ENTANGLEMENT IN SUPERFLUID ^{4}He

Understanding how quantum information is encoded in quantum matter

C. Herdman
UVM → IQC → Middlebury

P.-N. Roy
U Waterloo

R.G. Melko
U Waterloo and PI

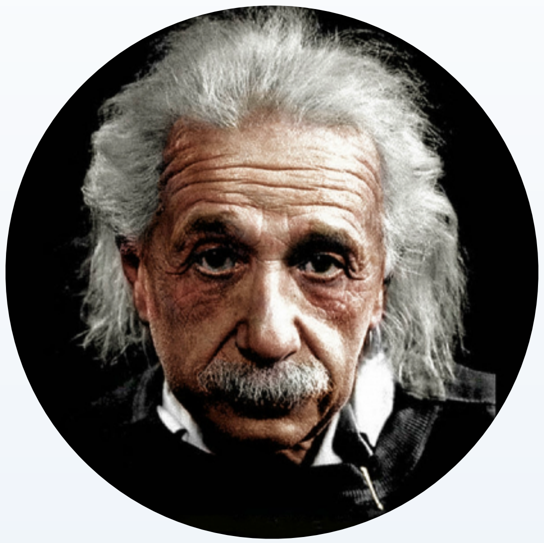
C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys. **13**, 556 (2017)

