The solid state of ^{4}He : to flow or not to flow

Lode Pollet
Minnesota,
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Phys. Rev. Lett. **97**, 080401 (2006).
Phys. Rev. Lett. **98**, 135301 (2007).
Phys. Rev. Lett. **99**, 035301 (2007).

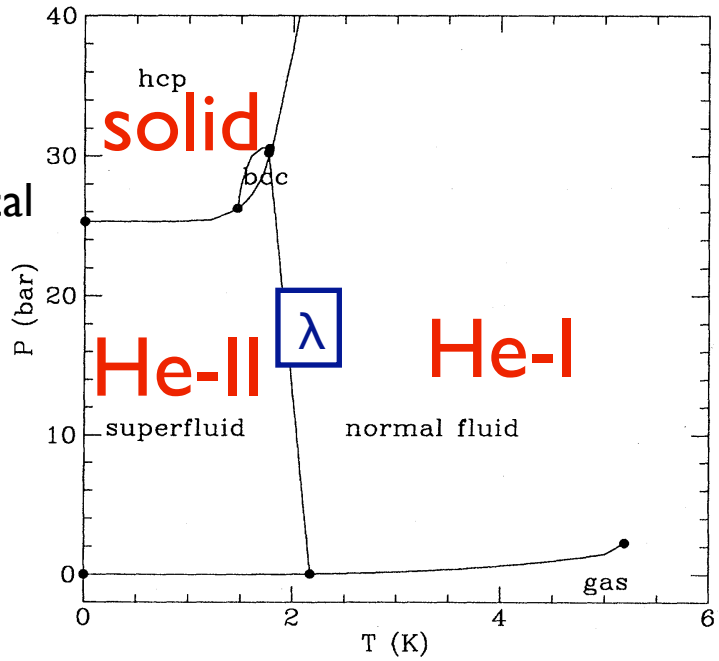
Outline

- Deformations of perfect crystal
- point defects : vacancies
- line defects : dislocations
- planar defects : grain boundaries

^{4}He

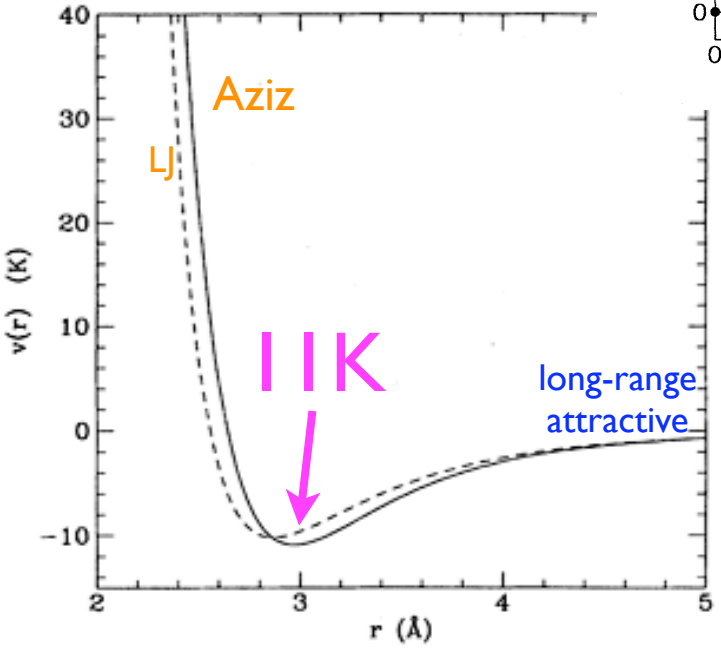
Debye-Waller
30%

most quantum-mechanical
solid known in nature

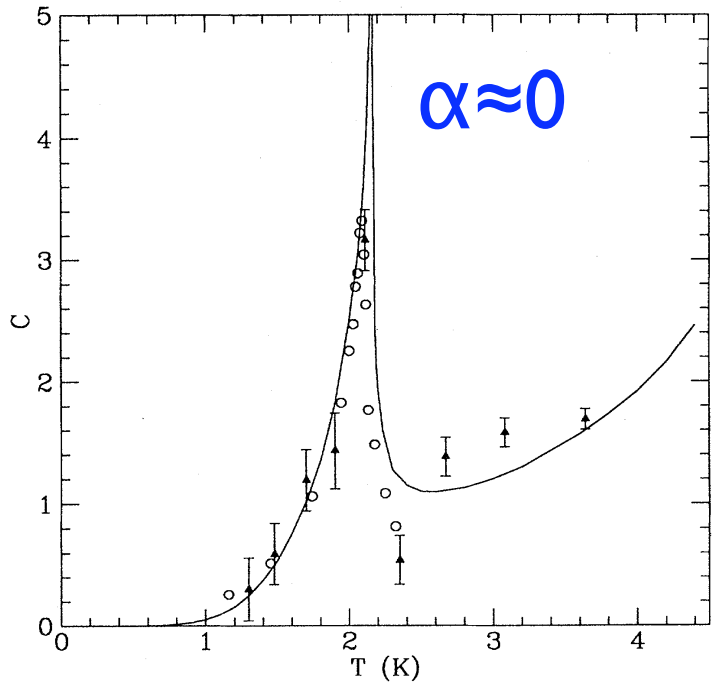


Kapitza (1938)

Aziz potential

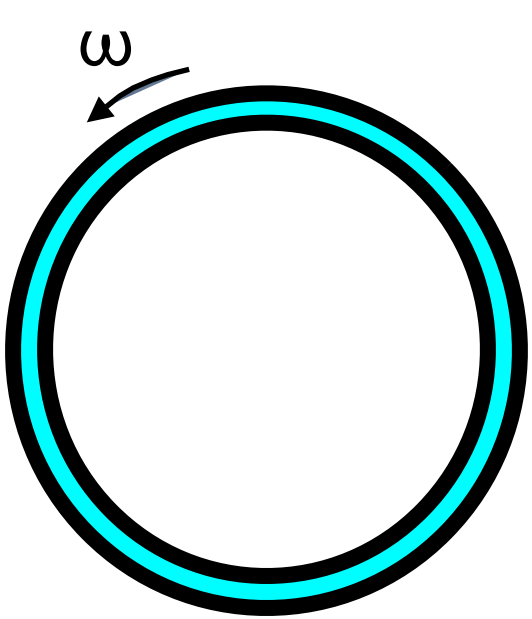


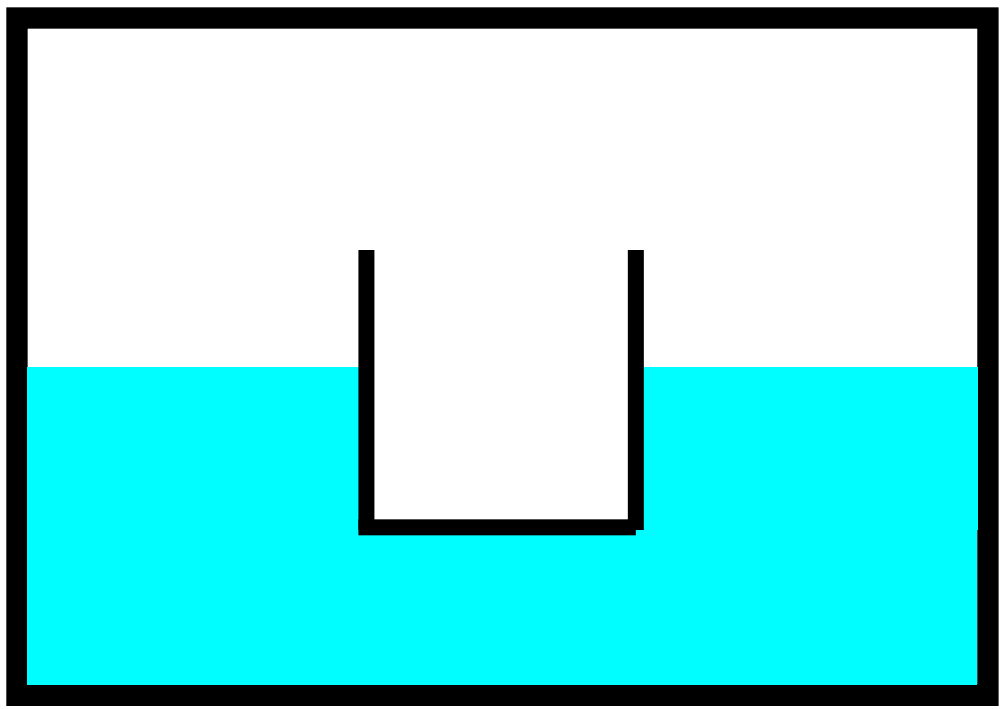
lambda transition



Superfluids

- zero viscosity
- zero entropy
- infinite thermal conductivity (fountain effect)
- does not rotate in slowly rotating cylinder;





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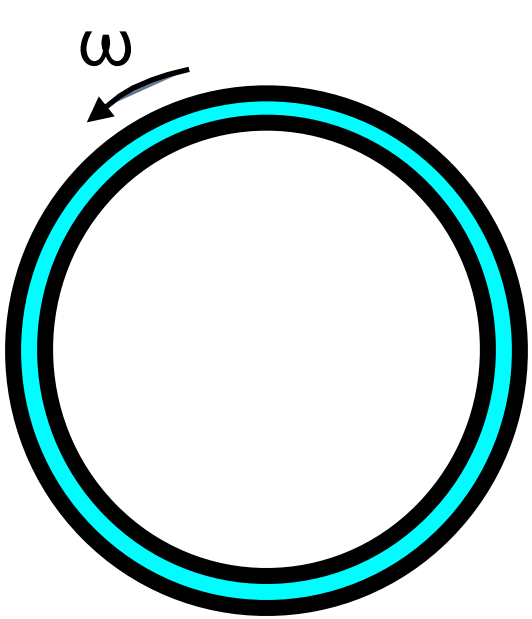

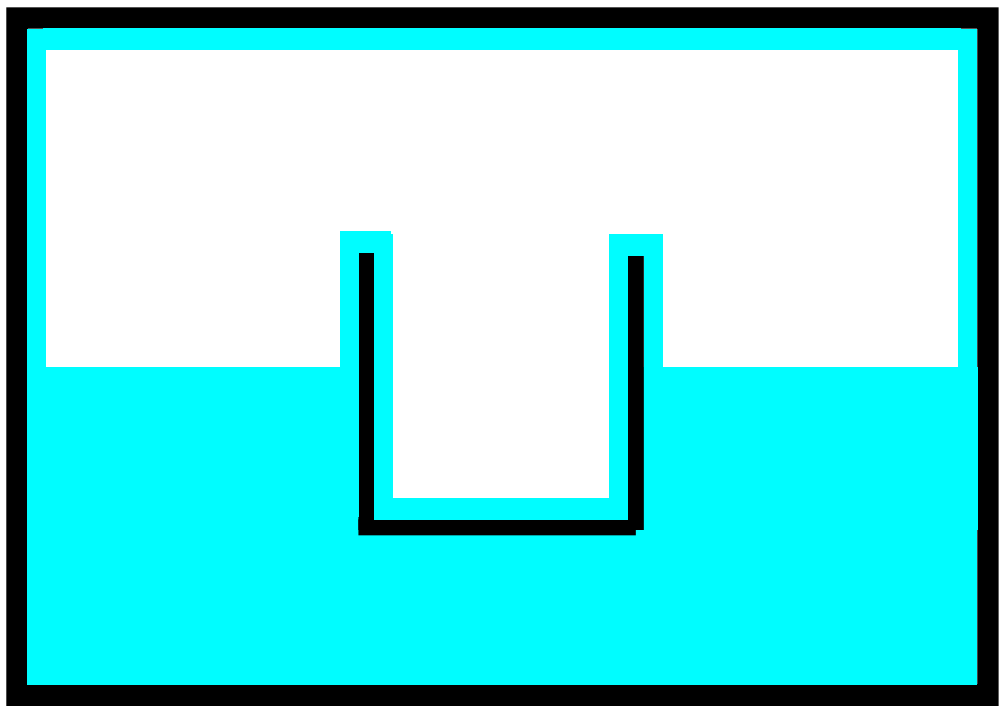

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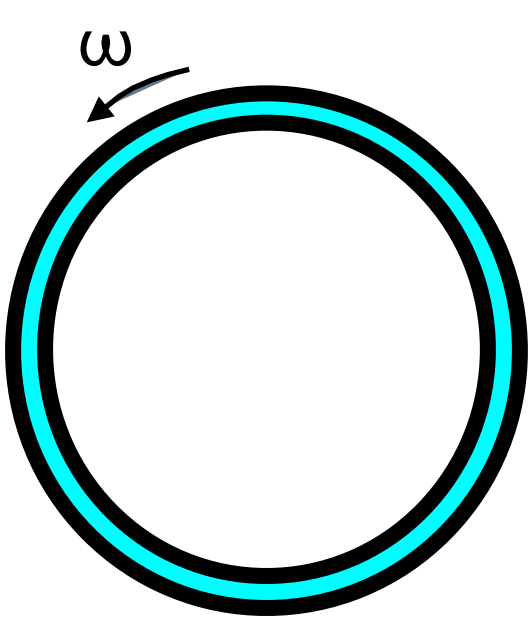
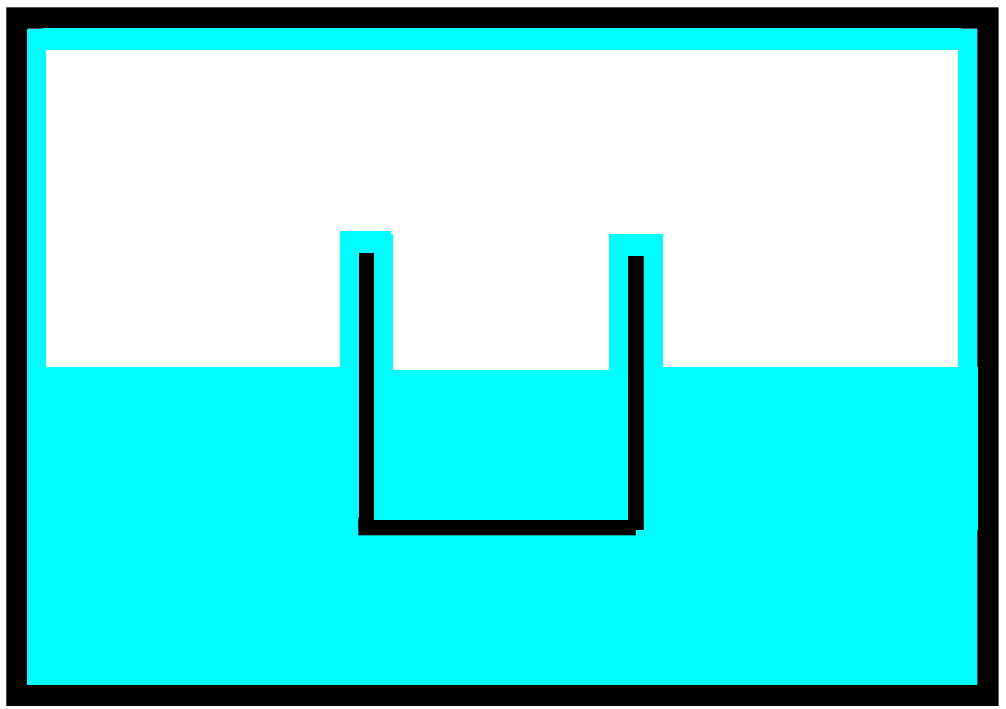
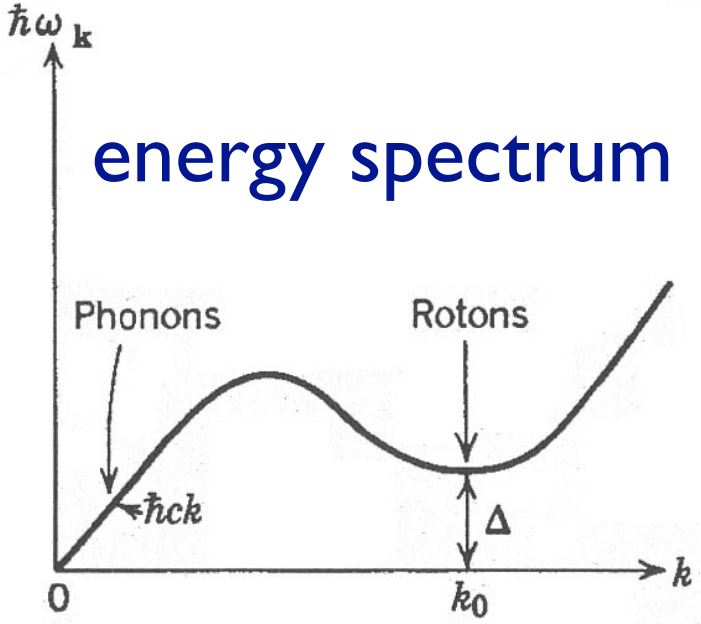




liquid ^{4}He



energy spectrum

Landau's two-fluid model

condensate fraction 10 %

superfluid fraction 100%

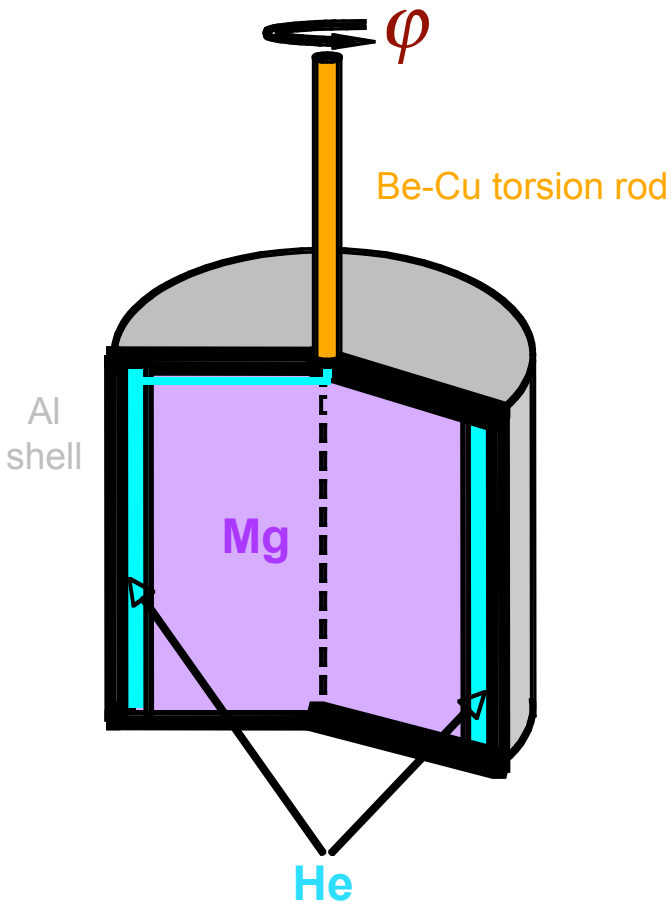
$$c = 239\mathrm{m/s}$$

$$\epsilon(k) = \Delta + \frac{\hbar^2(k-k_0)^2}{2\mu}$$

$$\Delta = 8.65\mathrm{K} \quad k_0 = 1.9\mathrm{A}^{-1}$$

$$\mu = 0.16 m_{\mathrm{He}}$$

Kim-Chan experiment



$$T_c \approx 0.2K$$

$$H = \frac{I\dot{\varphi}^2}{2} + \frac{\kappa\varphi^2}{2}$$

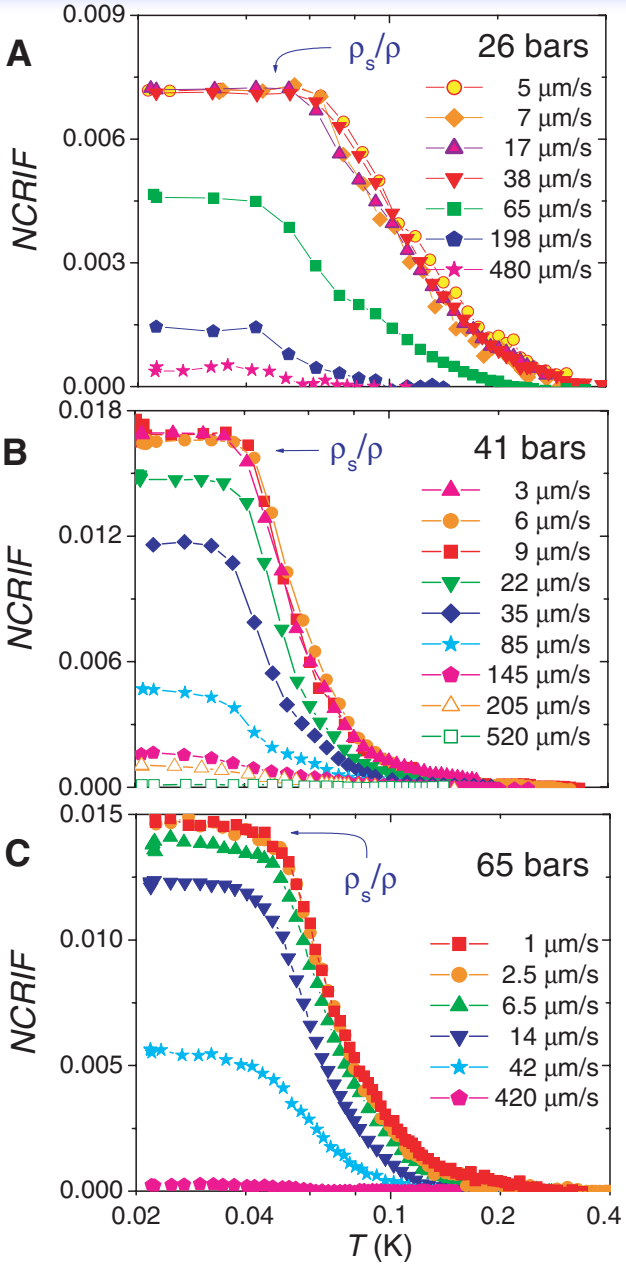
frequency of oscillations

$$\Omega = \sqrt{\kappa/I}$$

$$\frac{\rho_s}{\rho} = \frac{I_{\text{classical}} - I}{I_{\text{classical}}}$$

$$\rho_s(T \to 0)/\rho \approx 0.016$$

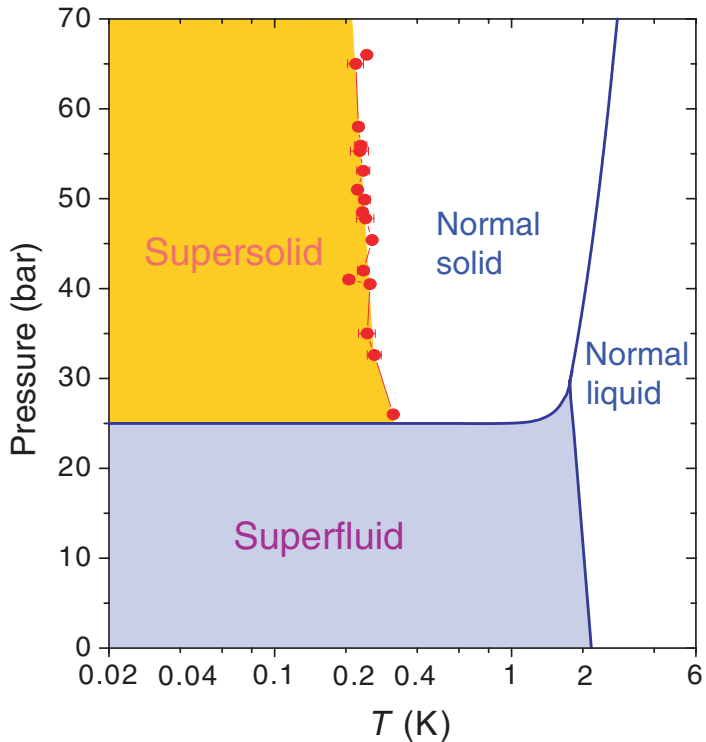
Kim-Chan experiment



data collapse
critical velocity
critical temperature

Simultaneously :
- ODLRO
- DLRO

Phase diagram



Why study vacancies in solid ^{4}He?

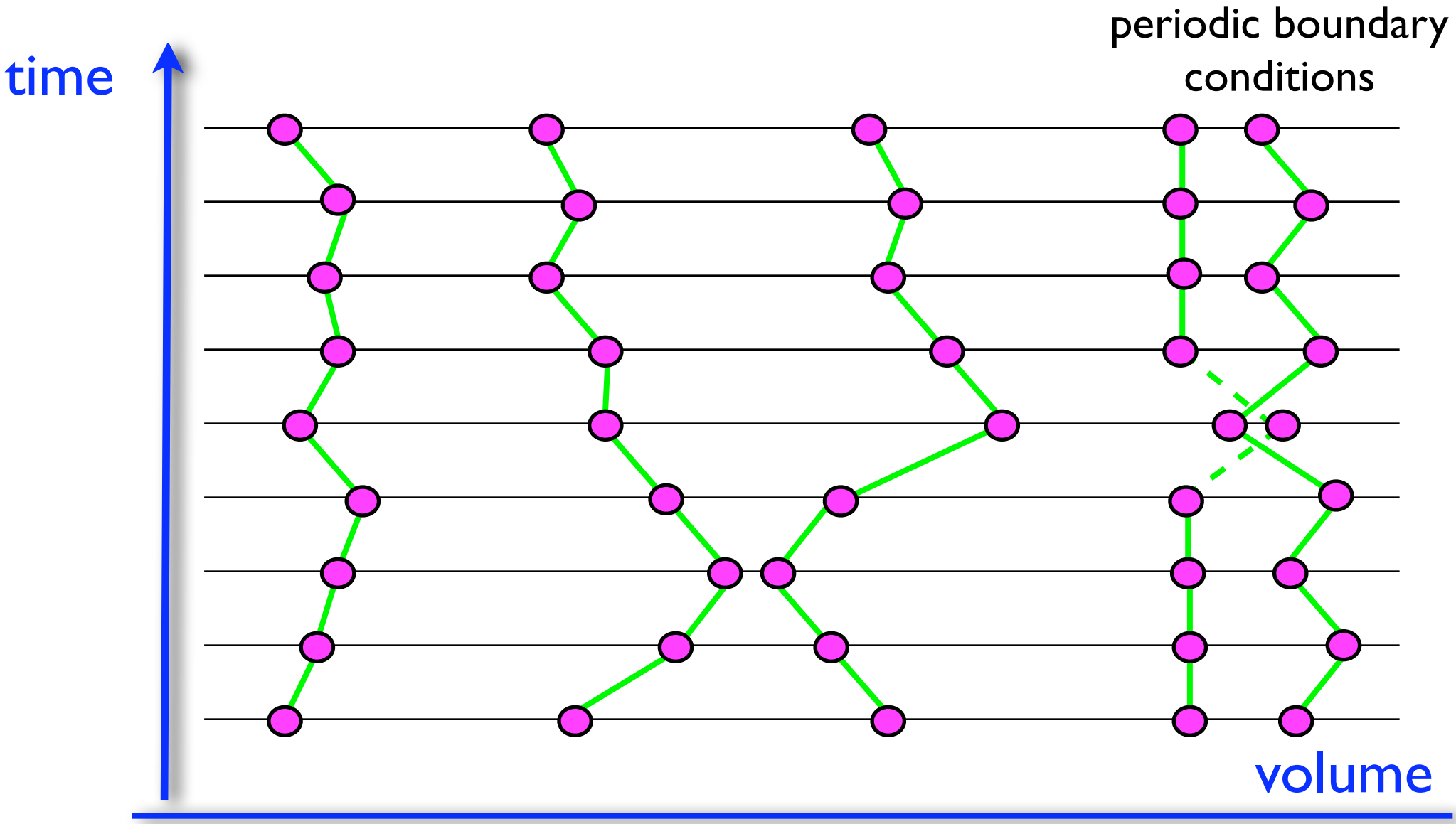
the necessary condition for a solid to be sf is to have zero-point vacancies and/or interstitials as an integral part of the ground state.

