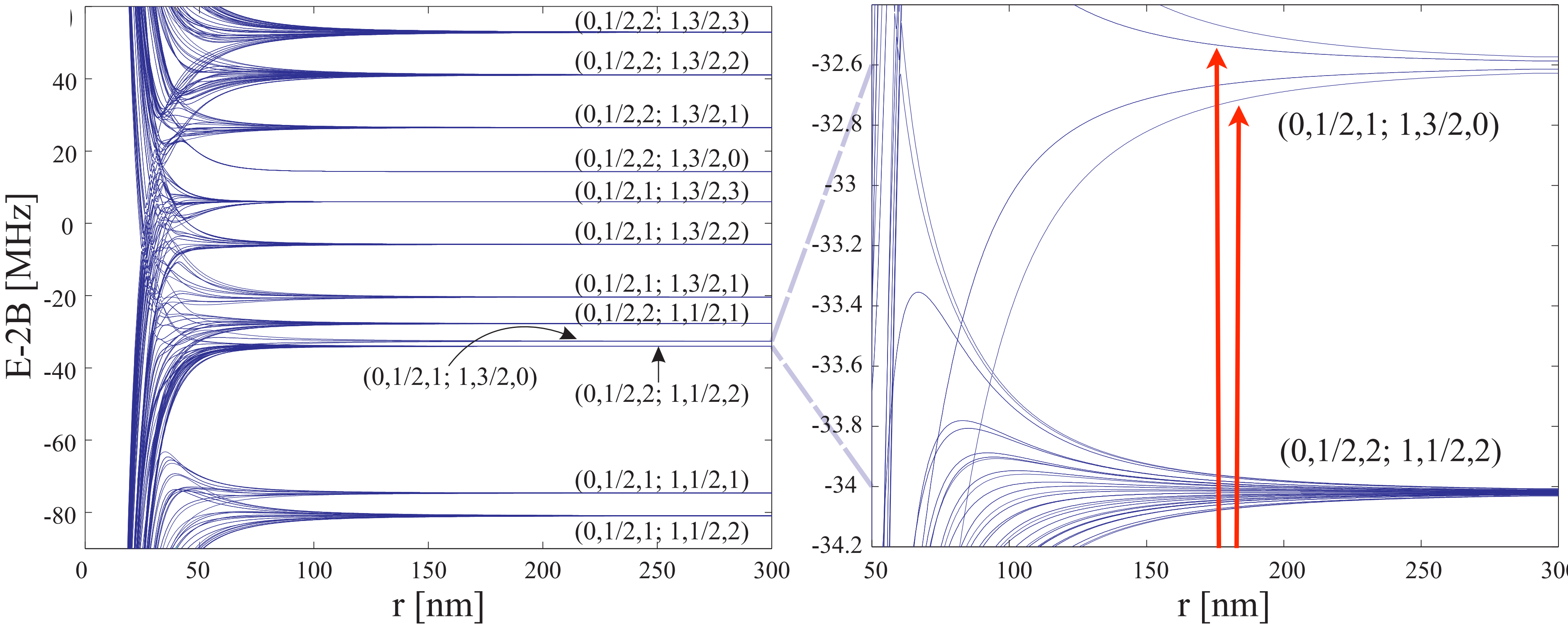


$F = 2$

$F = 1$

Encode here

hyperfine cont.



Asymptotic couplings exactly solvable

Can't build generic two body Hamiltonians but can build a large class

Example Hamiltonian in terms of spin-1 rep of su(2):

$$H_\beta = U(\mathbf{S}_1 \cdot \mathbf{S}_2 - \beta(\mathbf{S}_1 \cdot \mathbf{S}_2)^2)$$

Built with 6 microwave fields to allow tunable β

Continuing work....

Summary & Outlook

- Recipe for building a class of Hamiltonians with topologically ordered ground states

- We can design spin-spin interactions with polar molecules

  - Tunable range and anisotropy
  - Large coherence to decoherence ratio   Q~800-10000 for reasonable trapping parameters

- Examples of Lattice Spin Model with TO

  - The Kitaev Model
    - Gapped system with abelian excitations
    - Feasible technique for measuring quasiparticle statistics

- Can we increase the effective coupling (increase the gap)? Possible with self assembled lattices---->closer lattice spacings

- Building three body interactions