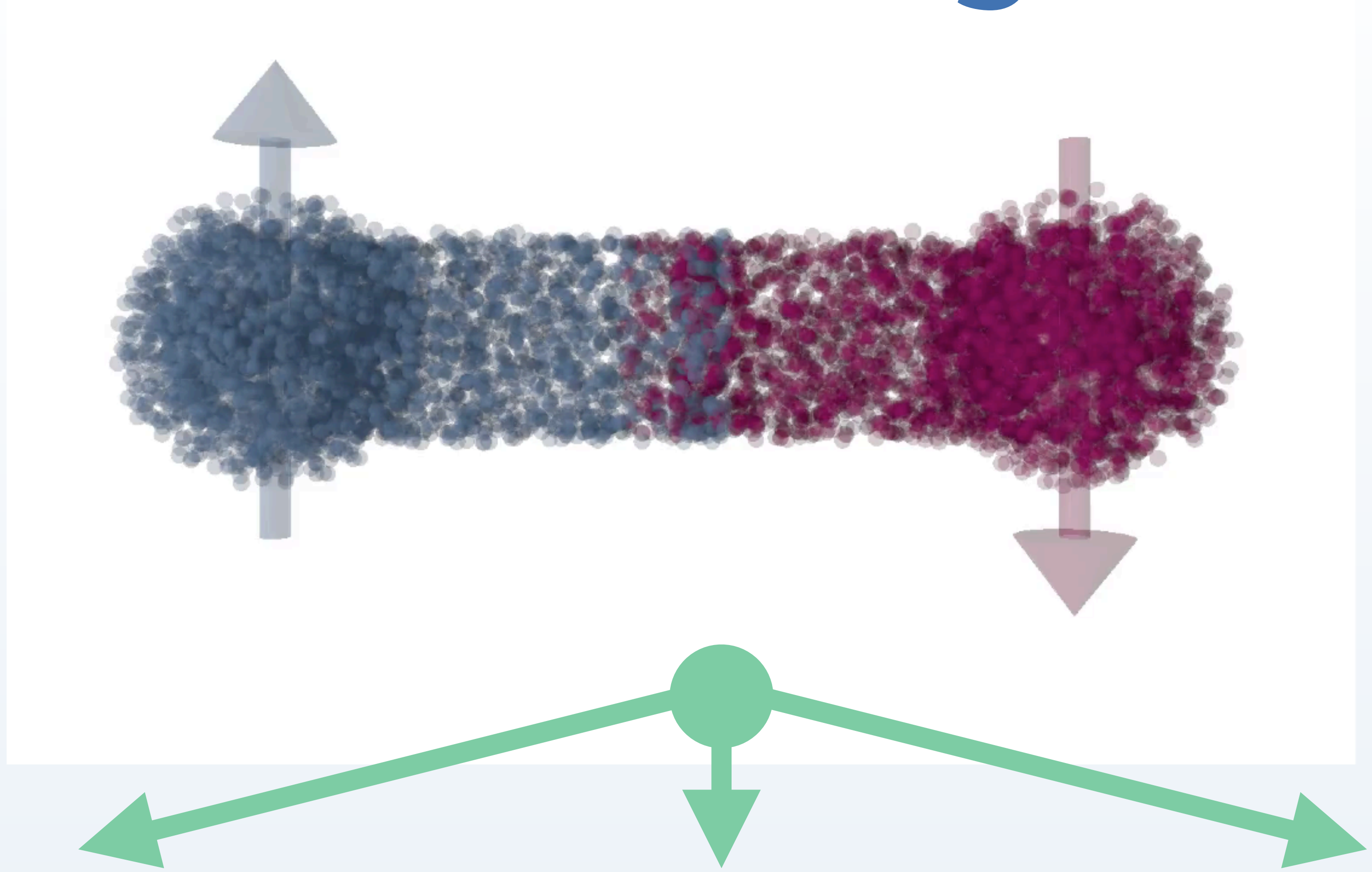
UNIVERSITÄT LEIPZIG

NSF

Adrian Del Maestro
University of Vermont

Quantum Entanglement

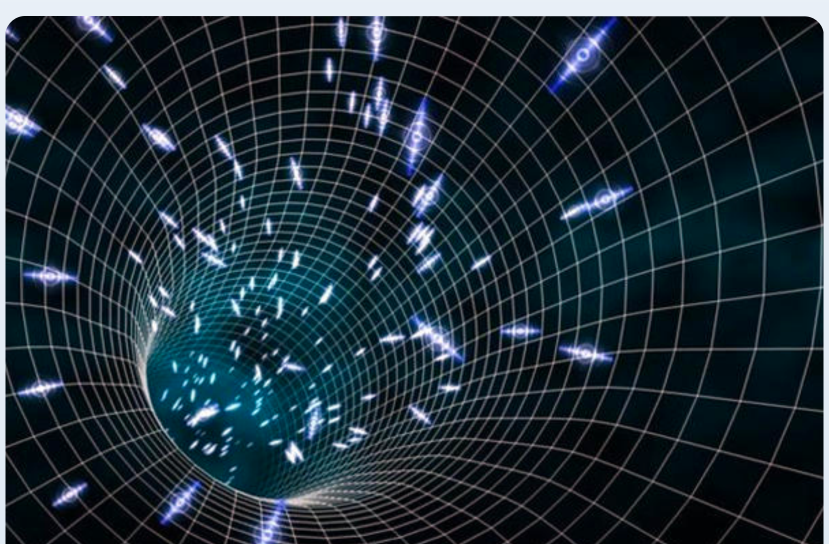


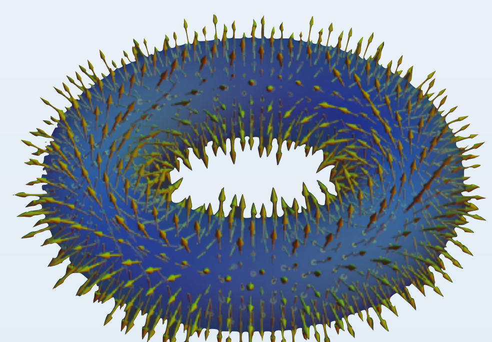


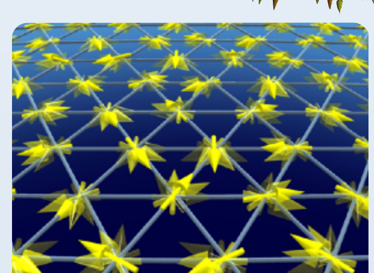
emergent spacetime

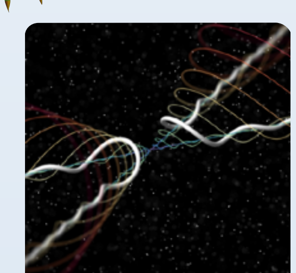
quantum matter

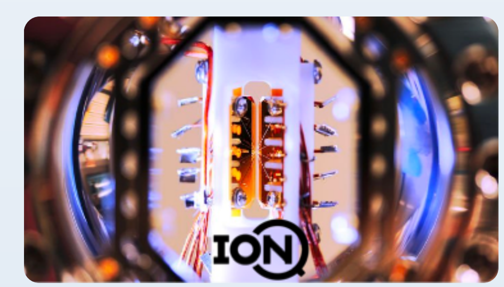
quantum technologies

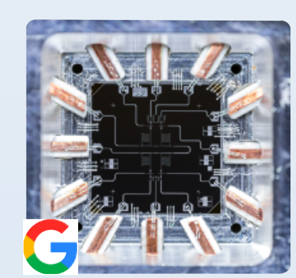


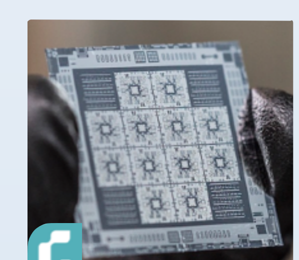






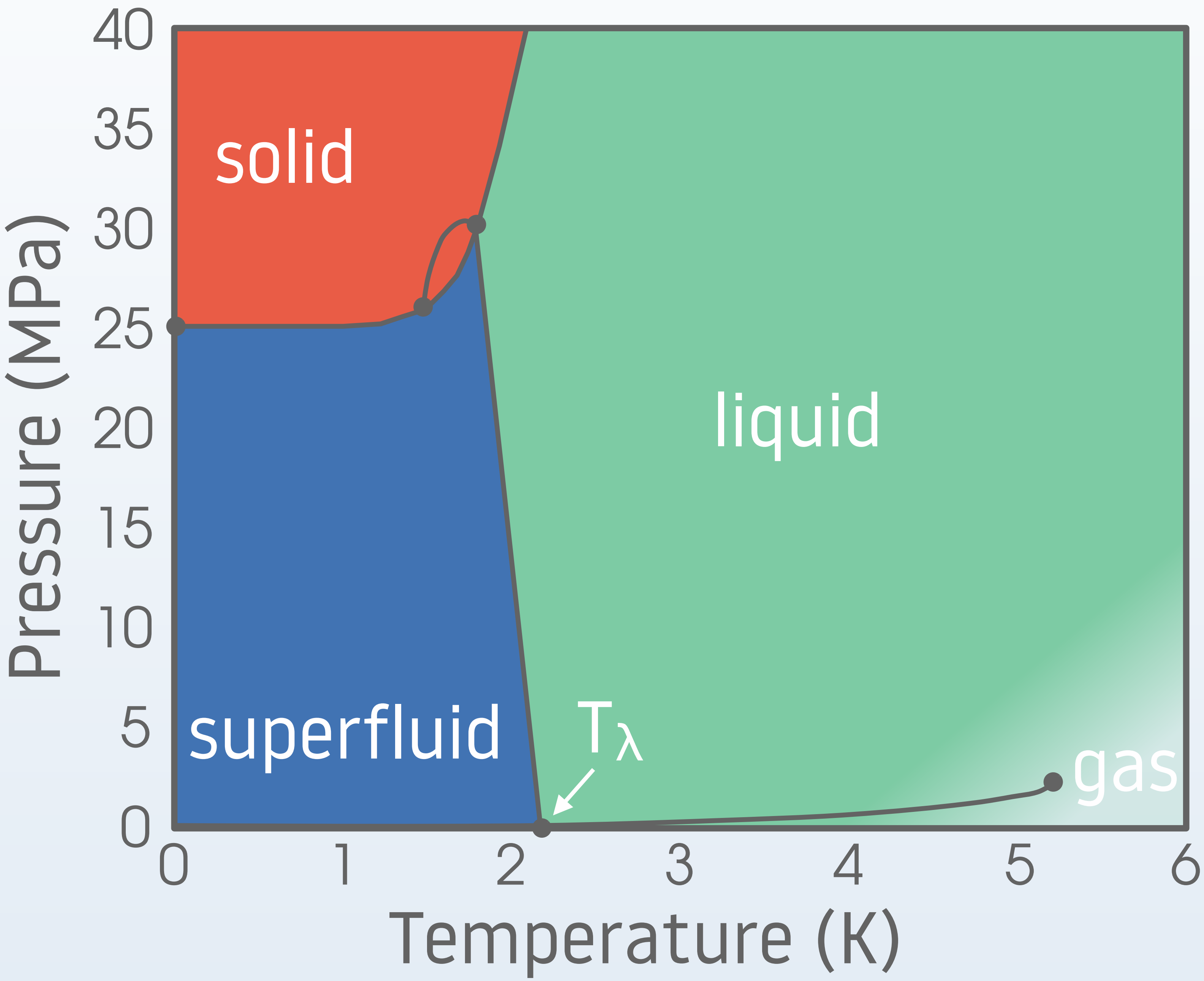






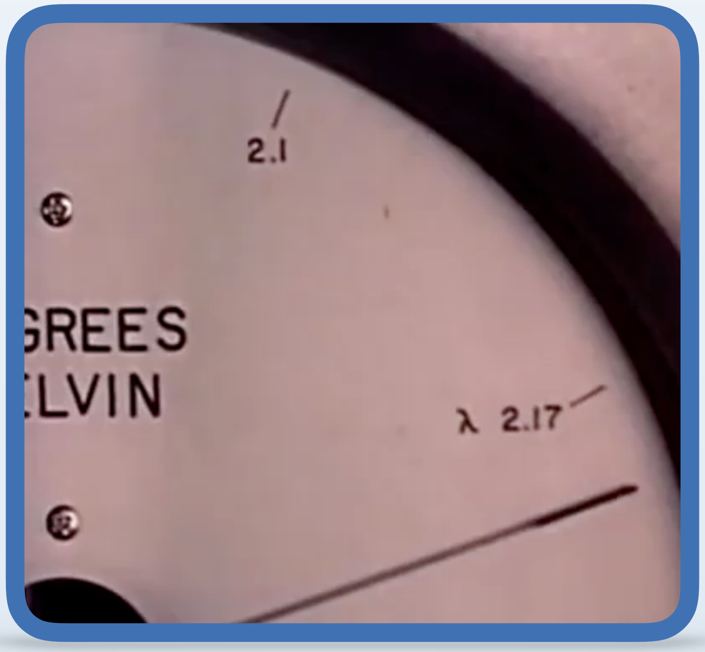

He

Helium is a Quantum Liquid



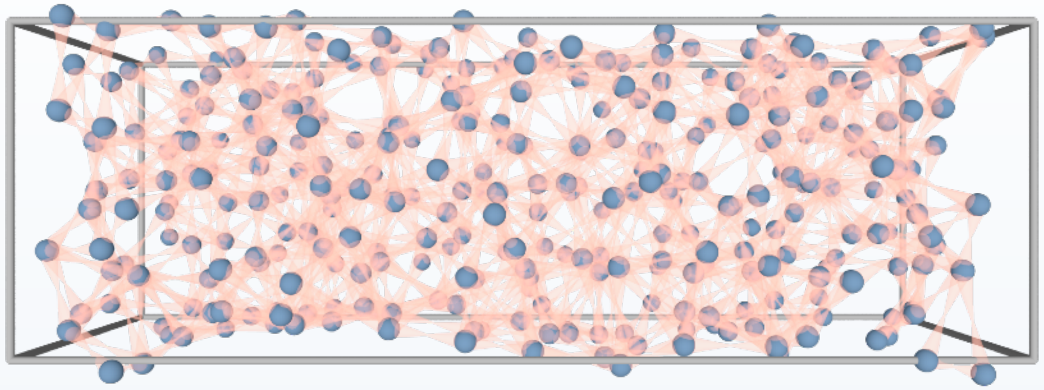
Superfluid is a fundamentally quantum state of matter

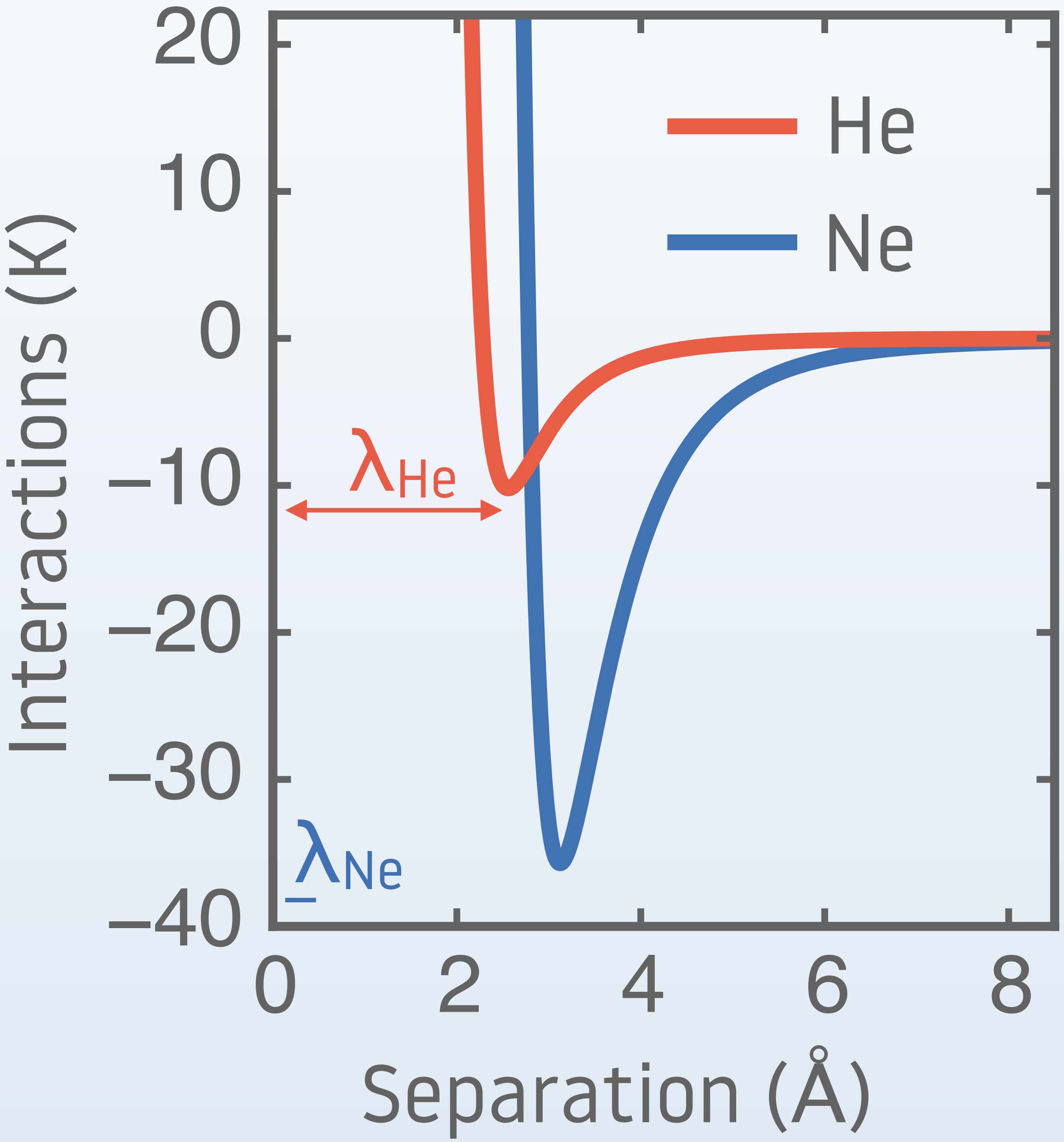
- dissipationless flow
- quantized vortices
- non-entropic flow