N.V. Prokof'ev and B.V. Svistunov, Phys. Rev. Lett. **94**, 155302 (2005).

superflow occurs generically in incommensurate solids

⟶ look at vacancies/interstitials

Path integral and worldlines



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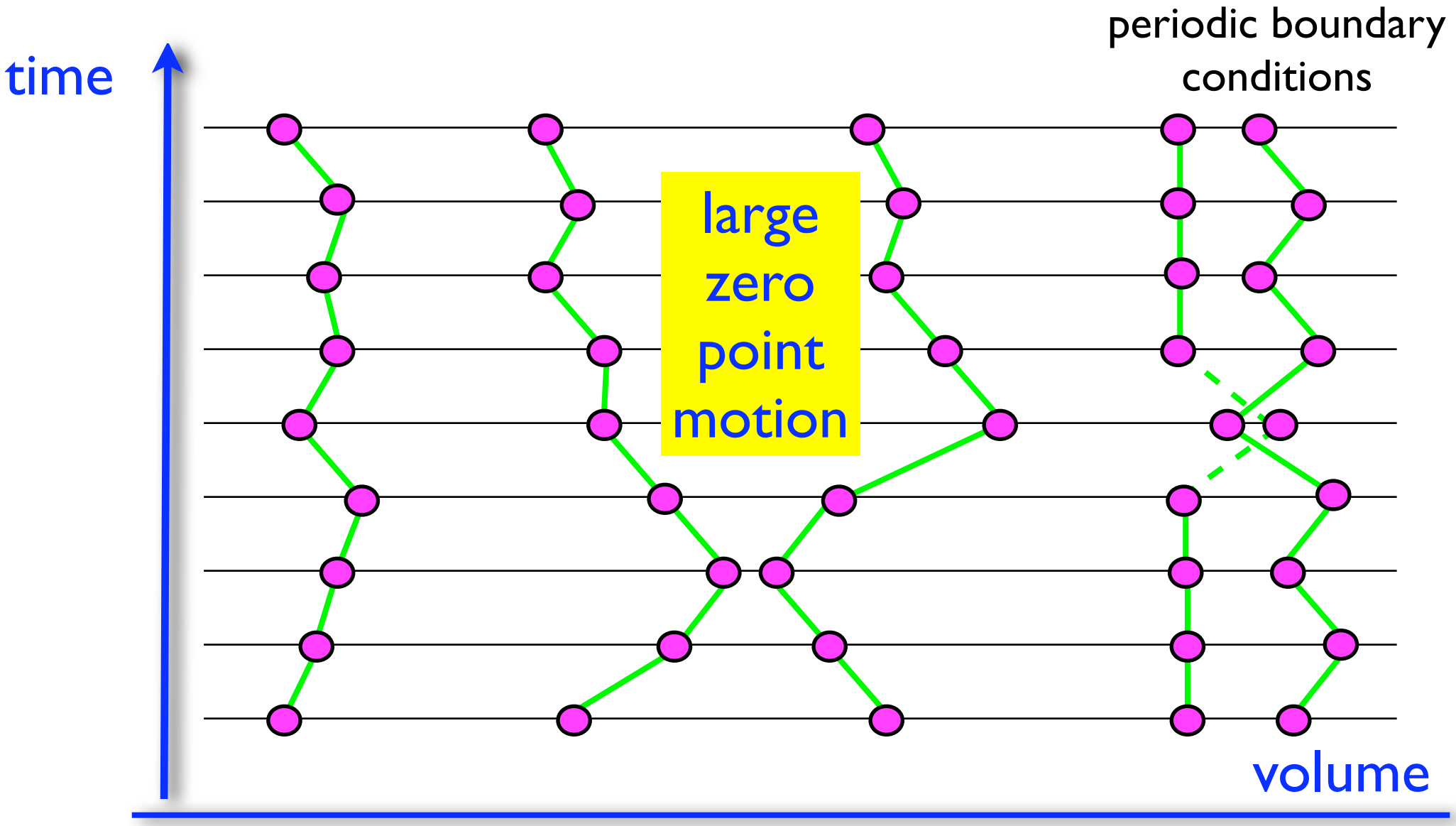

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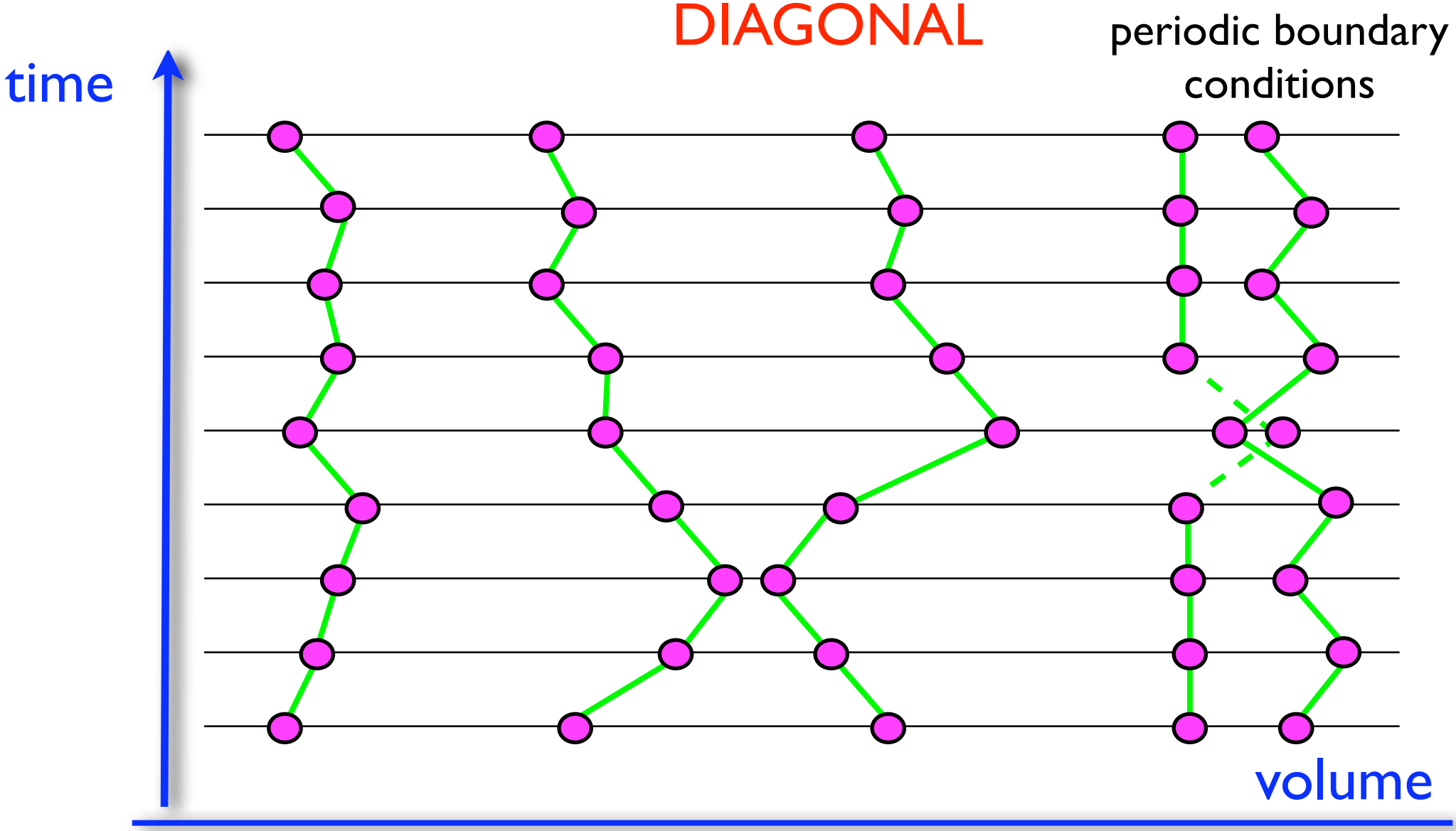

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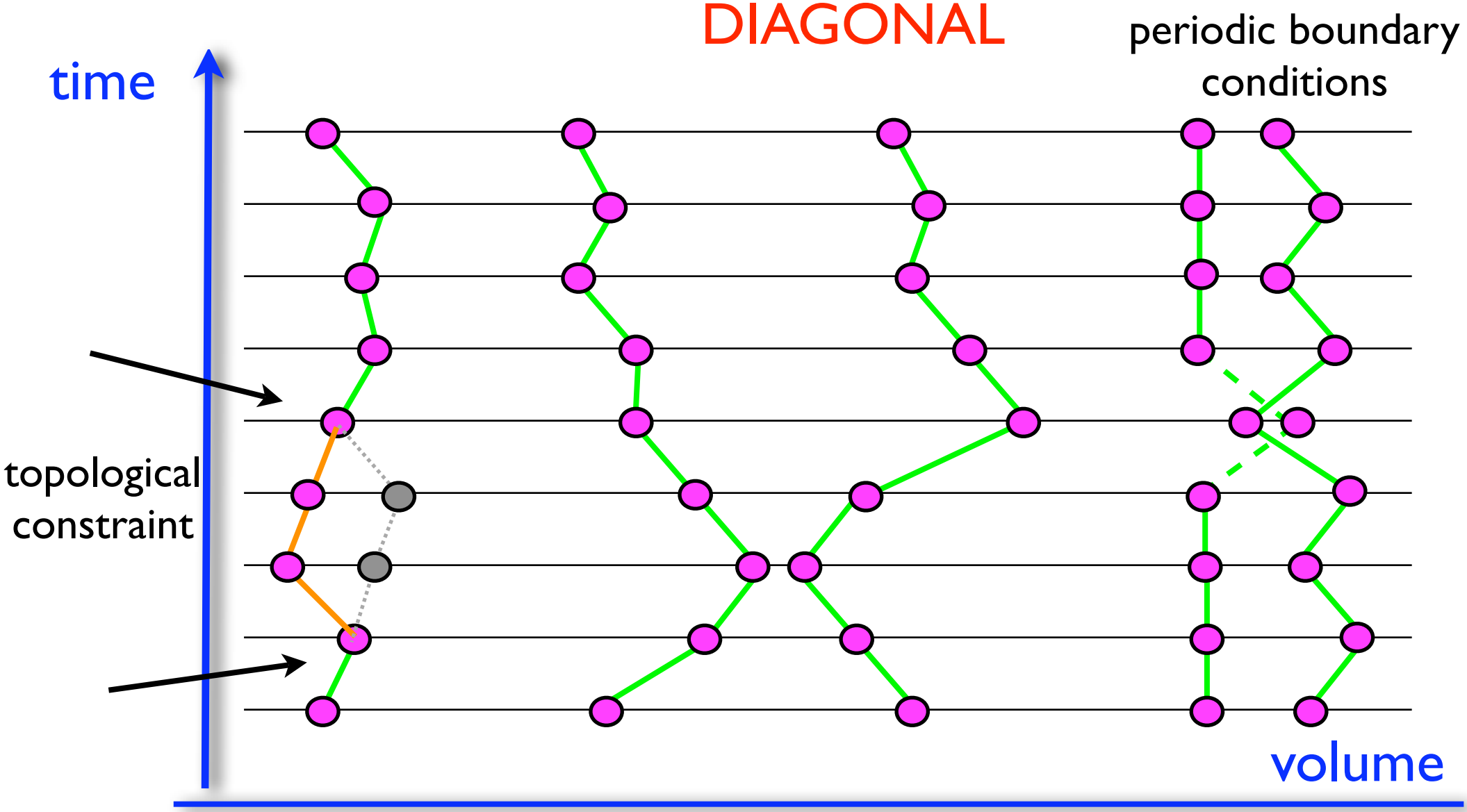
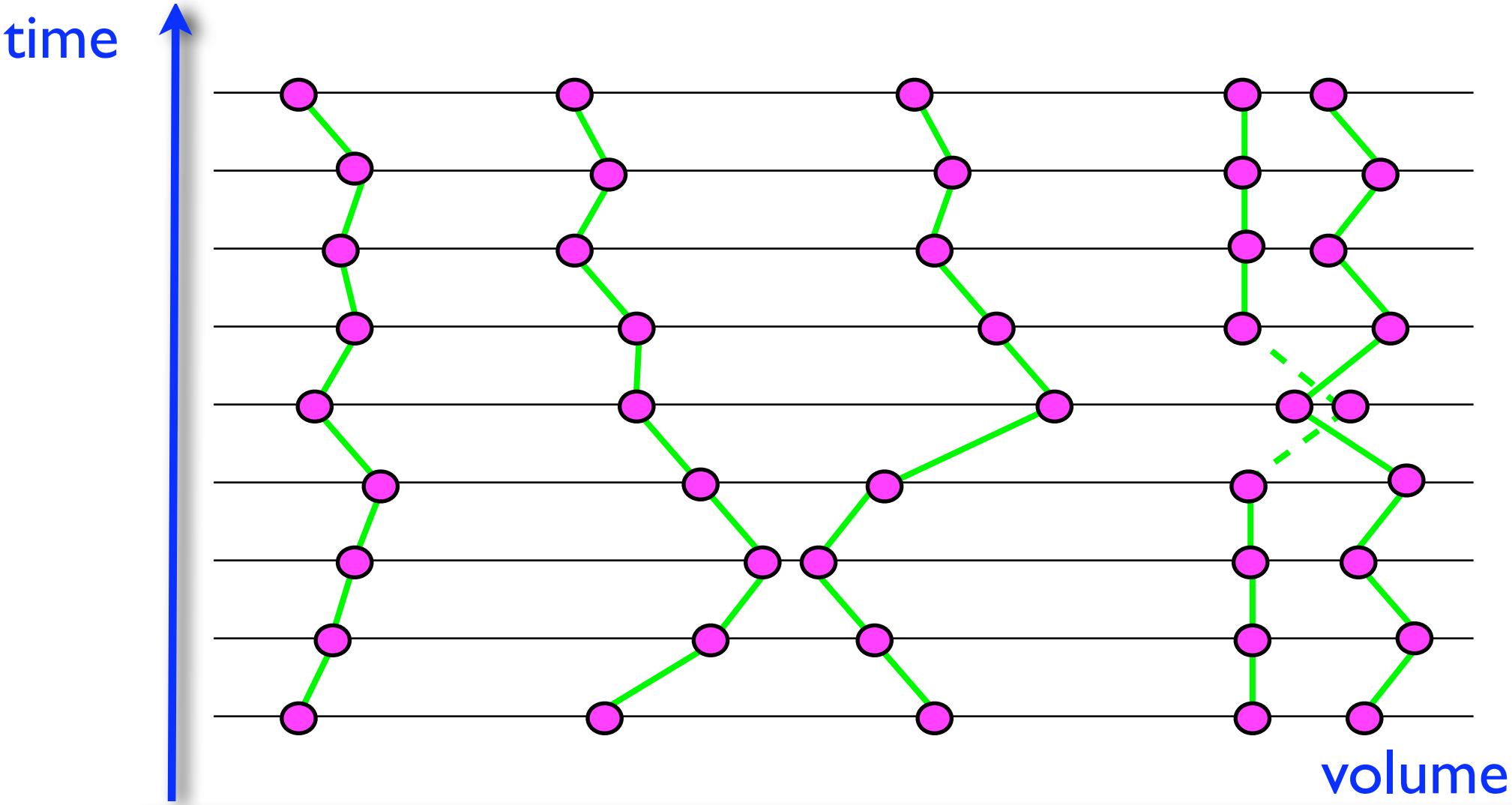


Path integral and worms



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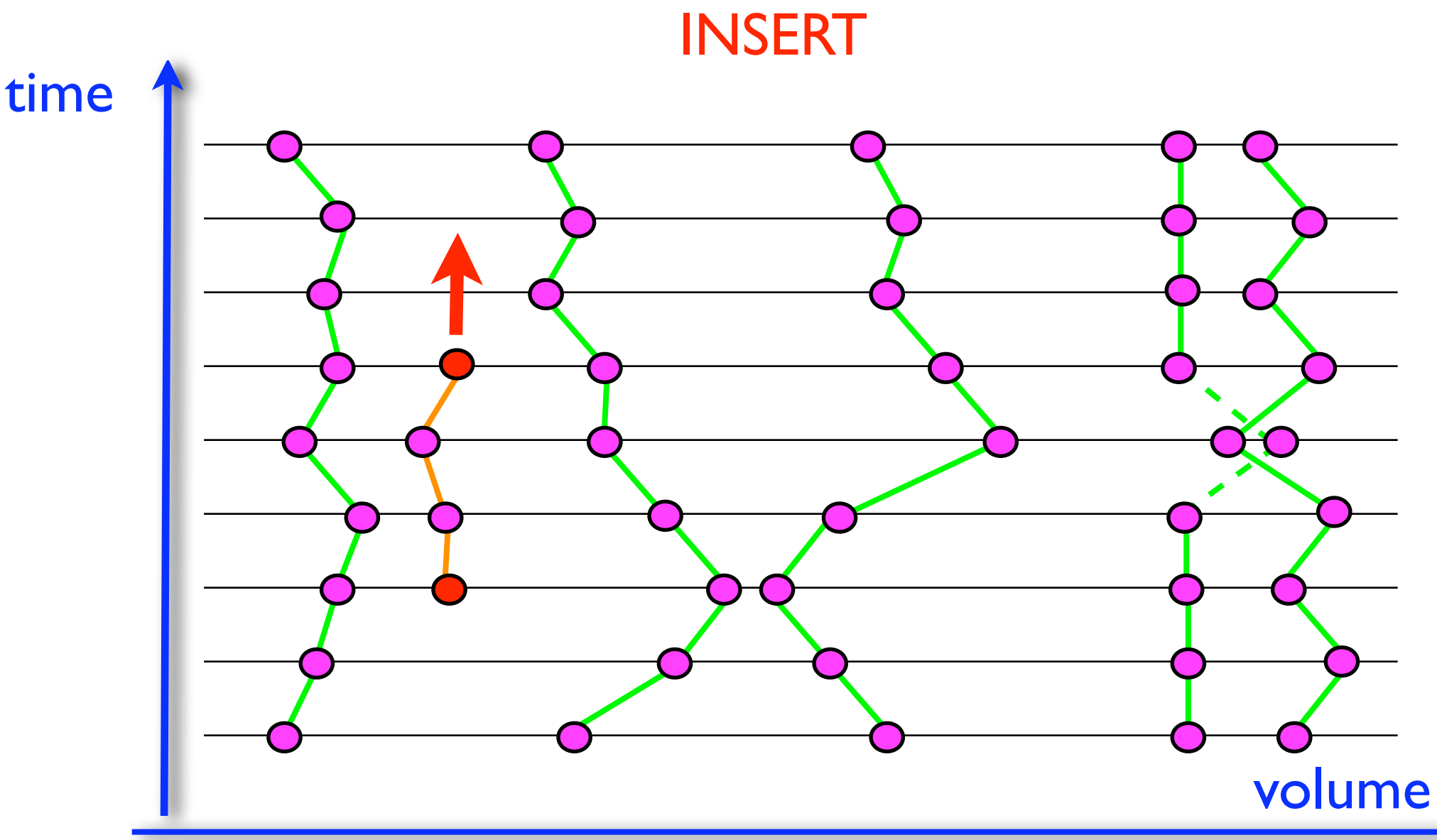
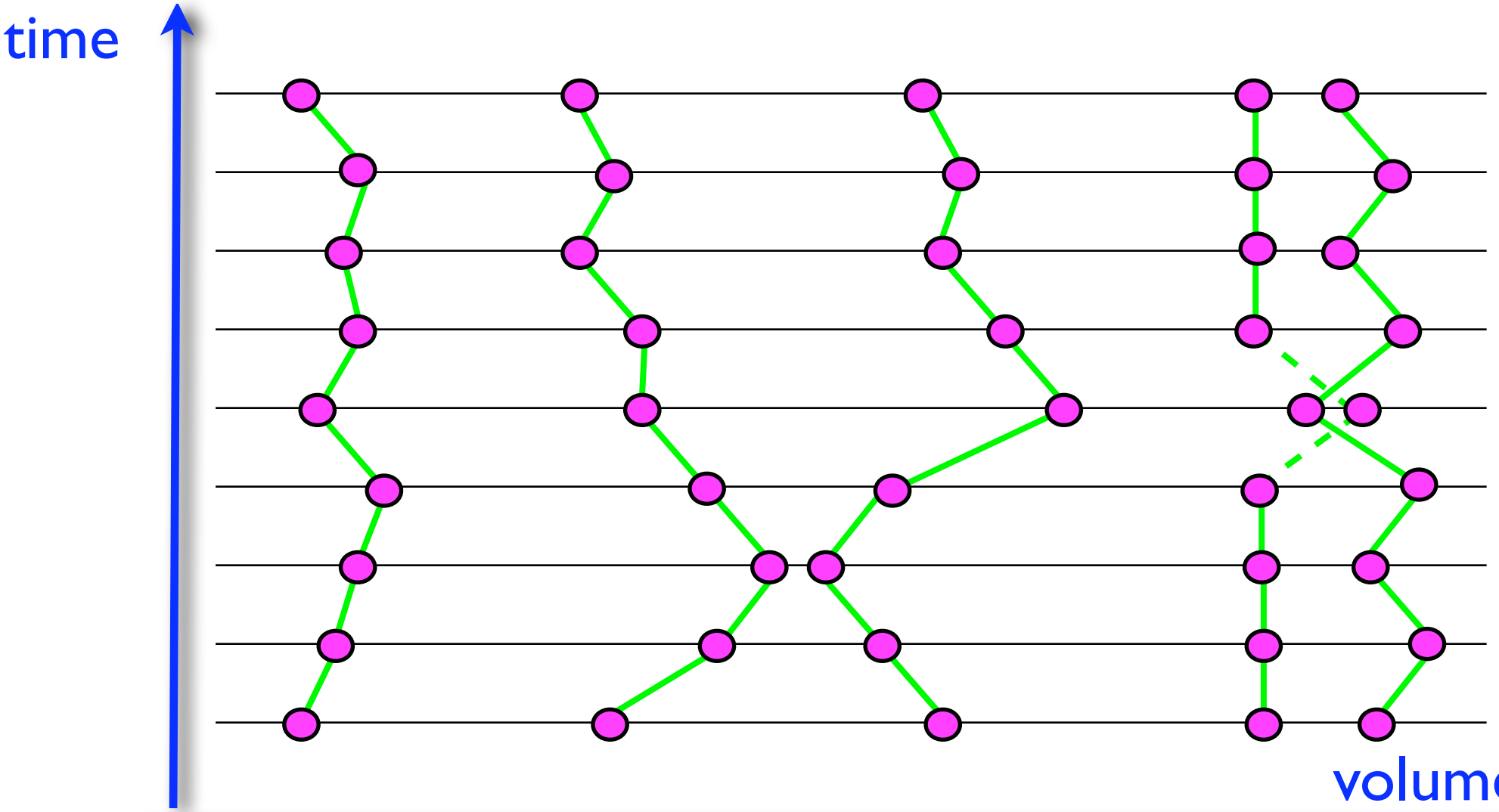


Path integral and worms

REMOVE



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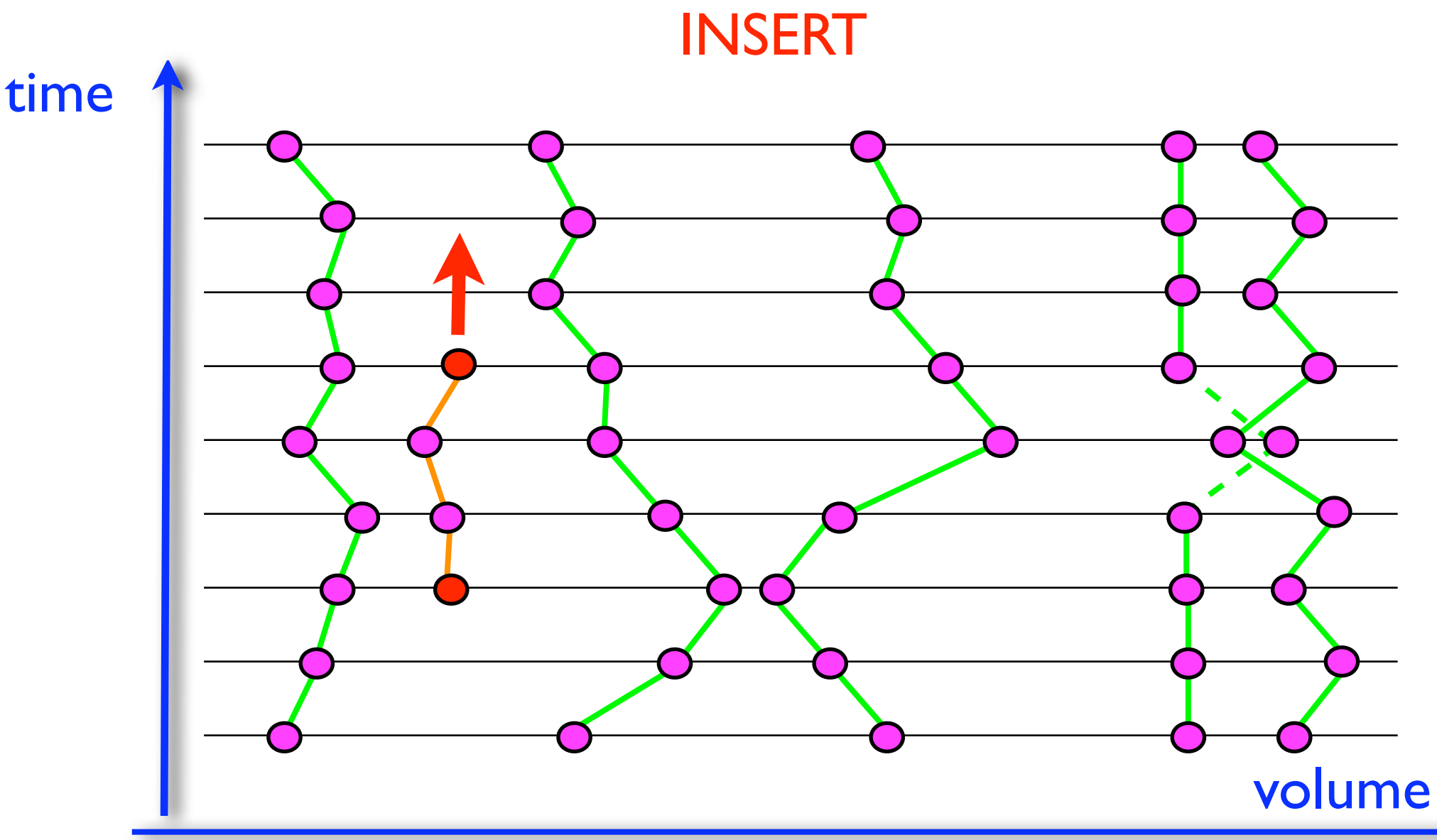

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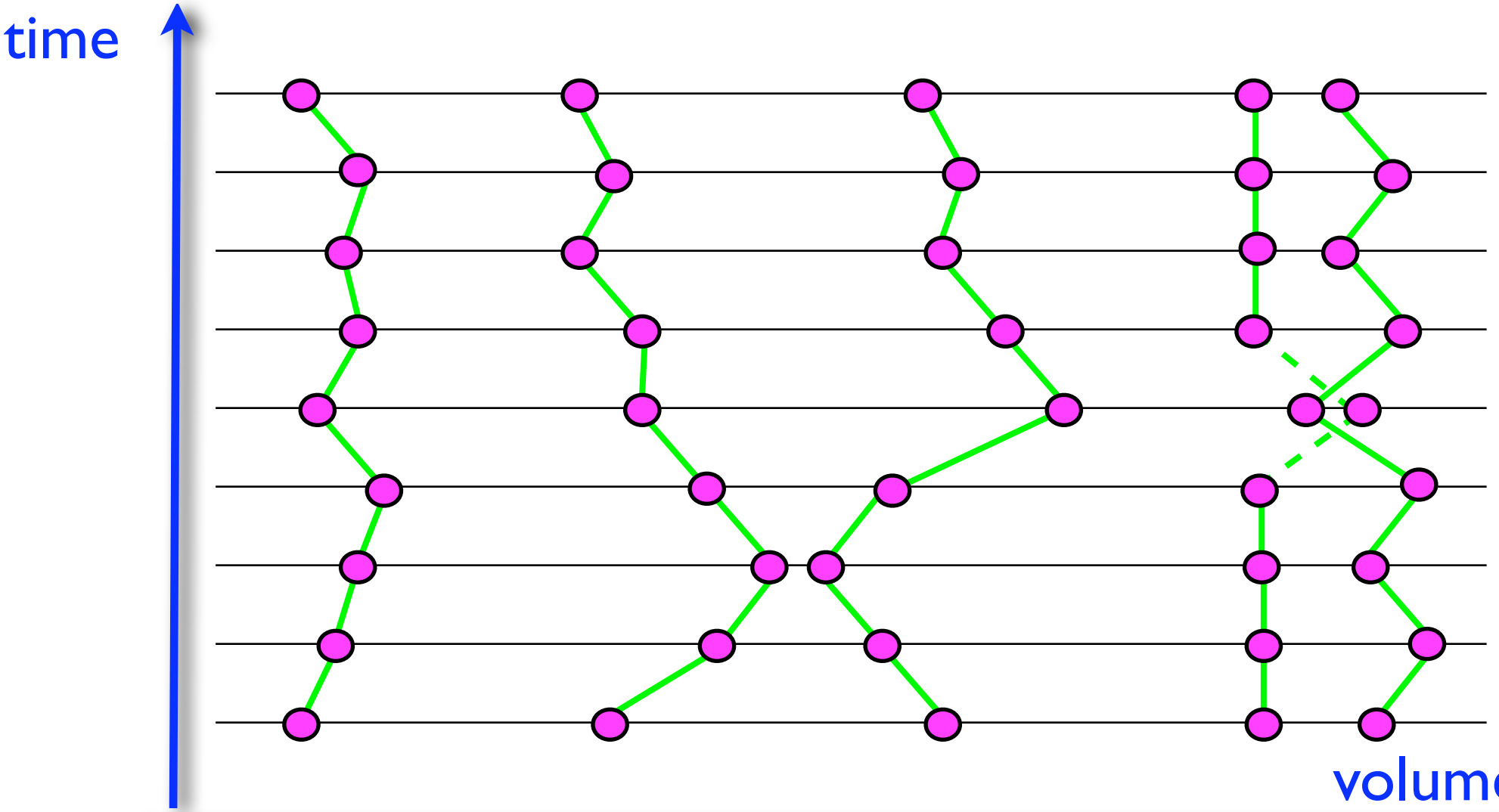

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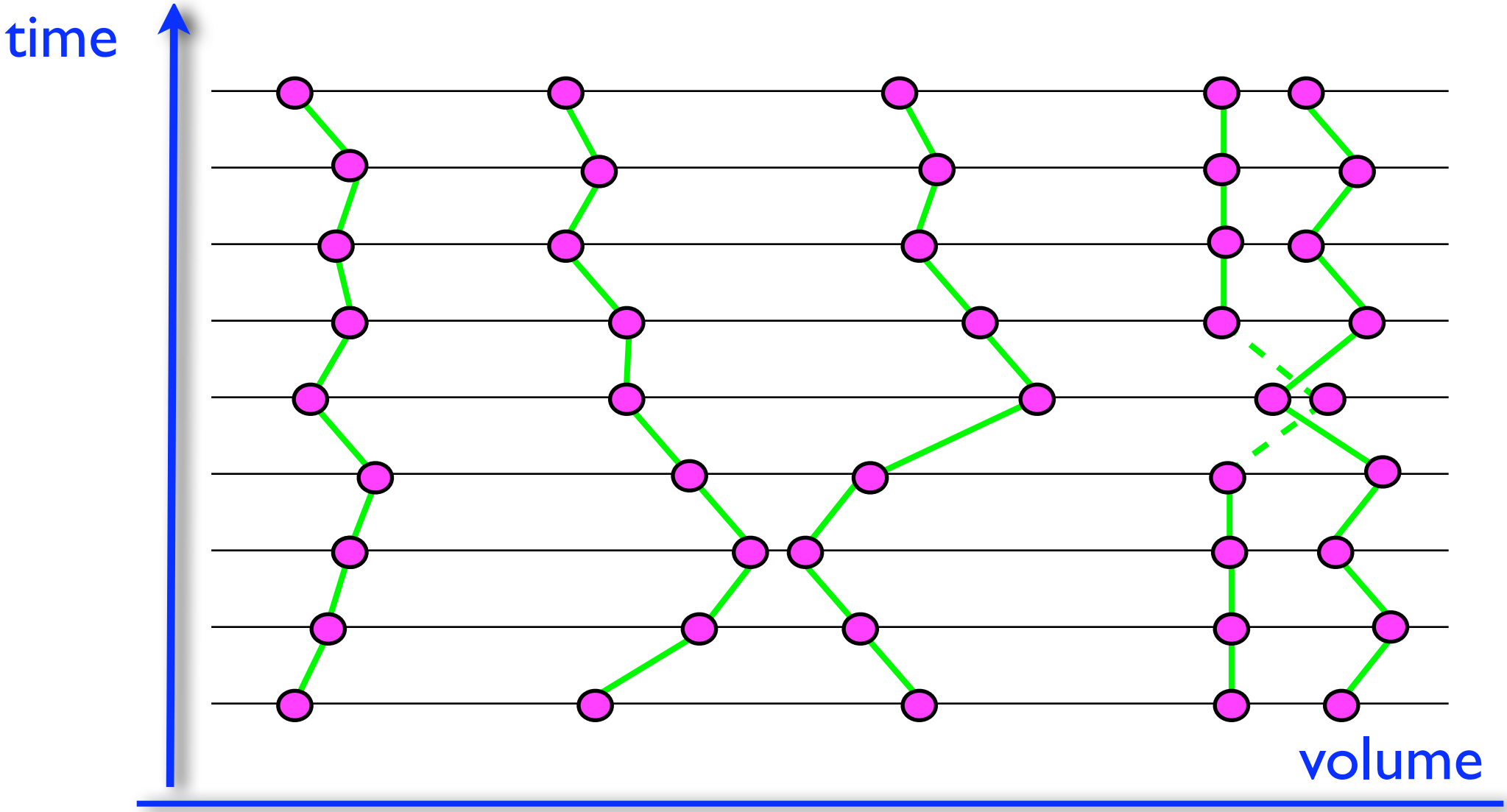

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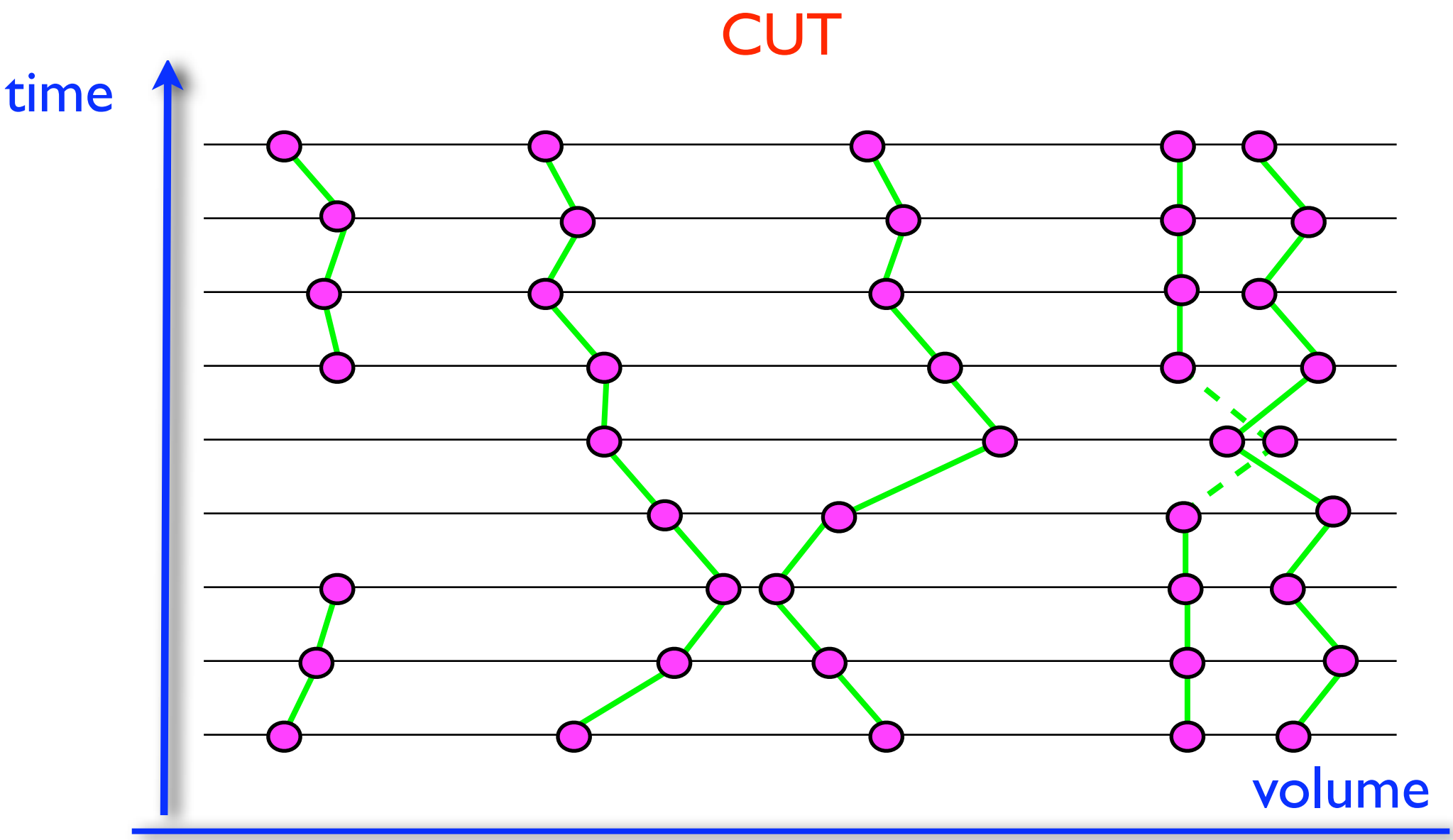

