DYNAMICAL CRITICAL SCALING IN QUANTUM PHASE TRANSITIONS

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Department of Physics - Indiana University

Shusa Deng: Dartmouth College

Lorenza Viola: Dartmouth College

Quantum Coherent Properties of Spins III

UCF (Orlando), December 2010

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NON-EQUILIBRIUM QUANTUM CRITICAL PHYSICS

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KIBBLE-ZUREK'S MOTIVATION

- Ideas were originated in speculations about phase transitions in the early universe -----T. W. B. Kibble, J. Phys. A 9, 1387 (1976)
- Topological defects such as cosmic strings may have been formed in early-universe phase transitions

Questions: How many defects would be formed in the phase transition?
How would they evolve as the Universe expands? ……

How to test ?

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Exciting developments in condensed matter physics-----
in particular, classical dynamical critical phenomena
(superfluids, superconductors, BEC……)

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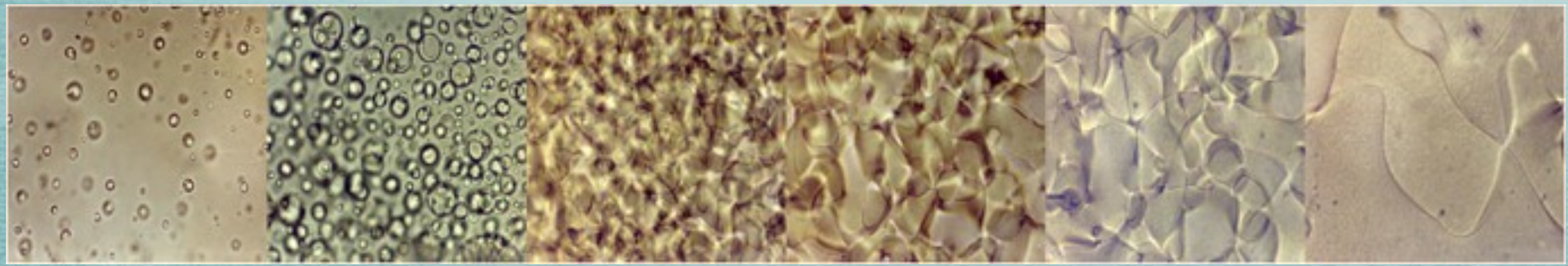

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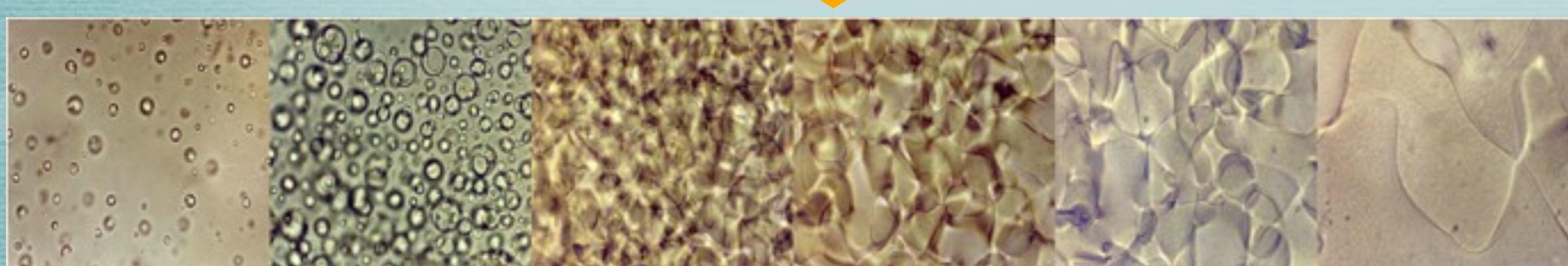

Quantum Version

HOW ABOUT QUANTUM?

Non-ergodic behavior of time-dependent T=0 magnetization of the anisotropic XY chain

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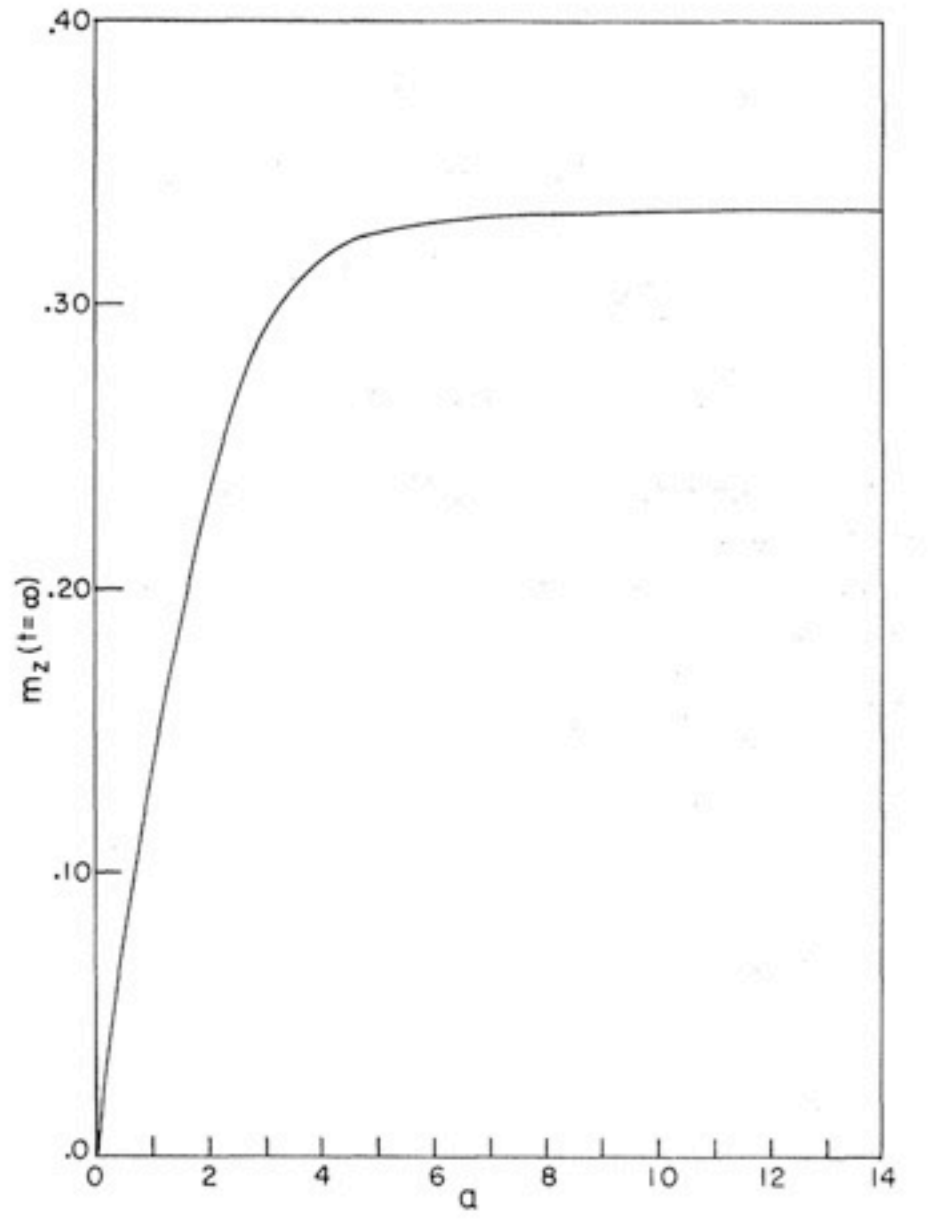

FIG. 1. Final magnetization versus initial field a for step function case: a – jumps to 0.

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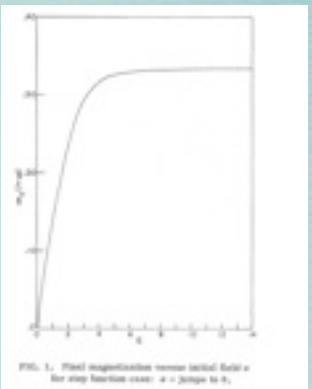
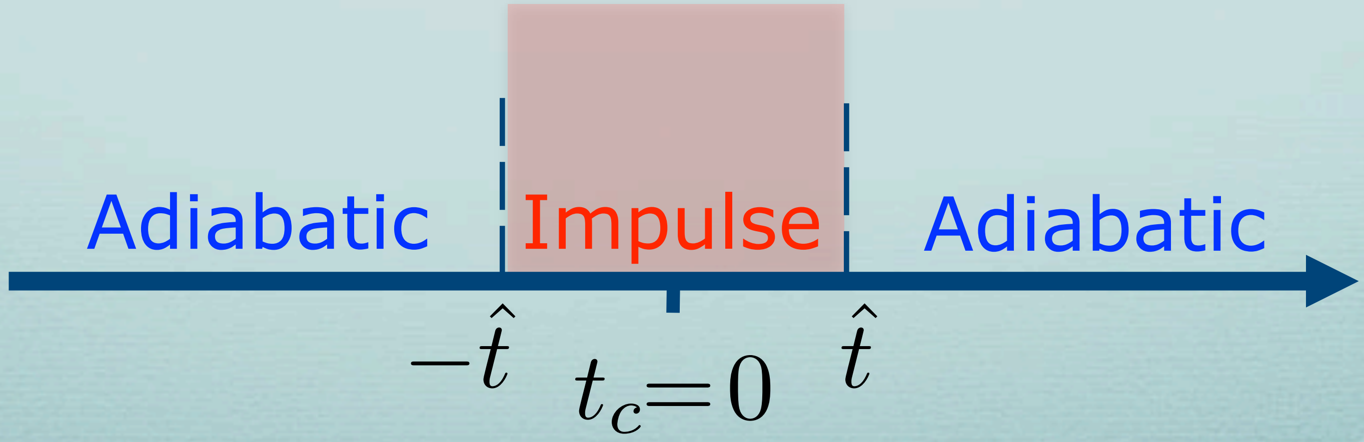


Kibble-Zurek Scaling (KZS)

--- W. H. Zurek, *et al.* PRL 95, 105701 (2005)

--- A. Polkovnikov, PRB, 72 (2005) 161201

Linear sweep of control parameter with constant speed τ



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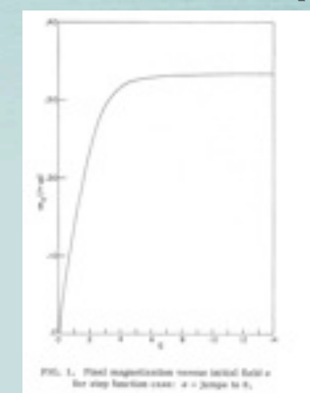

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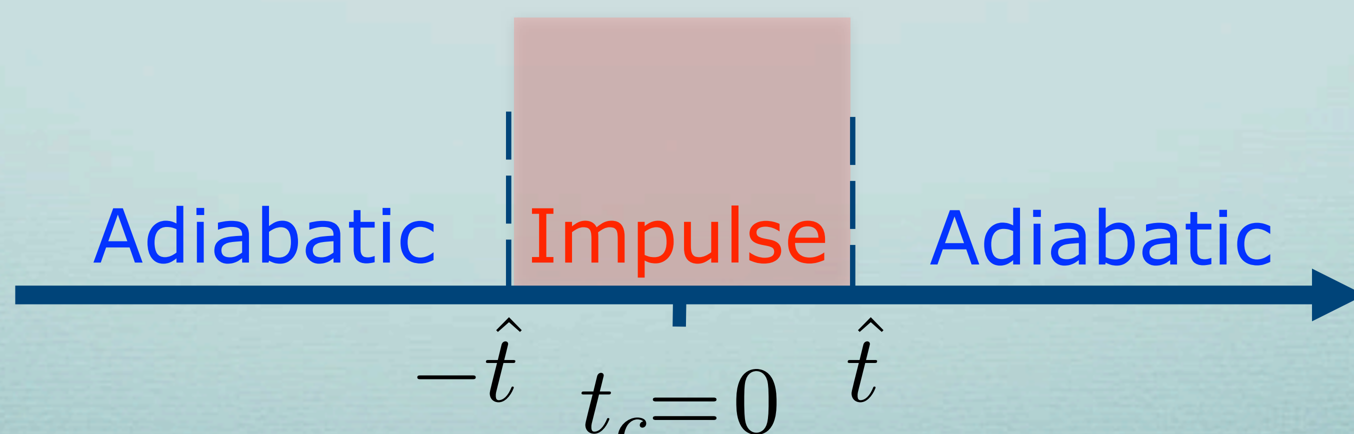

$$\hat{t} \sim \tau^{\nu z/(\nu z+1)} \quad , \quad n_{ex}(t_f) \sim \tau^{-d\nu/(\nu z+1)}$$

MOTIVATION

Tunable quantum systems:

--- Ultracold Atom systems
(quantum simulators)

Adiabatic quantum computation:
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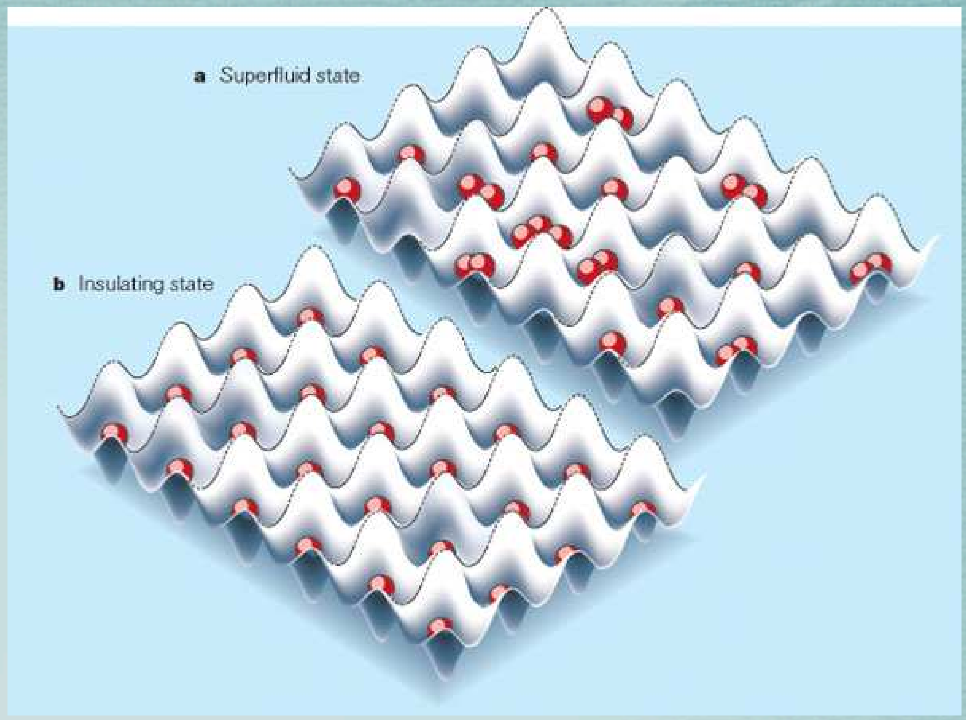