P. Kapitsa; J. Allen and A.D. Misener (1938)

What Makes ^{4}He so Quantum?





$$\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

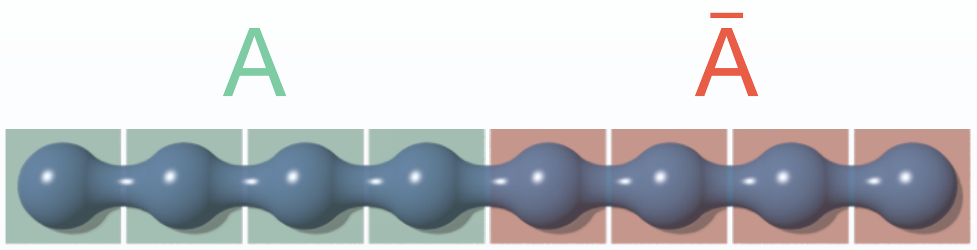
Helium-4 is the only atomic bosonic system with $\lambda_{dB} \sim r_s$ at $T \sim O(1\text{ K})$

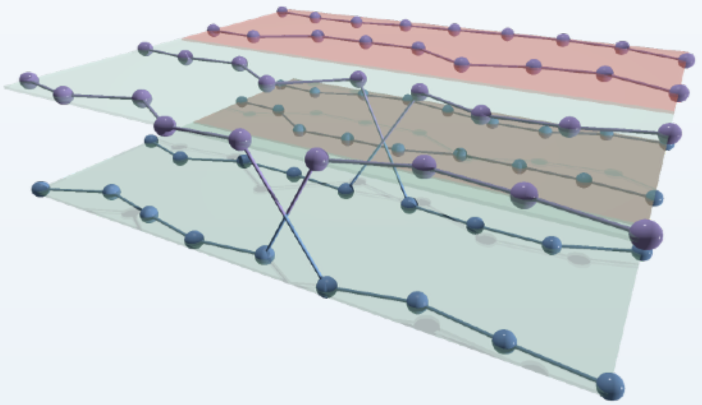
Superfluid ^4He is a macroscopic quantum phase of matter!

Is it entangled?

Entanglement and Entropy

quantifying uncertainty in many-body systems



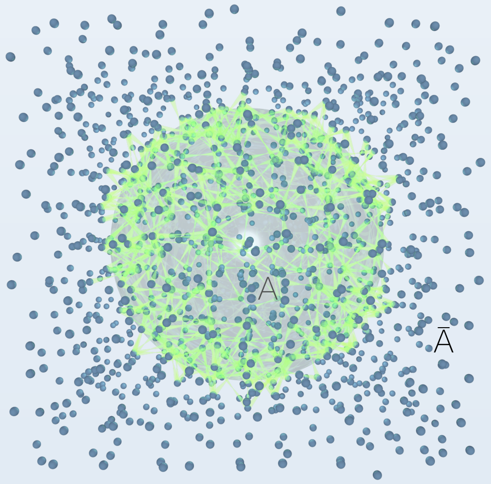


Measuring Entanglement

algorithmic development in d-dimensional continuous space

Area Law in ^{4}He

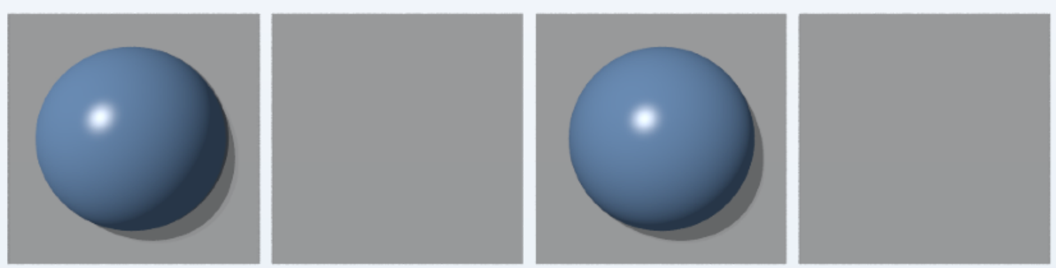
entanglement scaling in a real quantum liquid



Toy Quantum Matter

bosons with hard-cores on a 1d lattice

L = 4 sites



N = 2 bosons

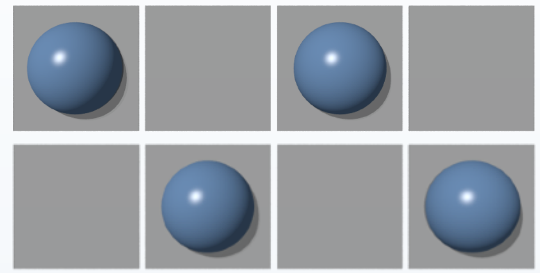
kinetic potential

$$H = -\sum_i \left(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i \right) + V \sum_i n_i n_{i+1}$$

Investigate the quantum ground state for different interaction strengths V

What are the ground states?

$V \gg 1$



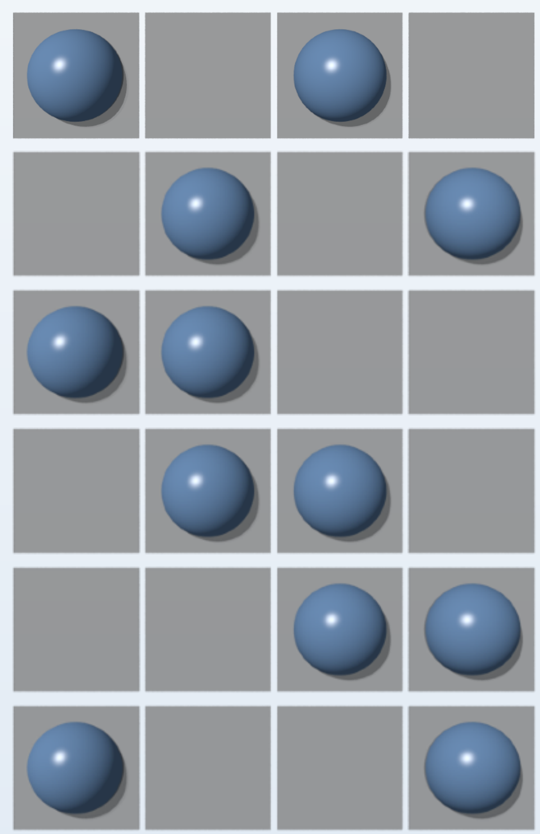
solid

$$T = -\sum_i \left(b_i^\dagger b_{i+1} + h.c. \right)$$

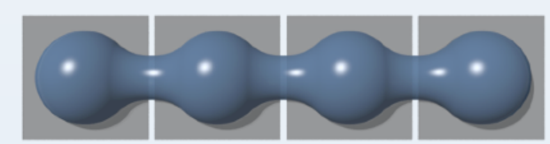
$$U = V \sum_i n_i n_{i+1}$$

$$|\Phi\rangle = \frac{1}{\sqrt{2}} \left(|1010\rangle + |0101\rangle \right)$$

$V \ll 1$



$\equiv$

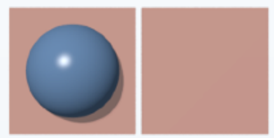


superfluid

$$|\Psi\rangle = \frac{1}{2} \left(|1010\rangle + |0101\rangle \right) + \frac{1}{2\sqrt{2}} \left(|1100\rangle + |0011\rangle + |1001\rangle + |0110\rangle \right)$$