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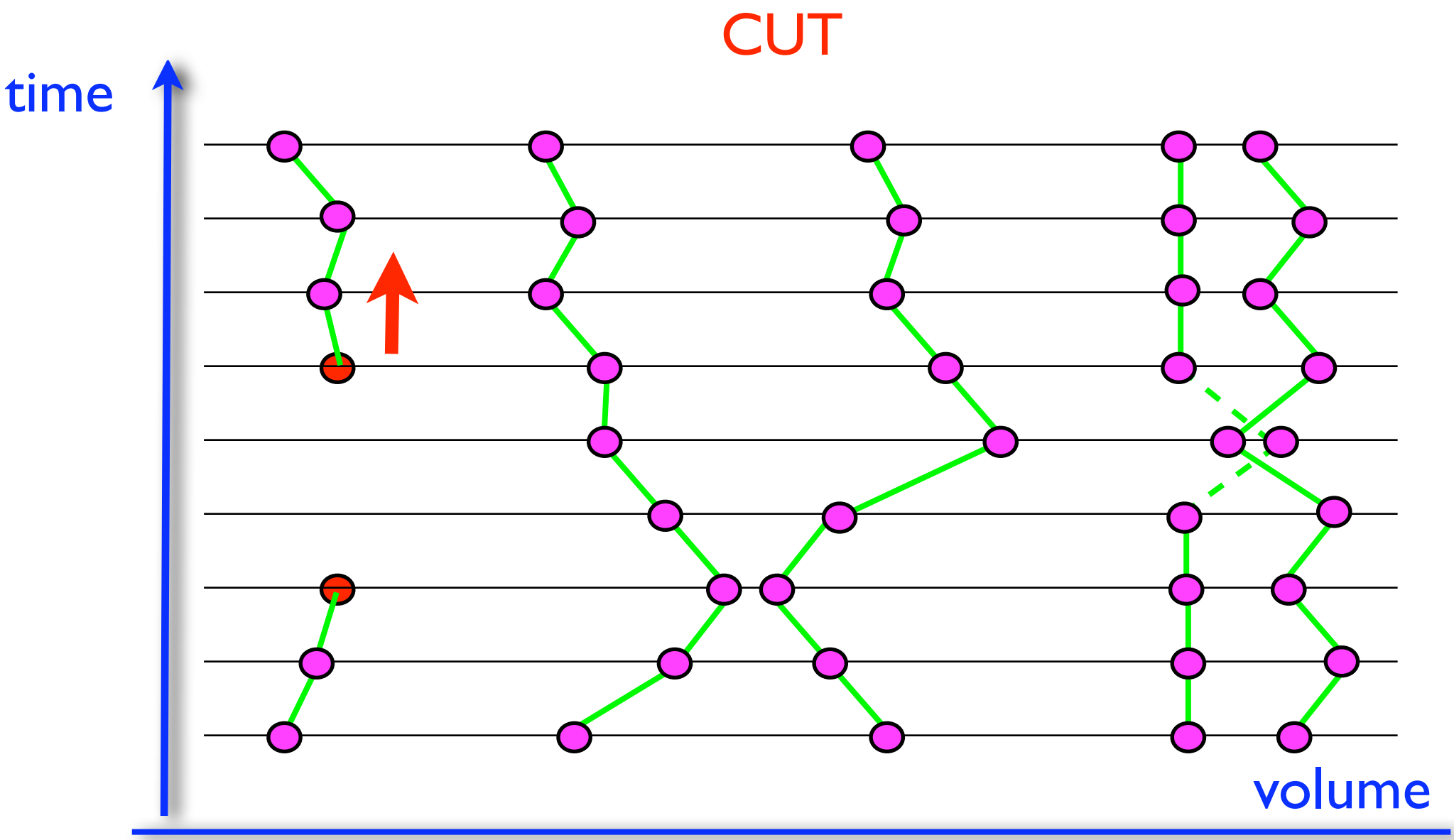

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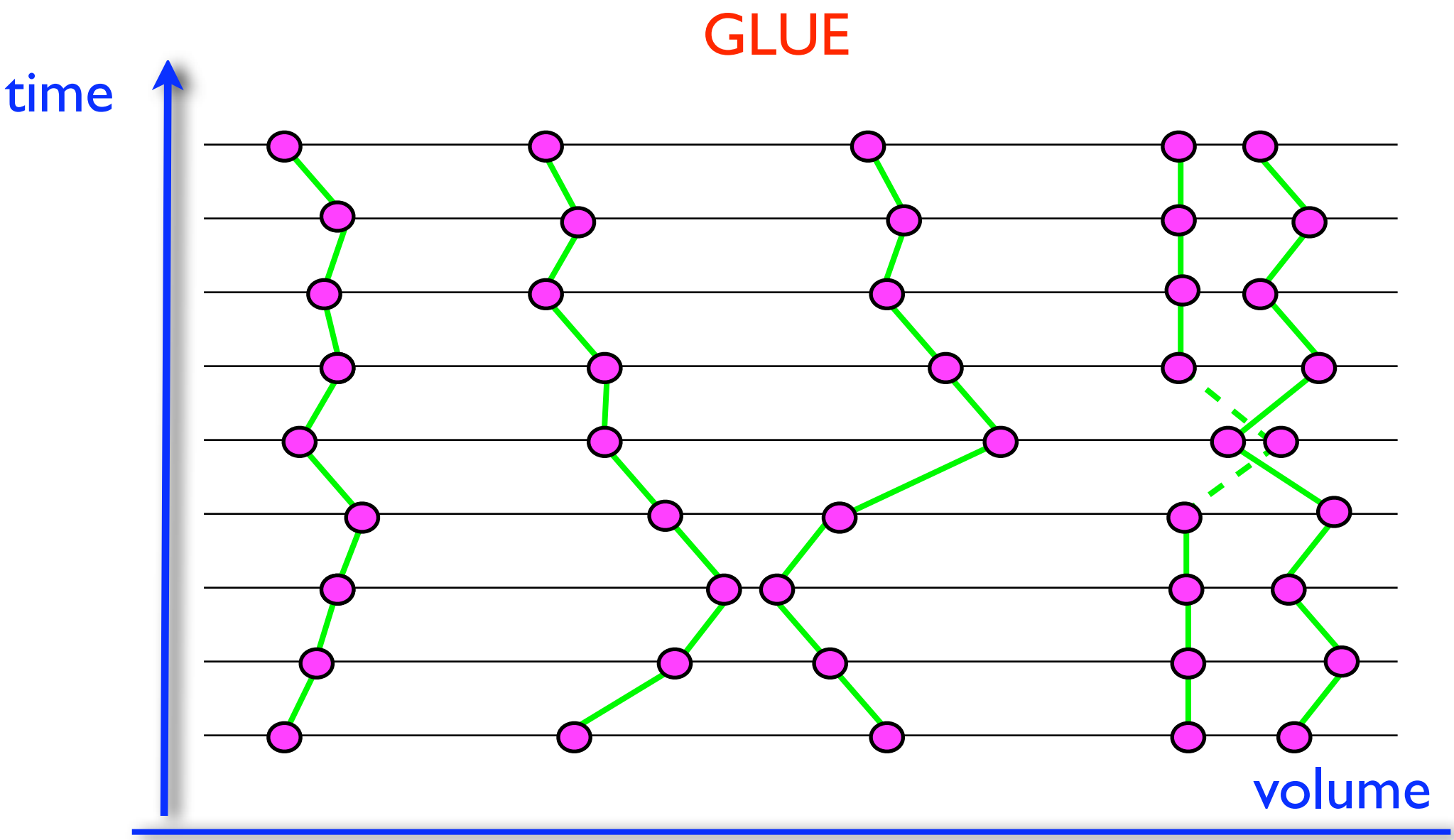

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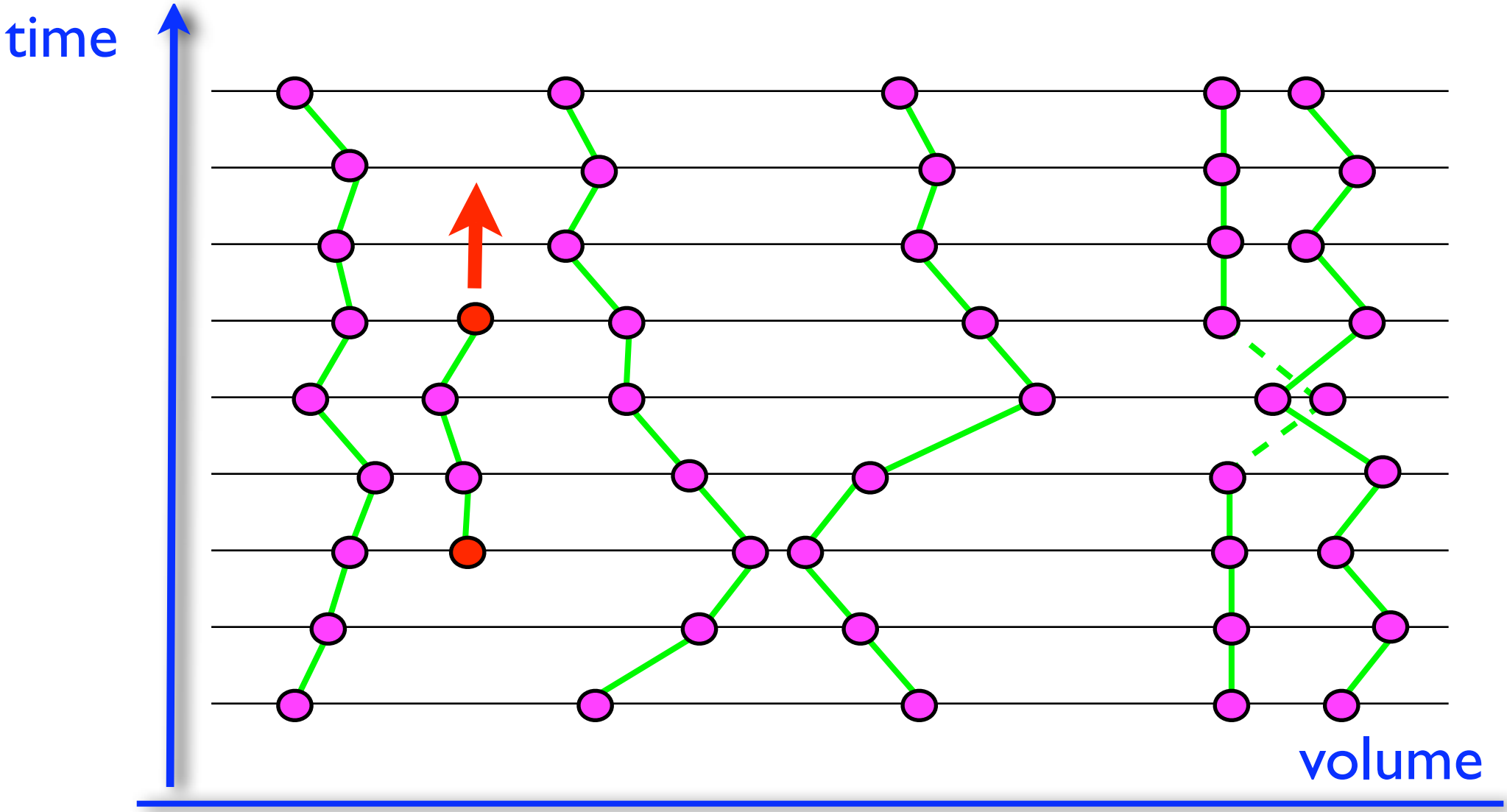

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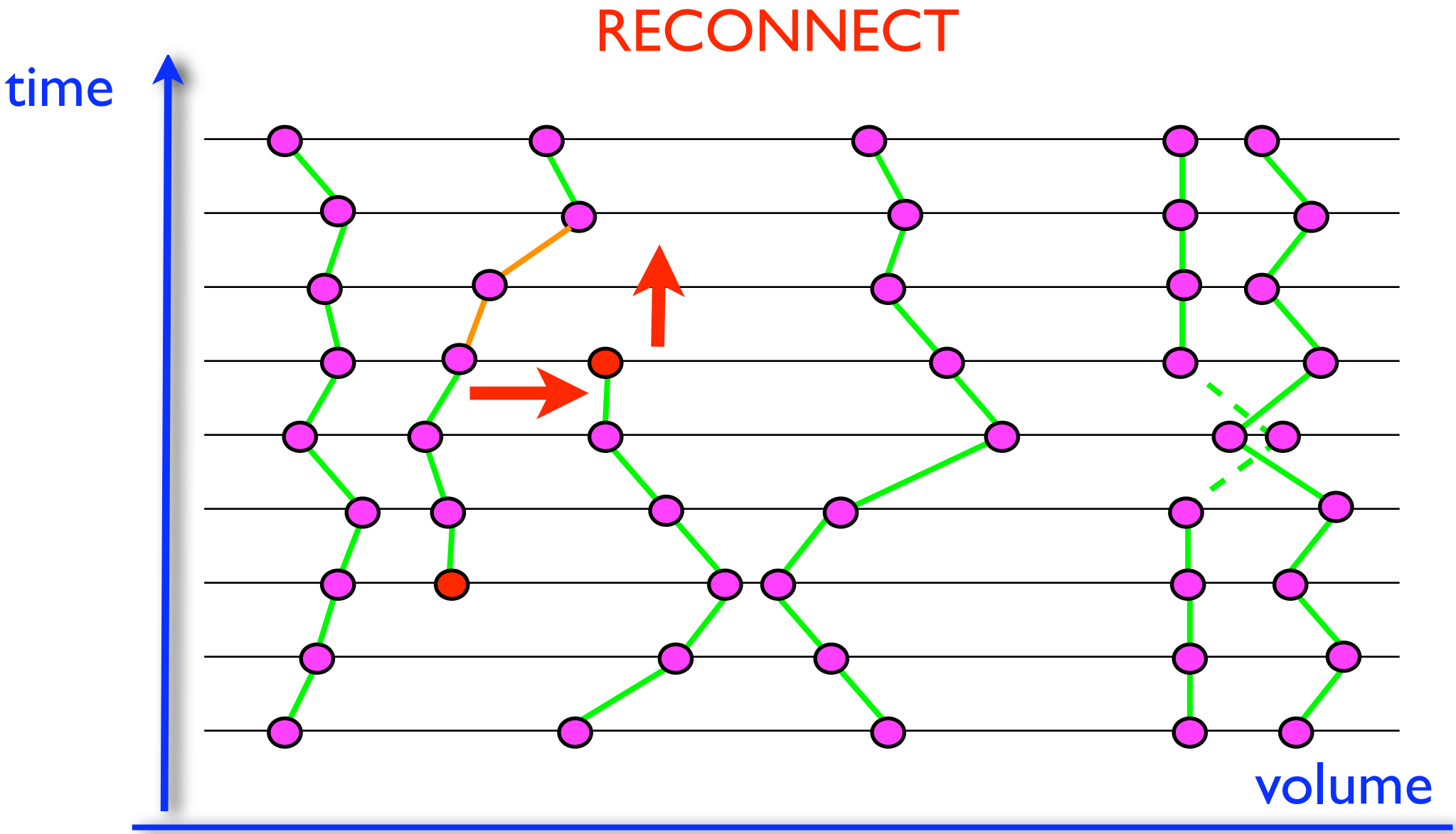
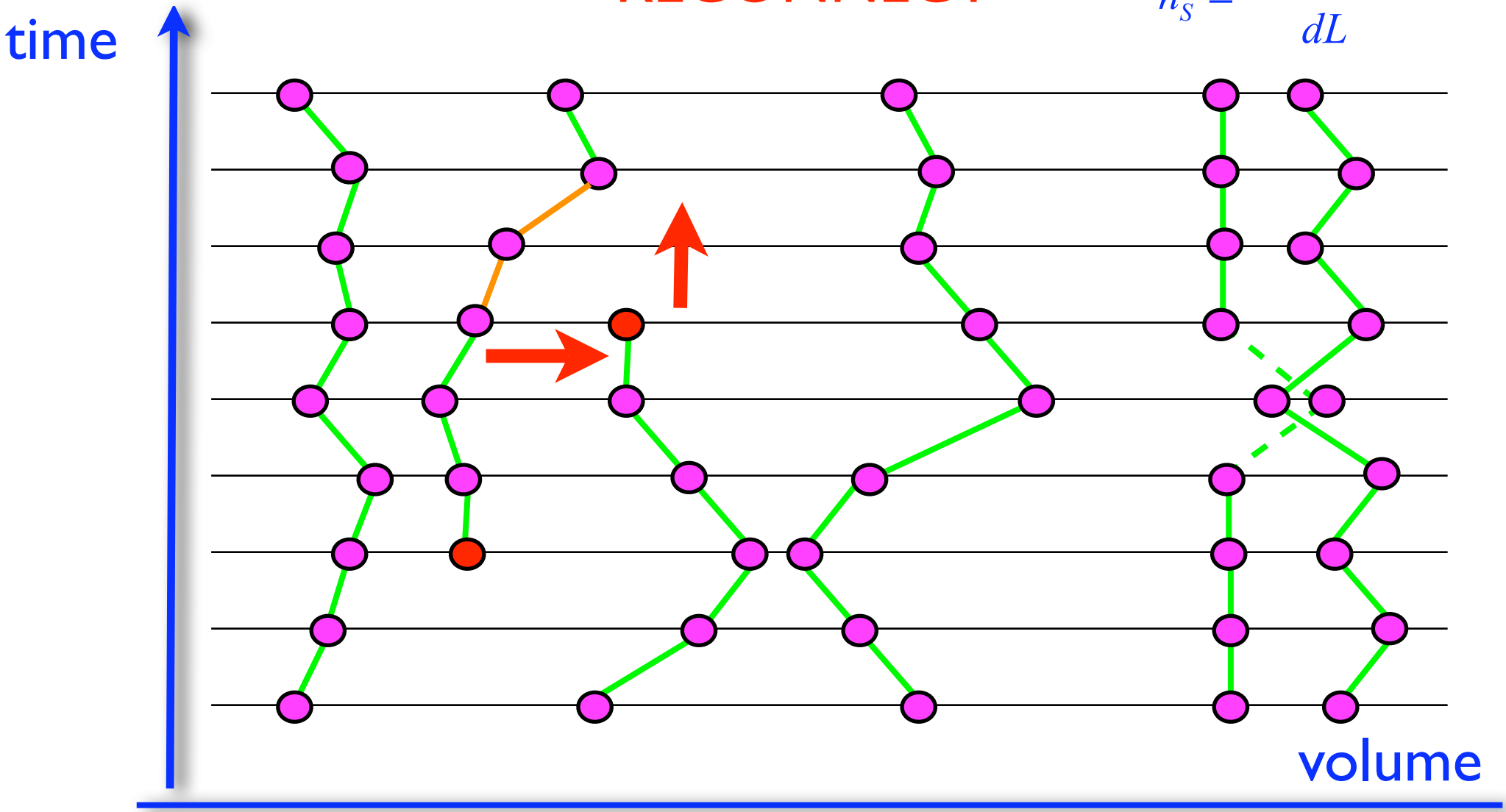


Path integral and worms

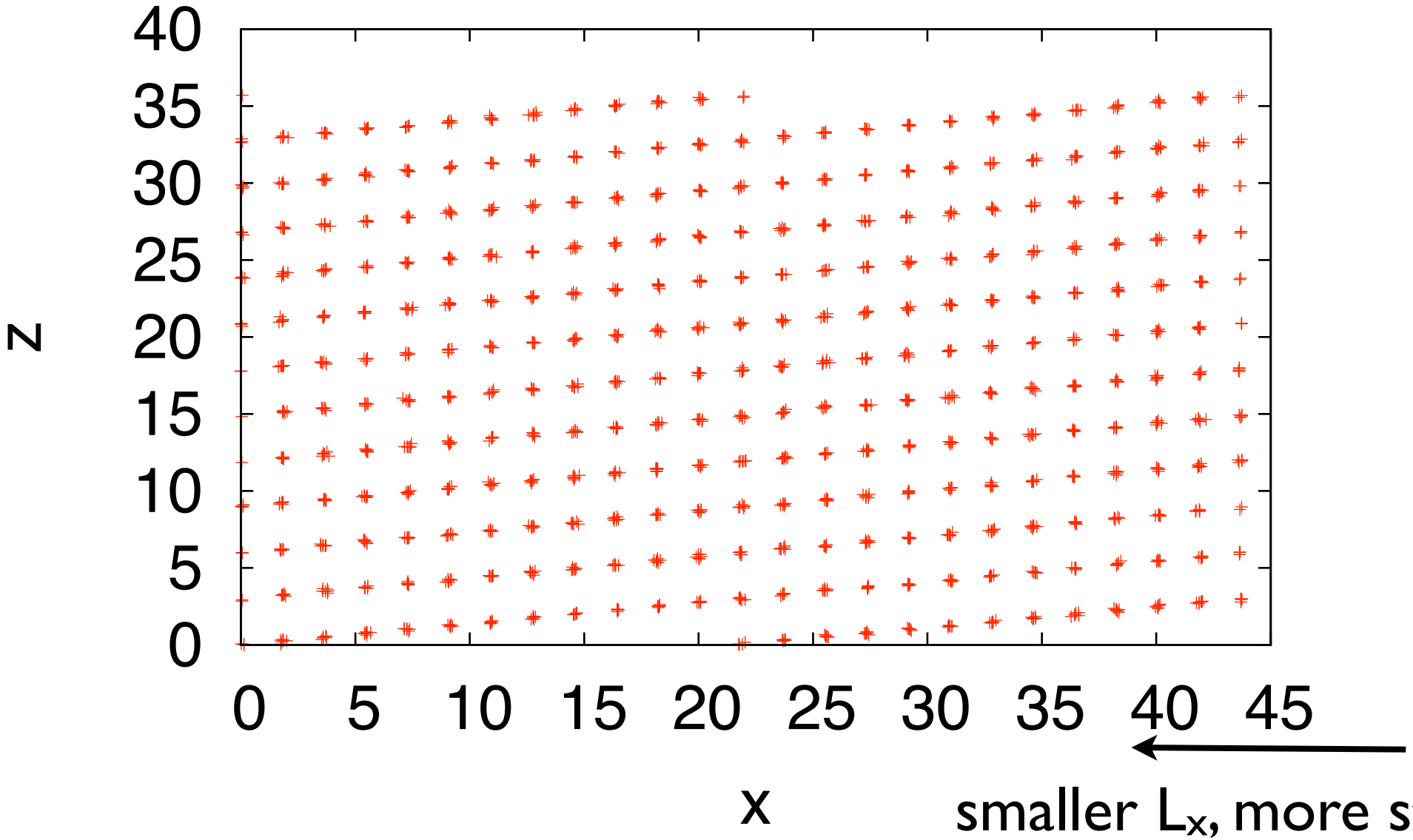
RECONNECT

$$n_S = \frac{mT\left\langle W^2 \right\rangle}{dL}$$



Shear stress

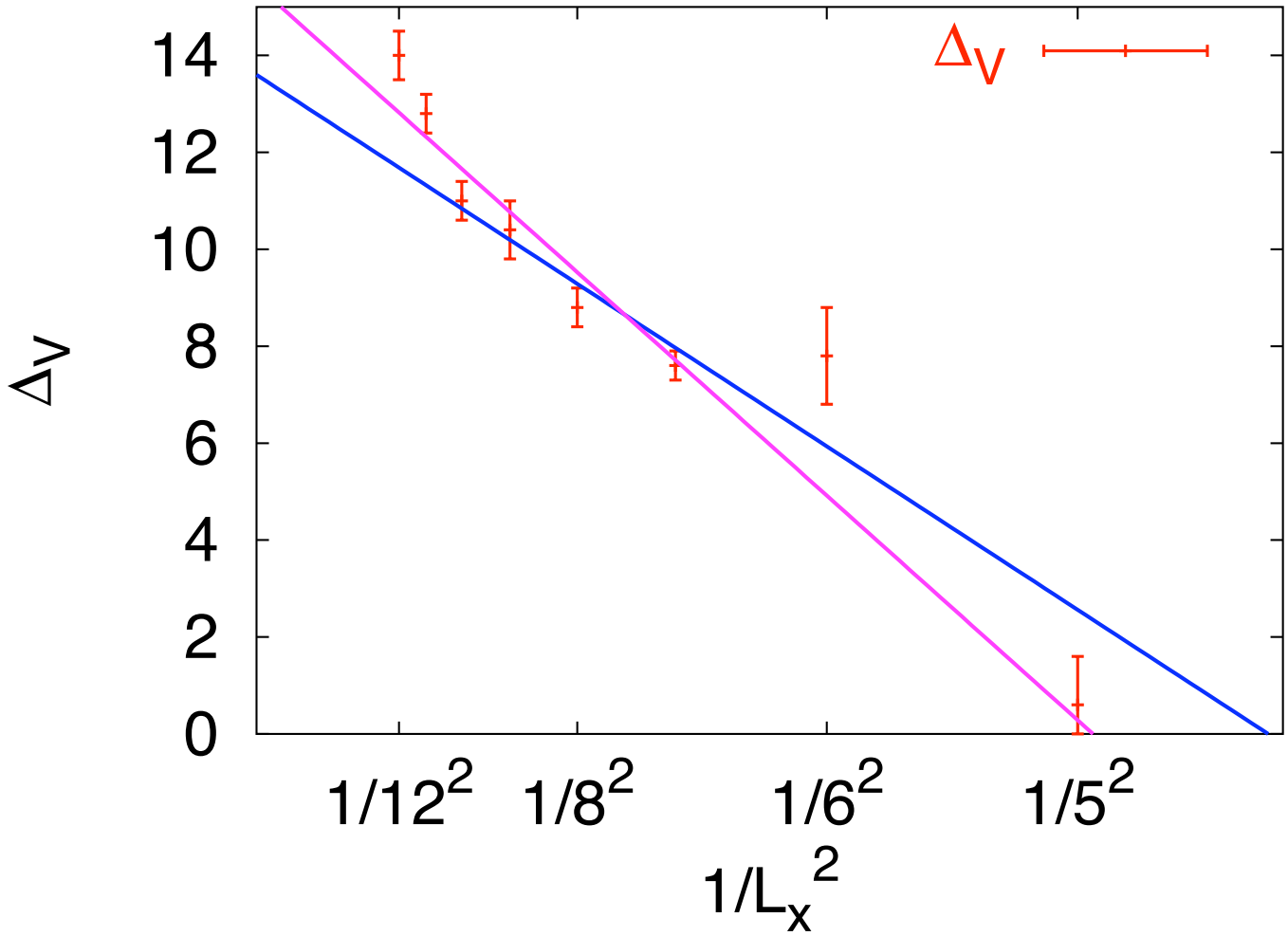
12x12x12, T=0.2K, n=0.0292 A^{-1}



smaller L_x, more stress

Shear stress

$$L = L_x \times 12 \times 12, n = 0.0292 \mathring{A}^{-1}, T = 0.2\mathrm{K}$$



$$\Delta - \Delta_0 \sim \frac{1}{L_x^2}$$

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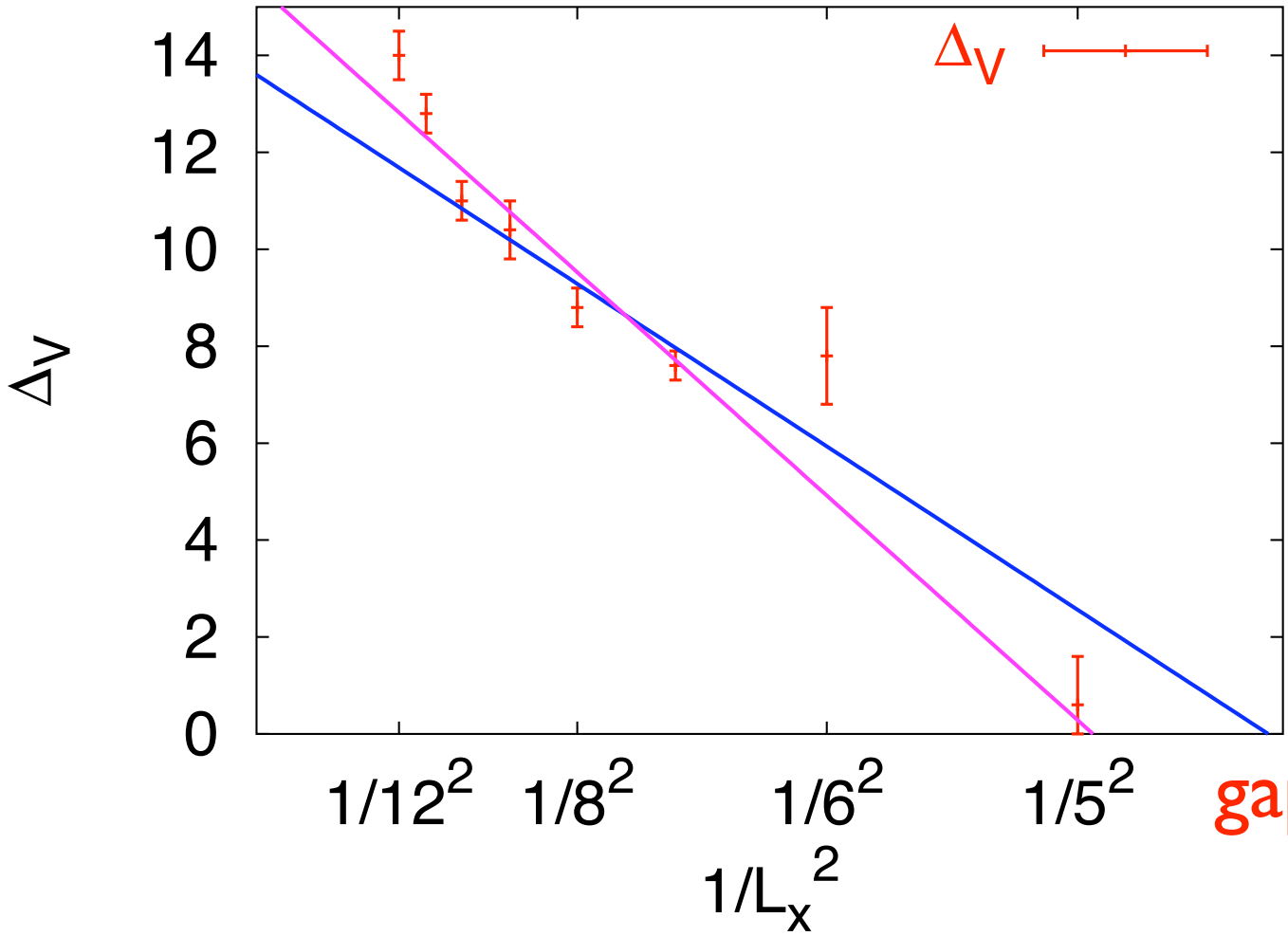
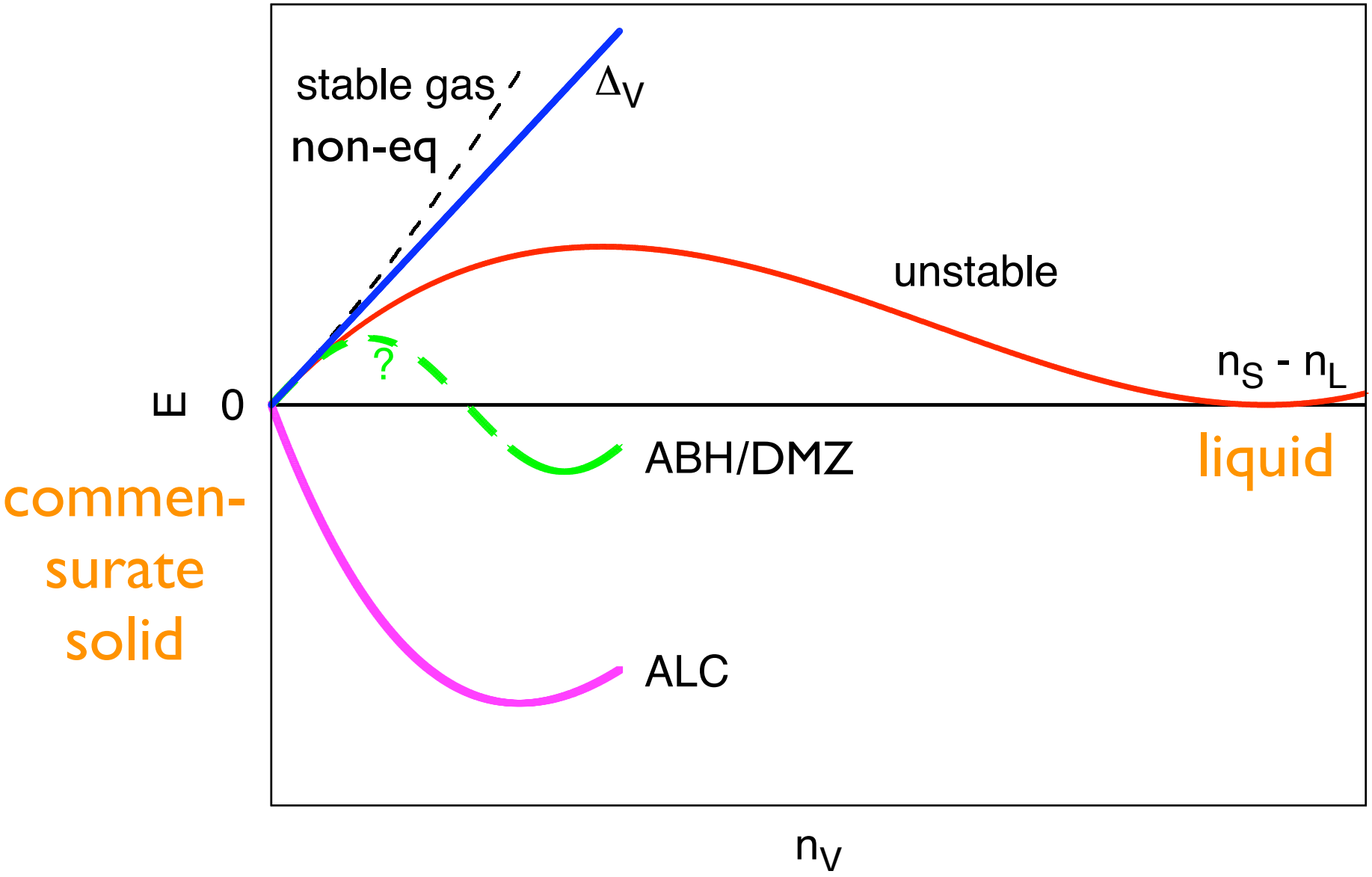