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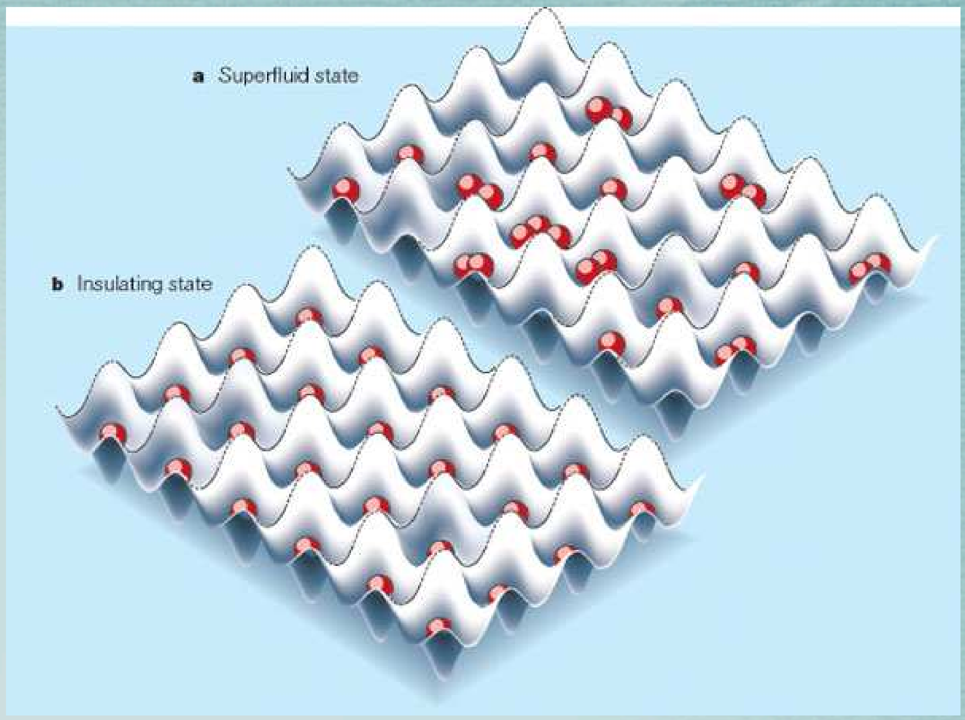

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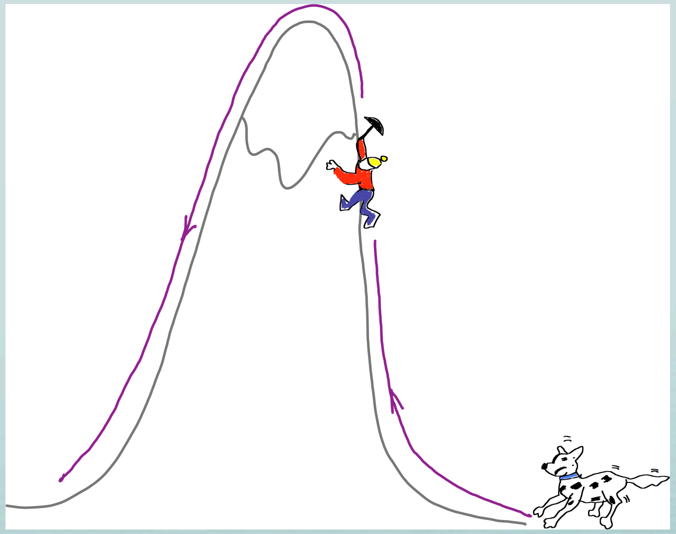

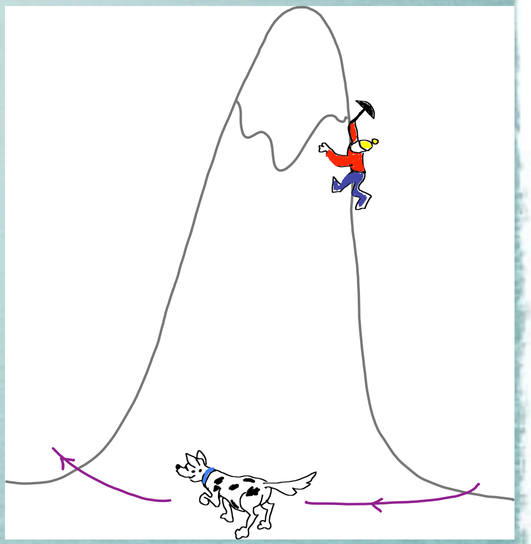

Increasing dimensions can be more efficient

KIBBLE-ZUREK (AND OTHERS) INTUITION

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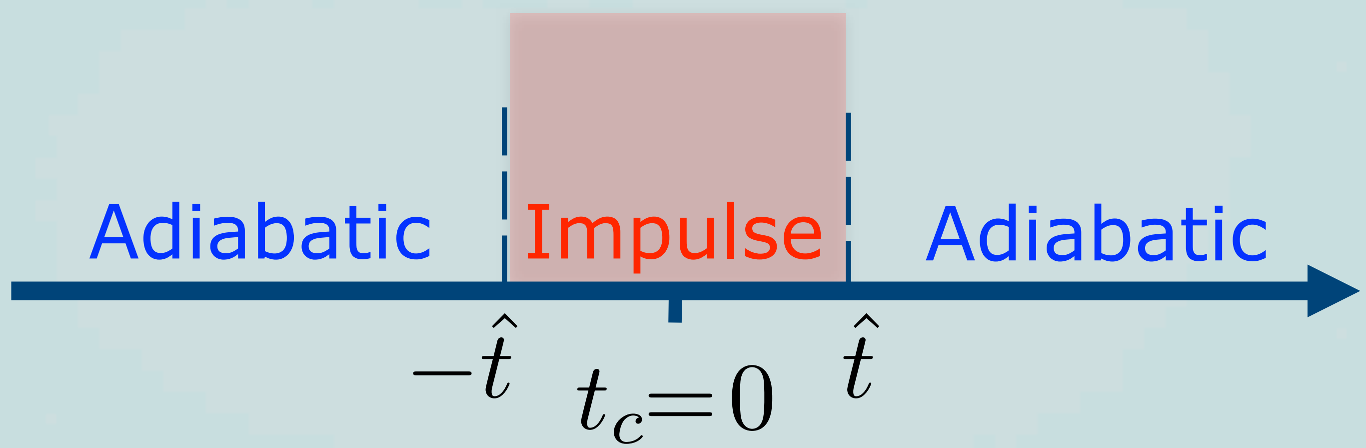
Near a QCP there is a vanishing energy scale: $\Delta \sim |\lambda(t) - \lambda_c|^{\nu z}$

Linear sweep of control parameter with constant speed τ

$$\lambda(t) - \lambda_c = \frac{t - t_c}{\tau}\ ,\ t_c = 0,\ \tau > 0$$



Relaxation time:

$$\tau_r(t) \sim \Delta^{-1} \sim |\lambda(t) - \lambda_c|^{-\nu z}$$

Transition rate:

$$T(t) = \left| \frac{\lambda(t)}{\dot{\lambda}(t)} \right|$$

Time scale of adiabaticity loss: $\hat{t}$

$$T(\hat{t}) = \tau_r(\hat{t})$$

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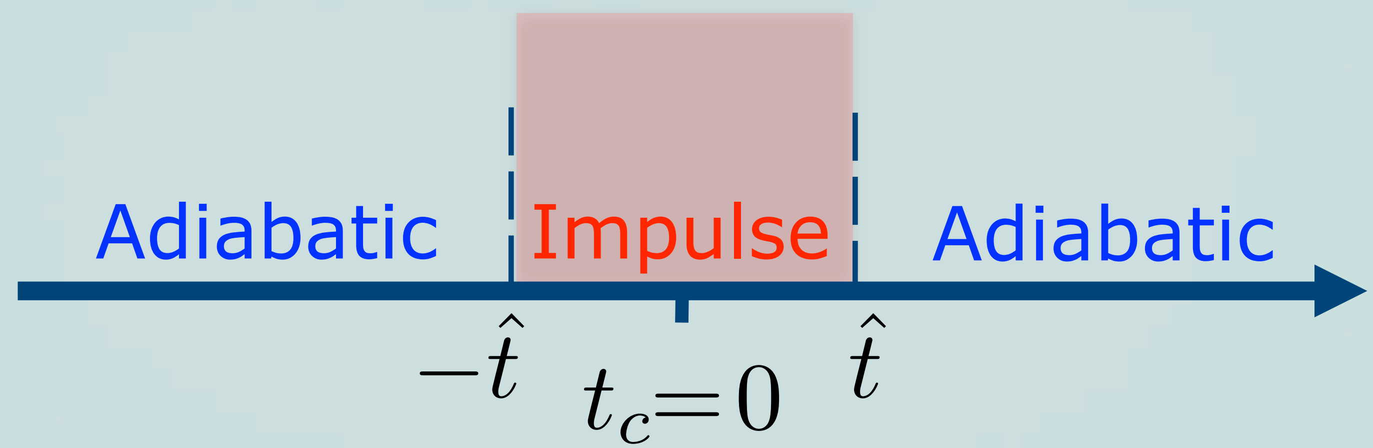
Symmetric impulse region



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ν and z

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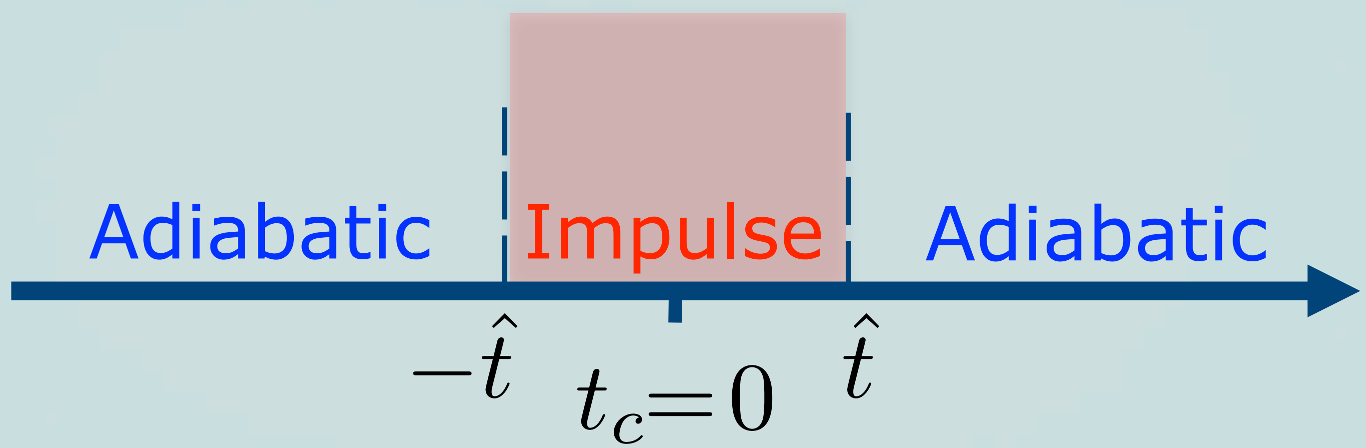

equilibrium critical exponents

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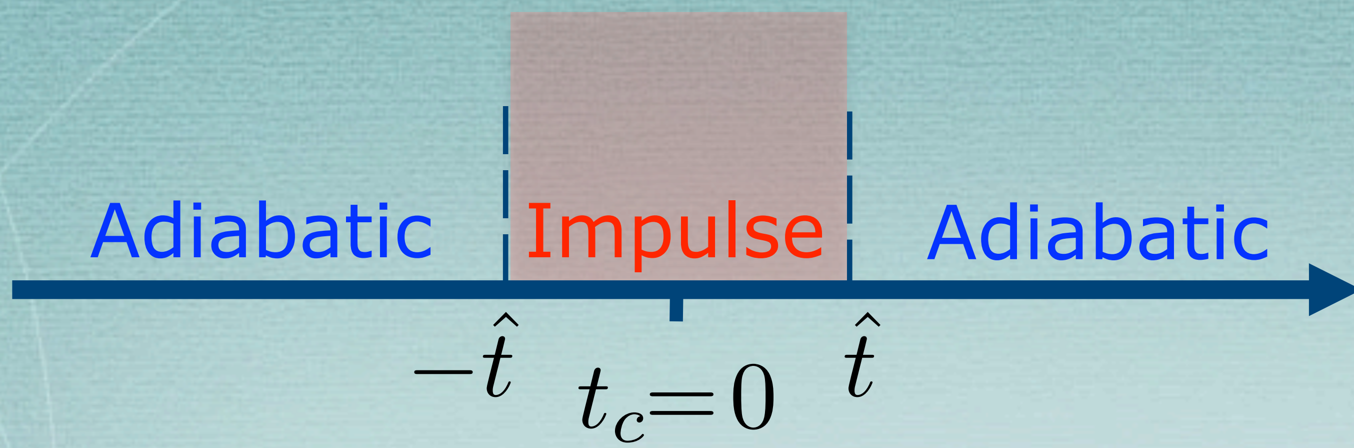
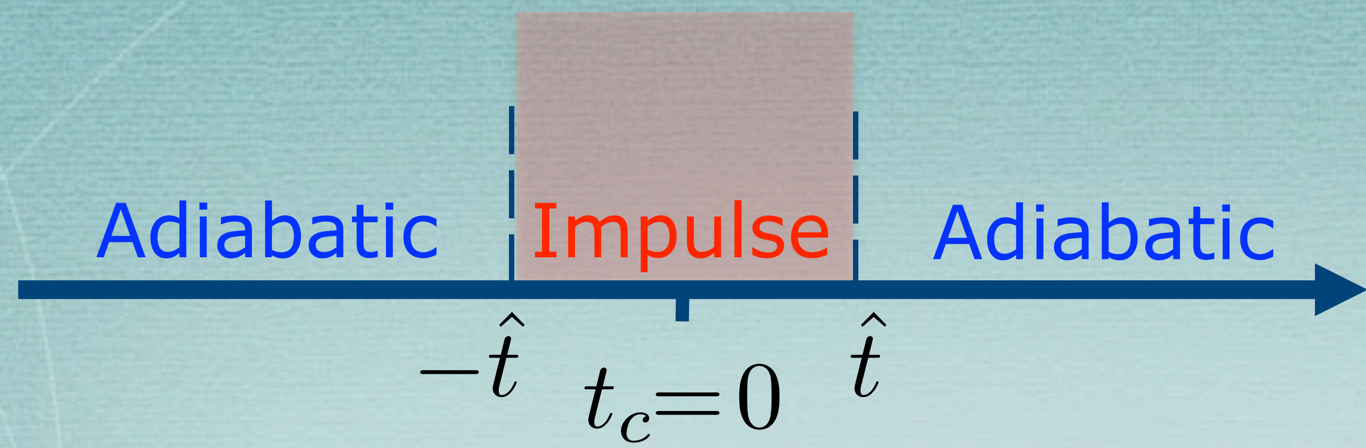


Defects

$$|\cdots\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\cdots\rangle \longrightarrow |\cdots\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\cdots\rangle$$

For a linear quench:



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For a linear quench:

$$\boxed{\hat{t}\sim\tau^{\nu z/(\nu z+1)}\ ,\ \xi(\hat{t})\sim\tau^{\nu/(\nu z+1)}\ ,\ n_{ex}(t_f)\sim\tau^{-d\nu/(\nu z+1)}}$$

density of excitations

A MAIN QUESTION

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To what extent universal quantum scaling persist out-of-equilibrium and encode information about the equilibrium phase diagram?

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How universal ?