A Quantum Bipartition

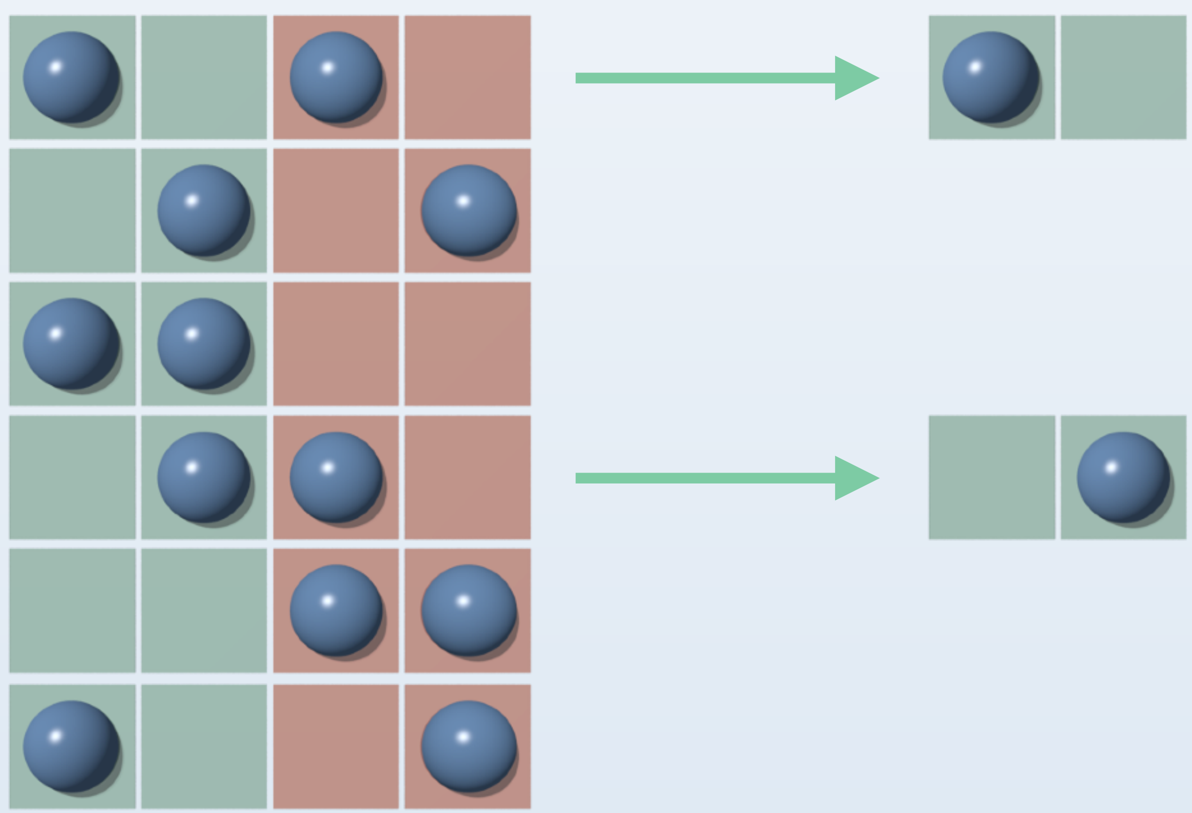
Break up the system into two parts
and make a local measurement on $\bar{A}$

Suppose we find:  in $\bar{A}$ what do we know about A?

$V \gg 1$

no uncertainty =
complete knowledge

$V \ll 1$



uncertainty =
incomplete knowledge

Quantifying Entanglement with Entropy

Entropy: A measure of encoded information

Entanglement: Non-locally encoded quantum information

Entanglement Entropy: A measure of entanglement

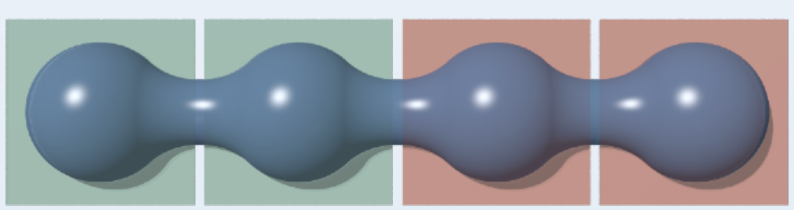
probabilities encoded in reduced density matrix $\rho_A = \mathrm{Tr}_{\overline{A}}\, \rho$

Shannon

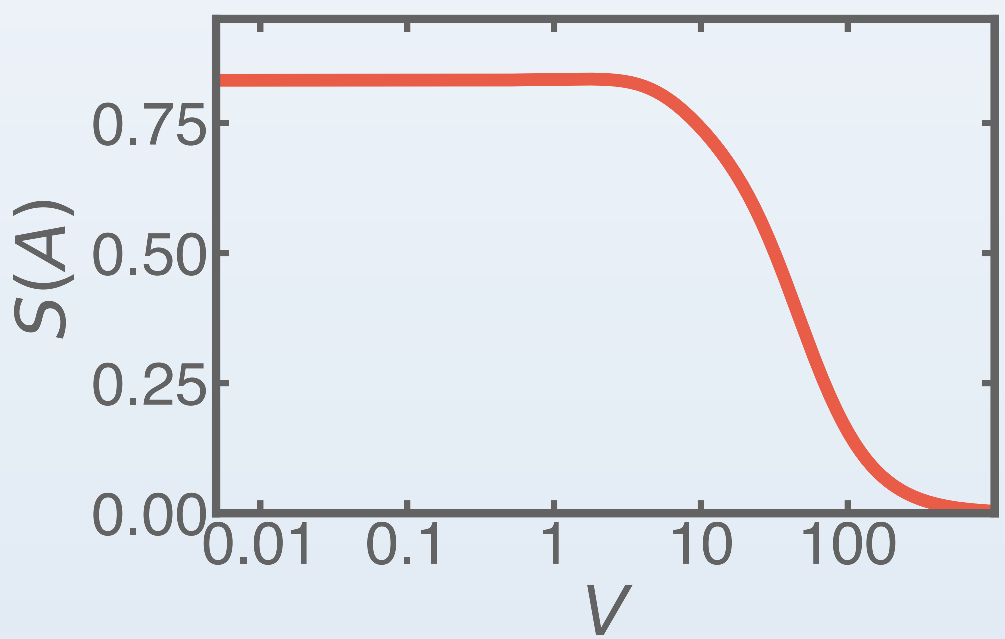
$$S = -\sum_a p_a \log p_a \;\rightarrow\; S(A) = -Tr \rho_A \log \rho_A$$

von Neumann

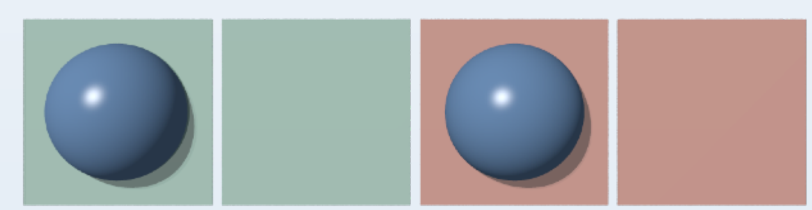
A $V \ll 1$ $\bar{A}$



$S(A) > 0$



A $V \gg 1$ $\bar{A}$

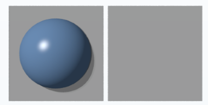
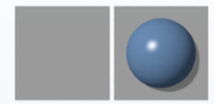