$$\Delta - \Delta_0 \sim \frac{1}{L_x^2}$$

gap disappears around

30bar

Vacancy scenarios



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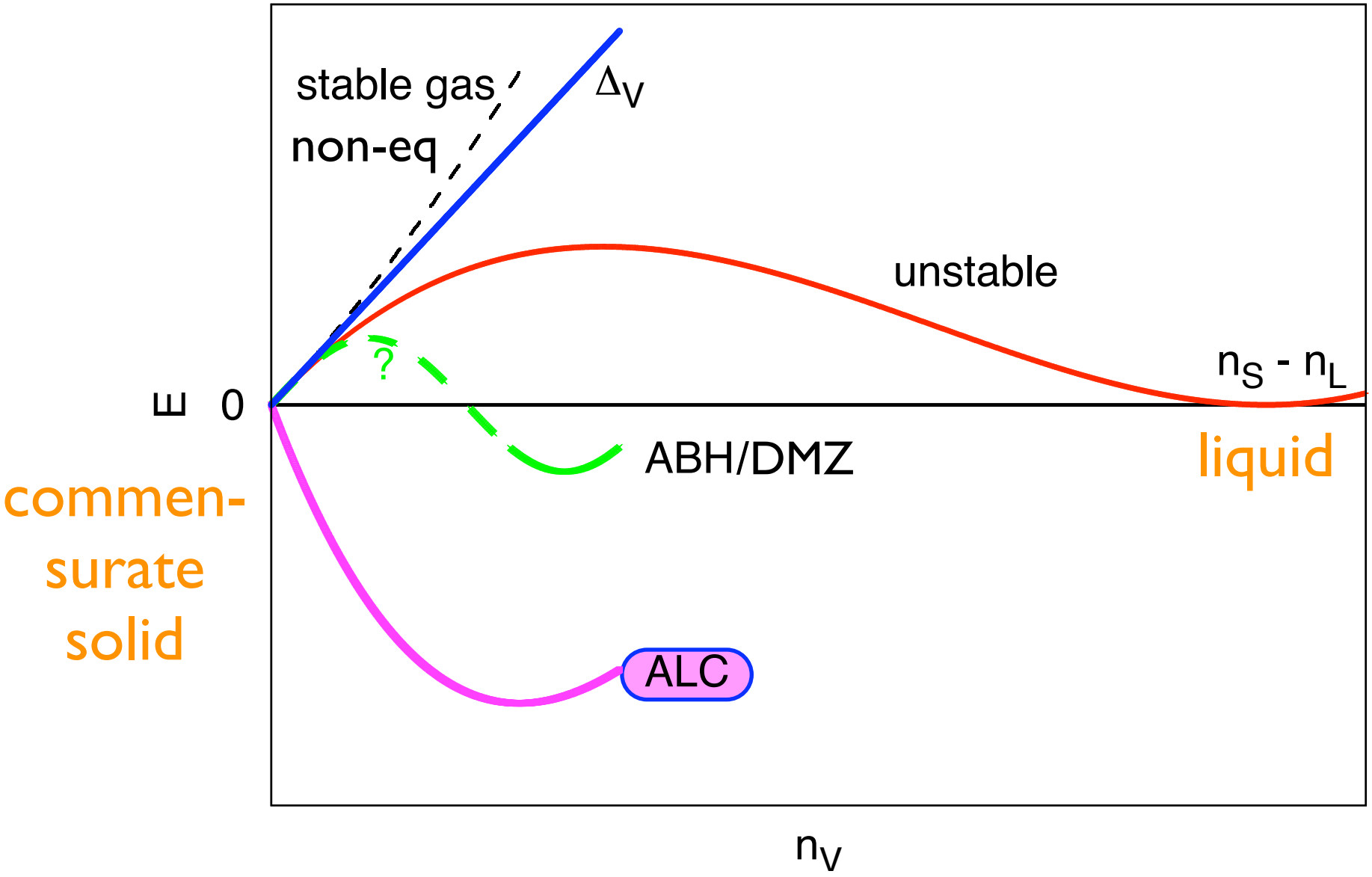
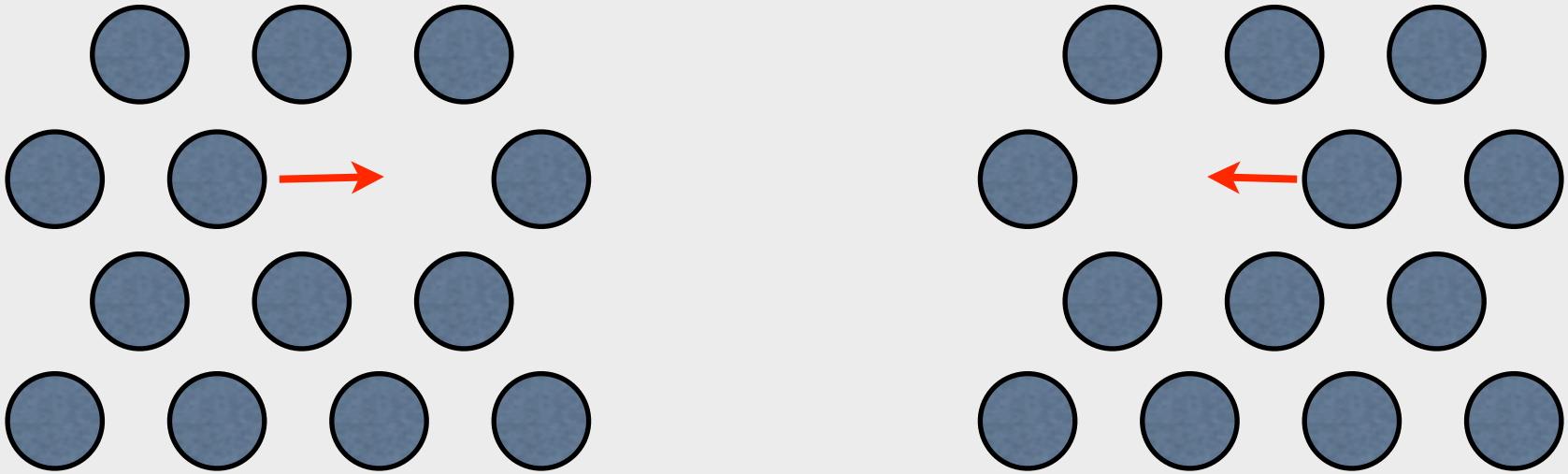


Vacancy scenarios

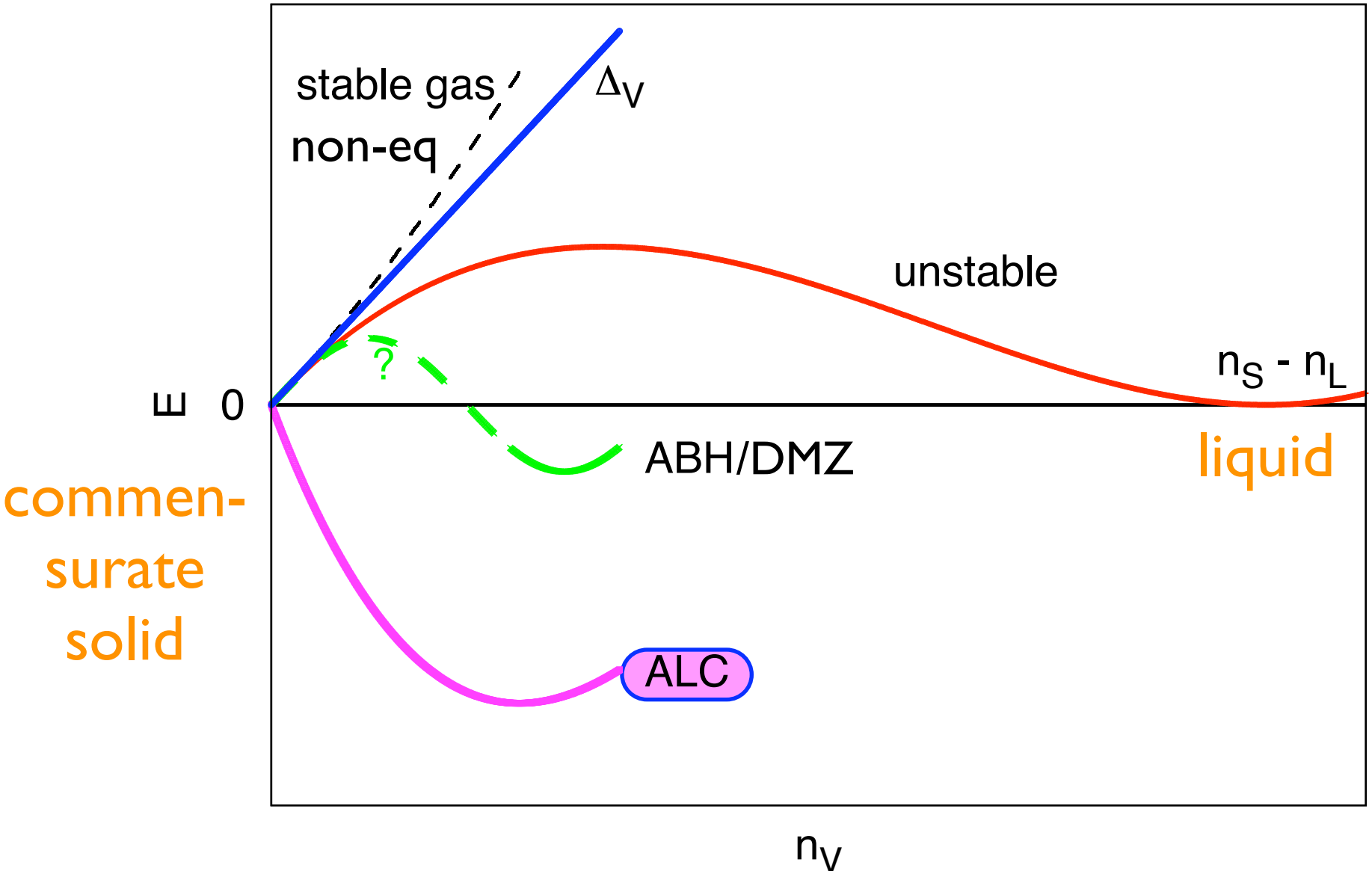
Andreev-Lifshitz-Chester

A. F. Andreev and I. M. Lifshitz, Sovjet. Phys. JETP **29**, 1107(1969).

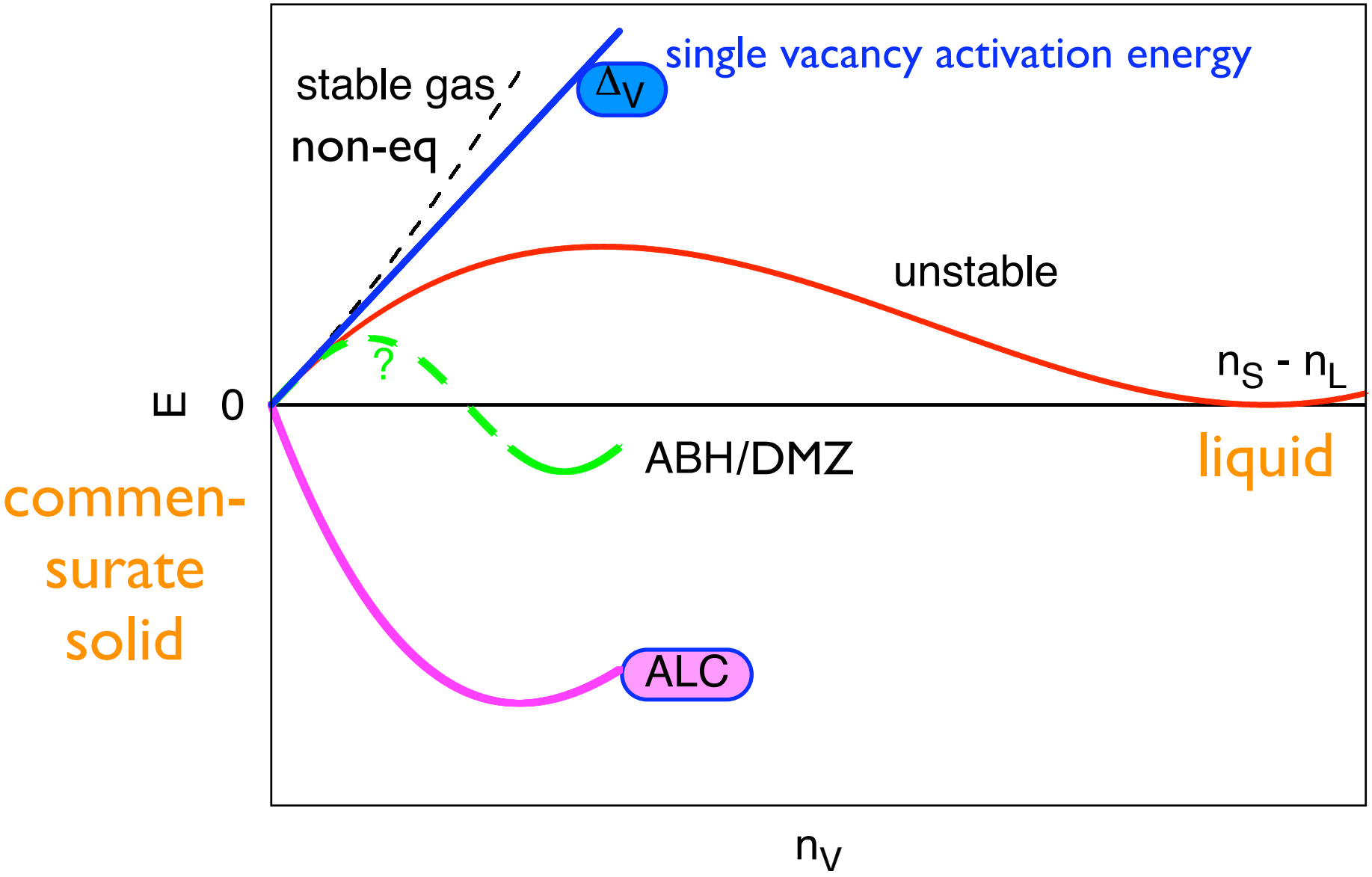


n_V

Vacancy scenarios



Vacancy scenarios



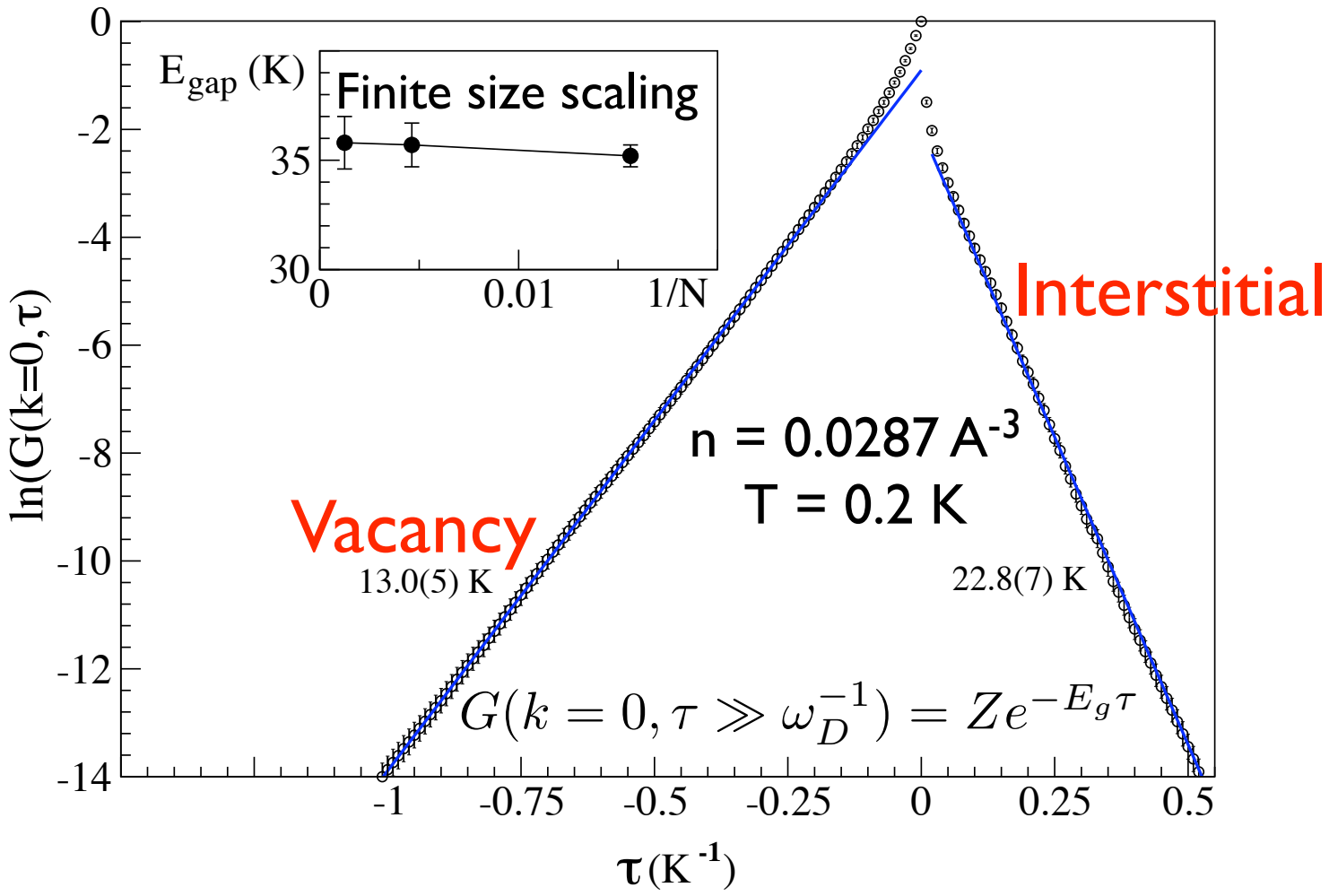
Vacancy scenarios

E

comme

surate

solid



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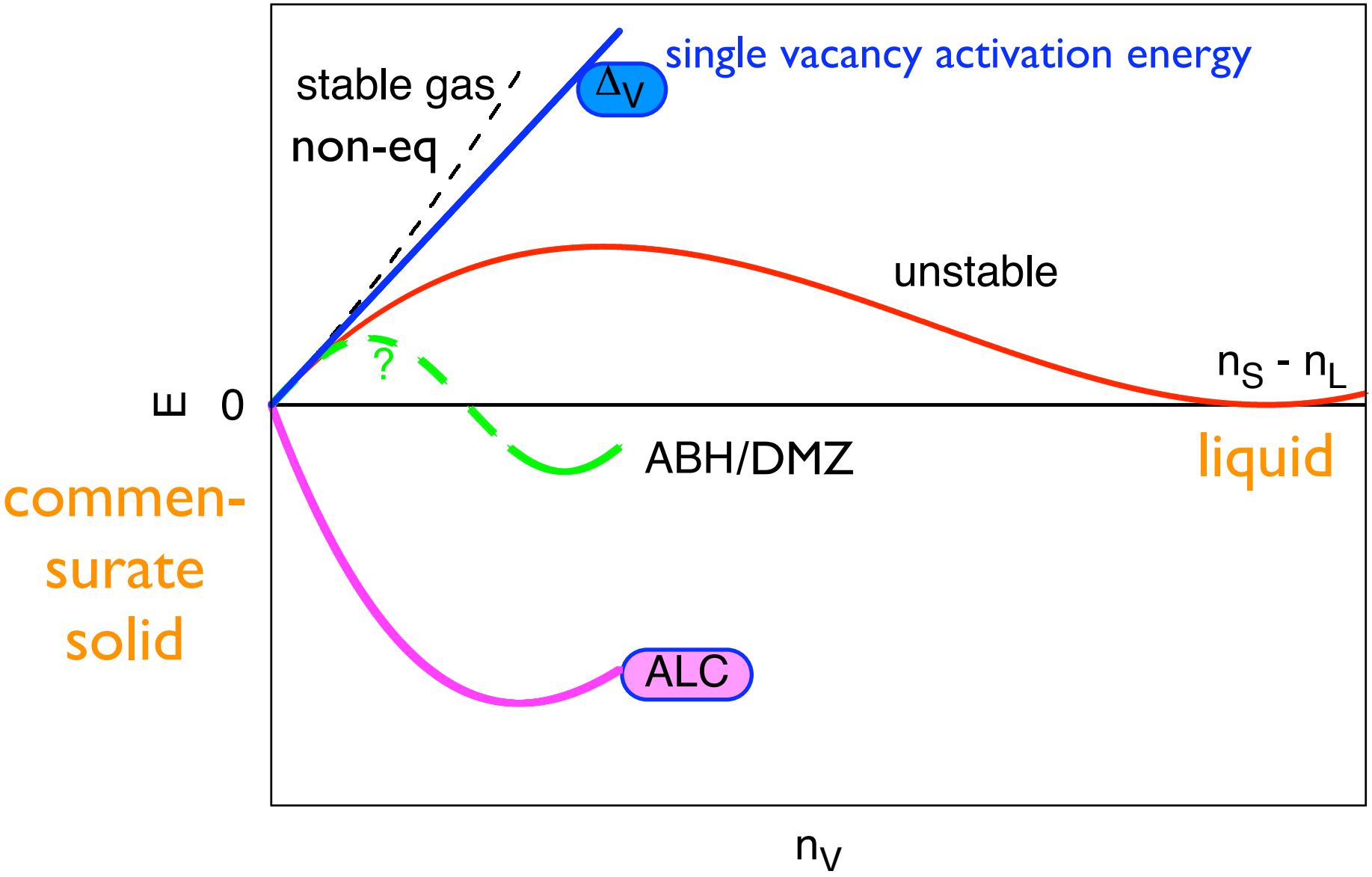

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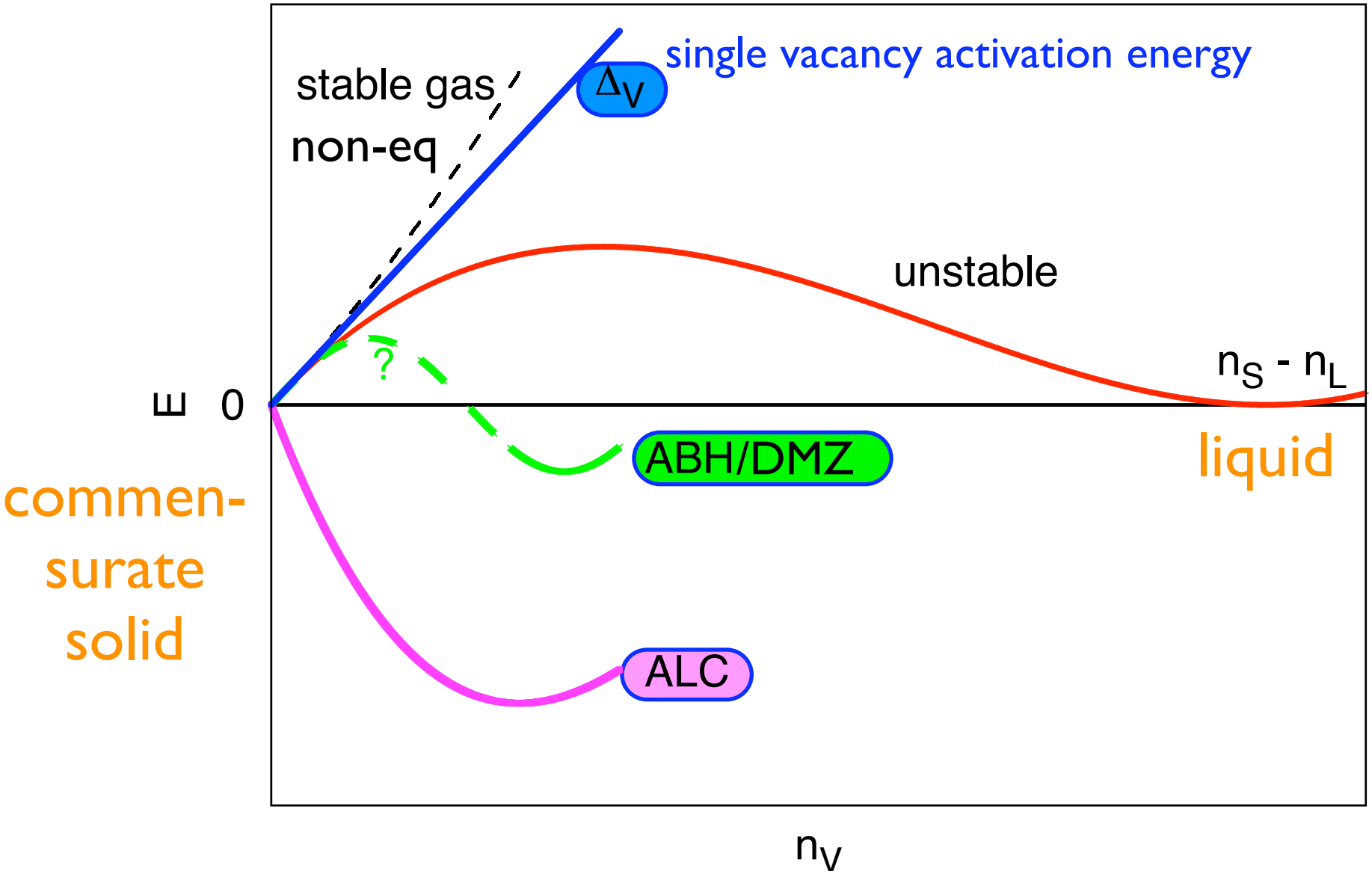
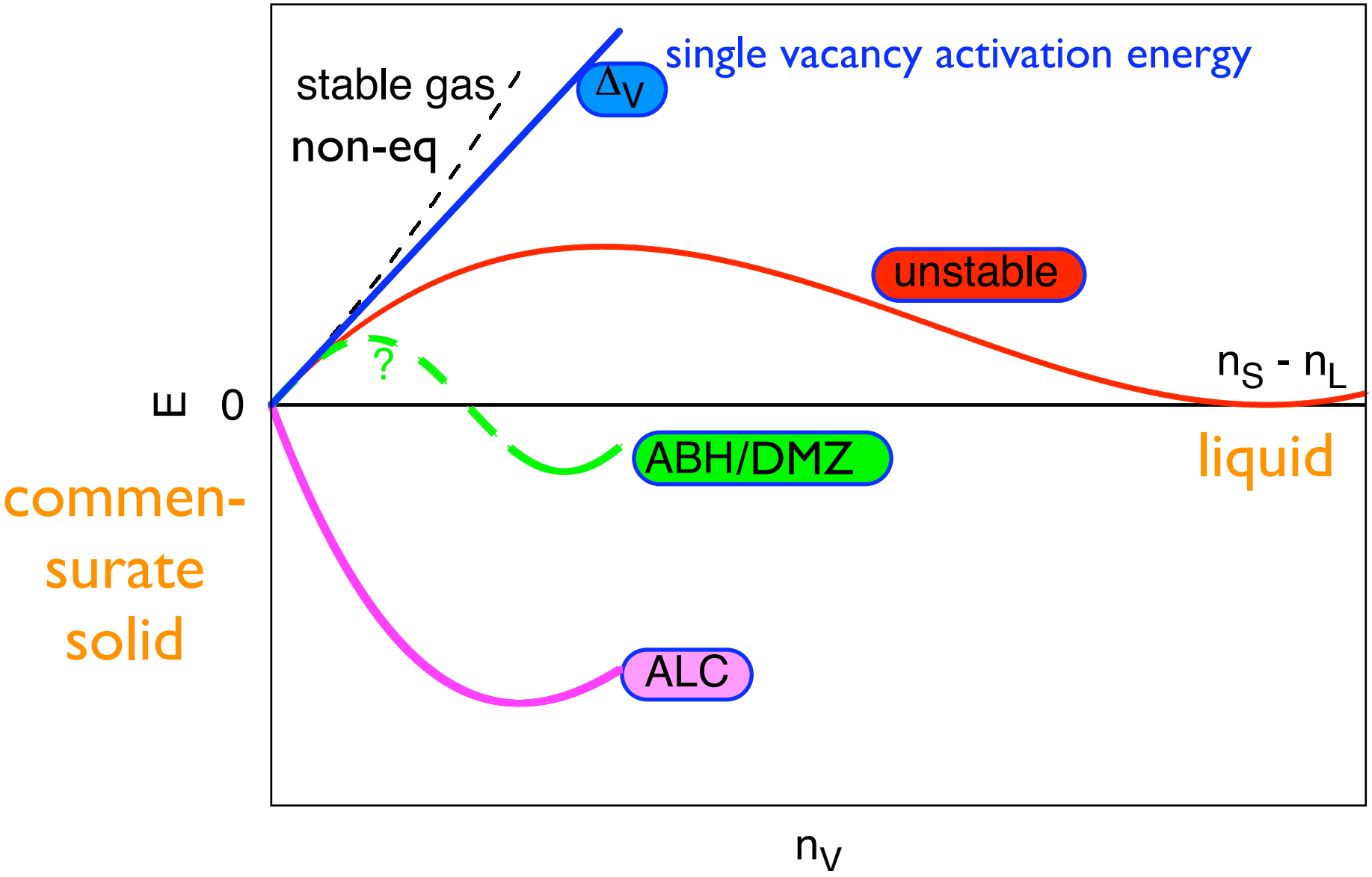
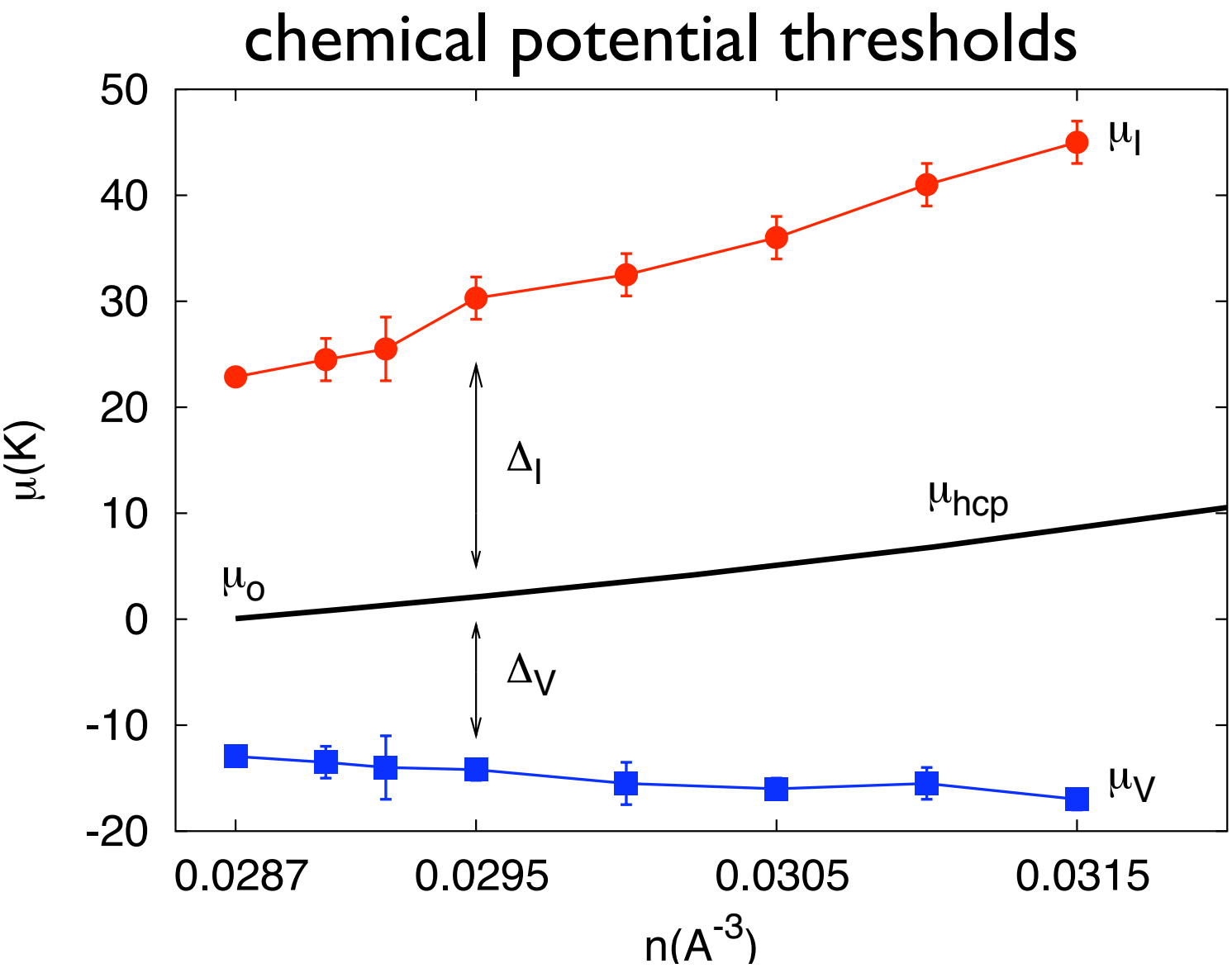