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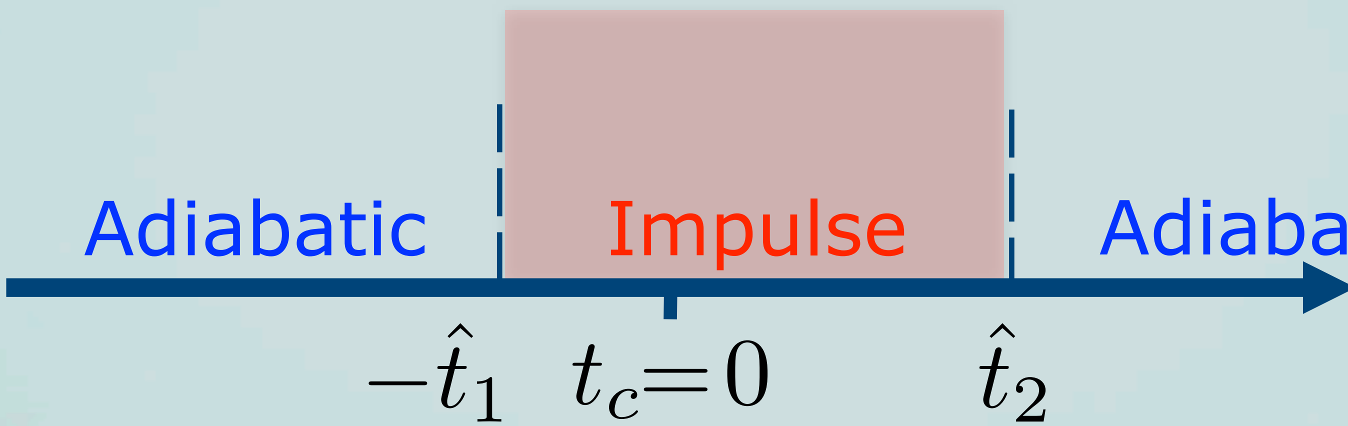

Dynamical asymmetry

A MAIN MESSAGE

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One needs in general path-dependent
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Important:

Time-dependent excitation pattern

Details about initial and final phases

Appropriate Landau-Zener analysis
when applicable is OK

ULTIMATE GOAL

Develop a theory and understanding of non-equilibrium scaling for quenches across quantum (multi)critical points and regions

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This talk:

Elementary integrable (Lie algebraic) toy model

Surprises emerge: Departure from KZS

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Elementary integrable (Lie algebraic) toy model

Surprises emerge: Departure from KZS

--- Isolated QCP
--- Isolated Multicritical QCP
--- Search for understanding

REFERENCES

Critical regions + Adiabatic renormalization

Europhys. Lett. 84, 67008 (2008)

Multicritical QCP + Anomalous path-dependent exponents

PRB 80, 241109(R) (2009)

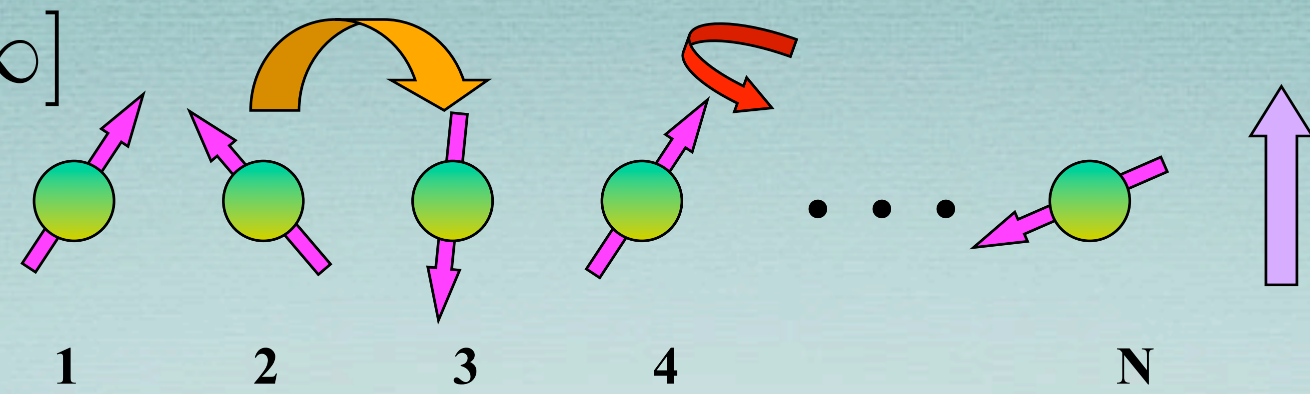
Initial excited and thermal states

arXiv:1011.0781

MODEL SYSTEM

$h, \delta \in [-\infty, \infty]$

$\gamma \in \mathbb{R}$



$B = h \pm \delta$

$$H = -\sum_{i=1}^{N} \left\{ \frac{1+\gamma}{2} \sigma_x^i \sigma_x^{i+1} + \frac{1-\gamma}{2} \sigma_y^i \sigma_y^{i+1} - [h - (-)^i \delta] \sigma_z^i \right\}$$

$$H = \sum_{k \in K_+} A_k^\dagger H_k A_k$$

$$H_k = 2 \begin{pmatrix} (h+\delta)\sigma_z & -\cos k\, \sigma_z + \gamma \sin k\, \sigma_y \\ -\cos k\, \sigma_z + \gamma \sin k\, \sigma_y & (h-\delta)\sigma_z \end{pmatrix}$$

$$A_k^\dagger = (a_k^\dagger, a_{-k}, b_k^\dagger, b_{-k}) \qquad K_+ = \{\frac{\pi}{N}, \frac{3\pi}{N}, \ldots \frac{\pi}{2} - \frac{\pi}{N}\}$$

MODEL SYSTEM

When $\gamma = 0 \quad \Rightarrow \quad U(1)$ Symmetry Multicritical QCP

$$H = -\sum_{i=1}^{N}\left\{\frac{1}{2}\sigma_x^i\sigma_x^{i+1} + \frac{1}{2}\sigma_y^i\sigma_y^{i+1} - [h - (-)^i\delta]\sigma_z^i\right\}$$

$$\hat{H}_{\pm k} = W_{\pm k}^{\dagger} H'_{\pm k} W_{\pm k}$$

$$W_k^{\dagger} = (a_k^{\dagger}, b_k^{\dagger}) \qquad\qquad W_{-k}^{\dagger} = (a_{-k}, b_{-k})$$

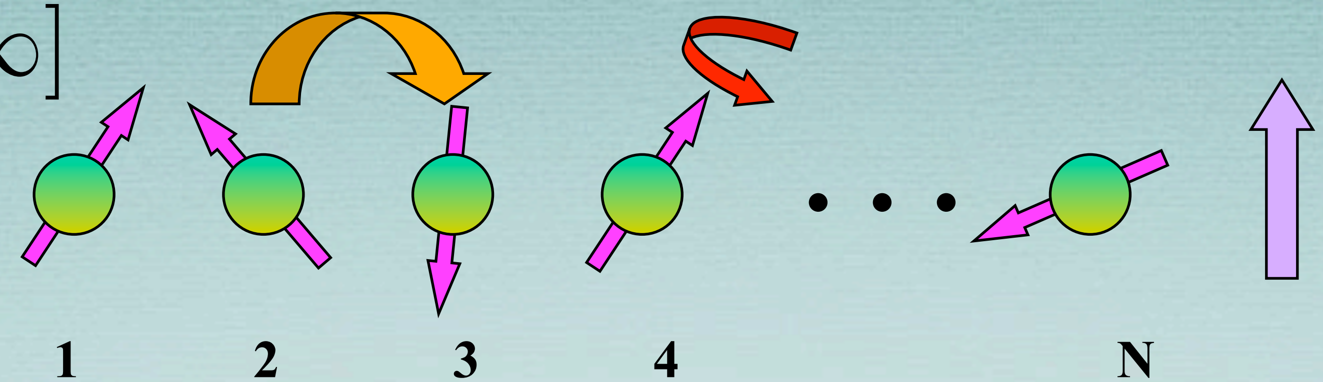
$$H'_{\pm k} = \pm 2h\mathbb{I}_2 + \begin{pmatrix} \pm 2\delta & \mp 2\cos k \\ \mp 2\cos k & \mp 2\delta \end{pmatrix}$$

Appropriate for a Landau-Zener analysis

MODEL SYSTEM

$$h, \delta \in [-\infty, \infty]$$

$$\gamma \in [0, 1]$$



$$B = h \pm \delta$$

$$|\Psi_{GS}\rangle = \prod_{k \in K^+} |\Psi_k\rangle$$

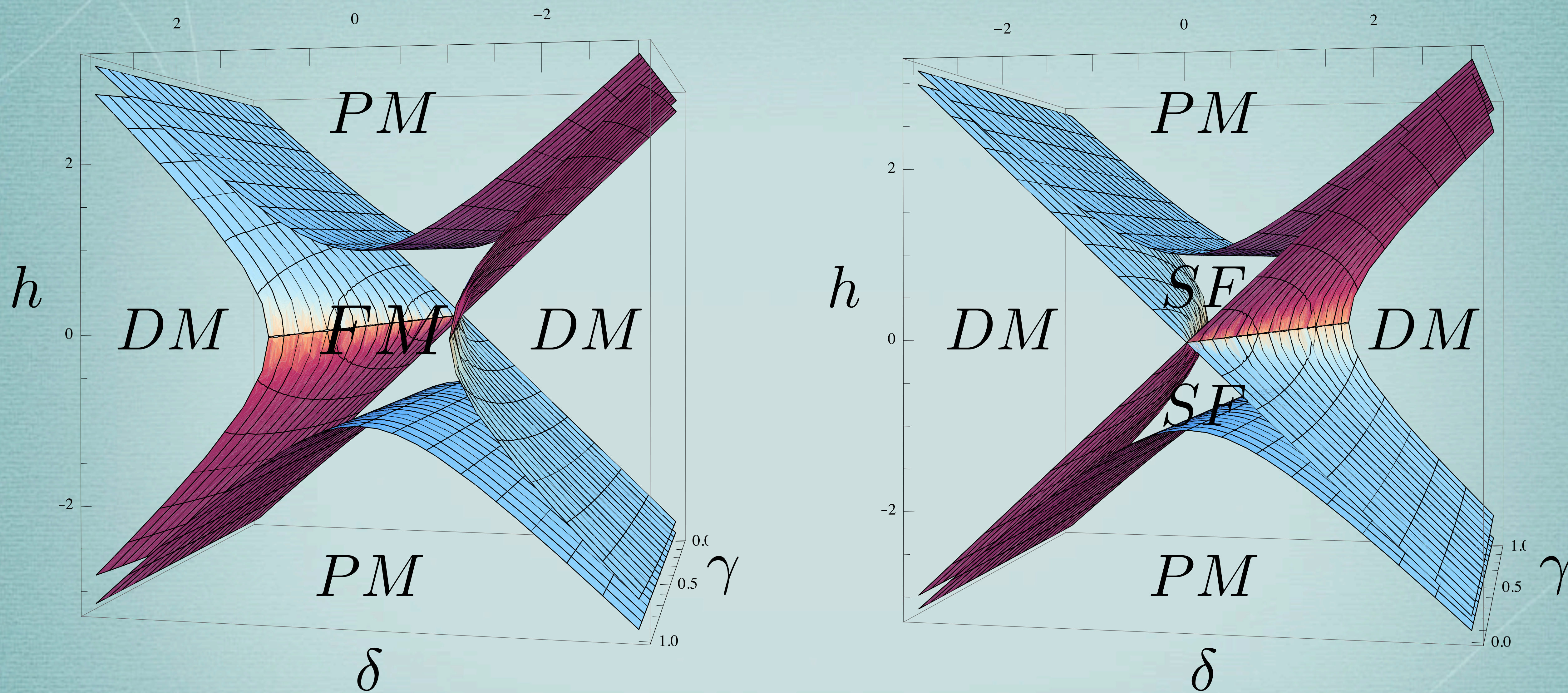
$$|\Psi_k\rangle = \left(u_k^{(1)} + u_k^{(2)} a_k^\dagger a_{-k}^\dagger + u_k^{(3)} b_k^\dagger b_{-k}^\dagger + u_k^{(4)} a_k^\dagger b_{-k}^\dagger + u_k^{(5)} a_{-k}^\dagger b_k^\dagger + u_k^{(6)} a_k^\dagger a_{-k}^\dagger b_k^\dagger b_{-k}^\dagger \right) |\mathrm{vac}\rangle$$

$$\epsilon_{k,1} \leq \epsilon_{k,2} \leq \epsilon_{k,3} \leq \epsilon_{k,4} \qquad \text{with} \quad \epsilon_{k,1}, \epsilon_{k,2} \leq 0 \quad \text{occupied}$$

$$\boxed{\Delta_k = 4\left[h^2 + \delta^2 + \cos^2 k + \gamma^2 \sin^2 k - 2\sqrt{h^2 \cos^2 k + \delta^2 (h^2 + \gamma^2 \sin^2 k)} \right]^{1/2}}$$

$$\Delta_k = \epsilon_{k,3} - \epsilon_{k,2}$$

QUANTUM PHASE DIAGRAM



Phase boundaries:

$$h^2 = \delta^2 + 1 \qquad \delta^2 = h^2 + \gamma^2$$

UNIVERSALITY CLASSES

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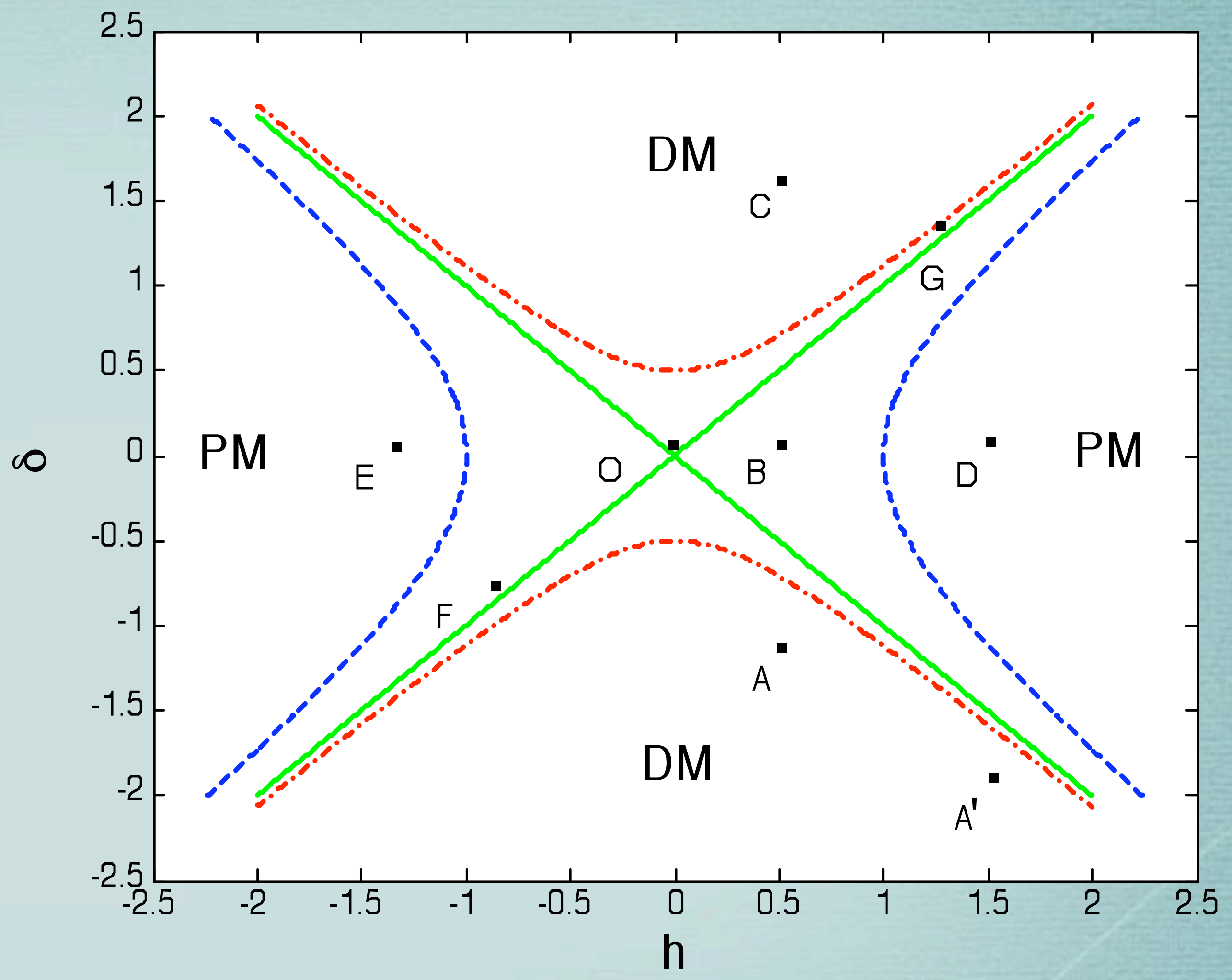
Four universality classes:

$$\gamma \neq 0 \quad \begin{cases} \nu = 1, z = 1 \\ \nu = 2, z = 1 \end{cases}$$

$$\gamma = 0 \quad \begin{cases} \nu = 1/2, z = 2 \\ \nu = 1, z = 2 \\ \nu = 1, z = 1 \ (O) \end{cases}$$