$S(A) = 0$

S(A) : how entangled is A with $\bar{A}$

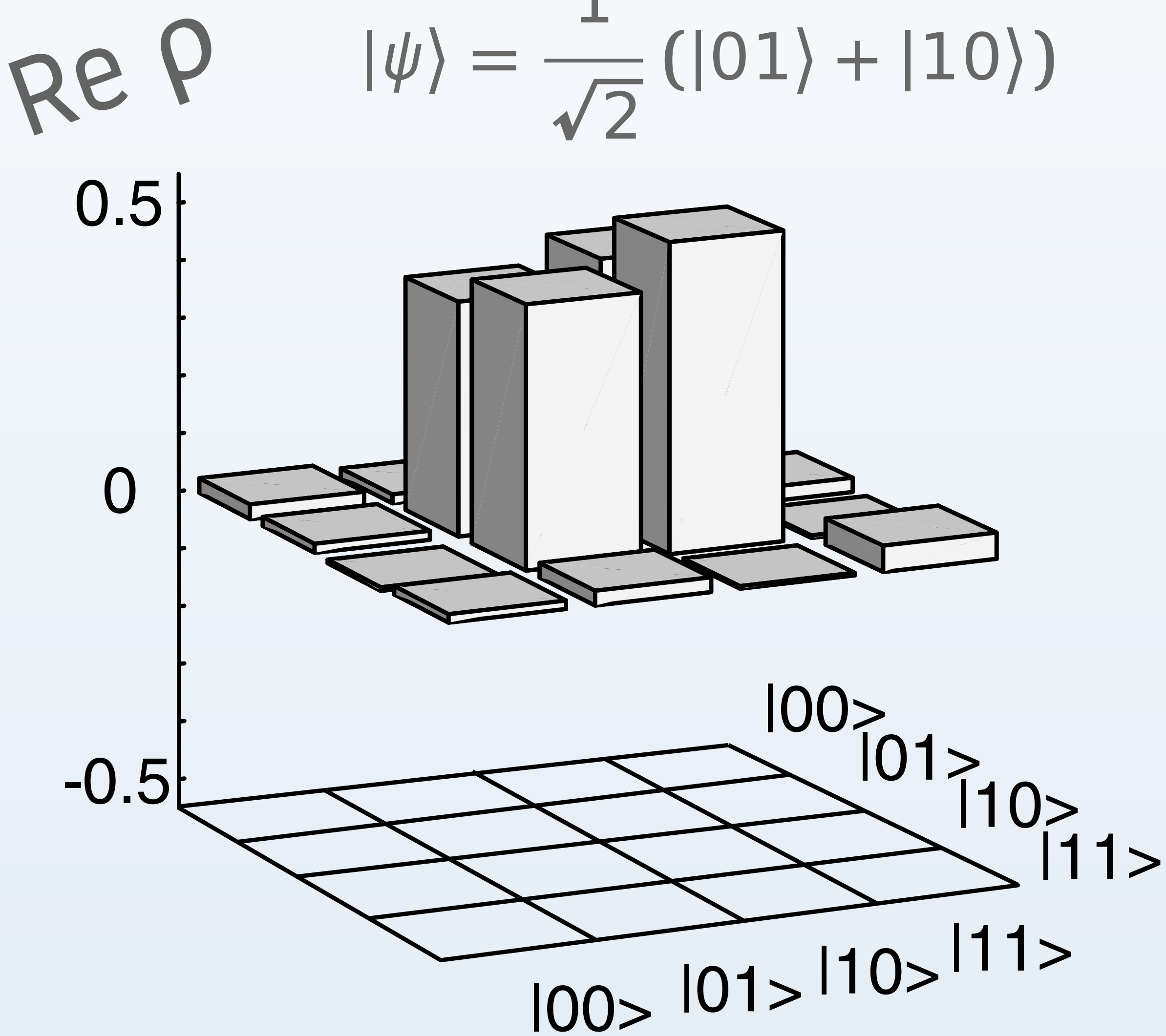
Experimental Measurement

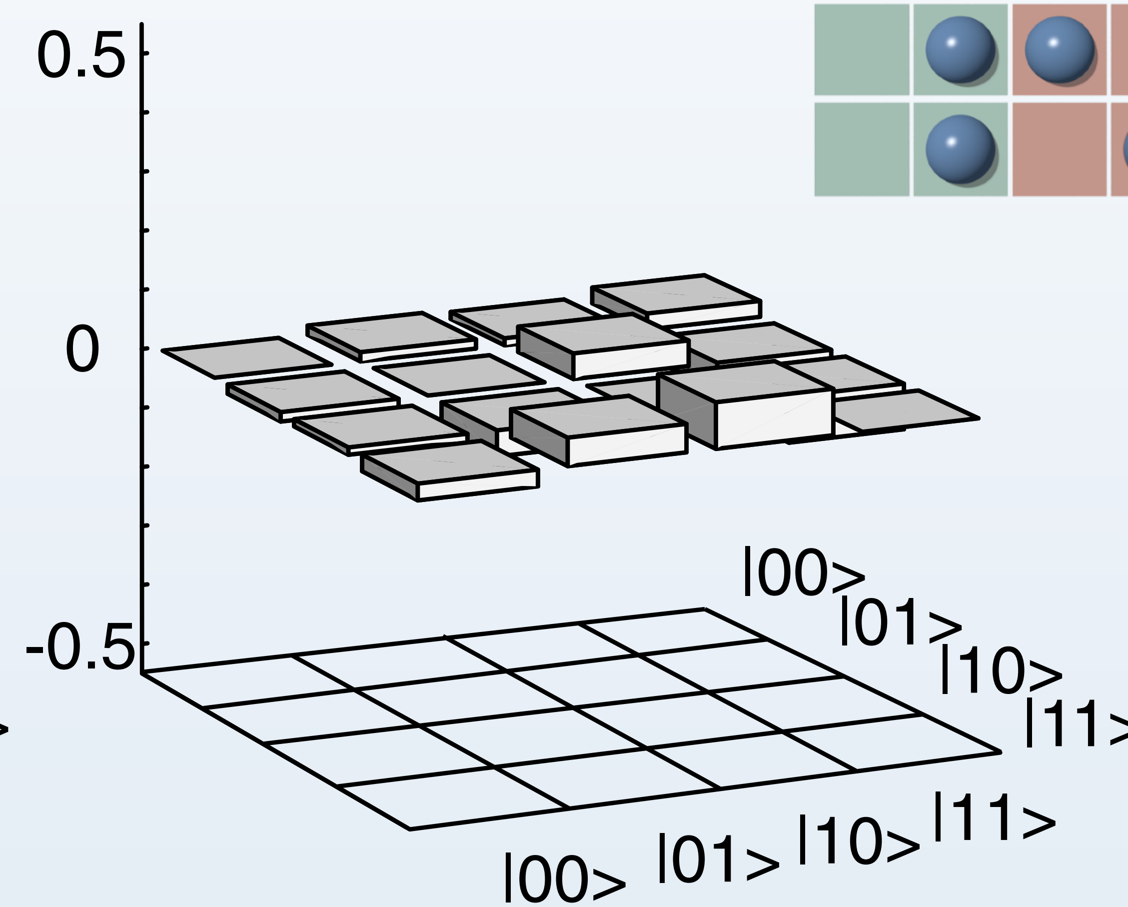
Density matrix is generally inaccessible

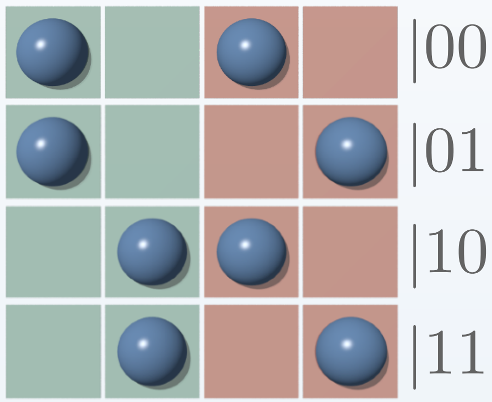
 $= |0\rangle$  $= |1\rangle$

C. F. Roos et al, PRL 92, 220402 (2004)

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



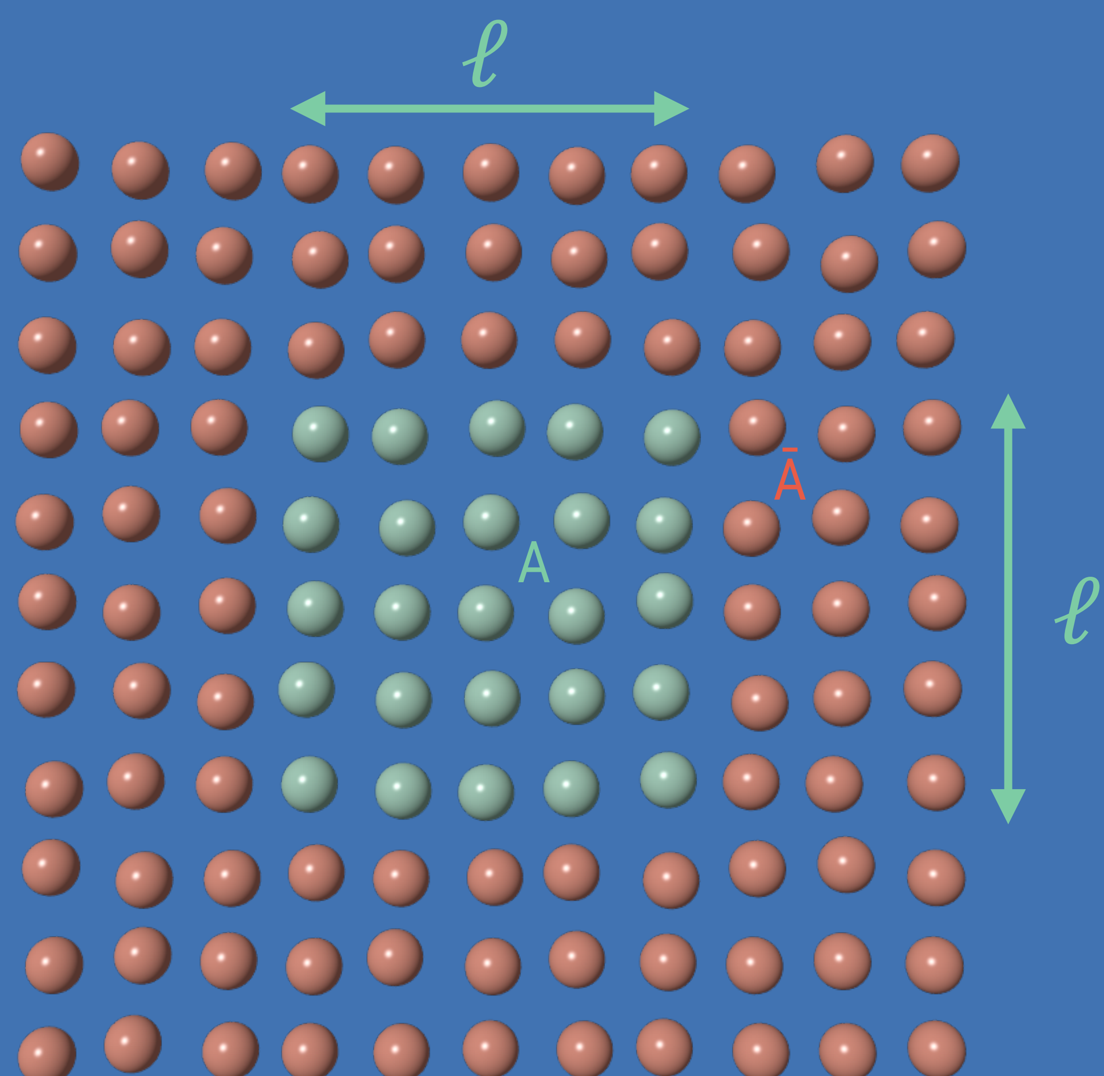




Measurement becomes exponentially difficult!