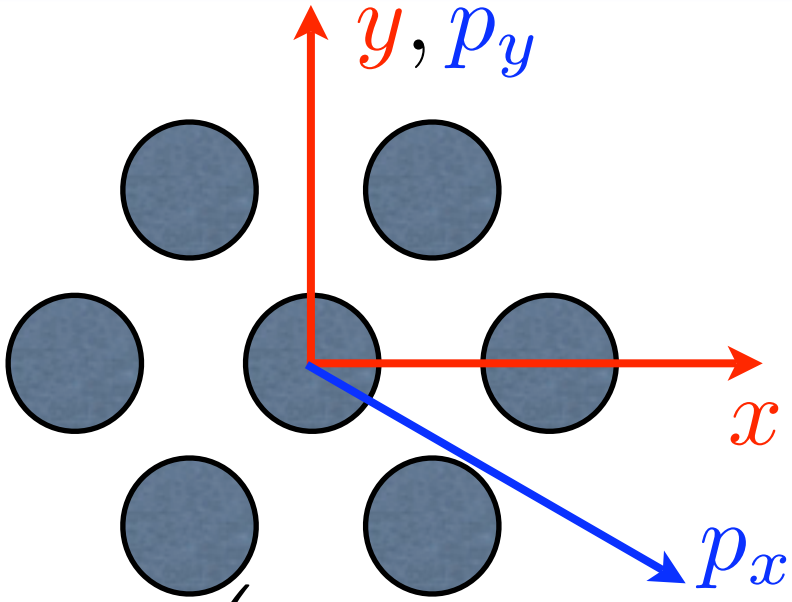
Vacancy scenarios



Higher density



Dispersion



tight binding
2 bands

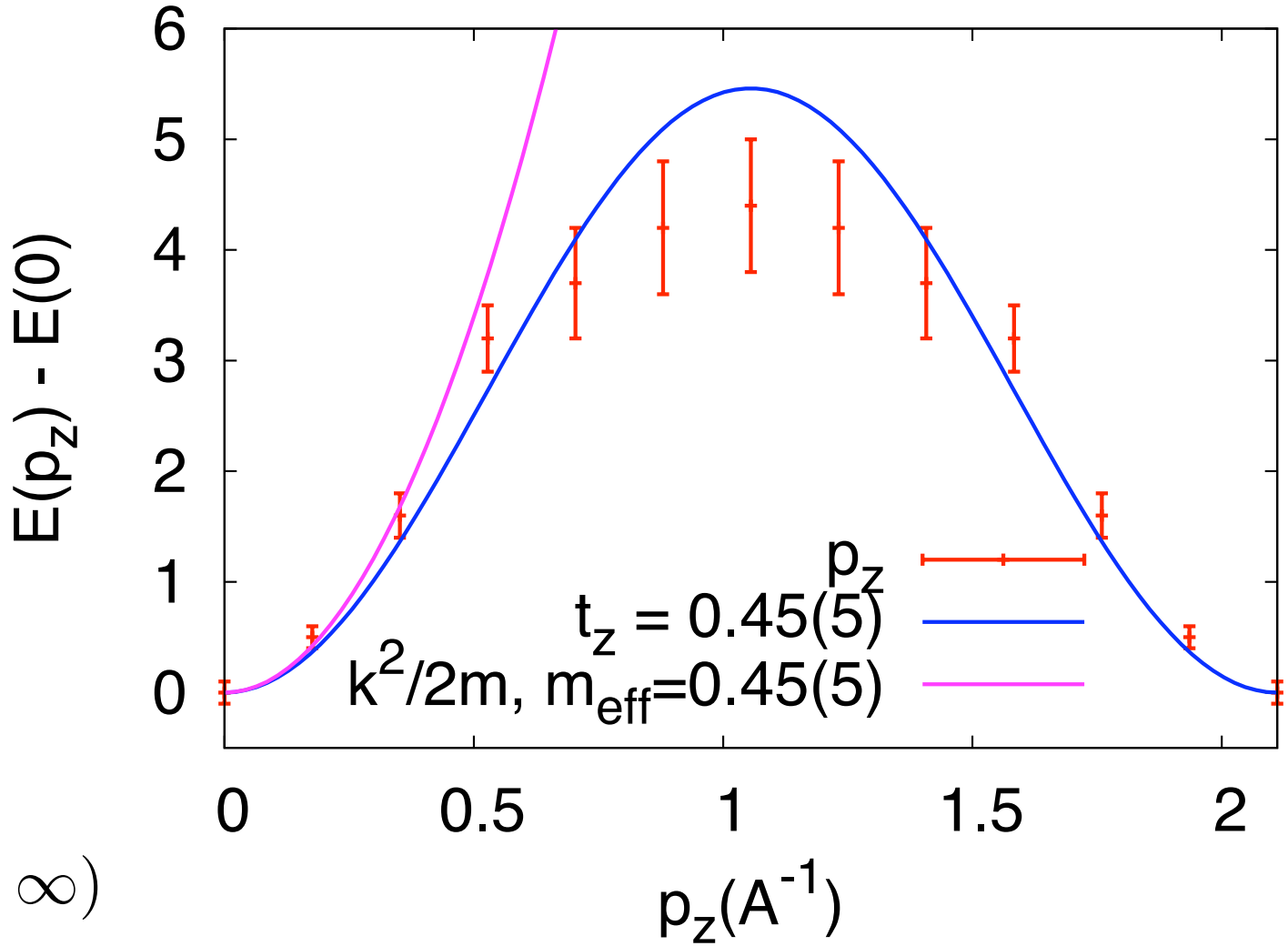
$$E = -2t_\perp \left(\cos k_x + 2 \cos \frac{k_x}{2} \cos \frac{\sqrt{3} k_y}{2} \right) \pm 2t_z \cos \frac{\sqrt{2} k_z}{\sqrt{3}} \sqrt{Q}$$

$$Q = 3 + 4 \cos \frac{k_x}{2} \cos \frac{\sqrt{3} k_y}{2} + 2 \cos k_x$$

Effective mass

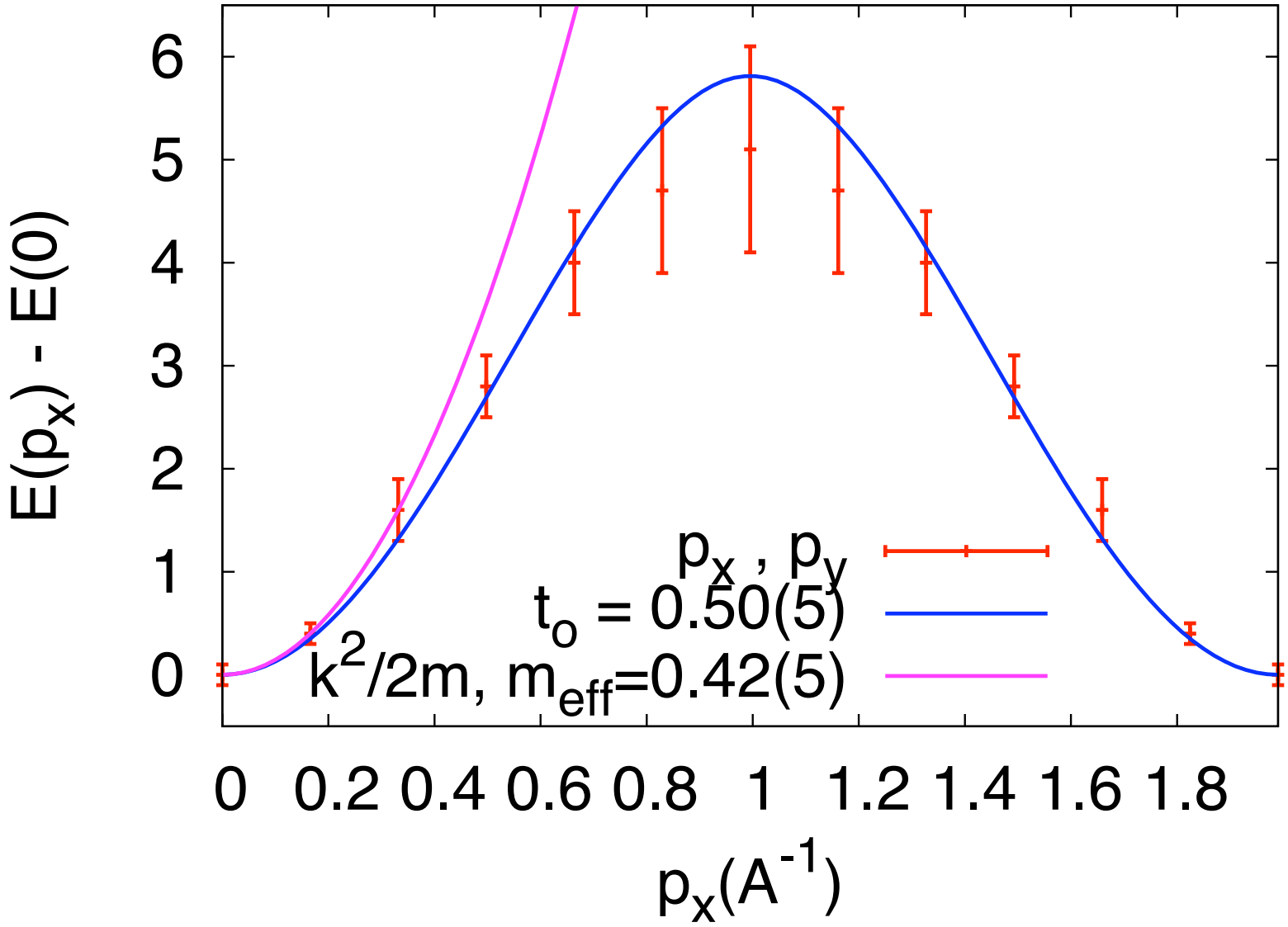
dispersion curve

$$L = 12 \times 12 \times 12, T = 0.2\mathrm{K}, n = 0.0292 \mathring{A}^{-1}$$

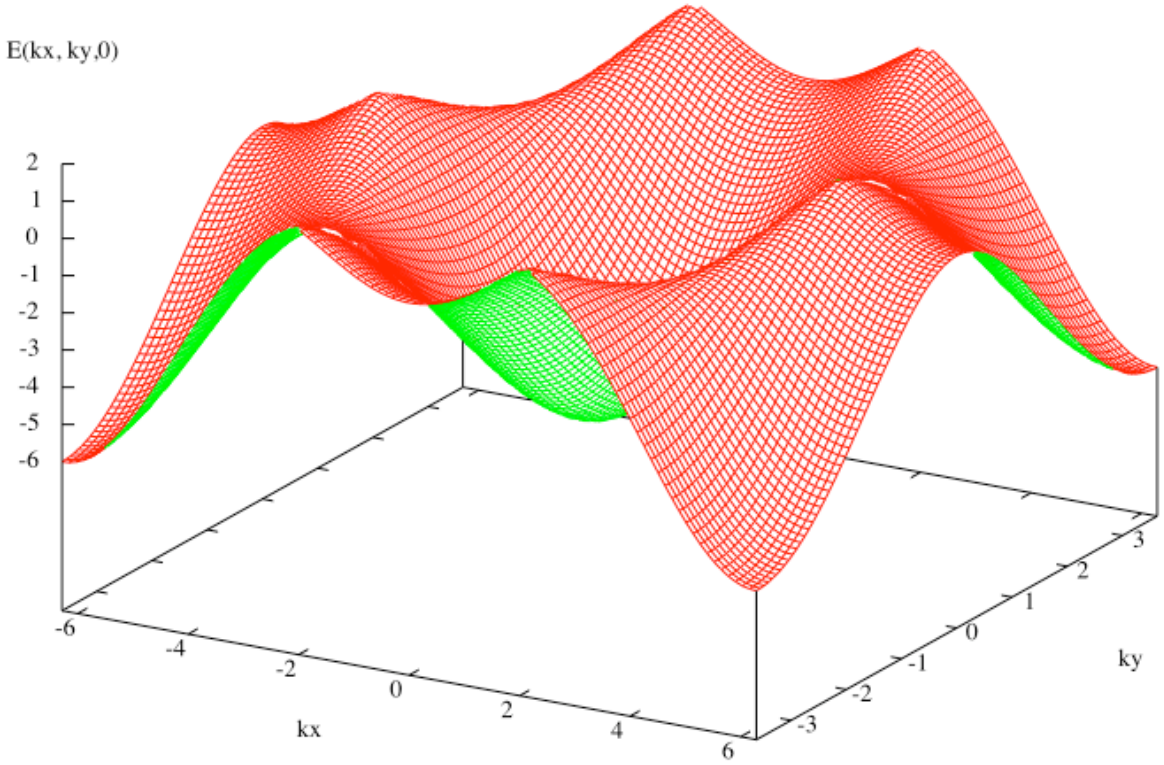


$G(p, t \to \infty)$

Effective mass



Effective mass



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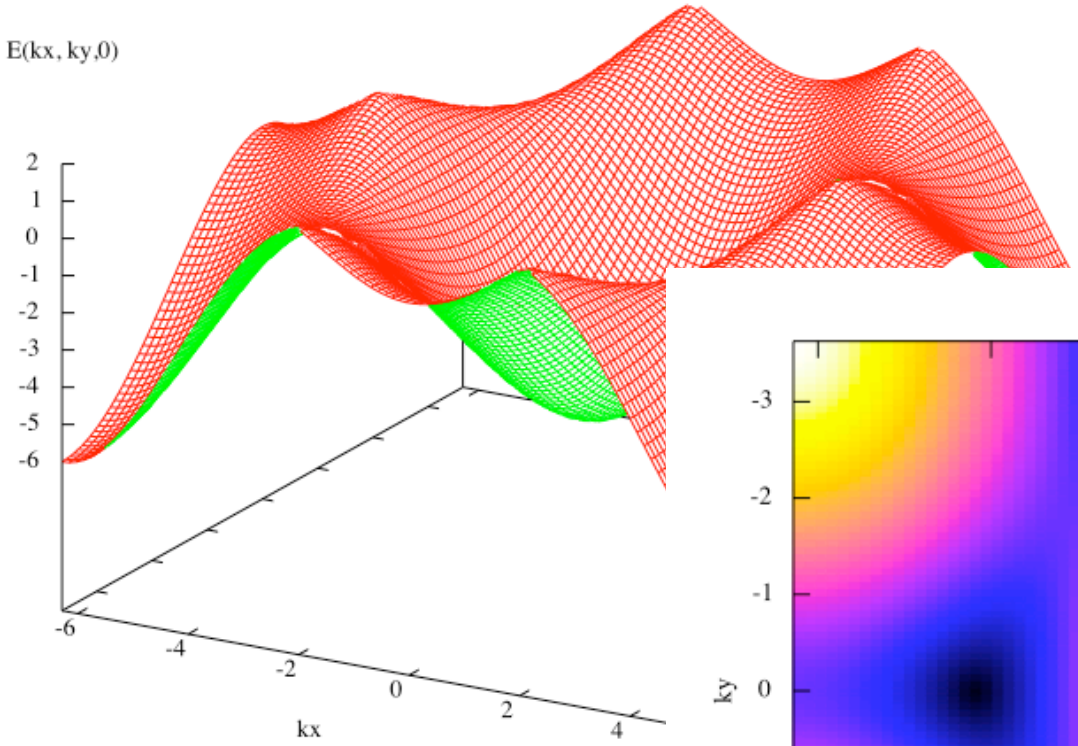
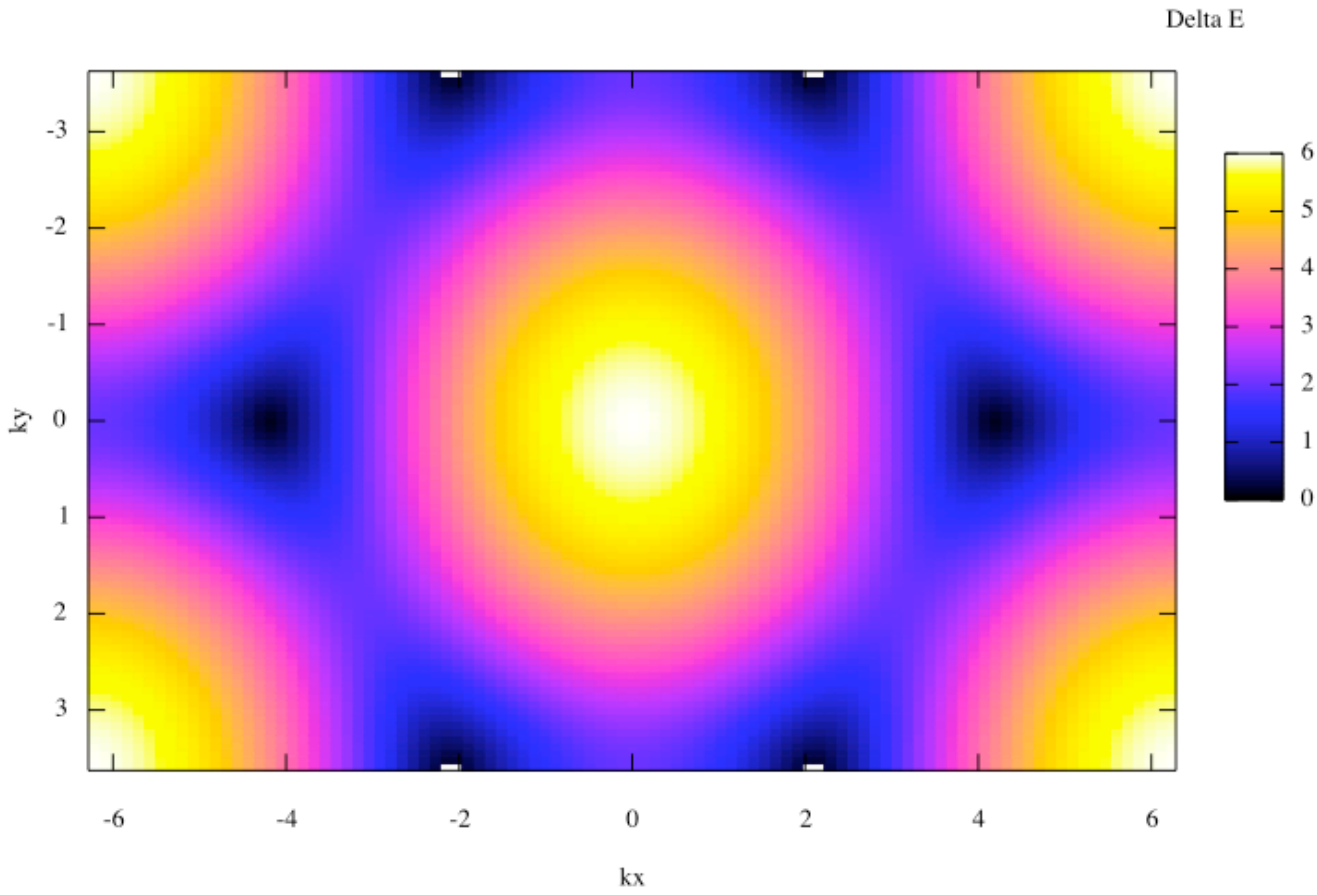




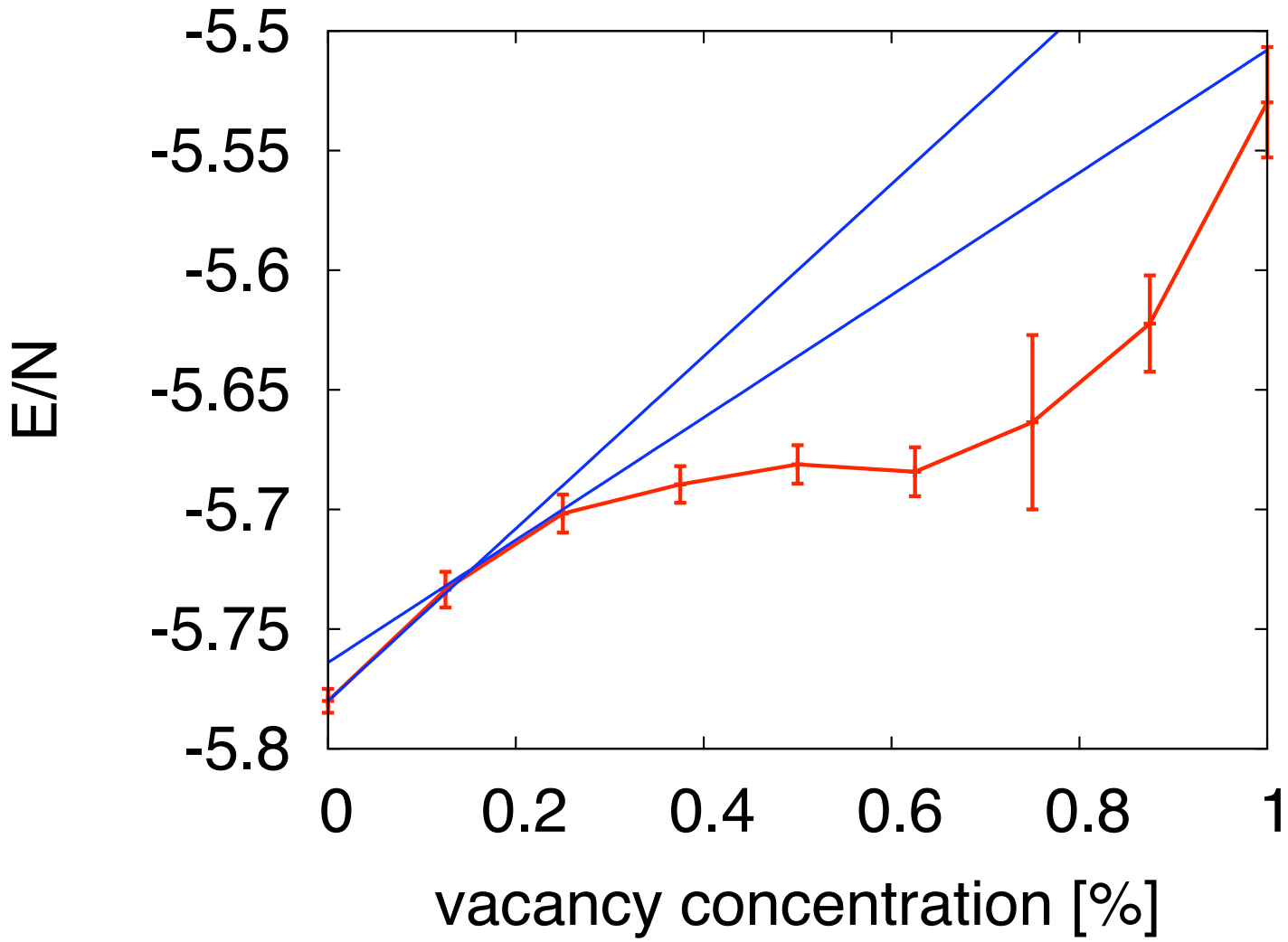
States with many vacancies

UNSTABLE

- grand-canonical ensemble : runaway in particle/vacancy number; thermodynamic unstable
- canonical ensemble : phase separation

effective vacancy-vacancy attraction

Energies



negative curvature

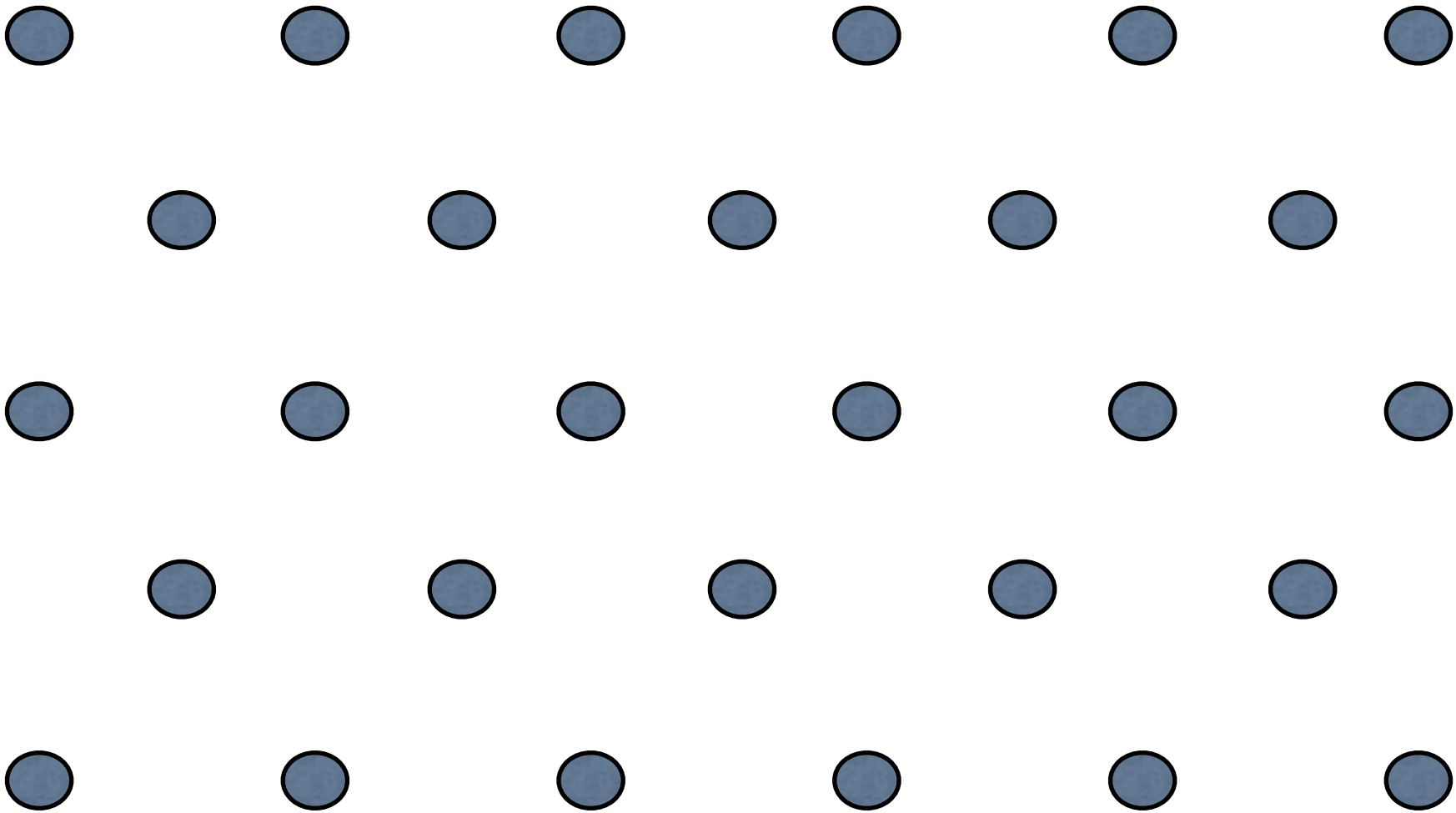
Vacancy -vacancy correlation function

How to localize vacancies/ What are vacancies?