$U(1)$ Symmetry



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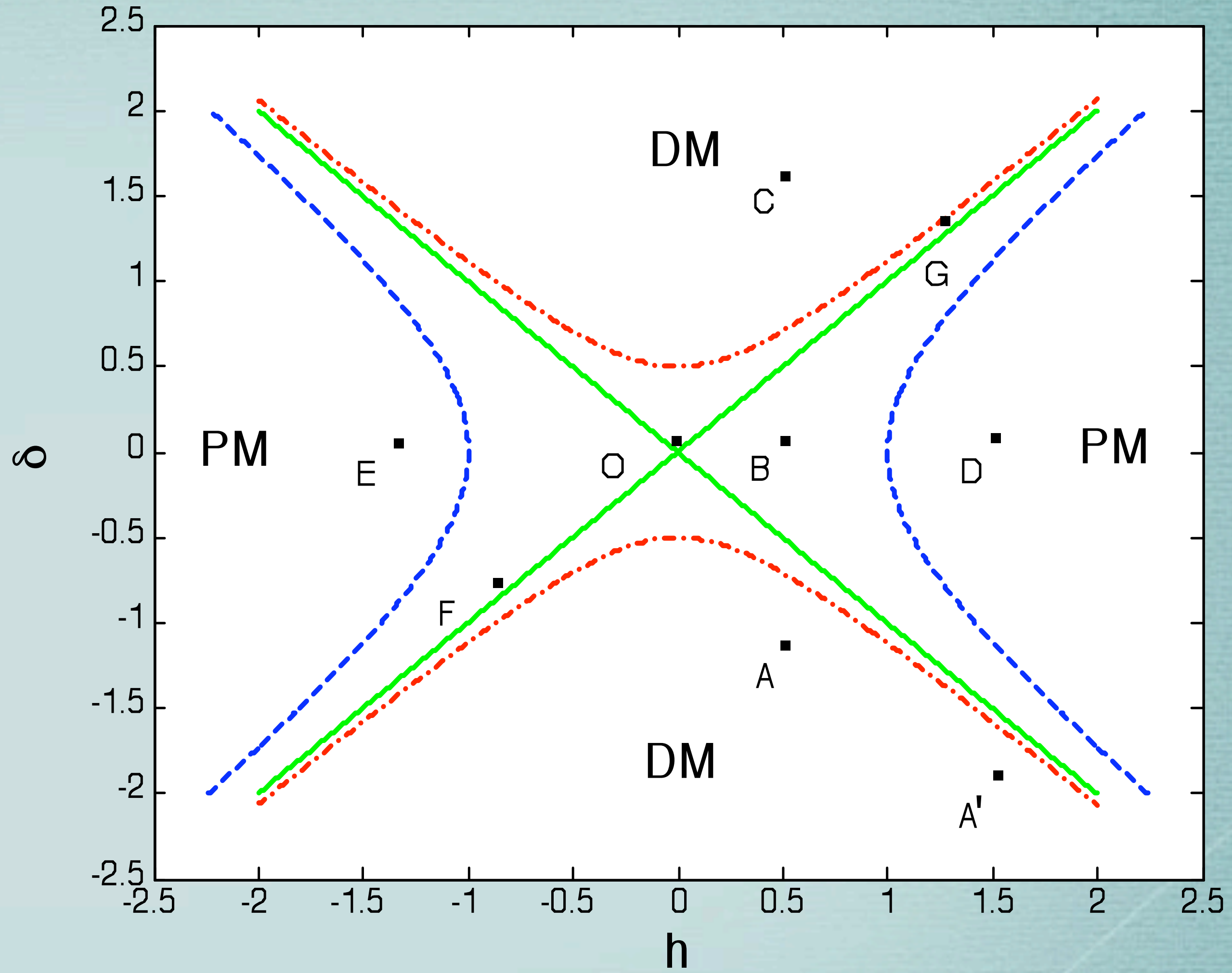

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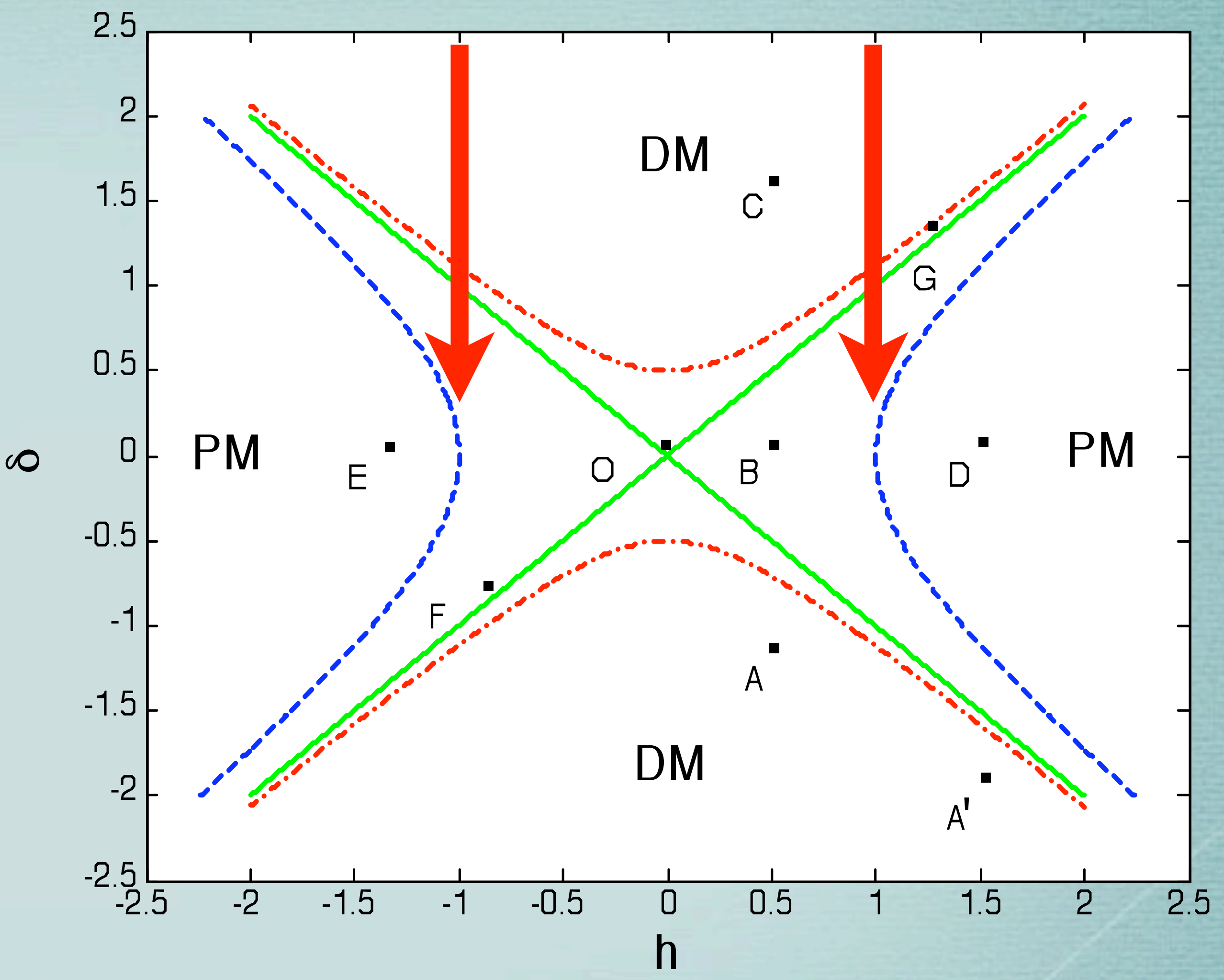

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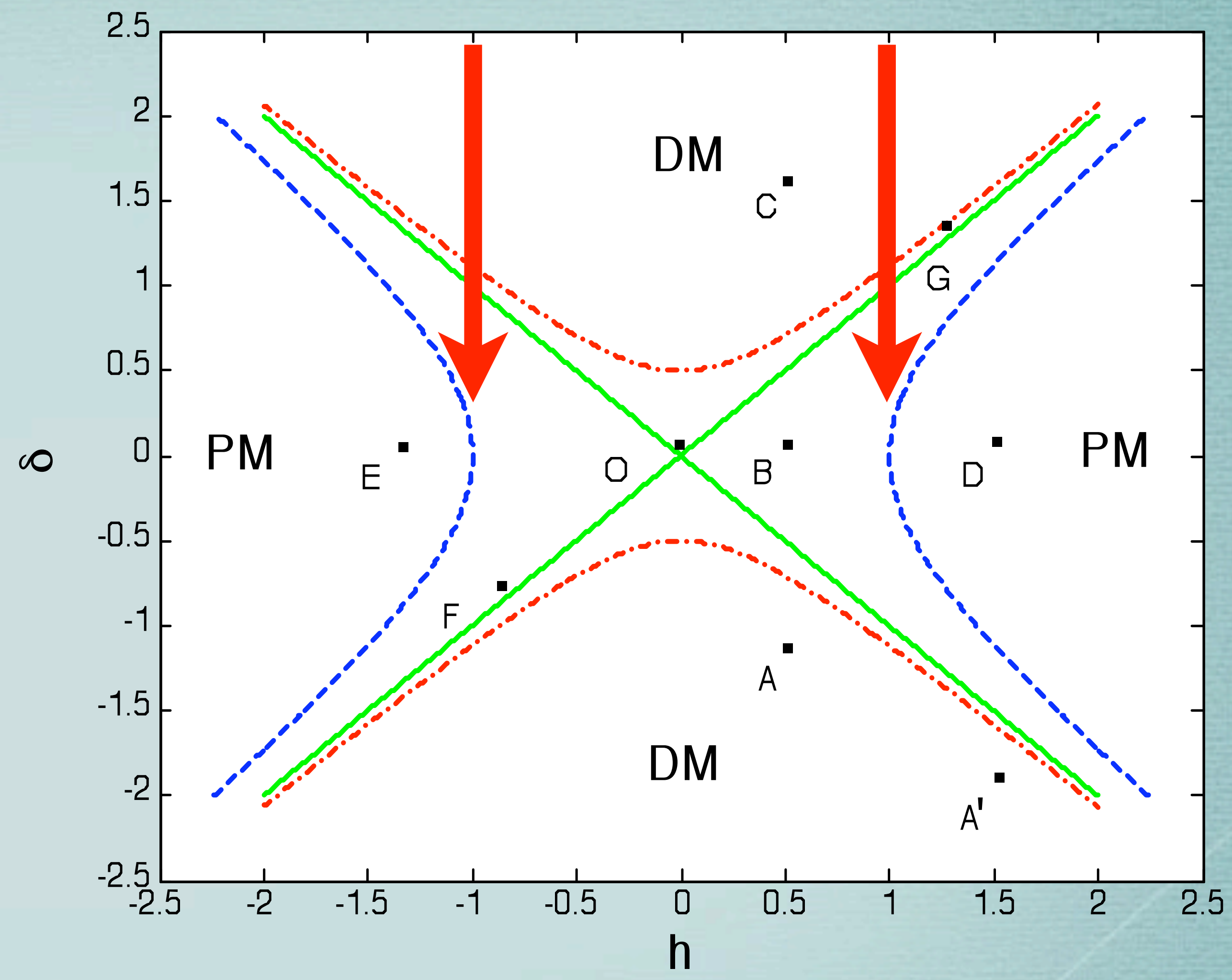

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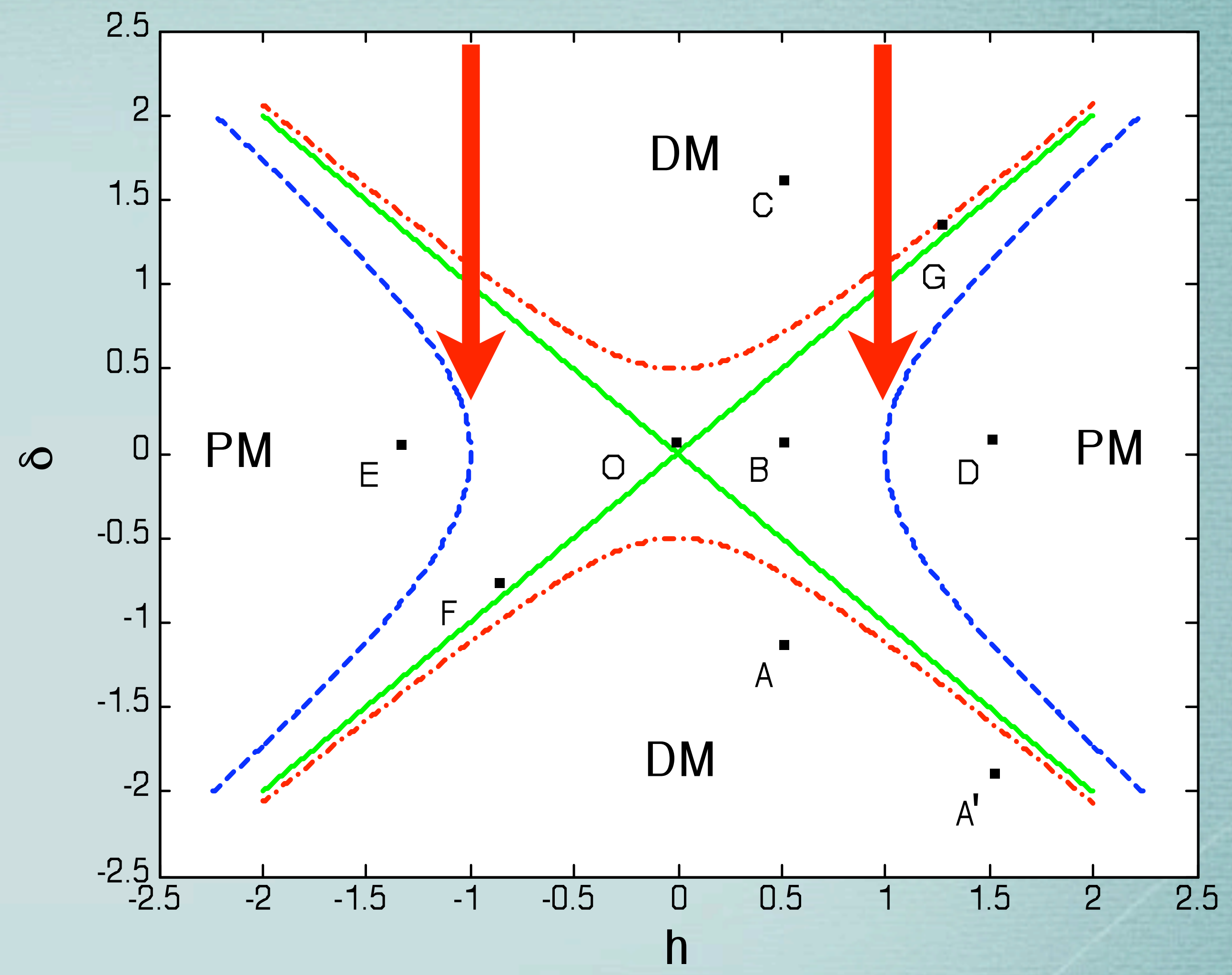

