Supersolids, Grain Boundary Flow, Vortex Glasses, and Other Candidates

David Thouless

University of Washington

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Which results look most interesting to me?

The reduced moment of inertia at low temperatures reported by Kim and Chan initially suggested the Andreev-Lifshitz mechanism, but the combination of relatively high onset temperature combined with relatively high superfluid fraction made that seem unlikely. It also seemed particularly unlikely that spontaneous formation of vacancies could persist to high pressures, because of the big change in density.

Sensitivity to the state of annealing and to ^{3}He impurities was a relief to those who were uncomfortable with superfluidity in an ideal crystal.

Irrotational flow in the solid would be indicated by a frequency-independent response of the fluid to oscillation. Sloppy oscillators would have a response that tends to zero in the low-frequency limit.

Balibar's study of the flow of solid out of an inverted beaker suggests creep of the solid, analogous to the creep of a superfluid film, which is not stopped by the visocosity of the normal component.

Why do we expect a Bose solid to behave like a rigid body?

Bose condensate has a single-particle wave function occupied by finite fraction of bosons in the system. Phase $S(\mathbf{r})$ of this wave function determines superfluid velocity by $\mathbf{v}_s = (\hbar/M)\nabla S$. Quantized vortex arises because condensate wave function is one-particle wave function and therefore single valued, and so S is single-valued modulo 2π. This leads to quantization of circulation, and to the inability of Bose condensate to respond to a slow rotation of its container.

Penrose and Onsager defined condensate wave function as the eigenvector of Dirac density matrix

$$< \phi^{\dagger}(\mathbf{r}')\phi(\mathbf{r}) > .$$

Largest eigenvalue is typically close to the number N of particles for a condensed atomic system, and around $0.07N$ for liquid helium. This is not the whole story, as this does not work for two-dimensional systems. Nevertheless, superfluidity is observed in two-dimensional systems, so real story of superfluid properties is more complicated than this. Superfluid density depends on the long-range correlation of the gradient of the phase, not disturbed by fluctuations of the phase (BKT theory).

Even in the high-temperature phase, where free vortices destroy the long-range velocity-velocity correlation function, there is still at least some short-range significance of the phase, as Anderson has recently pointed out.

For ideal single-crystal solid there seems to be no good reason to expect a single-particle condensate. The operation involved in the Penrose-Onsager criterion is the removal of one particle, creating a vacancy or destroying a thermally created interstitial, and adding a particle in a distant interstitial position or in an existing thermally excited vacancy. Why I should expect phase coherence for such a process, except by Andreev-Lifshitz-Chester mechanism — ground state with nonzero concentration of vacancies?

Solid neon should have dynamics of any other solid, and, even in helium, exchange is a small, short-ranged, effect.

I know wave functions for chain of hard-core bosons confined to a tube of length L. In the sector $0 < x_1 < x_2 < \ldots < x_N < L$ it is

$$\Psi \propto \det \exp(k_i x_j)\,.$$

Invariance under the transformation

$$x_n \to x_{n+1} \ \text{ for } \ 0 < n < N, \quad x_N \to x_1 + L\,,$$

gives $\exp(k_i L) = (-1)^{N+1}$ for all k_i.

Such a wave function has quantized circulation, since the sum of the wavenumbers k_i can only change by an integer multiple of $2\pi/L$. However, a long chain of atoms in a three-dimensional solid is restricted by the other atoms in their neighborhood, and so the matrix element of the splitting between the ground state and the first circulating state is reduced by the ratio of the hopping matrix element to the energy barrier to the power N. I therefore expect the quantization of circulation to be exponentially small in an ideal crystal.

What is going on?

I argue that even if we have vortex glass, there must be a fluid or glassy state to support the quantized vorticity.

Possibilities open at present include:

1. Bulk point defects. Why do they not just migrate to grain boundaries by quantum diffusion?
2. Glassy or fluid disorder at grain boundaries. I like a dense quantum fluid, because I have an old theory that I never found a use for. Glass does allow Penrose-Onsager long range order, because there are always low-cost sites for vacancies and interstitials. My theory does not look like a good fit to the experimental situation.
3. Fluid phase on line defects such as dislocations, as Svistunov described.
4. Mobile line defects or grain boundaries. How easy is it to move dislocations? Unless there are vacancies and interstitials around, dislocation is only free to glide, not climb; it can move in direction of Burgers vector, not perpendicular.

Fluid layers and their effect on solid

If we take NCRI literally there is *prima facie* evidence for considerable mass movement relative to the container – many nanometers per period.

Such movement is not large if it is some sloppy visco-elastic distortion, so evidence of persistence at lower frequencies is crucial. If the critical velocity and NCRI persist to low frequencies, then there is 'real' superfluid flow.

Why is there more loss of apparent moment of inertia for *thinner* samples, according to Rittner and Reppy?

Fluid grain boundary could produce limited amount of mass movement, as could fluid dislocation network, and could support quantized vortices.

Zero-point motion of grain boundaries can allow limited penetration of phase coherence into the ordered solid regions.

Glassy or fluid regions could explain the lower observe NCRIs, but seem implausible for figure of 20 or 30%. Samples giving such large numbers should show strong liquid-like X-ray signals.

Balibar's observation of the creep of solid out of an upturned beaker until it reaches the equilibrium level shows that liquid that flows out of the surface or linear fluid regions must be replaced by further melting until the excess solid is gone.

Perhaps part of the effect observed in torsional oscillator is due to mass transport by movement of the fluid boundaries through the solid, in a melting–regelation process.

Boris Spivak recently told me of an experiment, apparently unpublished, by Andreev and Sharvin, observing a solid helium sphere in superfluid, which fell must faster than Stokes formula would permit.

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