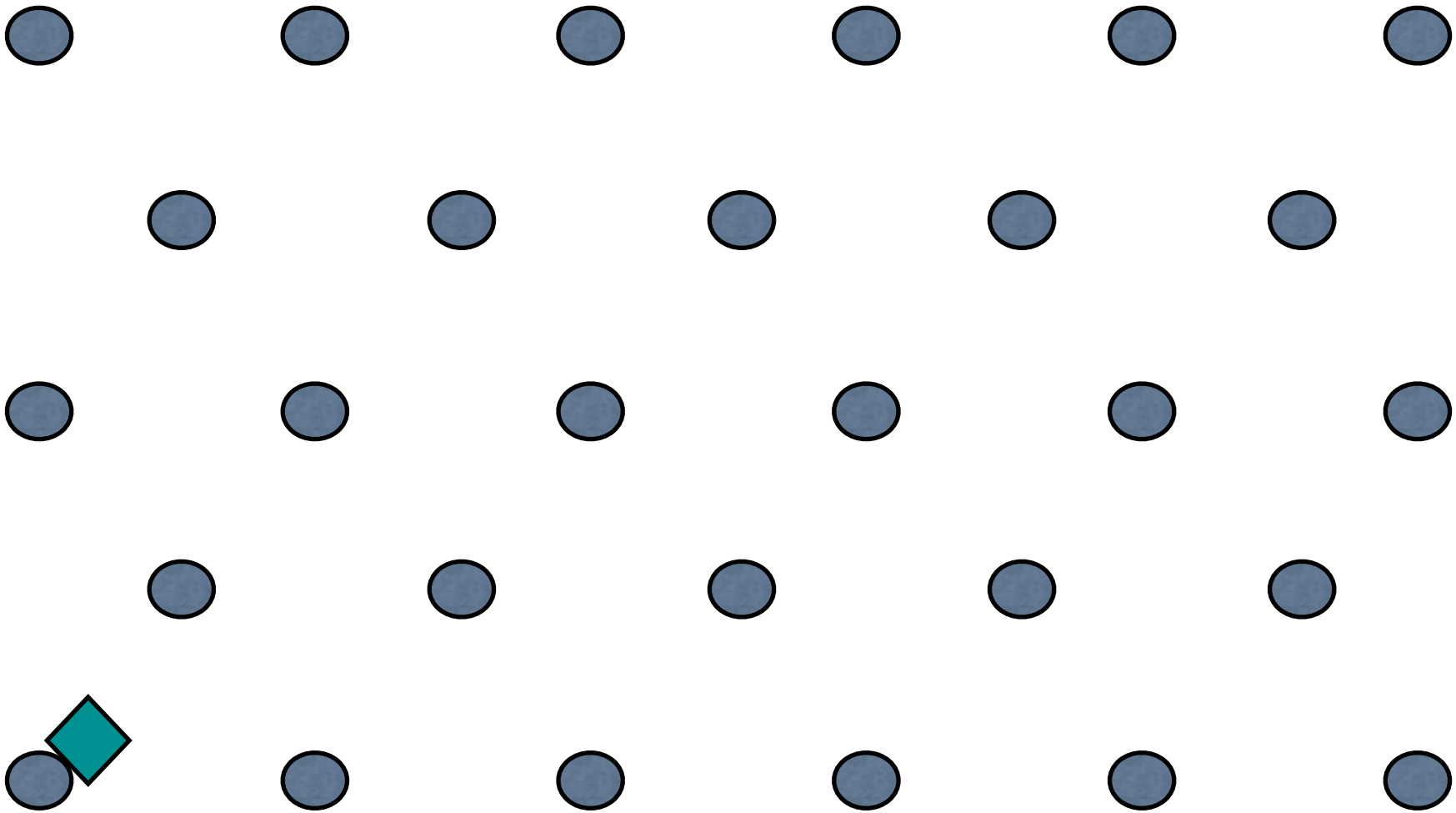
- separation in energy scale :
  - zero point fluctuation : Debye frequency 25 K^{-1}
  - vacancy exchange rate : ≈1 K^{-1}
- Renormalization procedure :

using partially averaged positions of atoms

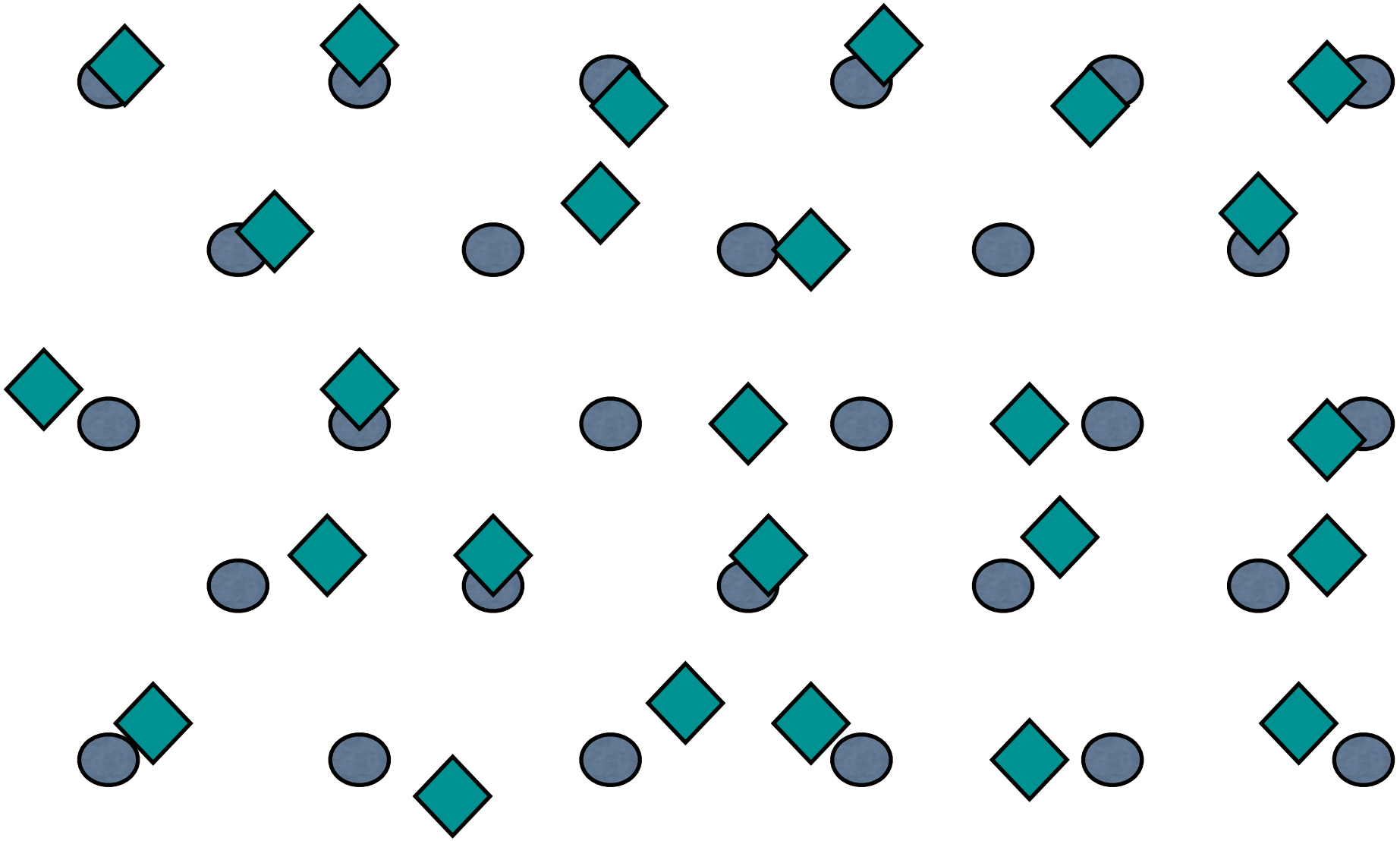
Renormalization procedure



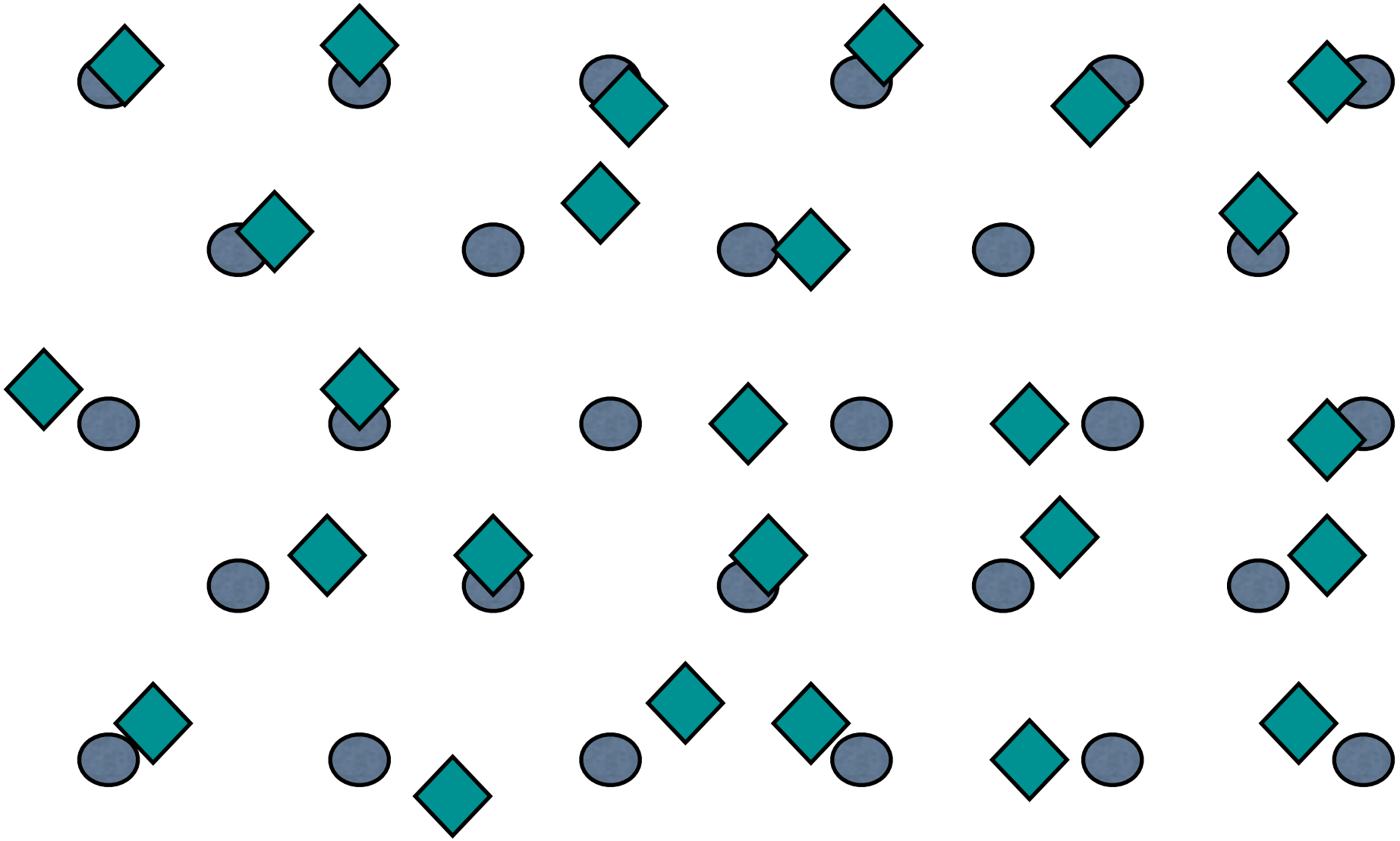
Renormalization procedure



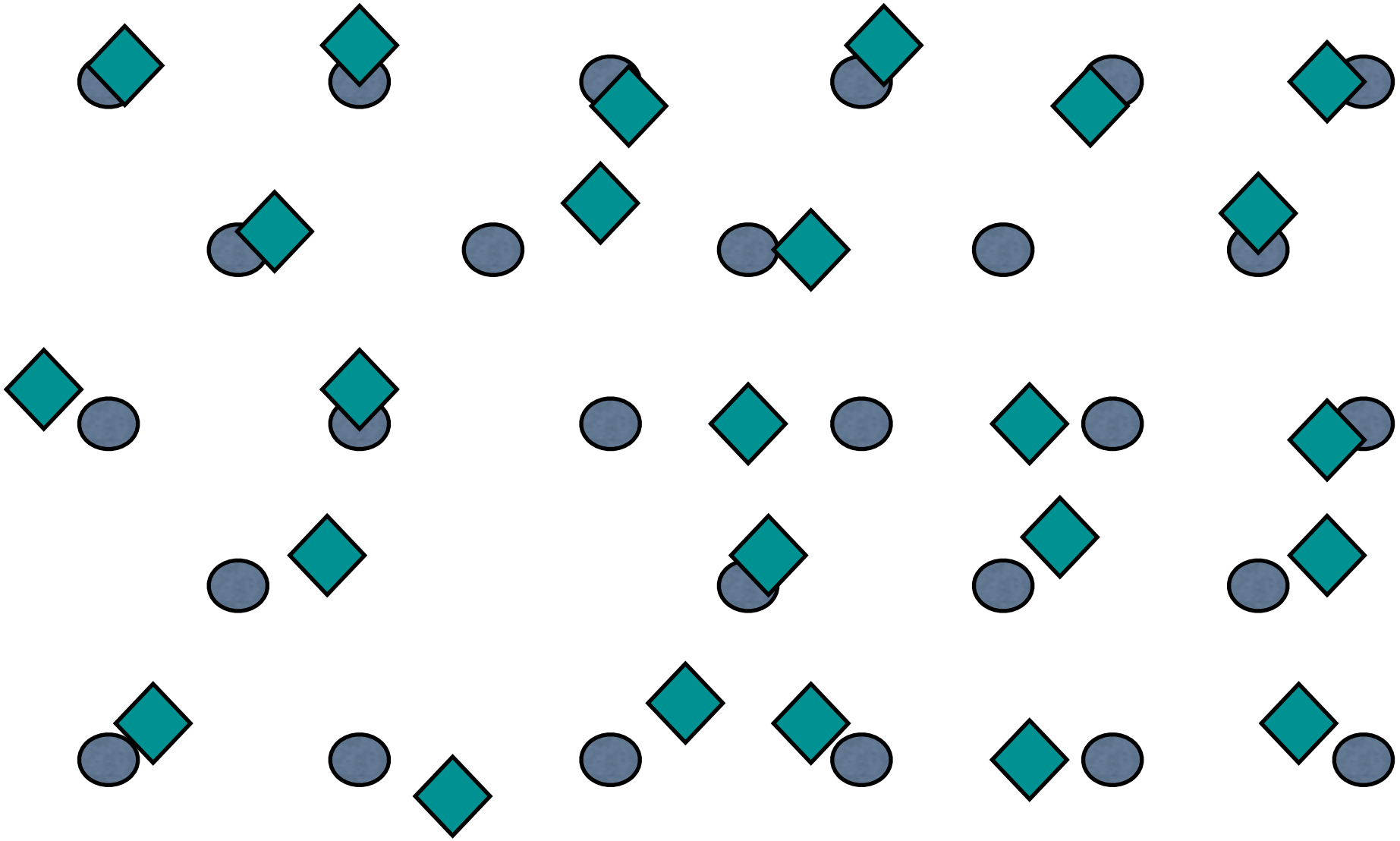
Renormalization procedure



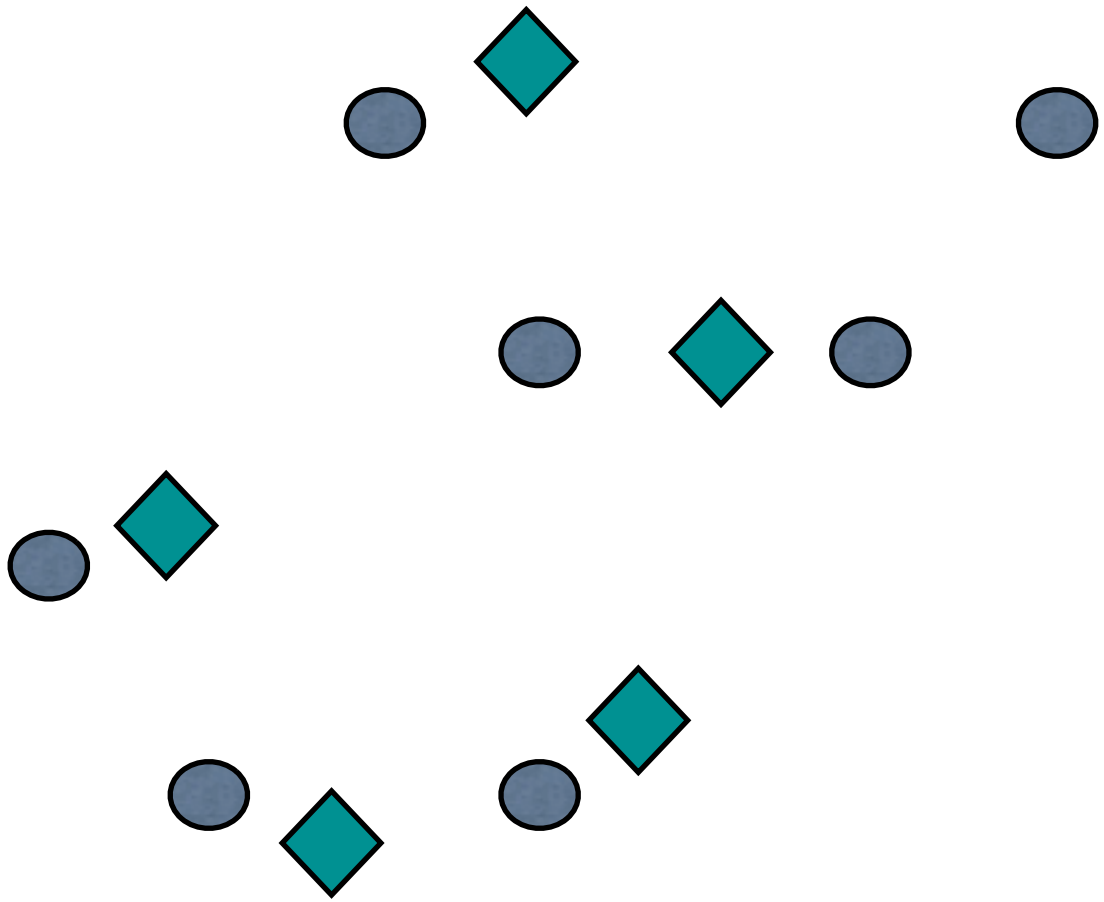
Renormalization procedure



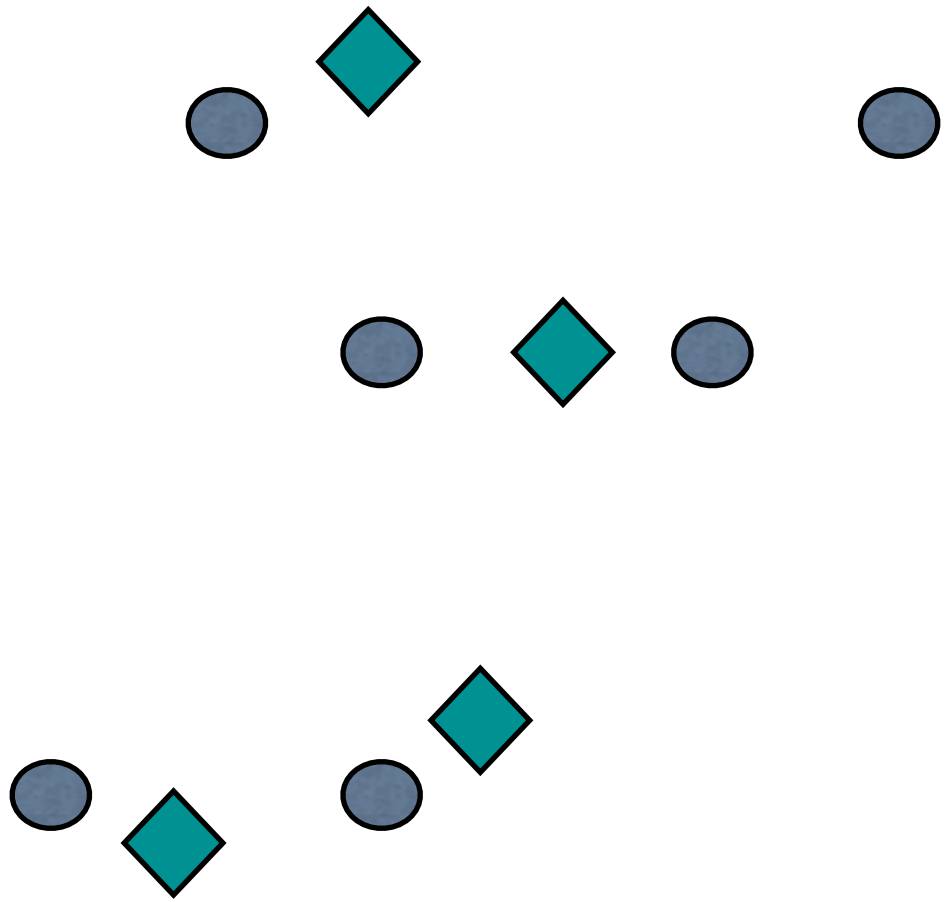
Renormalization procedure



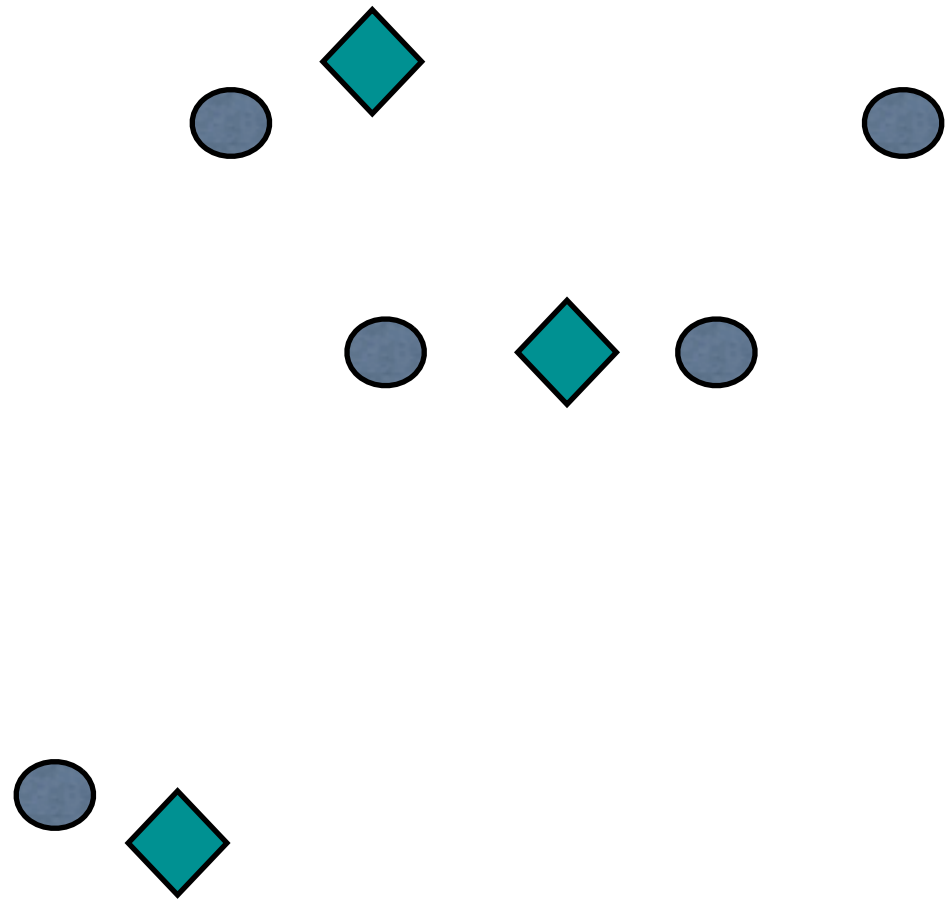
Renormalization procedure



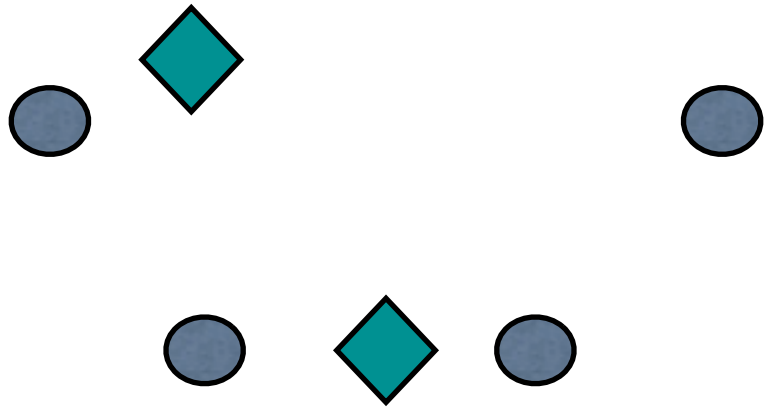
Renormalization procedure



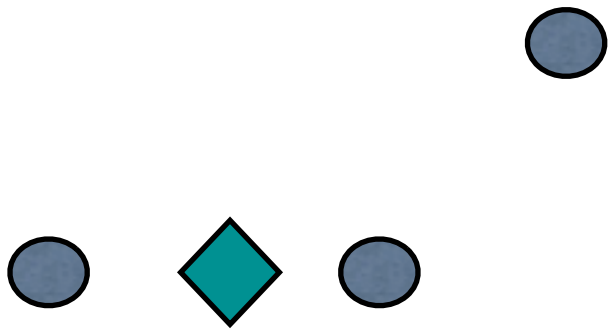
Renormalization procedure



Renormalization procedure

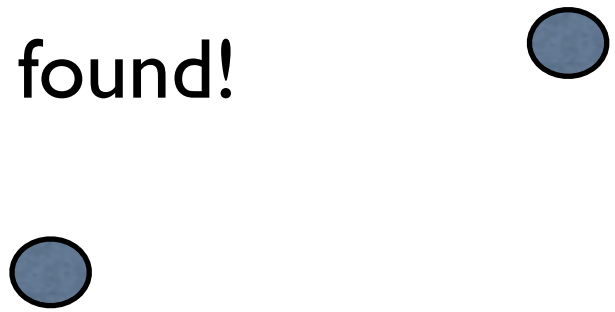


Renormalization procedure



Renormalization procedure

All vacancies found!

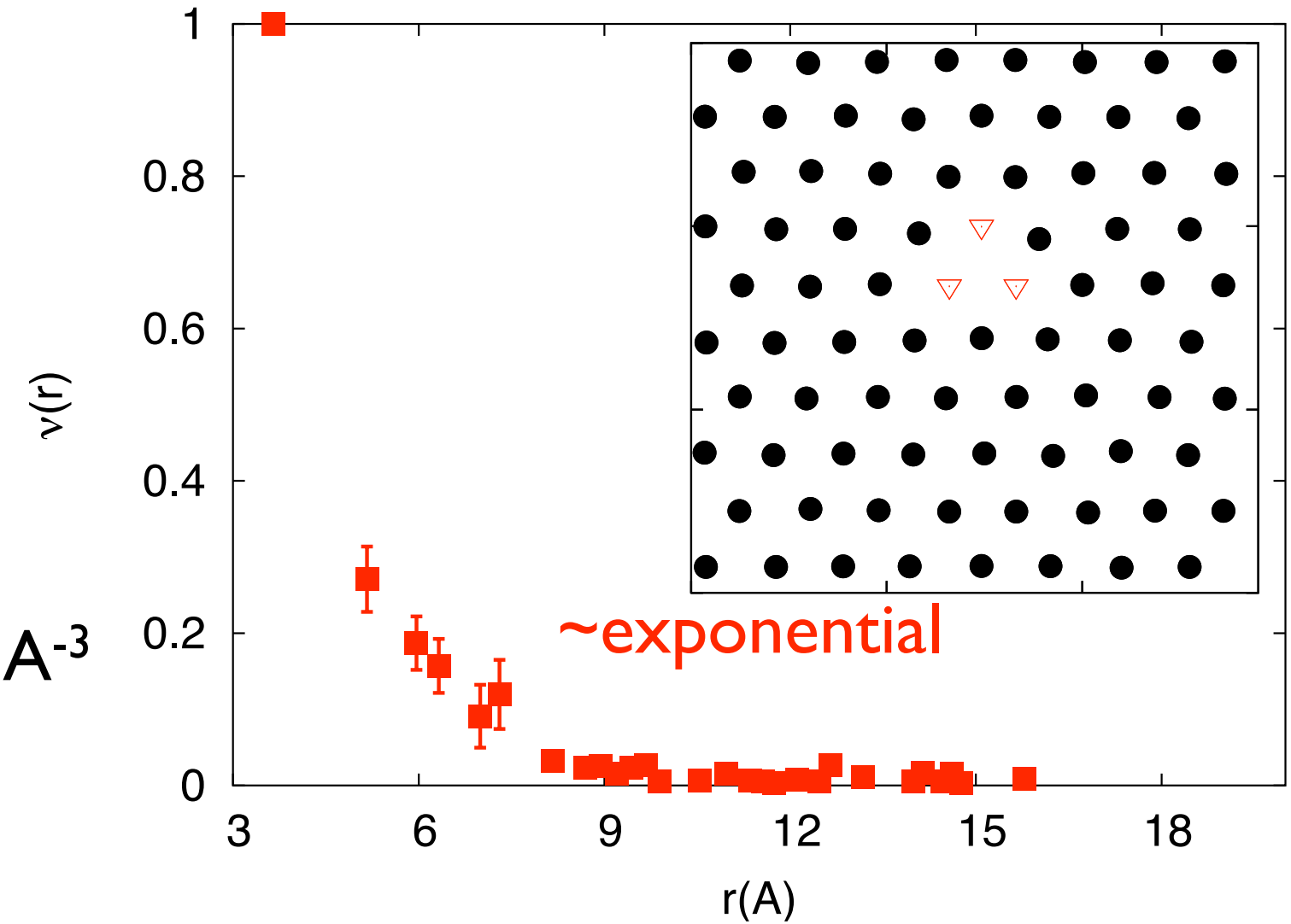


Phase separation

(canonical ensemble)

three vacancies

attractive interaction



$n = 0.0292\ A^{-3}$

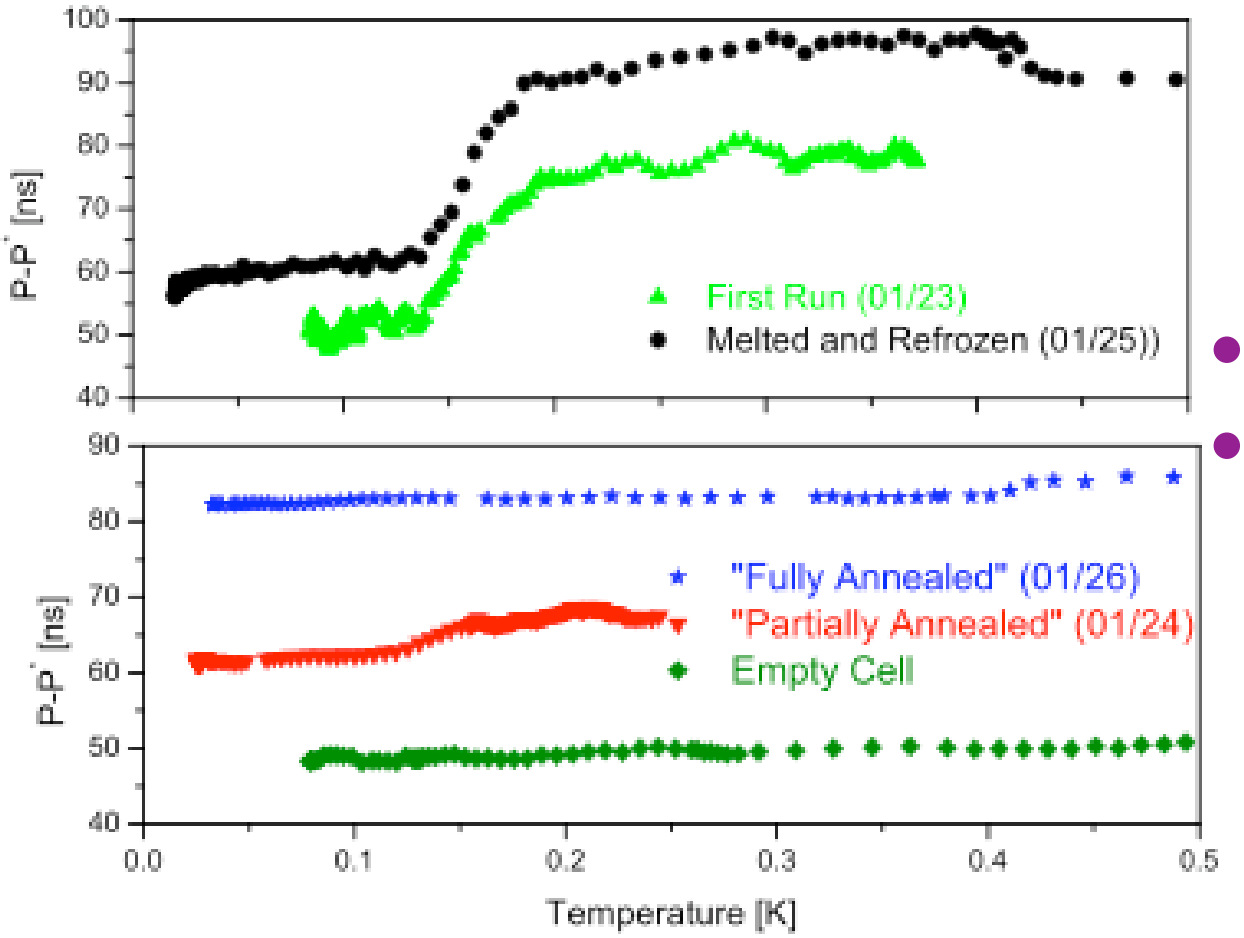
Outline

- Deformations of perfect crystal
- point defects : vacancies
- line defects : dislocations
- planar defects : grain boundaries

Elimination of the Supersolid State Through Crystal Annealing

Ann Sophie C. Rittner and John D. Reppy*
Laboratory of Atomic and Solid State Physics and the Cornell Center for Materials Research, Cornell University, Ithaca, New York 14853-2501
(Dated: April 22, 2006)

AnnSophie C. Rittner and J.D.Reppy, Phys. Rev. Lett. **97**, 165301 (2006).