Lifshitz universality class

QUENCH DYNAMICS:ADIABATIC AND SUDDEN QUENCHES

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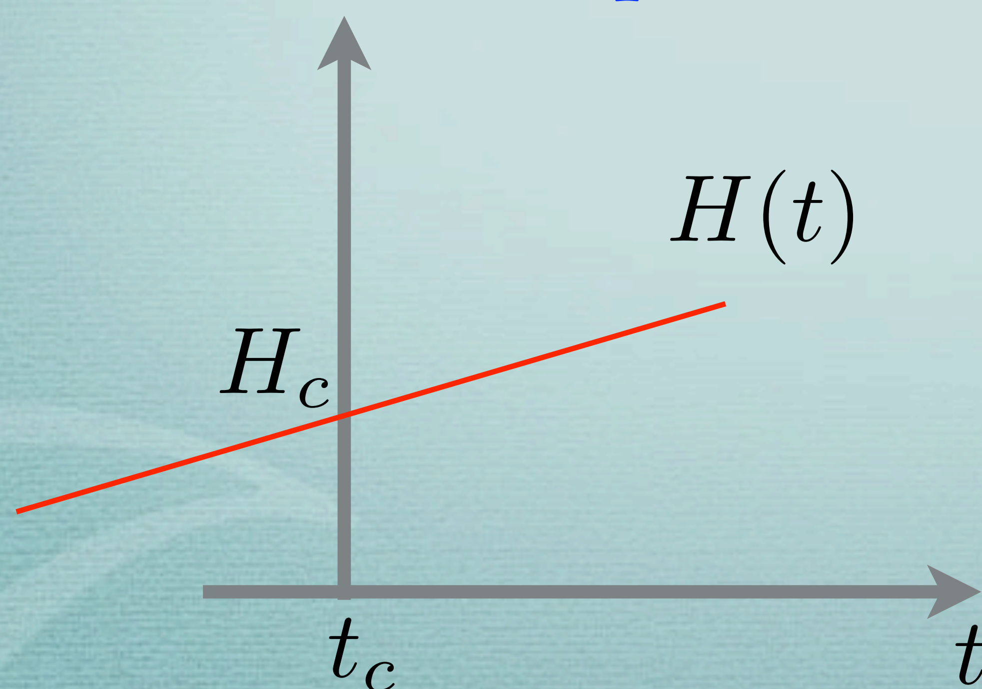
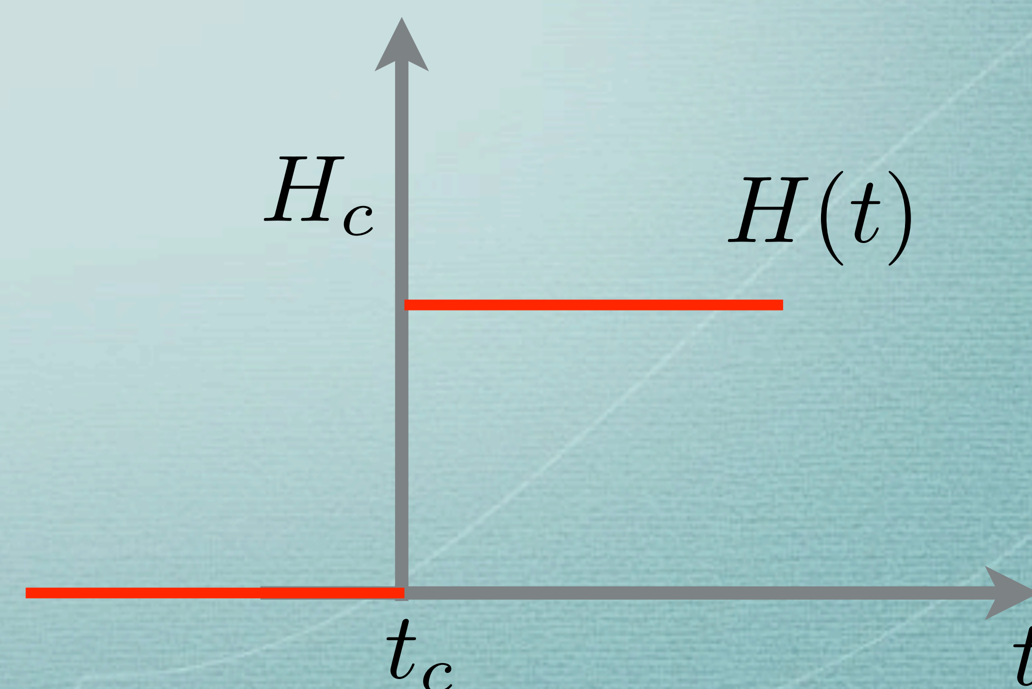
Adiabatic quenches	Sudden quenches
$H(t) = H_c + \left\lvert \frac{t - t_c}{\tau} \right\rvert^{\alpha} \mathrm{sign}(t - t_c) H_1$	$H(t) = H_c + \frac{\alpha}{\sqrt{\tau}} \theta(t - t_c) H_1$
$\alpha = 1 \rightarrow$ Linear quench	





PURE (GROUND AND EXCITED STATES) AND MIXED

$$H(t)|\psi_m(t)\rangle = E_m(t)|\psi_m(t)\rangle$$

snapshot

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$$H(t)|\psi_m(t)\rangle = E_m(t)|\psi_m(t)\rangle$$

snapshot

One may consider different classes of initial states: $t = t_i$

Ground state: $|\psi(t_i)\rangle = |\psi_{GS}(t_i)\rangle = |\psi_0(t_i)\rangle$

Excited (eigen)state: $|\psi(t_i)\rangle = |\psi_m(t_i)\rangle$

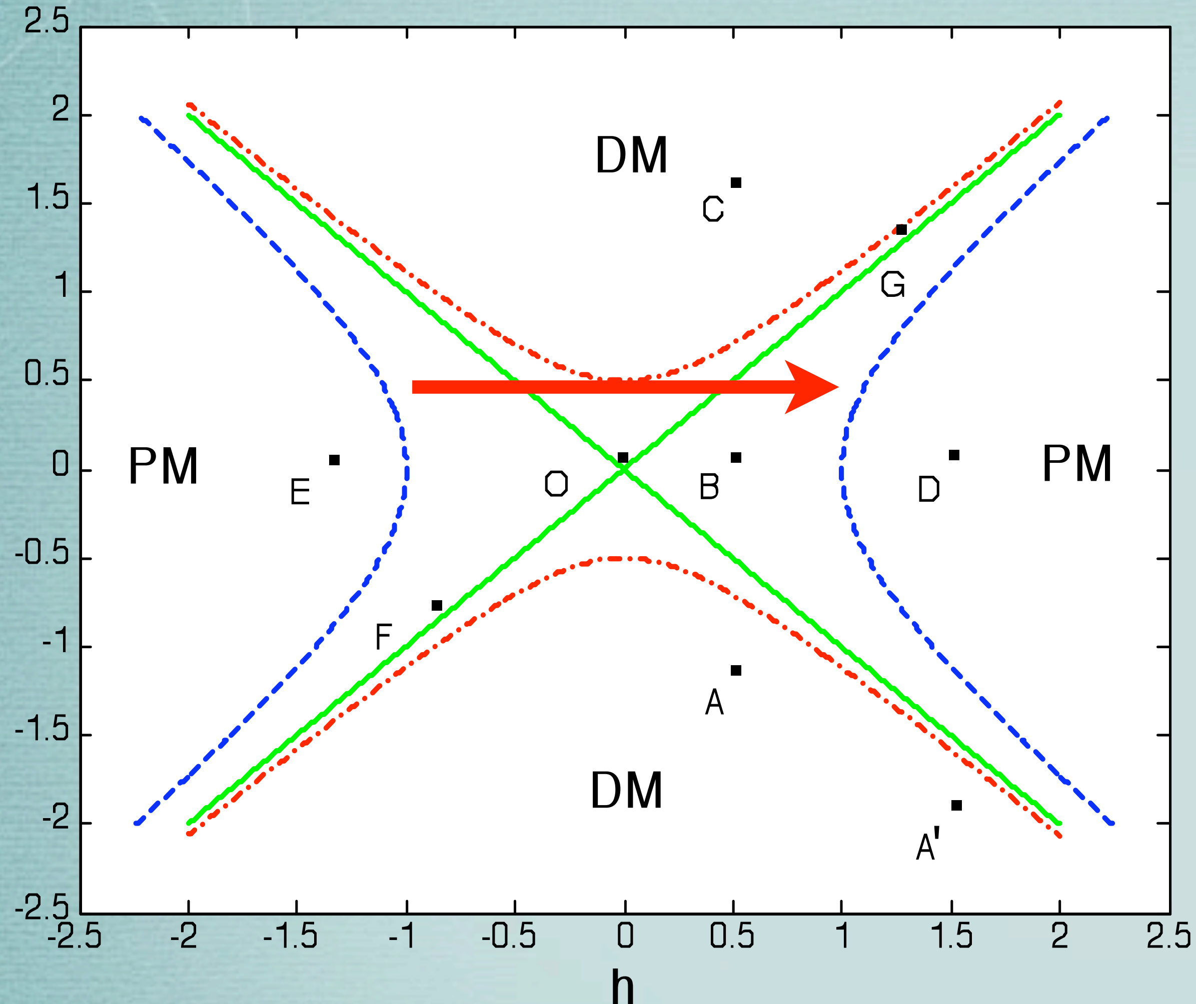
Excited state: $|\psi(t_i)\rangle = \sum_m a_m|\psi_m(t_i)\rangle$

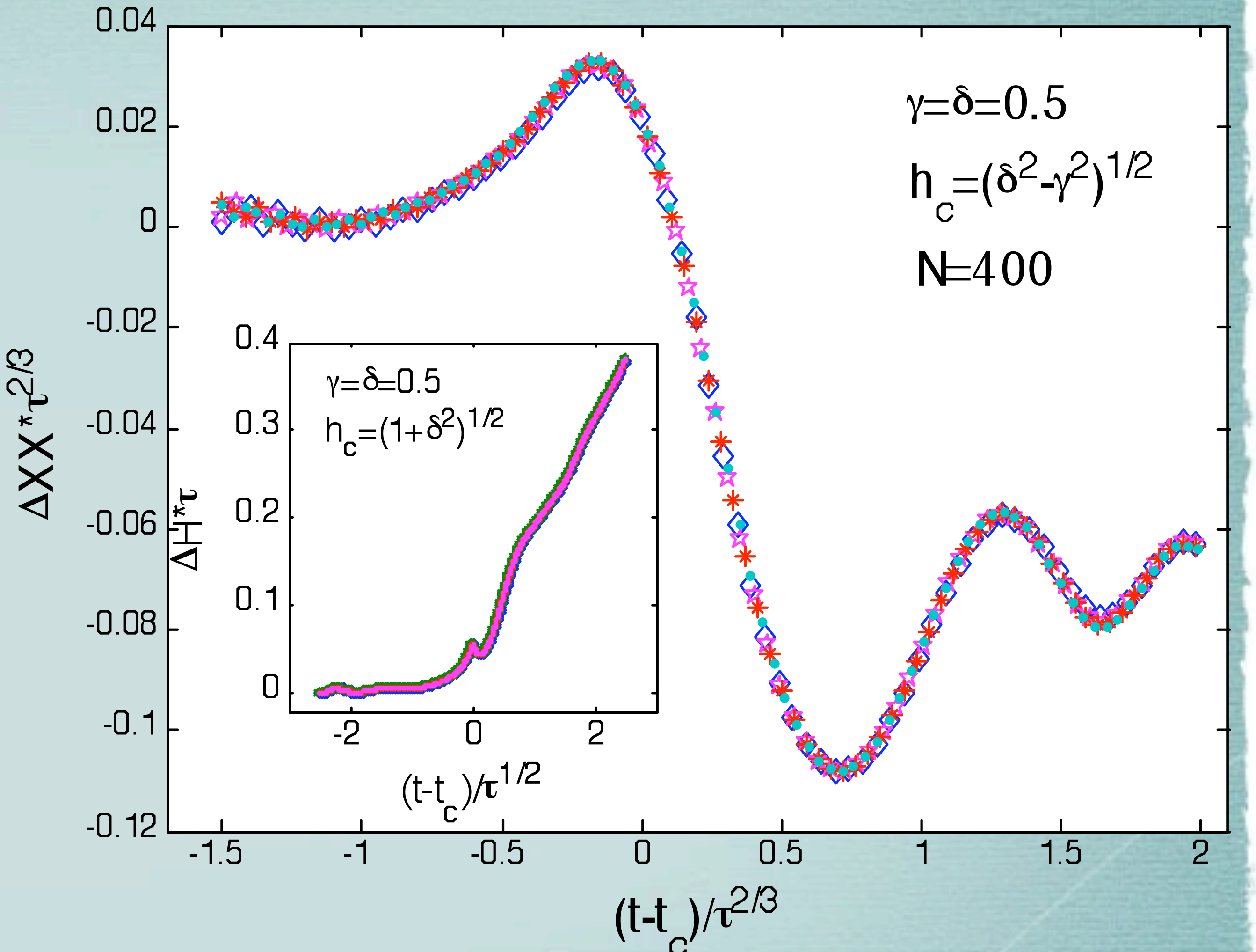
Thermal state: $\rho(t_i) = e^{-\beta H(t_i)}$

ISOLATED NON-MC QCP (ADIABATIC QUENCHES)

SIMPLE ISOLATED QCP

Alternating universality class





4th order QPT

$$\nu = 2, z = 1$$

$$n_{ex}^{\text{Alternating}}(t_f) \sim \tau^{-2/3}$$

SIMPLE ISOLATED QCP

$$n_{ex}(t) = \tau^{-\nu/(\nu z+1)} F_n\left(\frac{t - t_c}{\hat{t}}\right)$$

Scaling function

For a general observable:

$$\Delta\mathcal{O}(t) \equiv \langle\psi(t)|\,\mathcal{O}\,|\psi(t)\rangle - \langle\psi_{GS}(t)|\,\mathcal{O}\,|\psi_{GS}(t)\rangle = \tau^{(-\nu+\beta)/(\nu z+1)} F_{\mathcal{O}}\left(\frac{t - t_c}{\hat{t}}\right)$$