4 particles on 4 sites: $\rho \sim 10^5$ entries

How does entanglement scale with the size of the subregion?

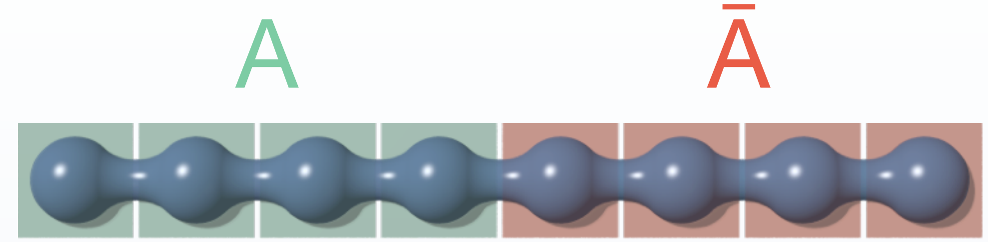


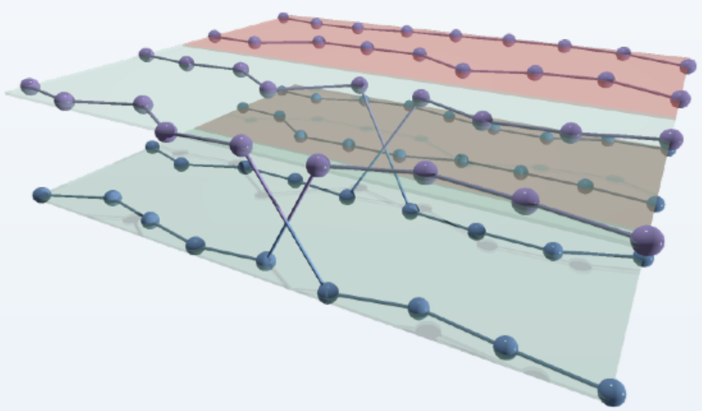
$$S(\ell) \sim \ell^{\lambda}$$

$$\lambda = ?$$

Entanglement and Entropy

quantifying uncertainty in many-body systems





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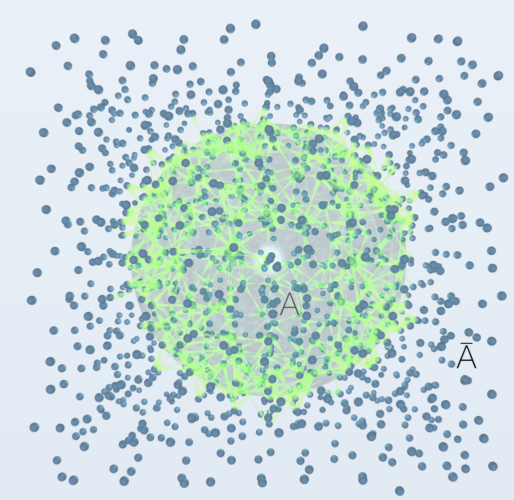
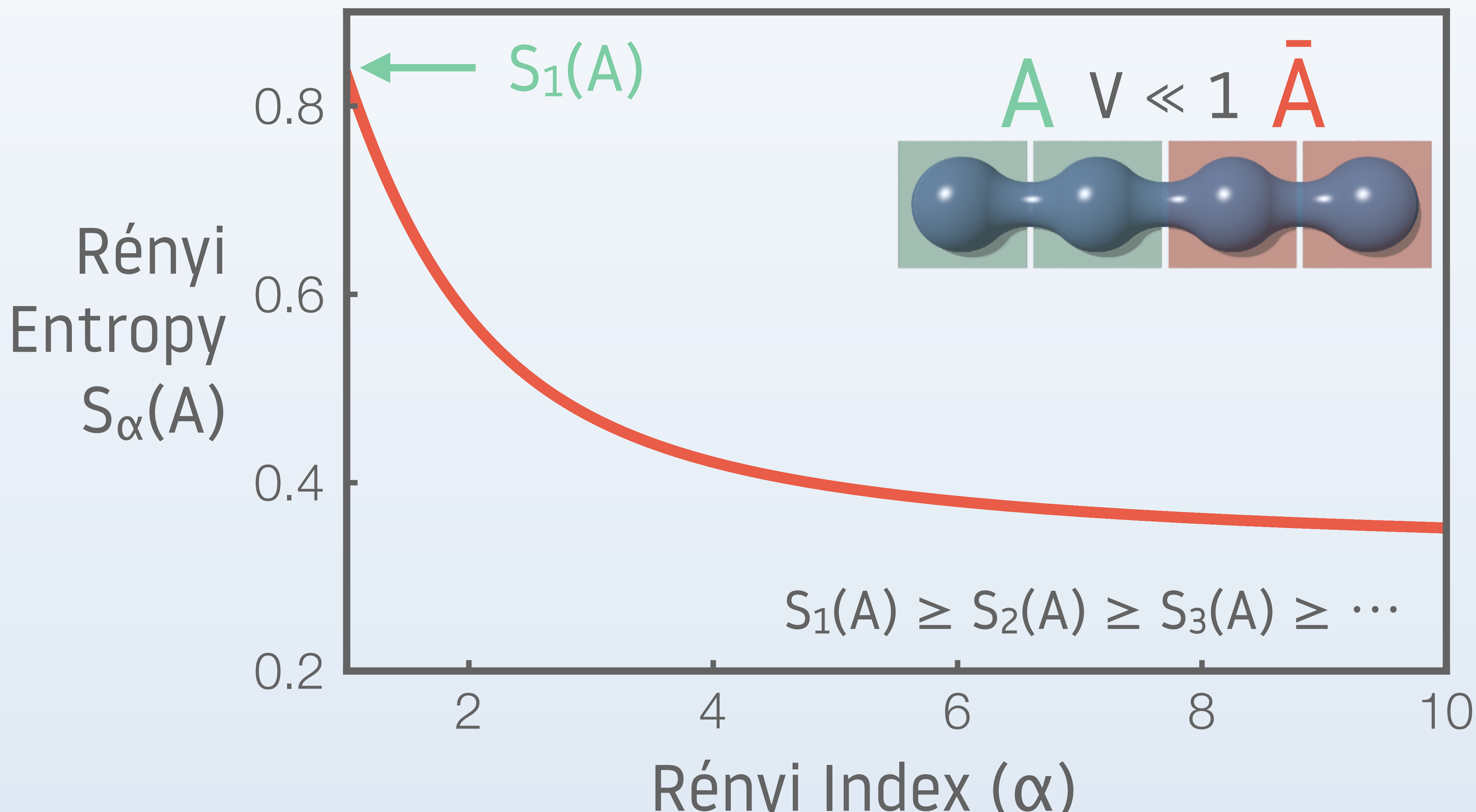


Rényi Entanglement Entropies

Alfréd Rényi

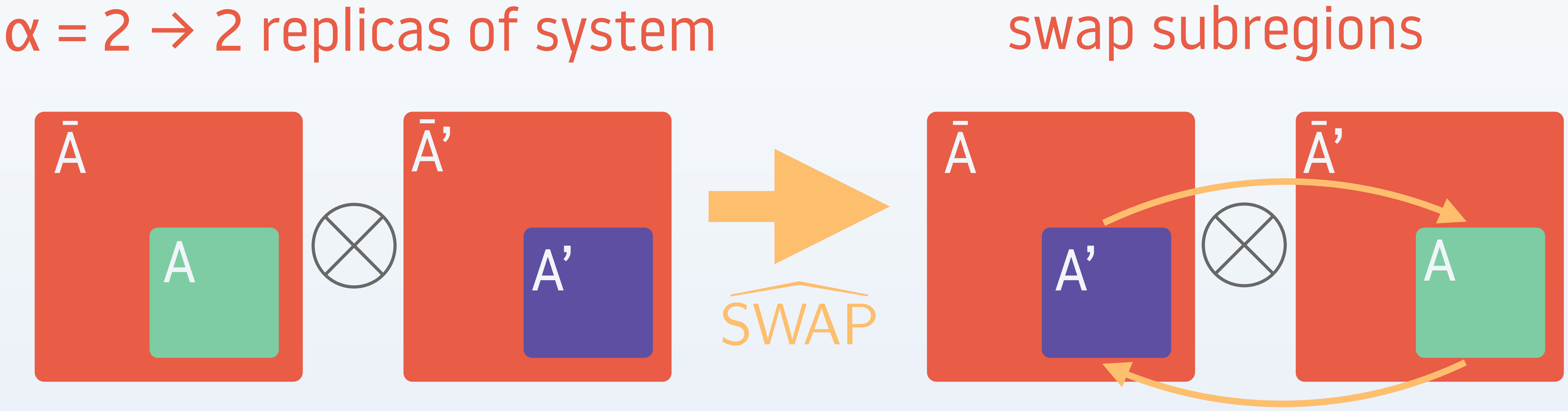
An alternate measure of entanglement

$$S_\alpha(A) = \frac{1}{1-\alpha} \log \mathrm{Tr}\, \rho_A^\alpha \;\Rightarrow\; \lim_{\alpha \to 1} S_\alpha(A) = -Tr \rho_A \log \rho_A$$