- square geometry
- bigger volume

no period drop

Experimental vacancy concentration

condensate fraction

$$n_0 = (0.08 \pm 0.78)\% \quad T = 120\mathrm{mK}$$
$$n_0 = (-0.10 \pm 1.20)\% \quad T = 80\mathrm{mK}$$

S. O. Diallo *et al.*, cond-mat/0702347

X-ray diffraction

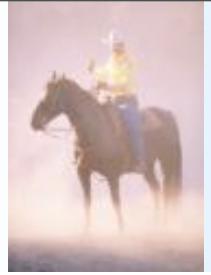
max. 1%
(interpretation in terms of thermal vacancies criticized by Anderson-Brinkman-Huse)

B. A. Fraass, P. R. Granfors, and R. O. Simmons, Phys. Rev. B **39**, 124 (1989)

Measurement of the melting curve

$p \sim T^4$

I. A. Todoshchenko *et al.*, cond-mat/0703743



non-equilibrium scenarios : the Roadmap

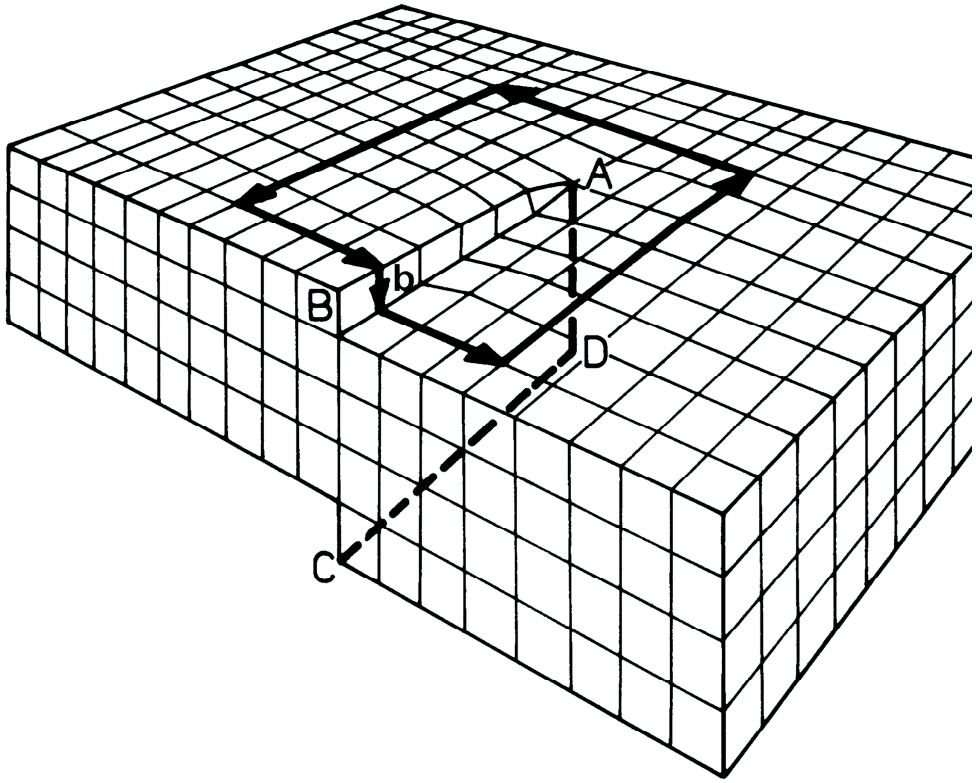
Superfluidity of Grain Boundaries and Supersolid Behavior

S. Sasaki,[1*] R. Ishiguro,[1†] F. Caupin,[1] H. J. Maris,[1,2] S. Balibar[1*]

Science **313**, no. 5790, p 1098. (2006)

- superglass M. Boninsegni, N.V. Prokof'ev, B.V. Svistunov, Phys. Rev. Lett. **96**, 105301 (2006).
- Migration of vacancies to grain boundaries / ridges, formation of dislocation loops
- 'ideal' domain walls, grain boundaries and dislocations lattice models
- 'rough' domain walls, grain boundaries, dislocations,...

Screw dislocation

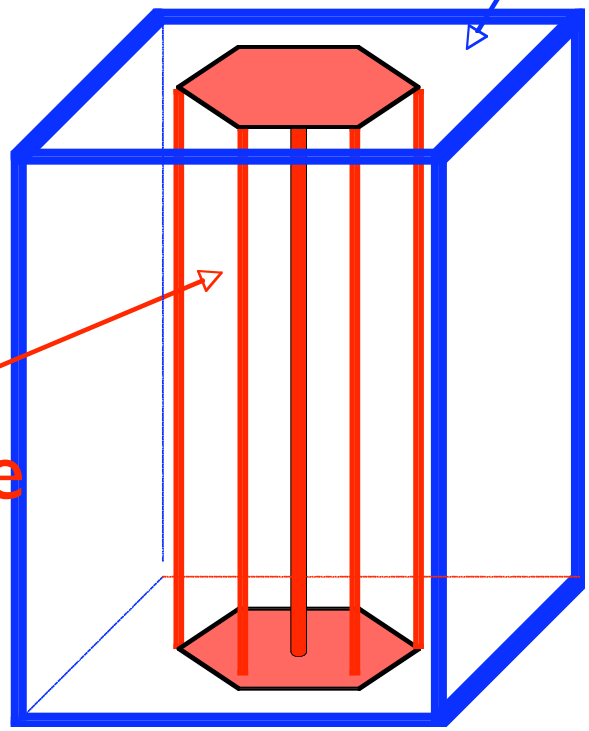


$$n = 0.0287 \mathring{A}^{-1}$$

$$T = 0.2 \rightarrow 1\mathrm{K}$$

1536 fixed

384 updatable

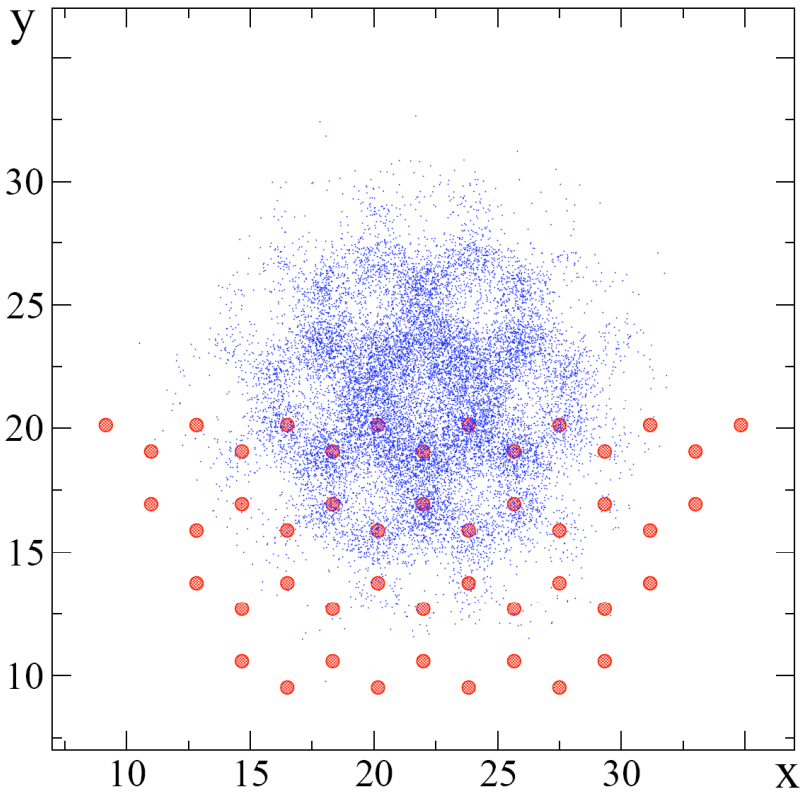


Screw dislocation

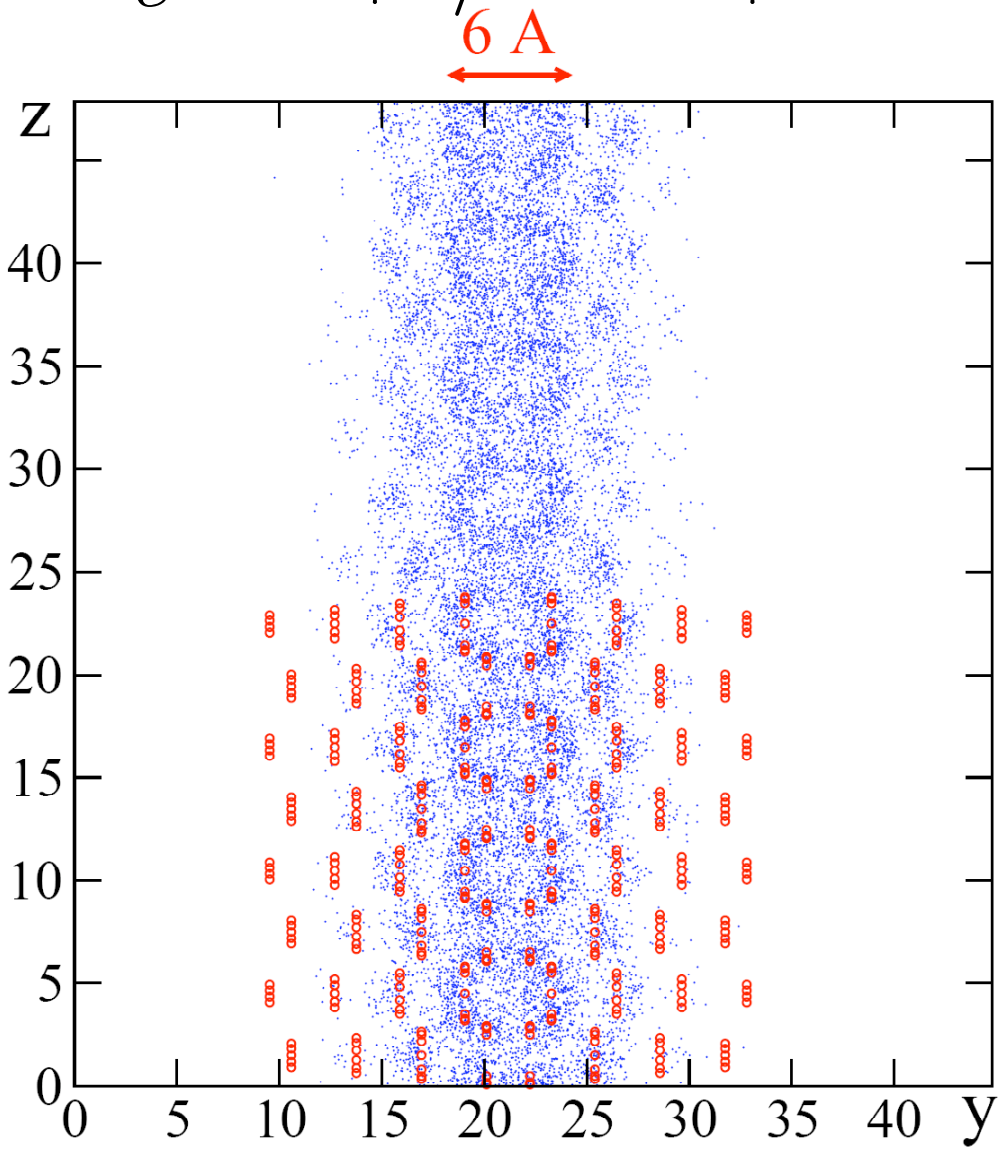
Shevchenko state

$$\tau = (T_*/T)^{2/K_{\rm L}-1}/T_*$$

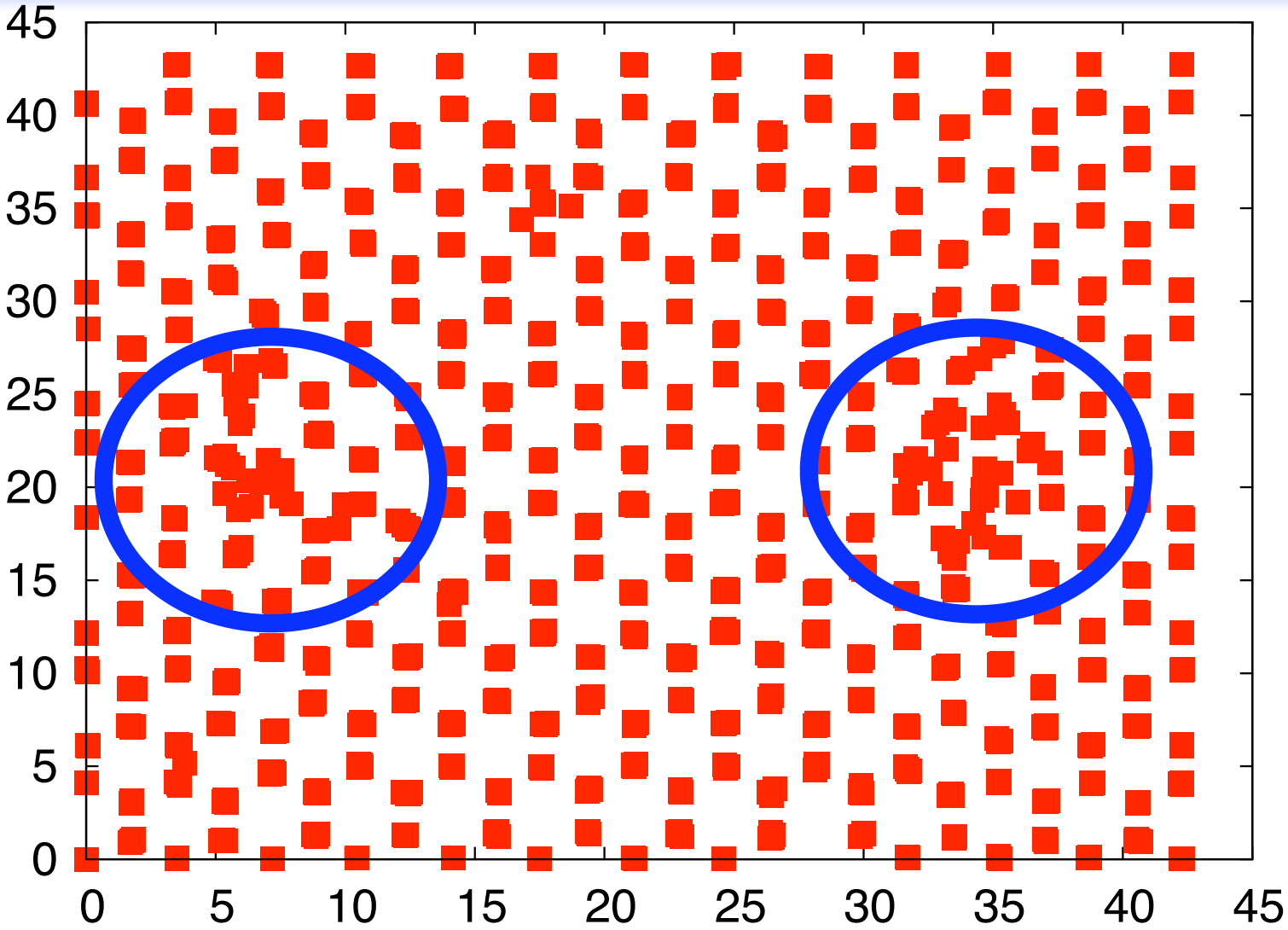
$$n_s \approx 1\mathring{A}^{-1}, K_{\rm L} = 0.205(20)$$



$$T_c \sim T_* a/l \ll T_*$$



edge dislocation



forms a glassy region, but can also be an insulator depending on the initial configuration

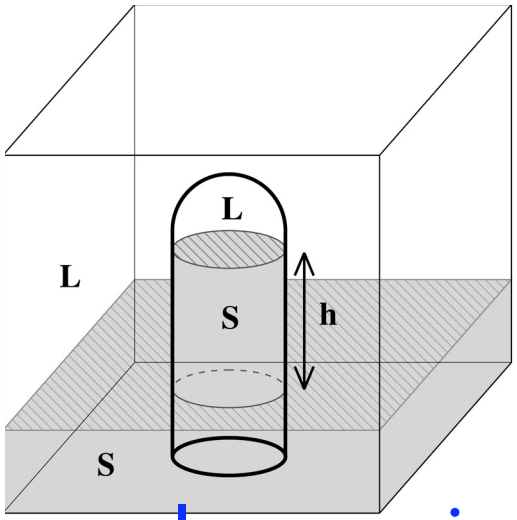
Outline

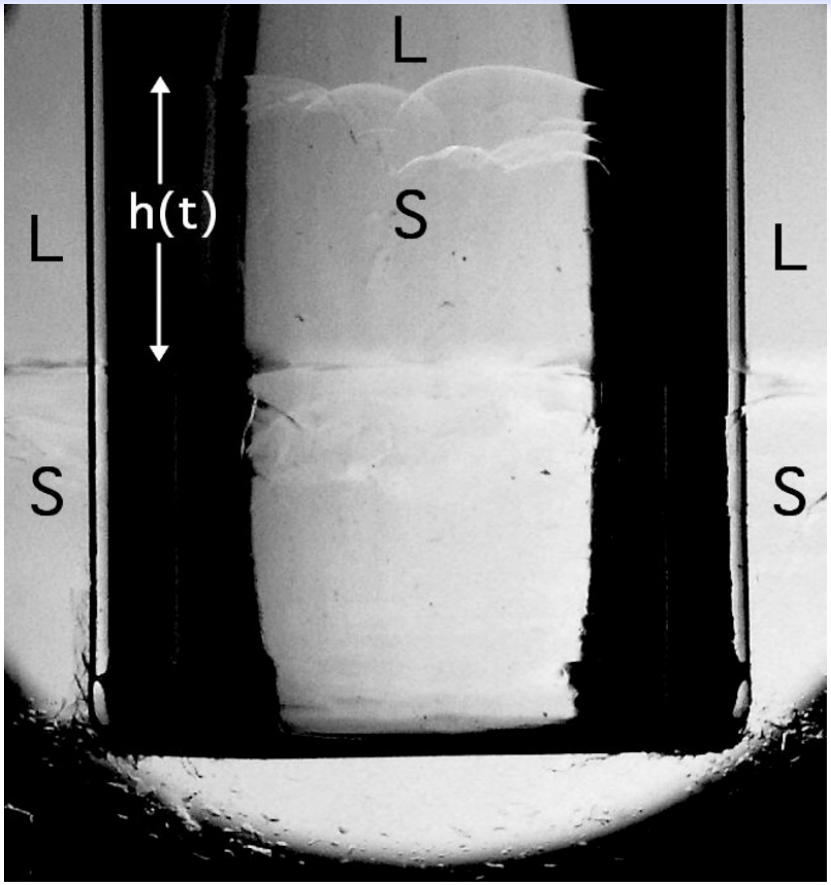
- Deformations of perfect crystal
- point defects : vacancies
- line defects : dislocations
- planar defects : grain boundaries

Superfluidity of Grain Boundaries and Supersolid Behavior

S. Sasaki,[1*] R. Ishiguro,[1†] F. Caupin,[1] H. J. Maris,[1,2] S. Balibar[1*]

"law of the communicating vessels"





clean samples: no signal

dirty samples : SS signal

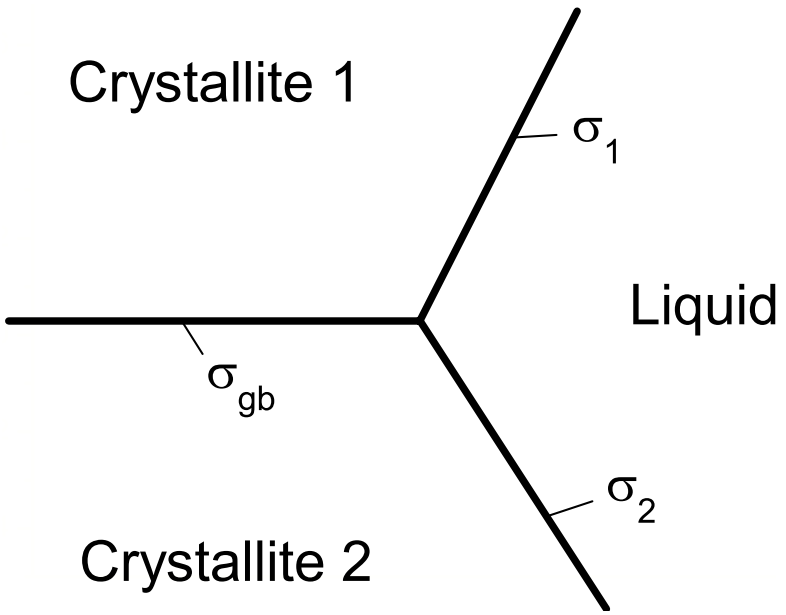
however :

- also signal above (1K) (cracks ? wetting?)
- vs. how can signal disappear for clean samples???
- liquid film caused by grain boundaries at the interface?

Science **313**, no. 5790, p 1098. (2006)

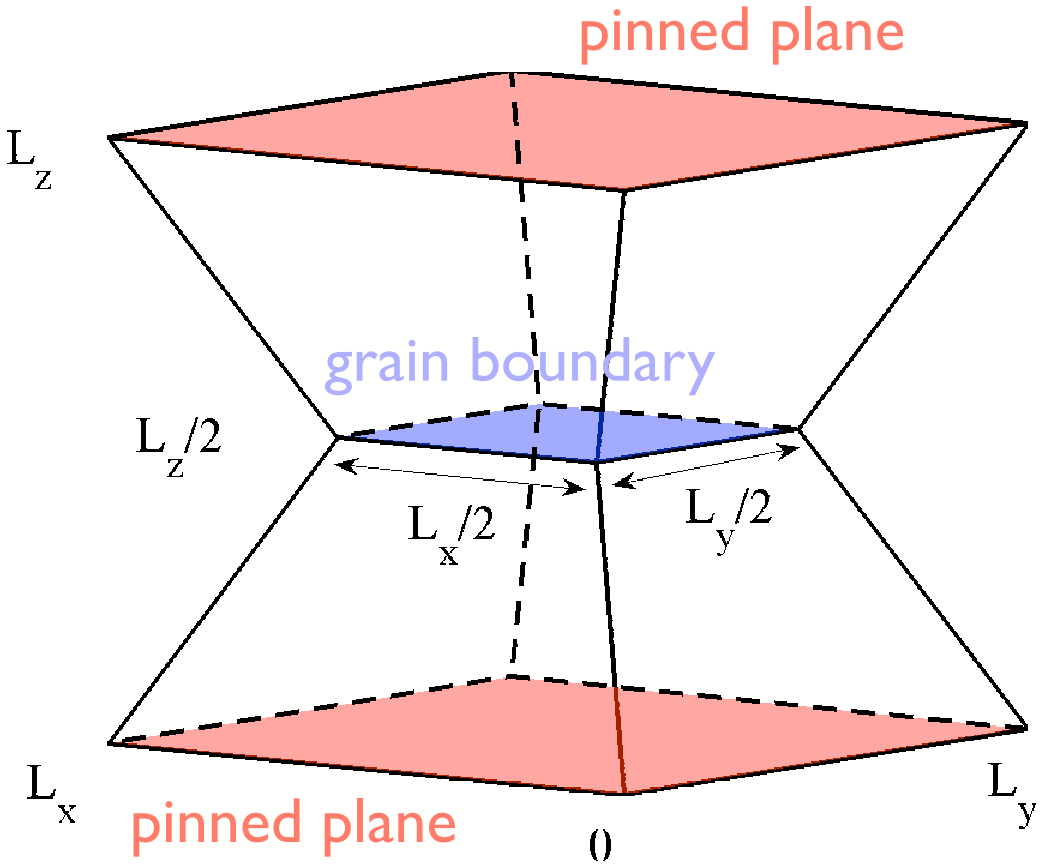
mechanical stability of grain boundaries

experimental conditions of phase coexistence :
true superfluid grain boundaries
or cracks filled with liquid?

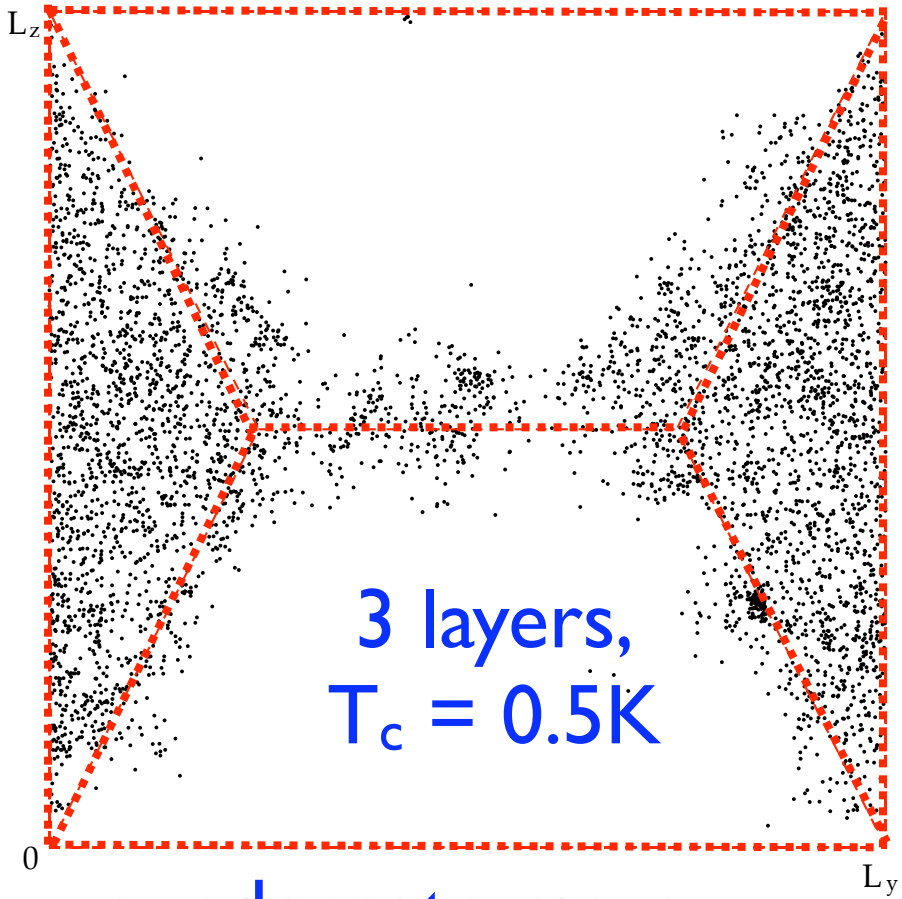


$$\sigma_{gb} < \sigma_1 + \sigma_2$$

grain boundaries are stable...

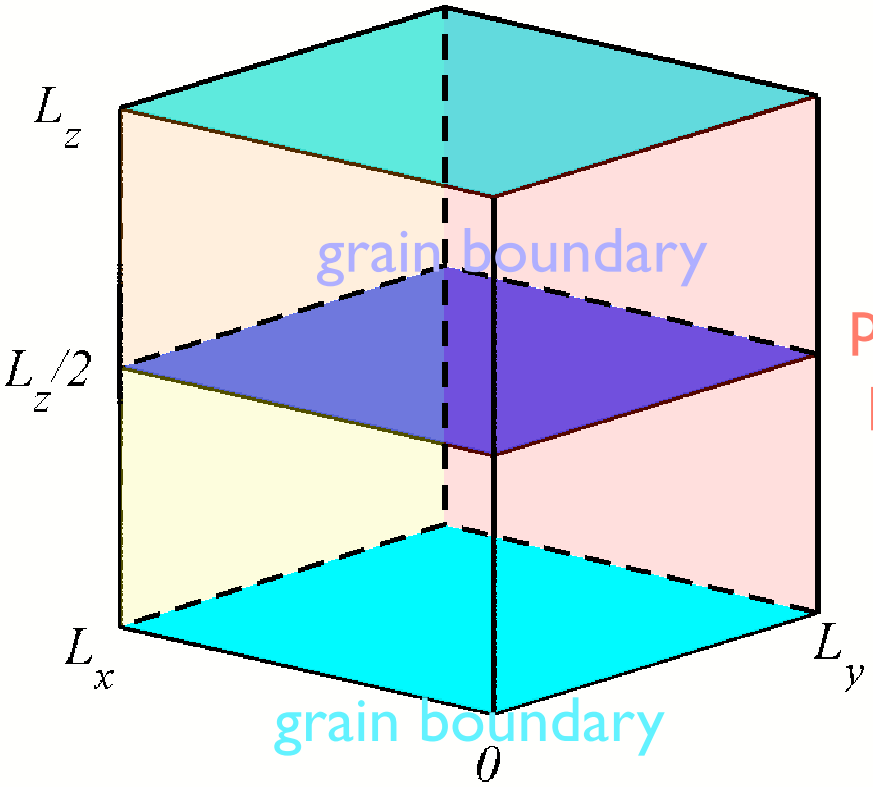


N = 13660, T = 0.8K

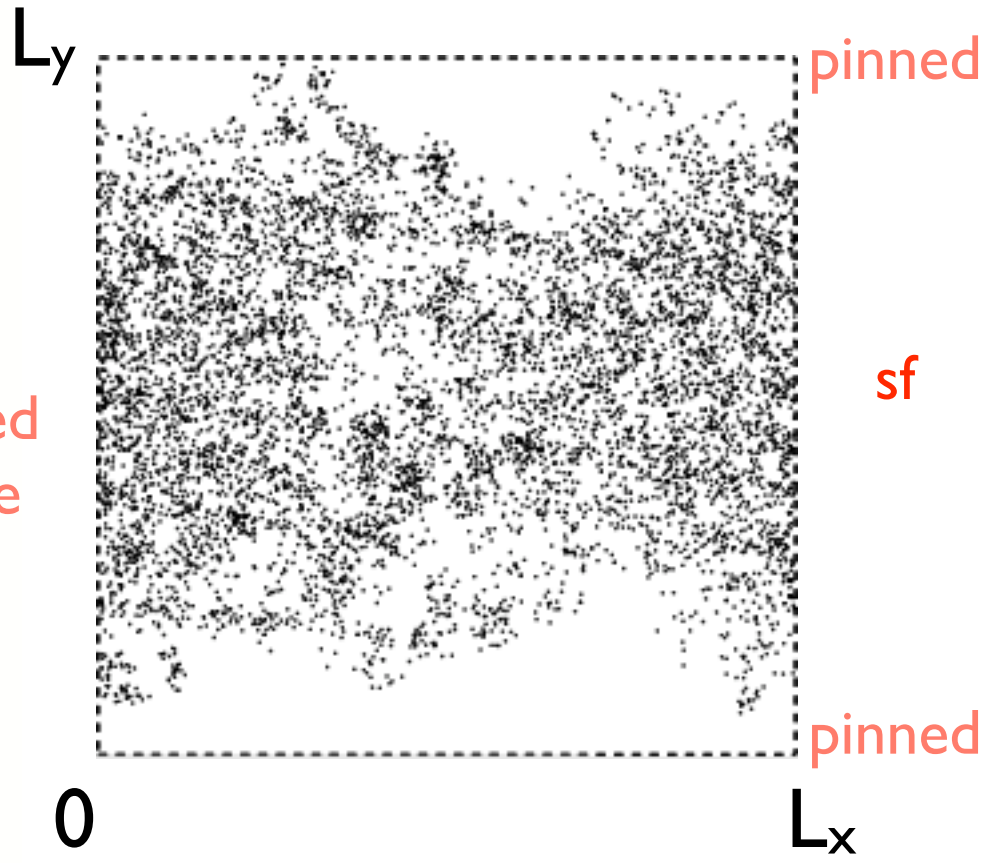


condensate map; map of the condensate wave function (spatial positions of the worm ends when they are sufficiently far away from each other)

...and generically superfluid...