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departure from adiabaticity

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departure from adiabaticity

Examples:

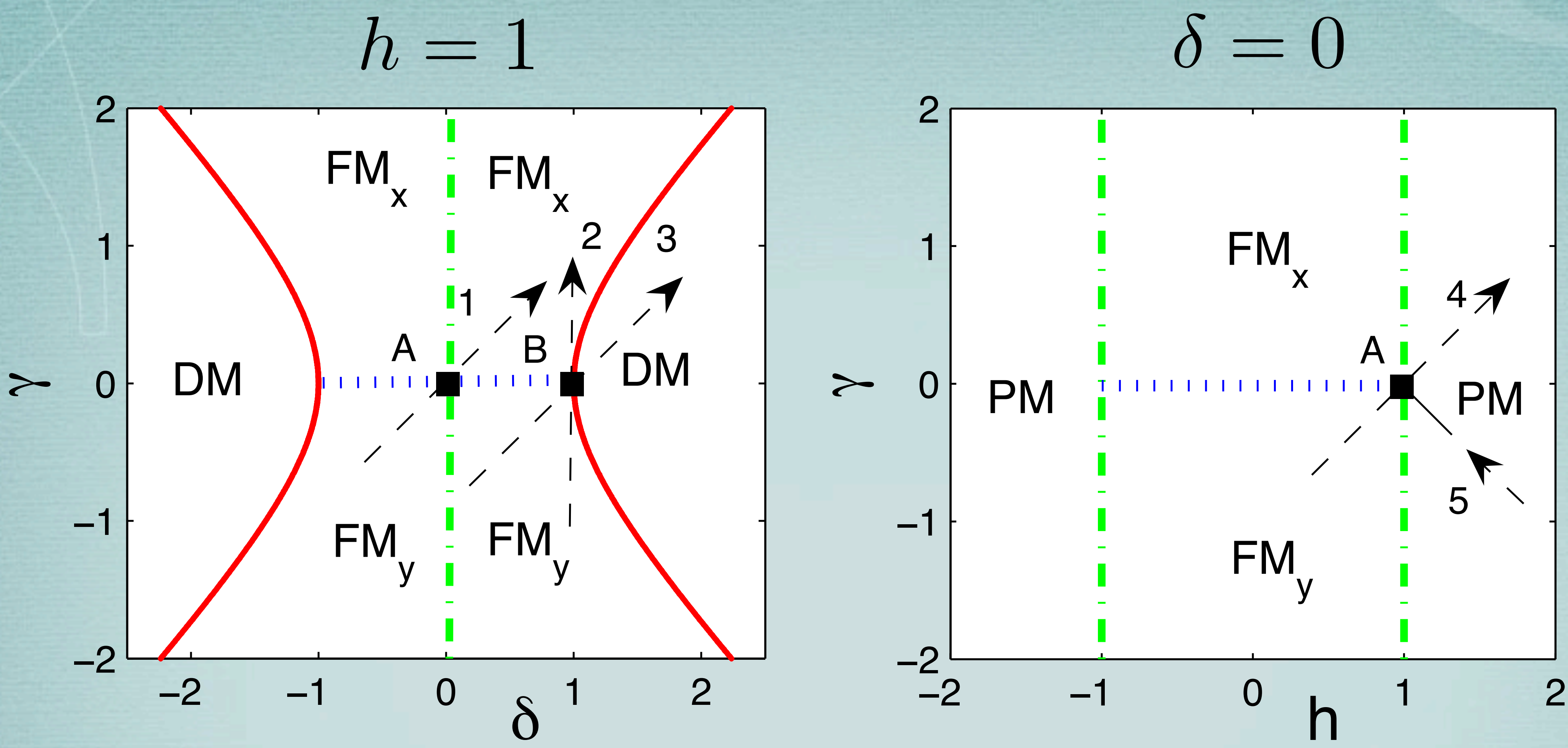
$$M_z = (\sum_{i=1}^{N} \sigma_z^i)/N \rightarrow \Delta M_z(t) = \tau^{(-\nu-\nu z+1)/(\nu z+1)} G((t-t_c)/\hat{t})$$

$$XX = (\sum_{i=1}^{N} \sigma_x^i \sigma_x^{i+1})/N \rightarrow \Delta XX(t) = \tau^{-\nu/(\nu z+1)} W((t-t_c)/\hat{t})$$

Monday, January 3, 2011

ISOLATED MULTICRITICAL QCP

Adiabatic Control Paths



Path	ν	z	Quench Scheme	Dynamical Scaling
1	1	2	$\gamma(t)=\delta(t)=t/\tau, h=1$	$n_{ex} \sim \tau^{-1/3} = \tau^{-\nu/(\nu z+1)}$
2	1	2	$\gamma(t)=t/\tau, h=\delta=1$	$n_{ex} \sim \tau^{-1/3} = \tau^{-\nu/(\nu z+1)}$
3	1/2	2	$\gamma(t)=\delta(t)-1=t/\tau, h=1$	$n_{ex} \sim \tau^{-1/6} \neq \tau^{-\nu/(\nu z+1)}$
4	1/2	2	$\gamma(t)=h(t)-1=t/\tau, \delta=0$	$n_{ex} \sim \tau^{-1/6} \neq \tau^{-\nu/(\nu z+1)}$

Important Observations

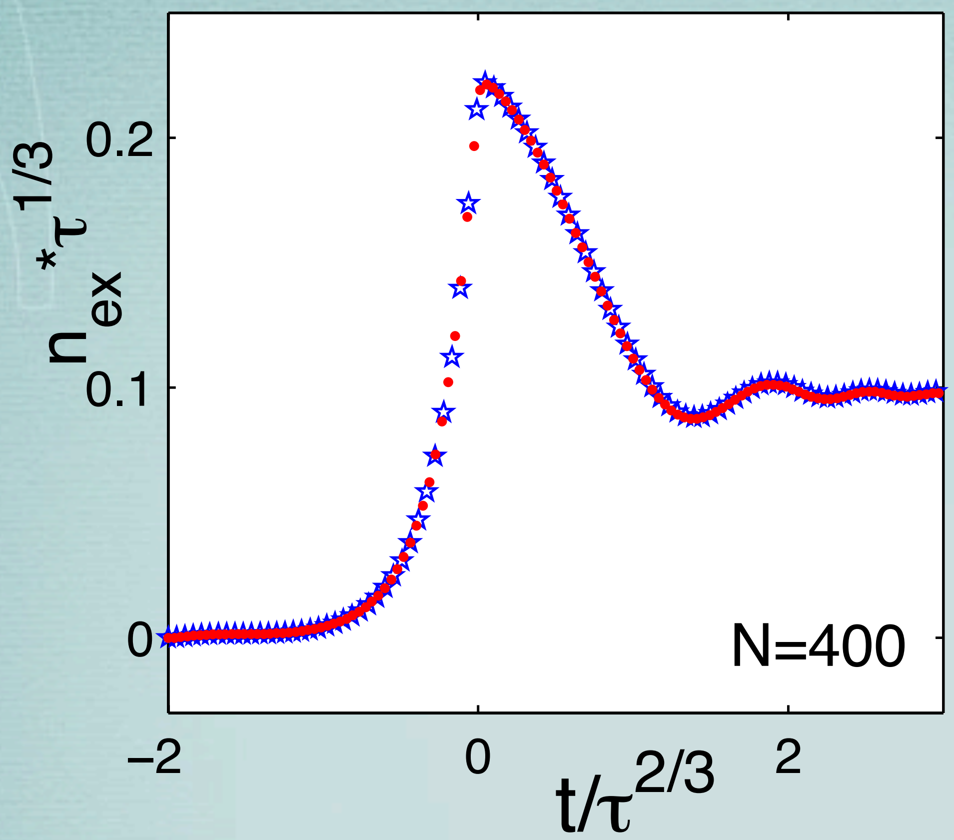
- Paths 1 and 2 start and end in the same phase.
  The excitation spectrum is symmetric under $\lambda \to -\lambda$

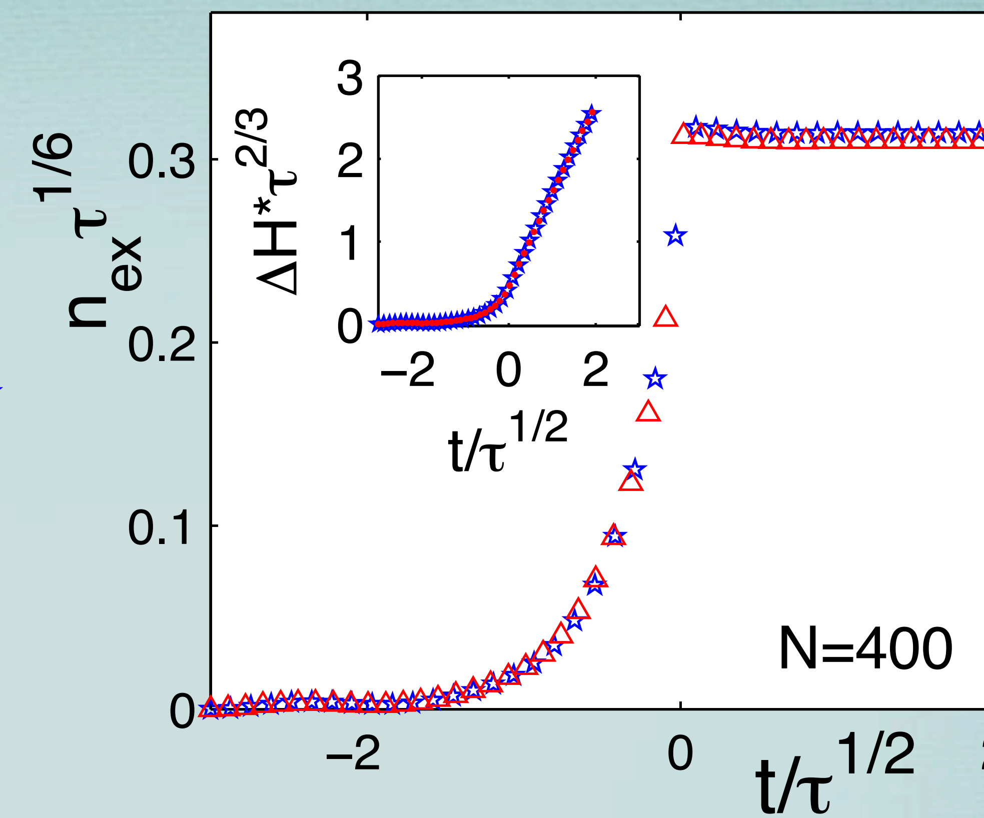
  KZS

- Along Paths 3 and 4 the MCPs A and B belong to the Lifshitz universality class

  non-KZS

Path 2-3





$$n_{ex} \sim \tau^{-1/3}$$

KZS

$$n_{ex} \sim \tau^{-1/6}$$

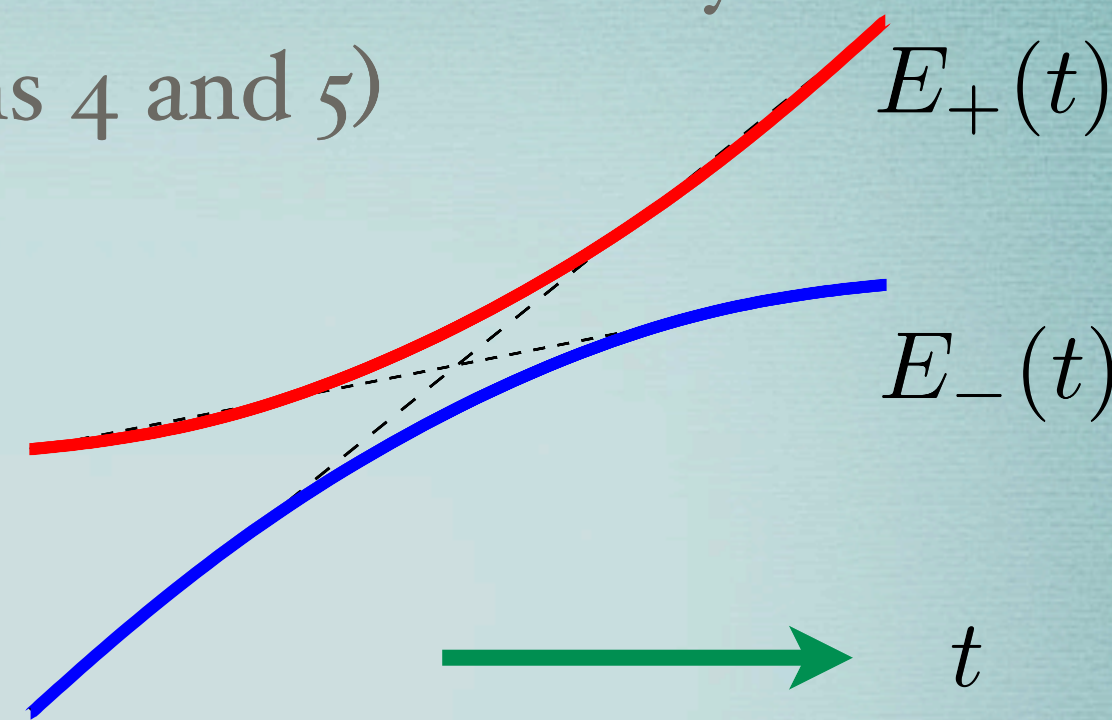
non-KZS

LANDAU-ZENER ANALYSIS

This analysis is sharp but limited to two-level systems

(paths 4 and 5)

$$H(t) = \begin{pmatrix} E + \dot{E}\,t & V \\ V & E \end{pmatrix}$$



Transition probability: $p = e^{-\frac{2\pi V^2}{\hbar|\dot{E}|}}$

Adiabaticity condition: $\dfrac{2V^2}{\hbar|\dot{E}|} \gg 1$

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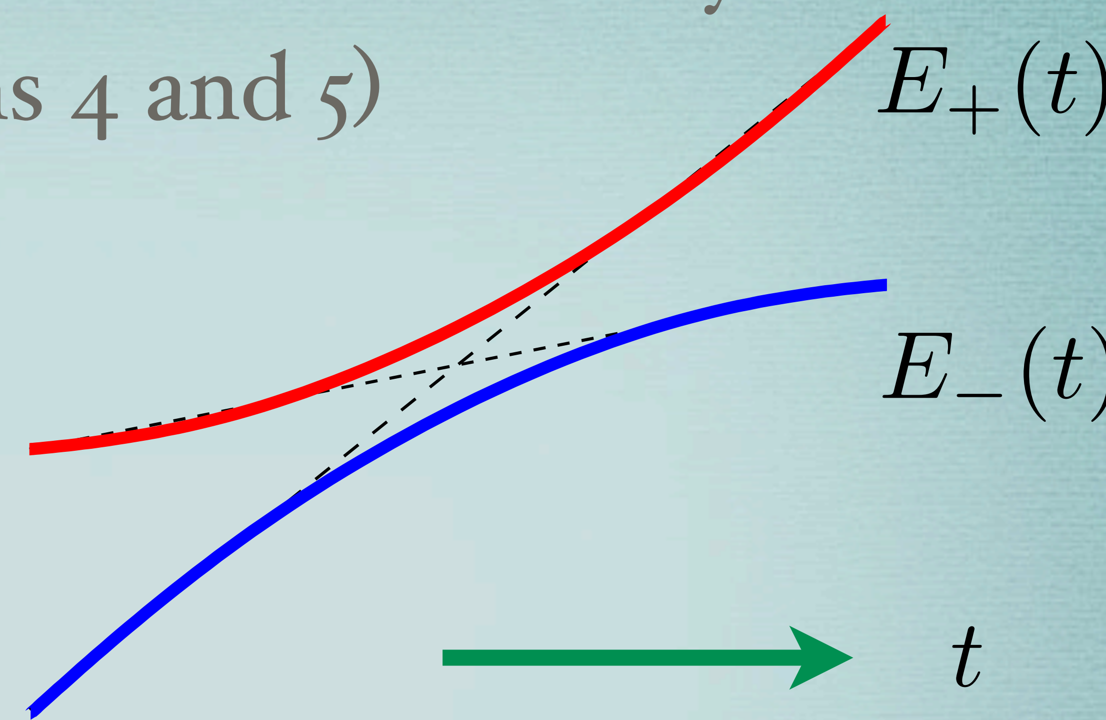