The Replica Method

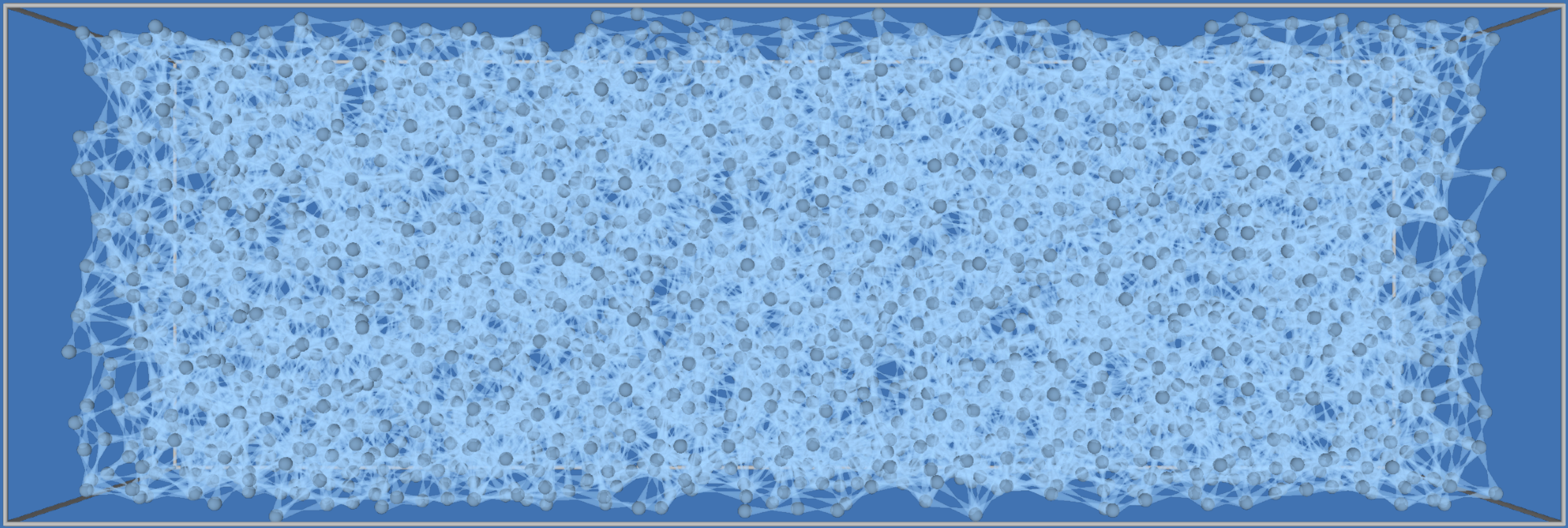
Computing Rényi entanglement entropies by swapping subregions between non-interacting identical copies



claim: $S_2(A) = -\log \mathrm{Tr}\, \rho_A^2 = -\log \left\langle \widehat{\mathrm{SWAP}} \right\rangle$

P. Calabrese and J. Cardy, J. Stat. Mech.: Theor. Exp. P06002 (2004)

Studying SWAP for quantum liquids

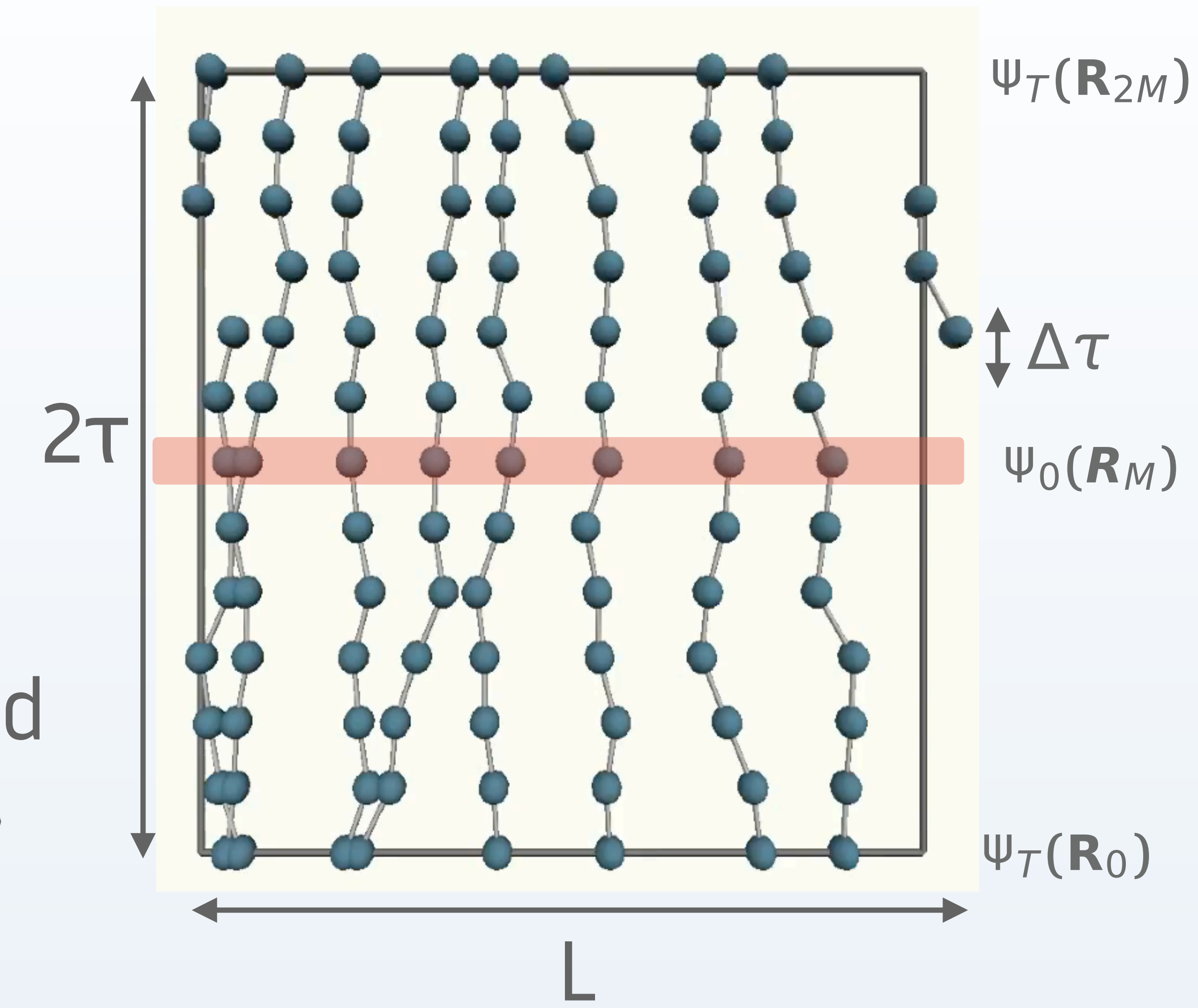


algorithmic development needed!

Path Integral Ground State QMC

Description

$$H = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + U_i\right) + \sum_{i<j} V_{ij}$$



example: 1d → (1+1)d

Configurations

project trial wavefunction to the ground state in **R** basis: $\Psi_0(\boldsymbol{R}) = \lim_{\tau\to\infty} \langle \boldsymbol{R}|e^{-\tau\hat{H}}|\Psi_T\rangle$

discrete imaginary time worldlines constructed from products of the short time propagator $G(\boldsymbol{R},\boldsymbol{R}';\Delta\tau) = \langle \boldsymbol{R}|e^{-\Delta\tau\hat{H}}|\boldsymbol{R}'\rangle$

Updates

Local and non-local worldline deformations with weights controlled by H and $\Psi_T(\boldsymbol{R})$