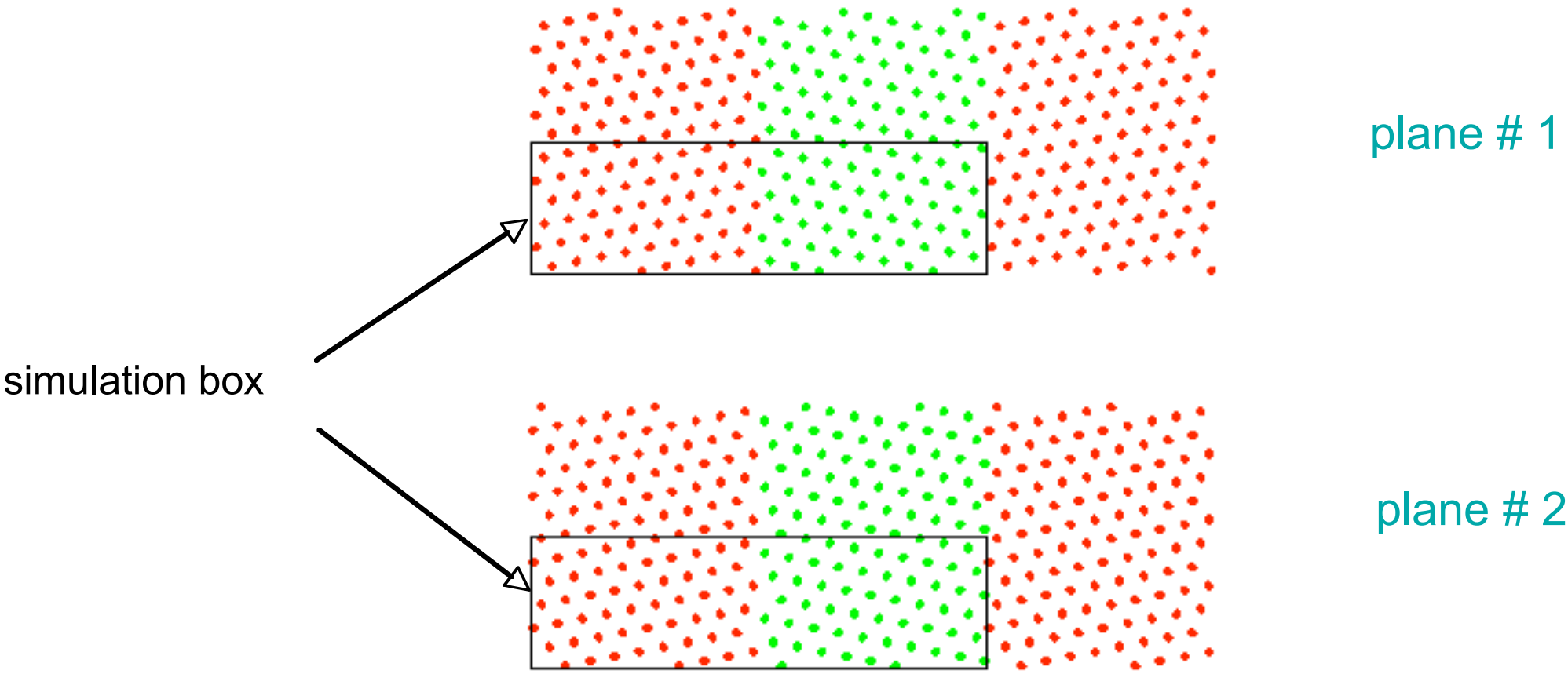


12x12x14



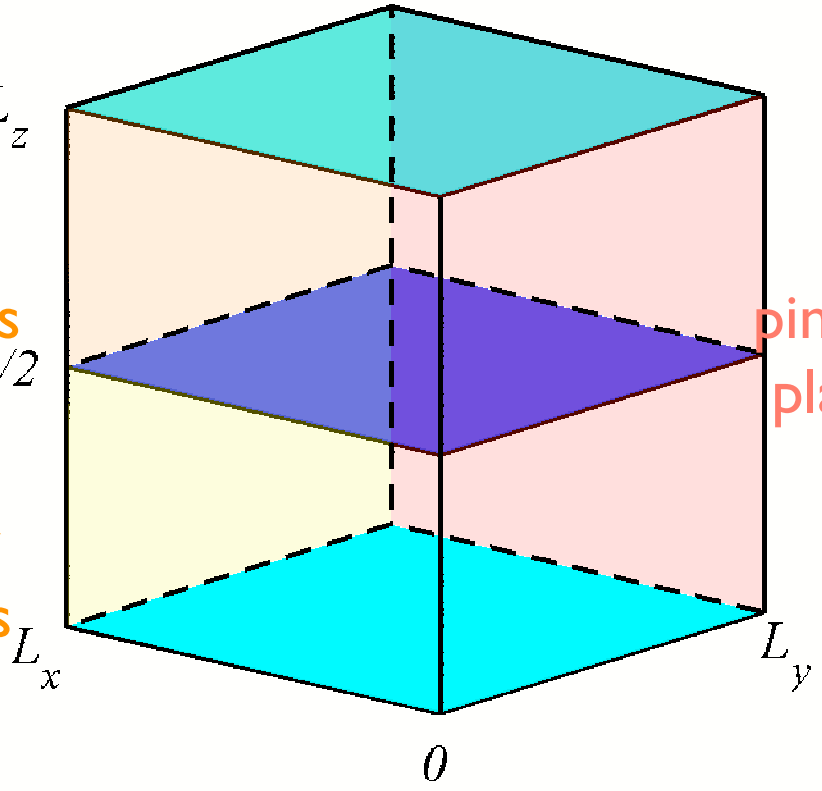
projection on xy plane of upper cuboid

...but not in all cases



Insulating

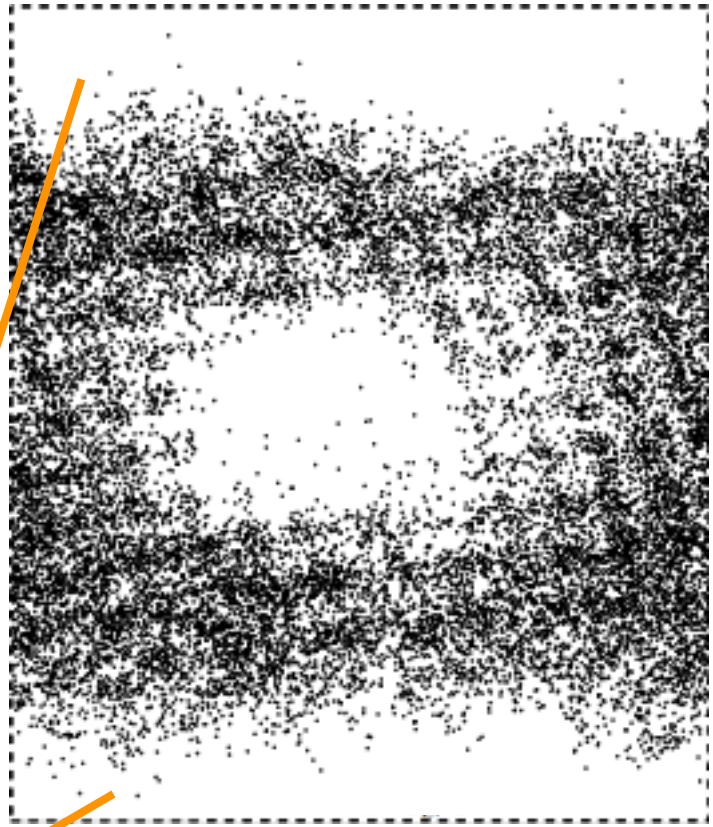
It happened also with the cuboids



additional grain boundaries due to periodic boundary conditions

pinned plane

$z = L_z + L_z/4$

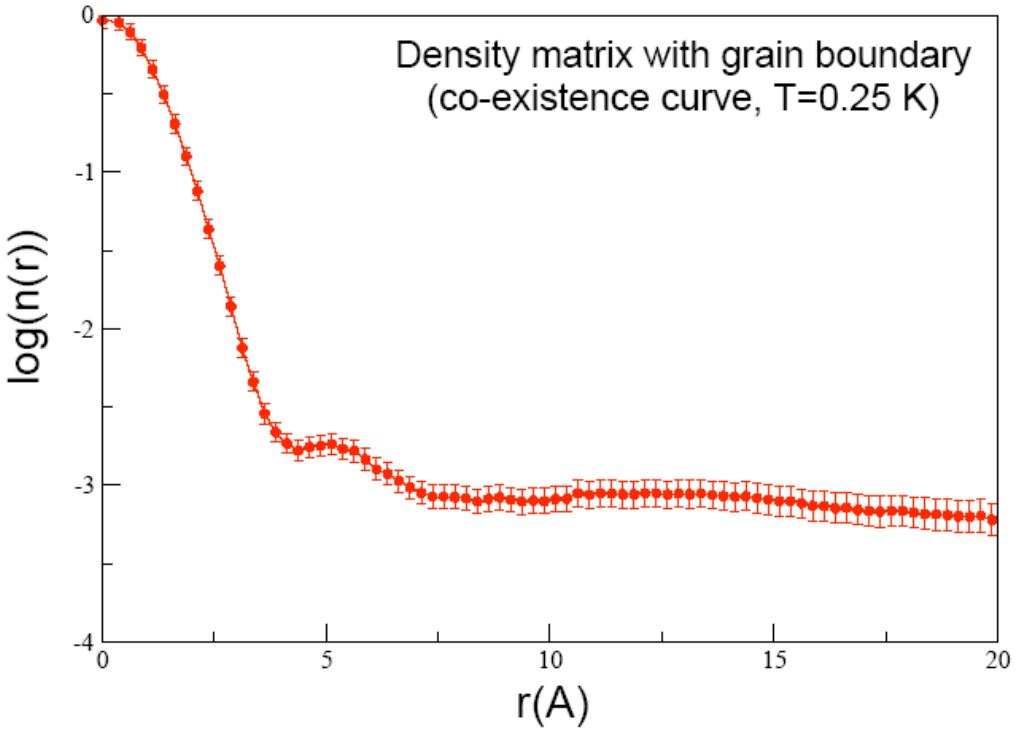


$z = L_z/4$

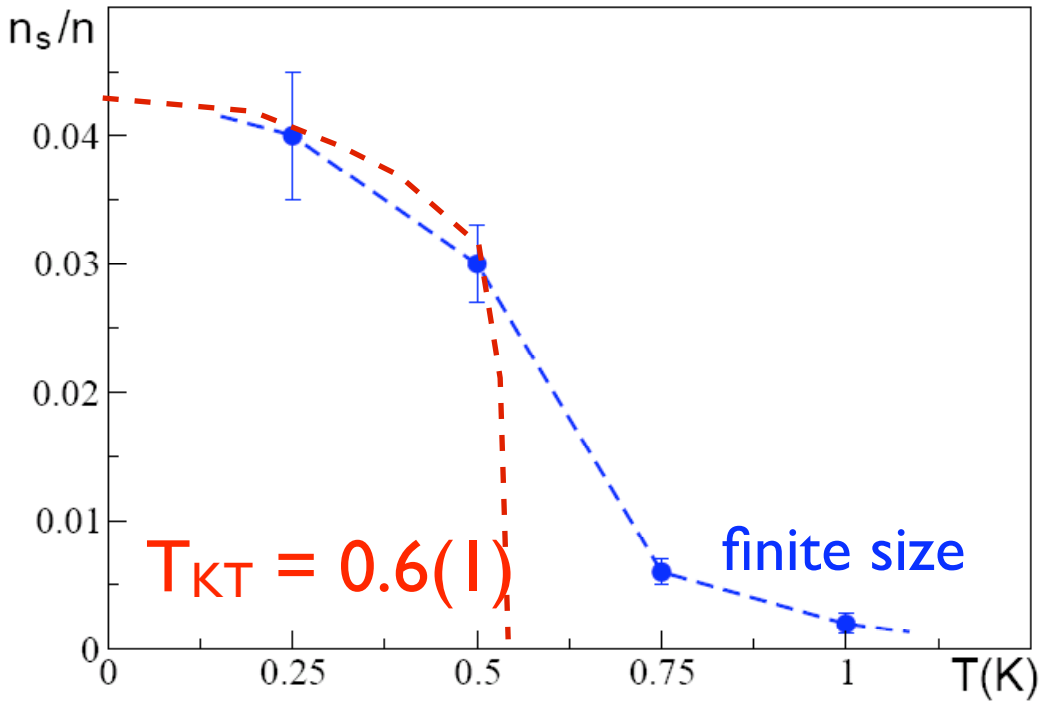
insulating additional grain boundary

projection on the xz plane

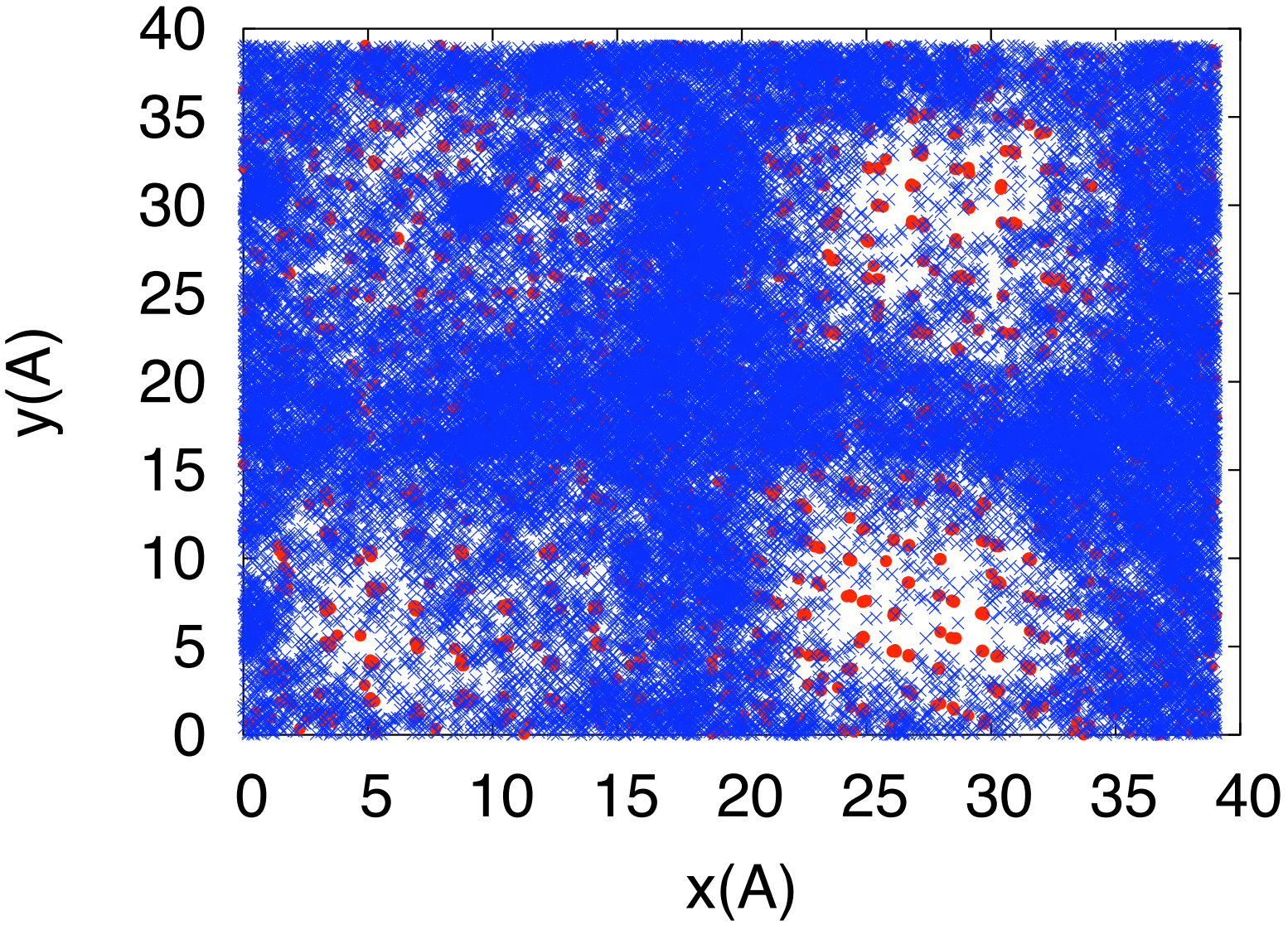
density matrix



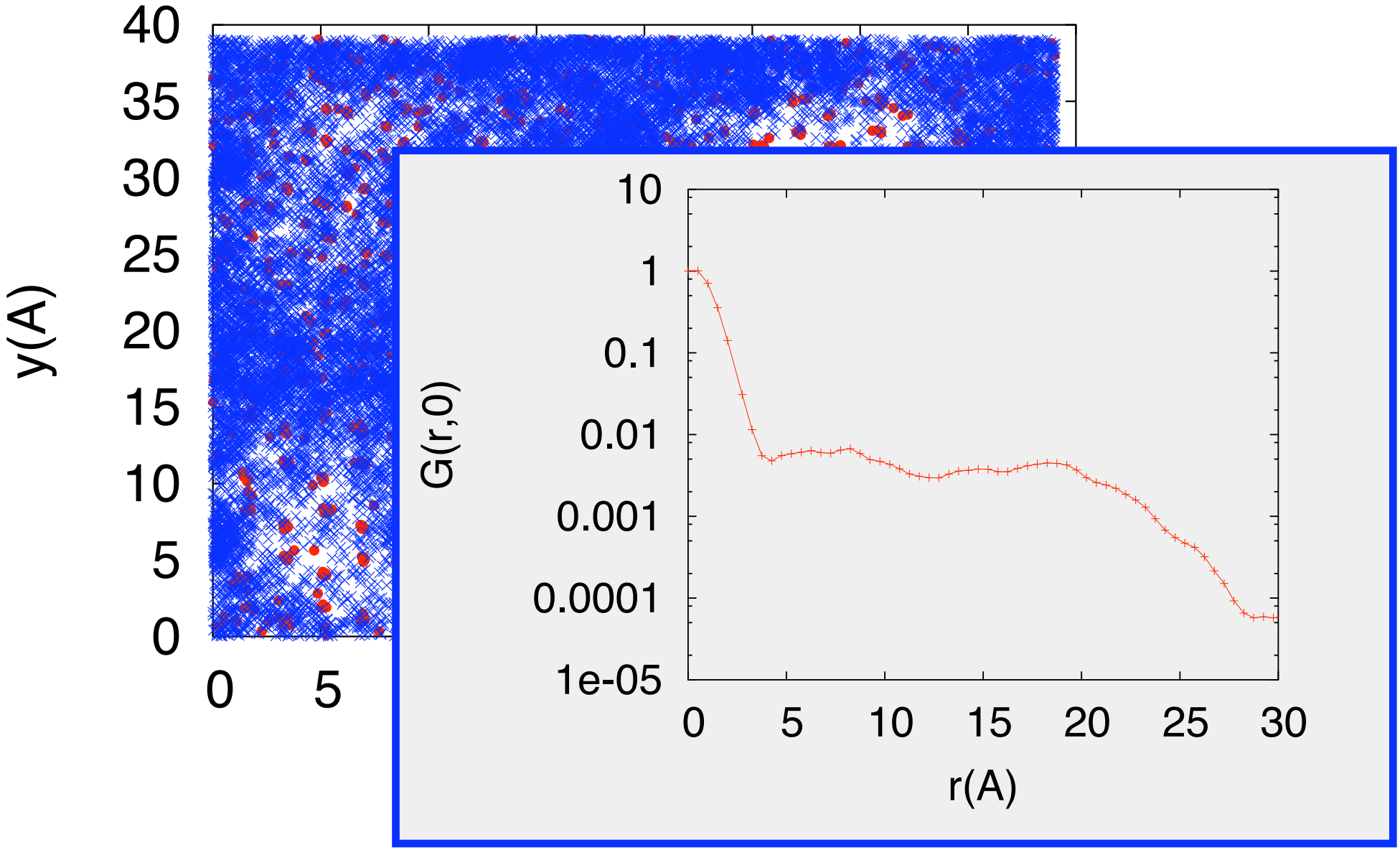
superfluid fraction



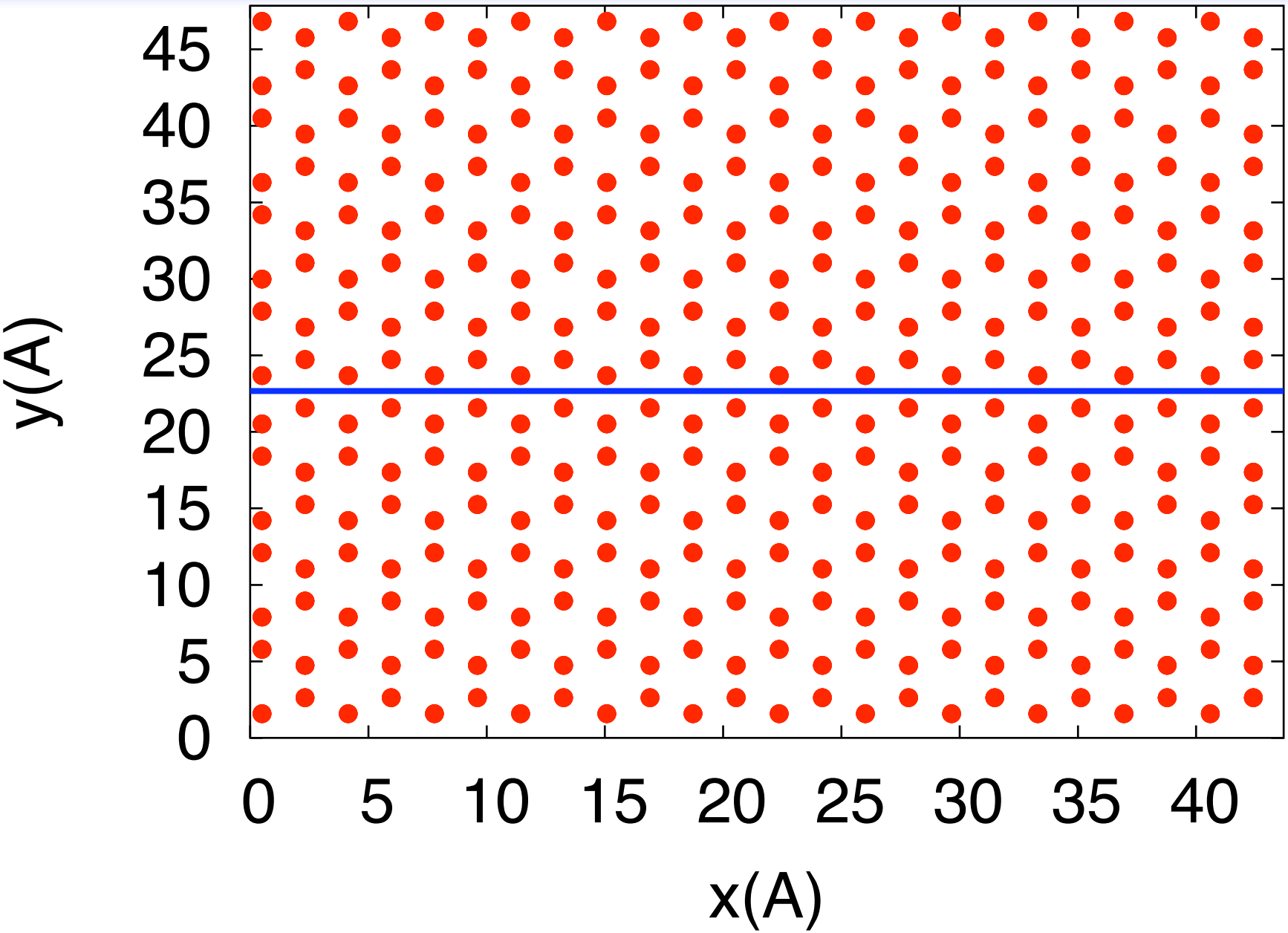
Glassy interfaces



Glassy interfaces

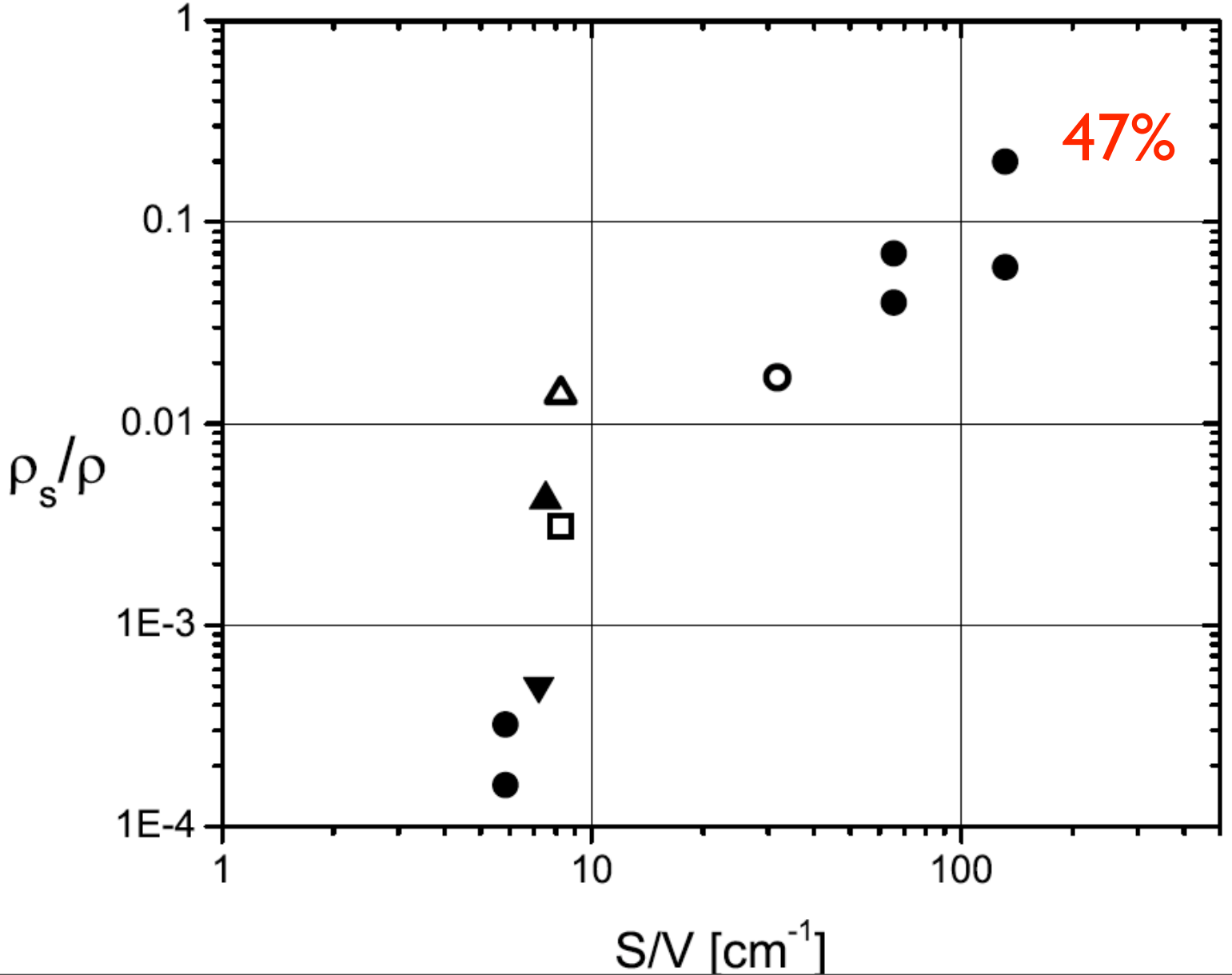


Stacking fault

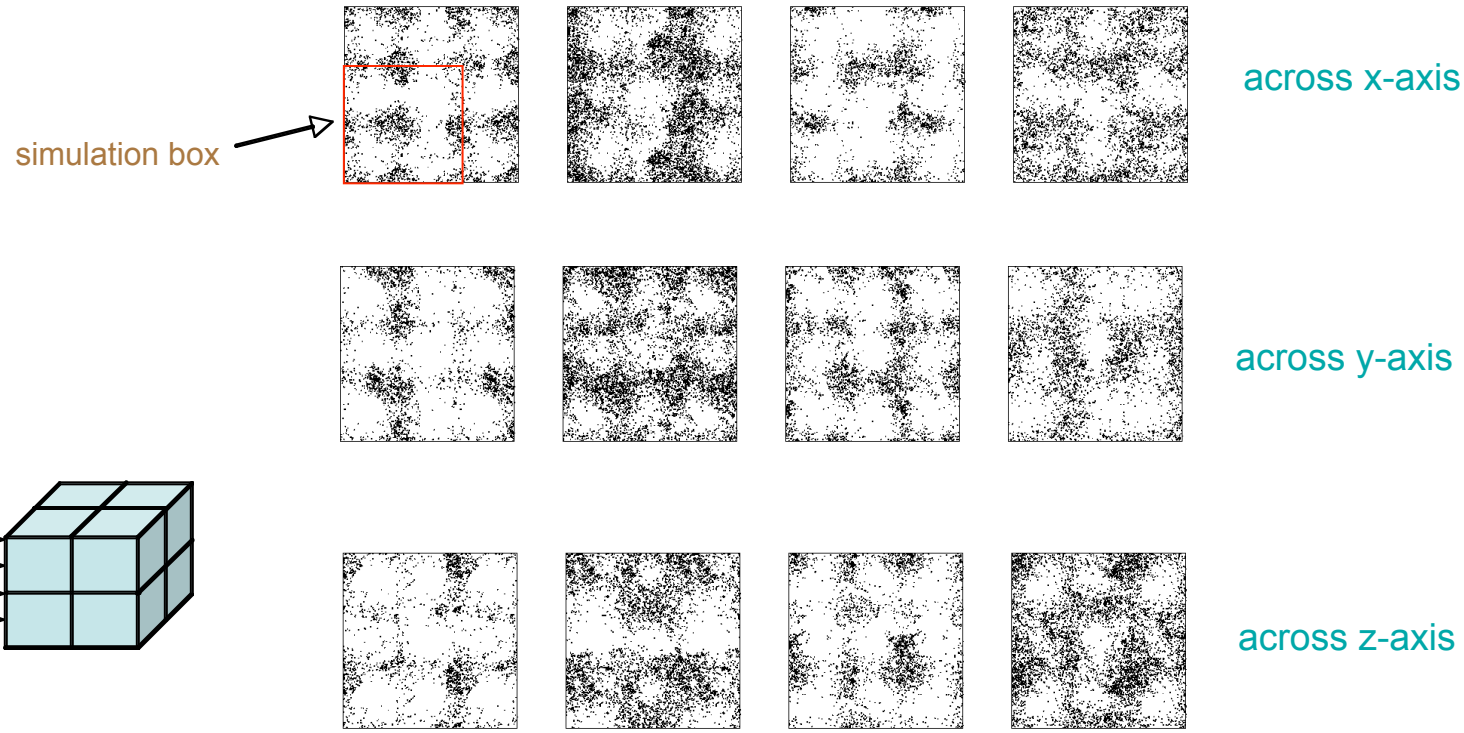


Latest results

AnnSophie C. Rittner and J.D.Reppy, cond-mat/0702665



Grain boundaries : condensate maps



Each of the 8 cubes is a randomly oriented crystallite

Conclusion

- perfect crystal is *insulating*
- phase separation of vacancies
- superfluid screw dislocations and Shevchenko state
- 'homeopathic' role of ^{3}He
- superfluid grain boundaries exist
- do supersolids exist for systems in continuous space (2D, 3D) with one type of particles interacting through a pair potential?