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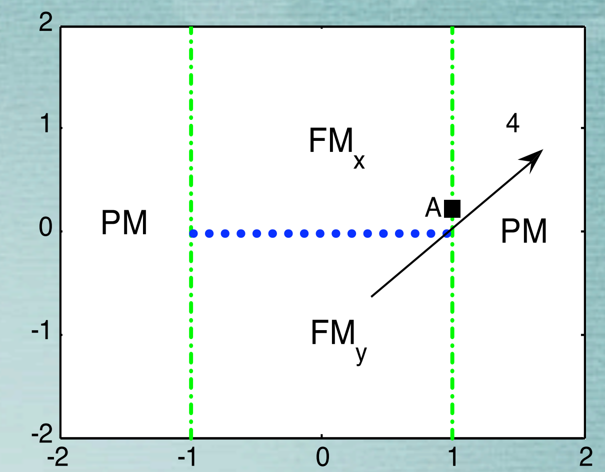
Infinite time expression (asymptotic $t_f \to \infty$)

Landau-Zener Analysis of Path 4

Path 4 $\quad \delta = 0, \quad \gamma(t) = t/\tau, \quad h(t) = 1 + \gamma(t)$

$$H = \sum_k A_k^\dagger H_k A_k, \quad \text{where } A_k = (c_{-k}, c_k^\dagger)$$

$$H_k = \begin{pmatrix} H_{k,11} & H_{k,12} \\ H_{k,12}^* & -H_{k,11} \end{pmatrix} = 2\begin{pmatrix} -h + \cos k & \gamma \sin k \\ \gamma \sin k & h - \cos k \end{pmatrix}$$



$$\xrightarrow{U_{q_k} H_k U_{q_k}^\dagger}$$

$$H'_{k,11} = -2(1 - \cos k)\cos 2q_k - 2t/\tau(\cos 2q_k - \sin k \sin 2q_k)$$
$$H'_{k,12} = 2(1 - \cos k)\sin 2q_k$$

where $\tan 2q_k = -\sin k$

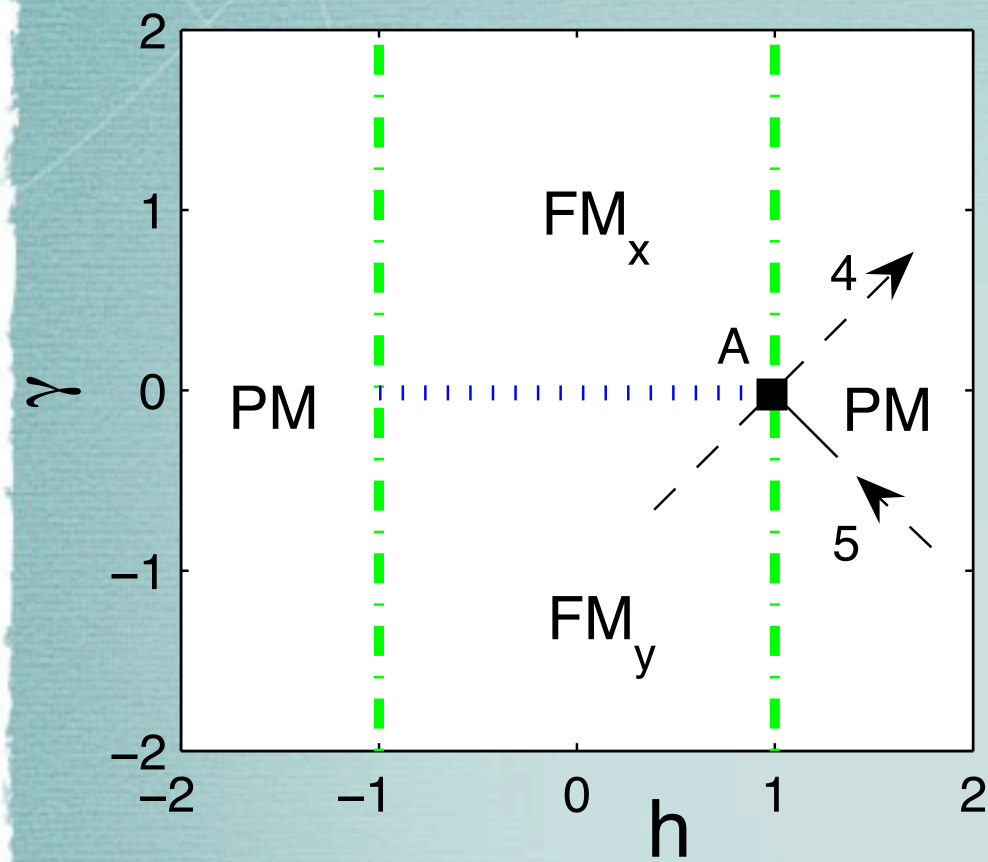
LZ formula: C. Zener, Proc. R. Soc. London, Ser. A 137, 696 (1932)

$$P_k = e^{-2\pi\tau(1-\cos k)^2 \sin^2 2q_k/(\cos 2q_k - \sin k \sin 2q_k)} \approx e^{-\pi\tau k^6/2} \qquad k_c = 0$$

$$n_{ex} = \frac{1}{\pi}\int_0^\pi P_k dk \approx \frac{1}{\pi}\int_0^\pi e^{-\pi\tau k^6/2} dk \sim \tau^{-1/6} \qquad k^6 = k^{2z_2}, z_2 = 3$$

Path 5

$\delta = 0$



$$n_{ex} \sim \tau^{-3/4}$$

non-KZS

Path	ν	z	Quench Scheme	Dynamical Scaling
5	1/2	2	$h(t) = 1 + \lvert\gamma(t)\rvert = 1 + \lvert t/\tau \rvert, \delta = 0$	$n_{ex} \sim \tau^{-3/4} \neq \tau^{-\nu/(\nu z+1)}, \tau^{-1/6}$

Path 5

$$n_{ex} \sim \tau^{-3/4}$$

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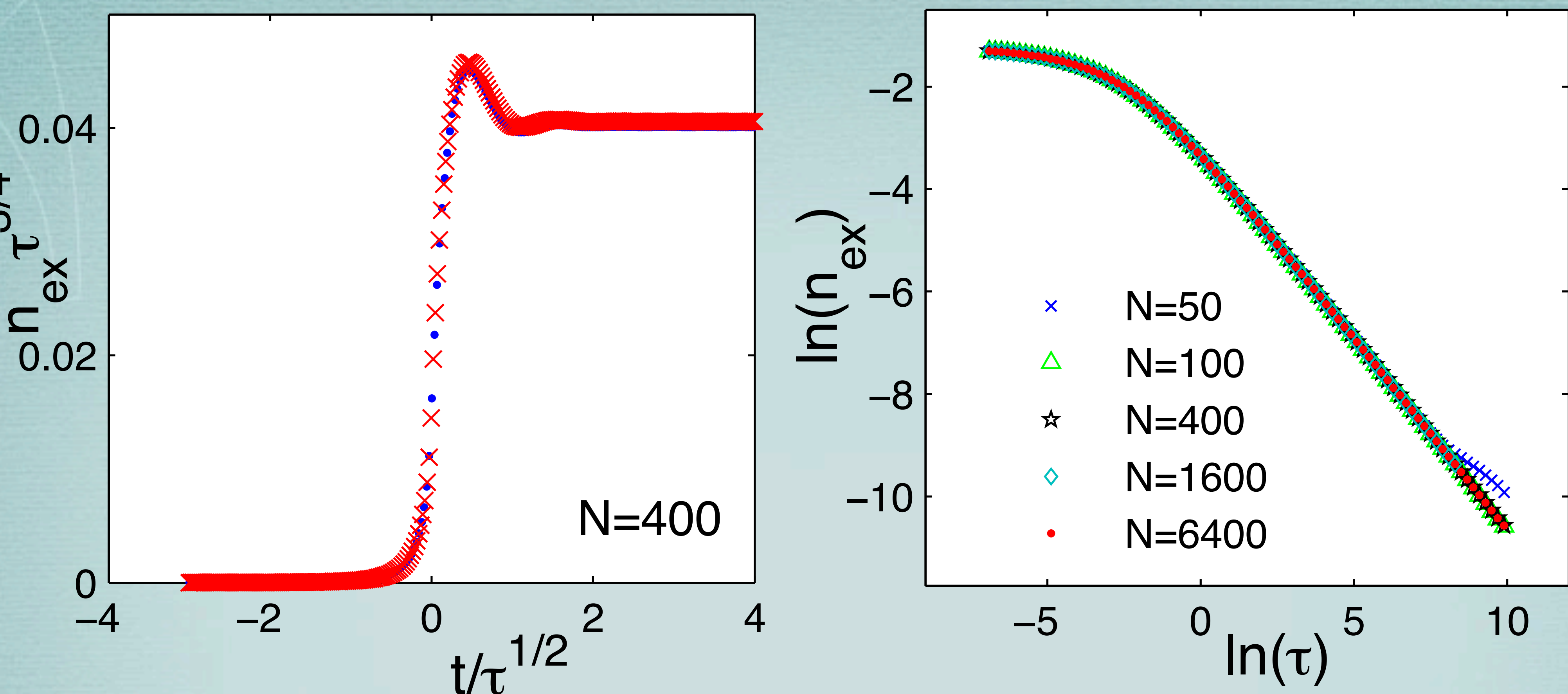
Path 5



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Path	ν	z	Quench Scheme	Dynamical Scaling
5	1/2	2	$h(t) = 1 + \lvert\gamma(t)\rvert = 1 + \lvert t/\tau \rvert, \delta = 0$	$n_{ex} \sim \tau^{-3/4} \neq \tau^{-\nu/(\nu z+1)}, \tau^{-1/6}$

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Finite Time Landau-Zener Analysis of Half-Path 5

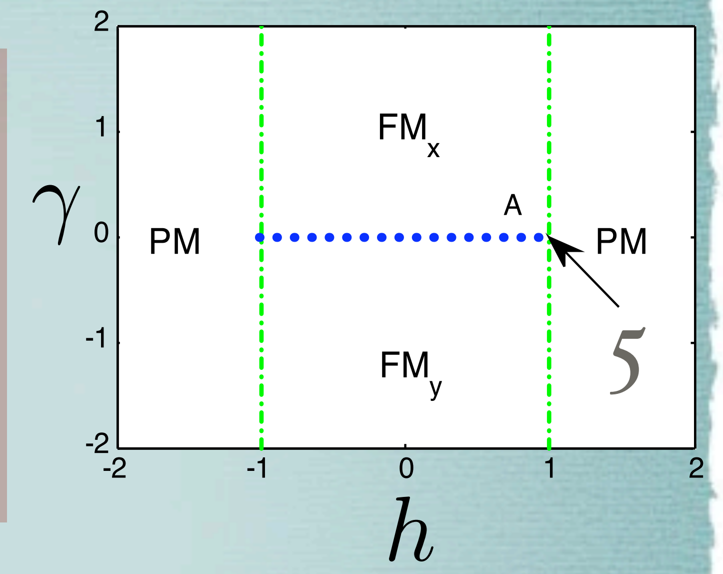
$$P_k(t_f) = e^{-\pi\omega^2/4}\left| D_{i\omega^2/2}(T_f\sqrt{2}e^{i3\pi/4})\cos\theta(T_f) - \frac{\omega}{\sqrt{2}} e^{-i\pi/4} D_{i\omega^2/2-1}(T_f\sqrt{2}e^{i3\pi/4})\sin\theta(T_f)\right|^2$$

--- N. V. Vitanov (1999)

$$\omega = (1-\cos k)\sin 2q_k\sqrt{\tau}\,/\sqrt{\cos 2q_k + \sin 2q_k \sin k} \sim k^3\sqrt{\tau}$$

$$T_f = -\omega/\sin k \sim -k^2\sqrt{\tau},$$

$$\theta(T_f) = 1/2\arctan(\omega/T_f) + \pi/2$$



Taylor expanding parabolic cylinder function around $T_f = 0$

$(\omega \ll |T_f| \ll 1)$

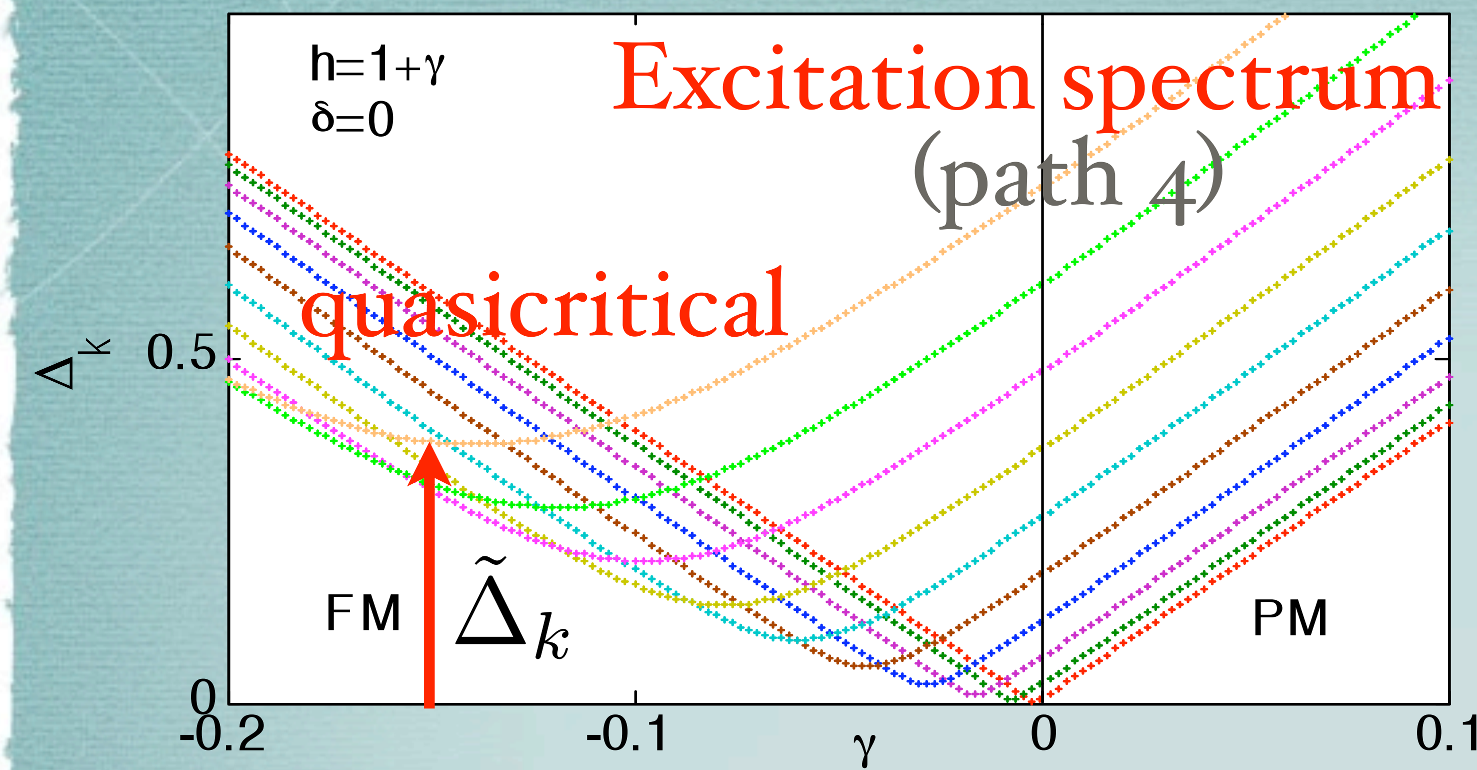
$$P_k(t_f) \approx (1-e^{-\pi\omega^2/2})/2 + \cos^2\theta(T_f)e^{-\pi\omega^2/2} - \sin 2\theta(T_f)/2\sin\chi_k\sqrt{(1-e^{-\pi\omega^2})}$$

$$\sim \cos^2\theta(T_f)e^{-\pi\omega^2/2} \sim k^2 \qquad \rightarrow (k-k_c)^{d_2} \qquad d_2 = 2$$

$$n_{ex}(t_f) = \int_0^{k_{\max}} \mathrm{P}_k(t_f)dk \sim \int_0^{\tau^{-1/4}} k^2 dk \sim \tau^{-3/4}$$

k_c is not excited

Physical Understanding



Minimum gap along the path:

$$\partial \Delta_k(\gamma, 1+\gamma, 0)/\partial\gamma = 0$$

$$\tilde{\gamma} = -(1-\cos k)/(1+\sin^2 k)$$

$$\Delta_k(\tilde{\gamma}) \equiv \tilde{\Delta}_k \sim (k-k_c)^3$$

Standard KZS (1 and 2):

energy scale $\hat{\Delta} \sim \tau^{-\nu z/(\nu z+1)}$

at QCP $\Delta_k \sim k^z$

determine $k_{\max}$ $\hat{\Delta} \sim k_{\max}^z$

$$n_{ex} \sim \int_0^{k_{\max}} d^d k \sim \tau^{-d\nu/(\nu z+1)}$$

Non-KZS (3 and 4):

energy scale $\hat{\Delta} \sim \tau^{-\nu z/(\nu z+1)}$

along the path $\tilde{\Delta}_k \sim \mathrm{k}^{z_2}$

determine $k_{\max}$ $\hat{\Delta} \sim k_{\max}^{z_2}$

$$n_{ex} \sim \int_0^{k_{\max}} d^d k \sim \tau^{-\nu z/[(\nu z+1)z_2]}$$

More Generally:

energy scale $\hat{\Delta} \sim \tau^{-\nu z/(\nu z+1)}$

along the path $\tilde{\Delta}_k \sim \mathrm{k}^{z_2}$

determine $k_{\max}$ $\hat{\Delta} \sim k_{\max}^{z_2}$

$$n_{ex} \sim \int_0^{k_{\max}} k^{d_2} d^d k \sim \tau^{-(d+d_2)\nu z/[(\nu z+1)z_2]}$$

Typical gap: $\hat{\Delta}$

$$n_{ex} = \int_0^{k_{max}} P_k(t_f) d^d k$$

WHAT HAVE WE LEARNED?

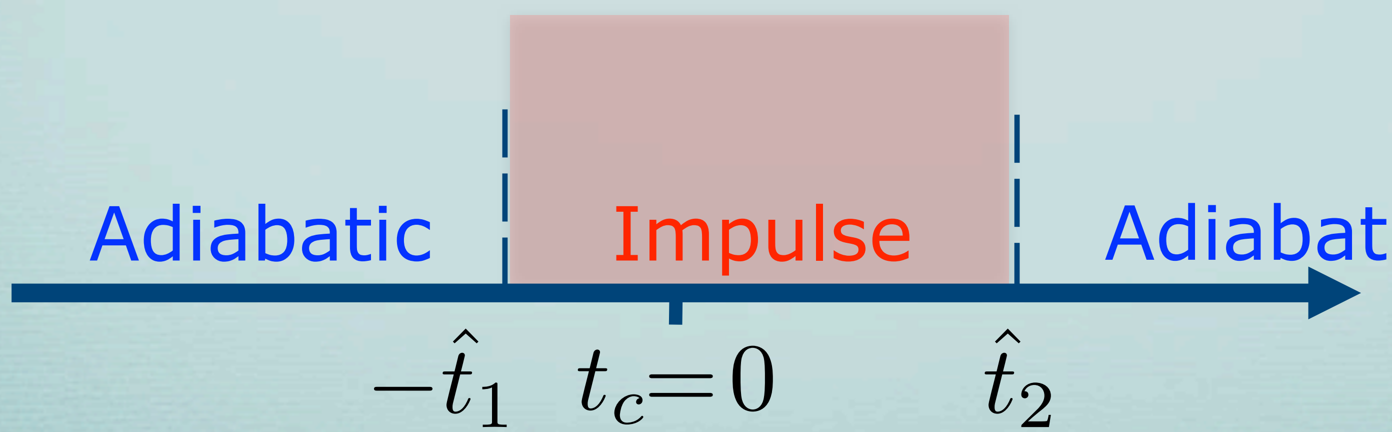
- Only relevant modes matter in a dynamical process

  Non-critical energy modes may dominate scaling

  Anomalous (non-static) critical exponents may emerge

- Path-dependent minimum gap determines z_2

  Effective dimensionality exponent $d_2 \neq 0$



Dynamical asymmetry

(shift of the center of impulse region)

SEARCH FOR UNDERSTANDING

Is it possible to develop some general framework for dynamical critical scaling?

ADIABATIC RENORMALIZATION

$$H(t) = H_c + [\lambda(t) - \lambda_c]H_1 = H_c + (t - t_c)/\tau H_1$$

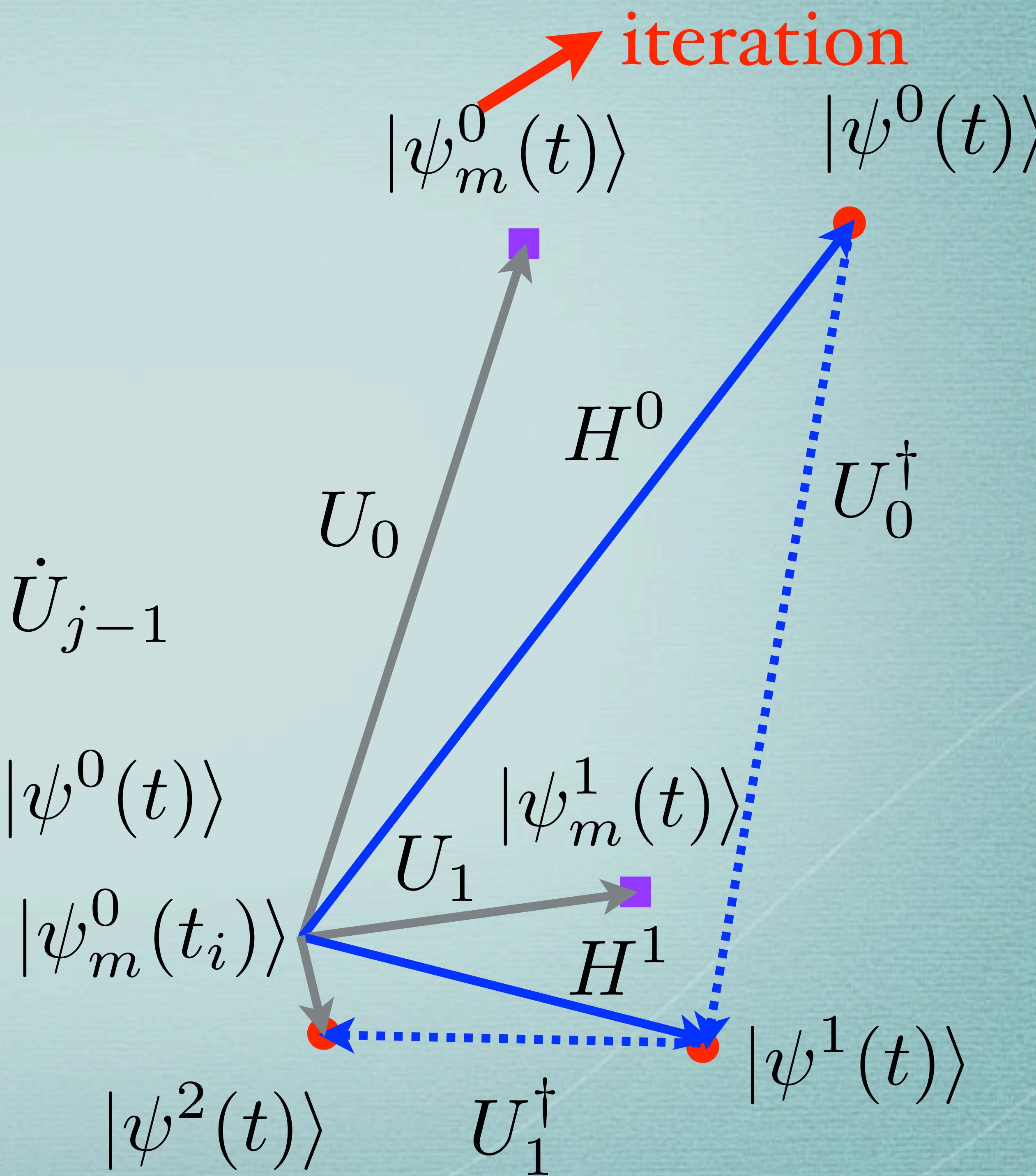
$$H(t)|\psi_m(t)\rangle = E_m(t)|\psi_m(t)\rangle$$

$$|\psi^j_m(t)\rangle = U_j(t)|\psi^0_m(t_i)\rangle$$

$$i\hbar\partial_t|\psi^j(t)\rangle = H^j(t)|\psi^j(t)\rangle$$

$$H^j(t) = U^\dagger_{j-1}H^jU_{j-1} - i\hbar U^\dagger_{j-1}\dot{U}_{j-1}$$

$$|\psi^j(t)\rangle = U^\dagger_{j-1}(t)\cdots U^\dagger_1(t)U^\dagger_0(t)|\psi^0(t)\rangle$$



Hope is: time-dependence disappears

ADIABATIC RENORMALIZATION

$$H(t) = H_c + [\lambda(t) - \lambda_c]H_1 = H_c + (t - t_c)/\tau H_1$$

$$H(t)|\psi_m(t)\rangle = E_m(t)|\psi_m(t)\rangle$$

Non-adiabatic correction:

$$|\psi(t)\rangle = c_0(t)|\psi_0(t)\rangle + \sum_{m\neq 0} c_m(t)|\psi_m(t)\rangle$$

$$c_0^{(1)}(t) = e^{-i\Gamma_0(t)}$$

$$c_m^{(1)}(t) = e^{-i\Gamma_m(t)} \int_{t_{\rm in}}^{t} dt' \dot{\lambda}(t') \frac{\langle\psi_m(t')|H_1|\psi_0(t')\rangle}{E_m(t') - E_0(t')} e^{i\int_{t_{\rm in}}^{t'} ds \Delta_m(s)}$$

$$\Delta_m(t) = E_m(t) - E_0(t)$$

--- A. Polkovnikov, PRB, 72 (2005) 161201

Main scaling assumptions:

$$E_m(t) - E_0(t) = \delta\lambda(t)^{\nu z} f_m(\Delta_m(t_{min})/\delta\lambda(t)^{\nu z})$$

$$\langle\psi_m(t)|H_1|\psi_0(t)\rangle = \delta\lambda(t)^{\nu z-1} g_m(\Delta_m(t_{min})^{1+\frac{d_2}{2z_2}}/\delta\lambda(t)^{\nu z})$$

$\Delta_m(t_{min})$: minimum gap of mode m at t_{min}

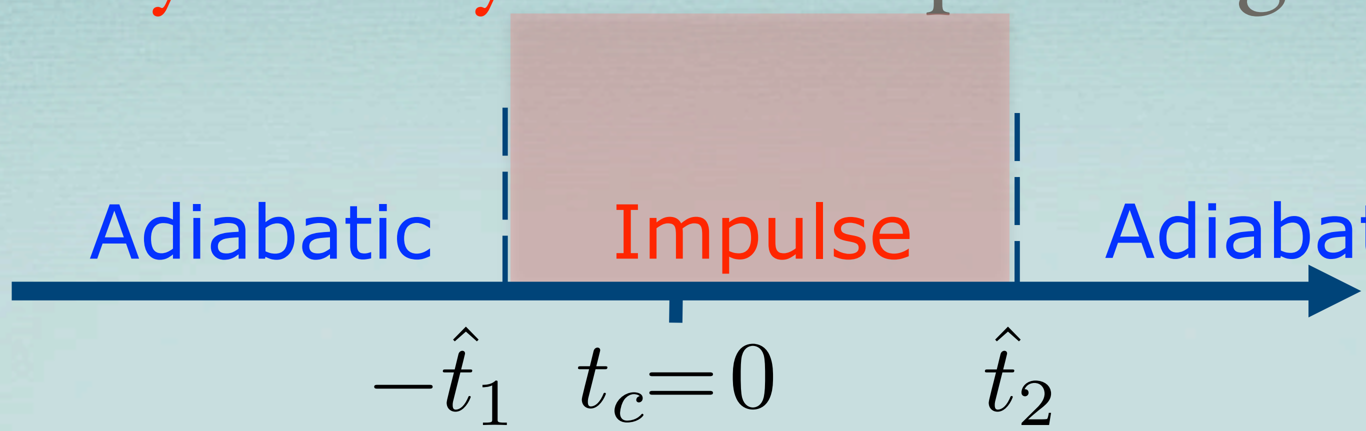
$$\rho(E) \sim E^{d/z_2 - 1}$$

$$f_m(x), g_m(x)(x \to 0) \to \text{constant}$$

$$f_m(x), g_m(x)(x \to \infty) \to x$$

CONCLUSIONS

- Dynamical Asymmetry of the Impulse region (non-KZ)



Relevant modes versus critical mode: minimum gap

Dynamical critical exponents not determined from equilibrium

● Knowledge about the path-dependent excitation process may be crucial and non-equilibrium exponents cannot be fully predicted from equilibrium ones

- Quench across critical regions:

--- Dominant critical point

--- Cancellation mechanism

- For quenches involving isolated QCPs,non-ergodic dynamical scaling is fully captured by first-order adiabatic renormalization with appropriate scaling assumptions

- Role of Initial State

What happens away from integrability?