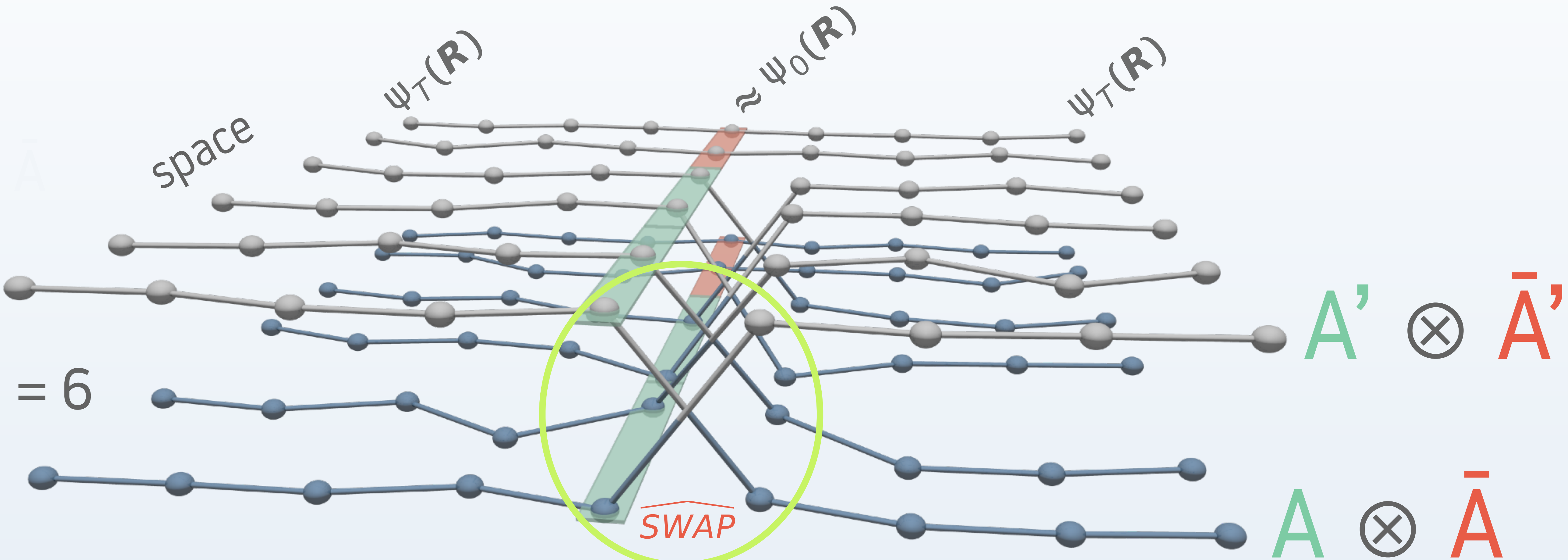
Observables

exact method for computing ground state expectation values

$$O_\tau = \frac{\langle\Psi_T|e^{-\tau\hat{H}}\hat{O}e^{-\tau\hat{H}}|\Psi_T\rangle}{\langle\Psi_T|e^{-2\tau\hat{H}}|\Psi_T\rangle}$$

Porting the Replica Method to PIGS

Break paths at the center time slice τ, measure $\widehat{SWAP}$ when replicas are linked via short time propagator G.



$$\langle \widehat{SWAP} \rangle = \langle G\left(\mathbf{R}_\tau \otimes \mathbf{R}'_{\tau'}, \widehat{SWAP}\left[\mathbf{R}_{\tau+\Delta\tau} \otimes \mathbf{R}'_{\tau+\Delta\tau}\right]; \Delta\tau\right)\rangle$$

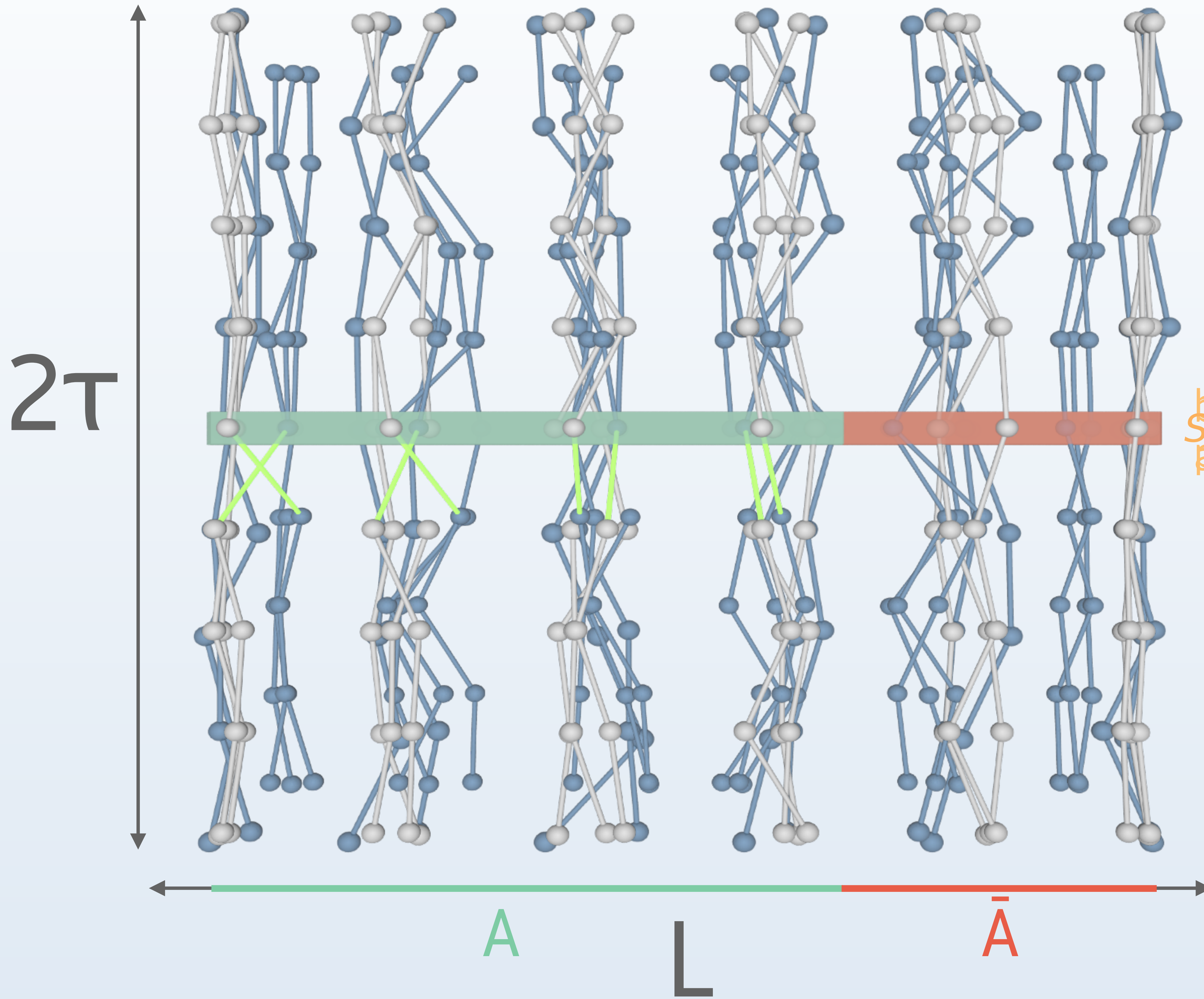
Technology adapted from other QMC flavors

M. B. Hastings, I. González, A. B. Kallin, and R. G. Melko, PRL 104, 157201 (2010)
R. Melko, A. Kallin, and M. Hastings, PRB 82, 100409 (2010)
C. Herdman, R. Melko and A.D. Phys. Rev. B, 89, 140501 (2014)
C. M. Herdman, S. Inglis, P. N. Roy, R. G. Melko, and A.D., PRE 90, 013308 (2014)
T. Grover, Phys. Rev. Lett. 111, 130402 (2013)
Assaad, Lang, Toldin, Phys. Rev. B 89, 125121 (2014)
Broecker and Trebst, J. Stat. Mech. (2014) P08015
J. E. Drut and W. J. Porter, PRB 92, 125126 (2015)

SWAP Simulation Details

connect paths between

$\langle \widehat{SWAP} \rangle = \left\langle G\left(\boldsymbol{R}_{\tau} \otimes \boldsymbol{R}'_{\tau}, \widehat{SWAP}\left[\boldsymbol{R}_{\tau+\Delta\tau} \otimes \boldsymbol{R}'_{\tau+\Delta\tau}\right]; \Delta\tau\right)\right\rangle$



break paths

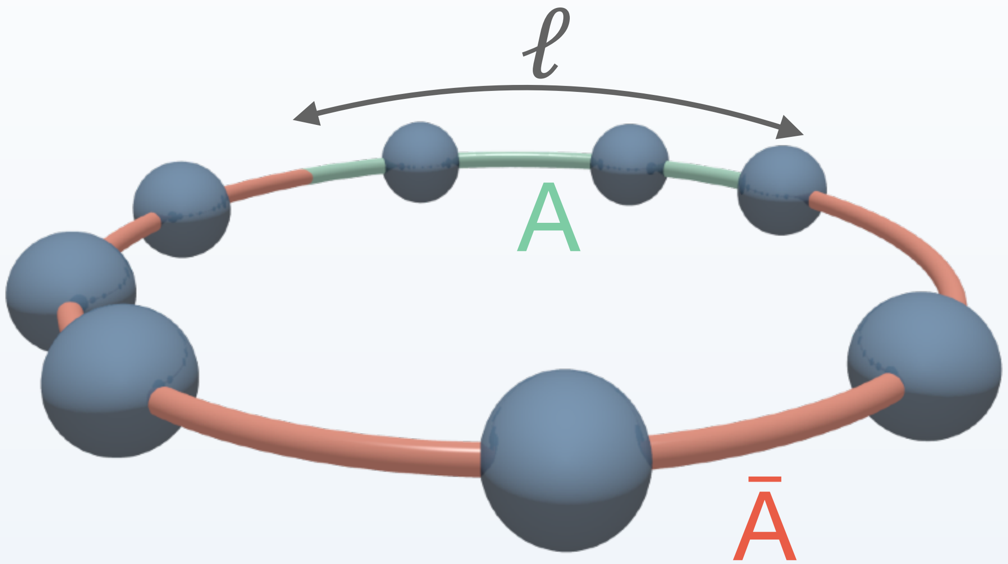
$S_2 = -\log\langle \widehat{SWAP} \rangle$

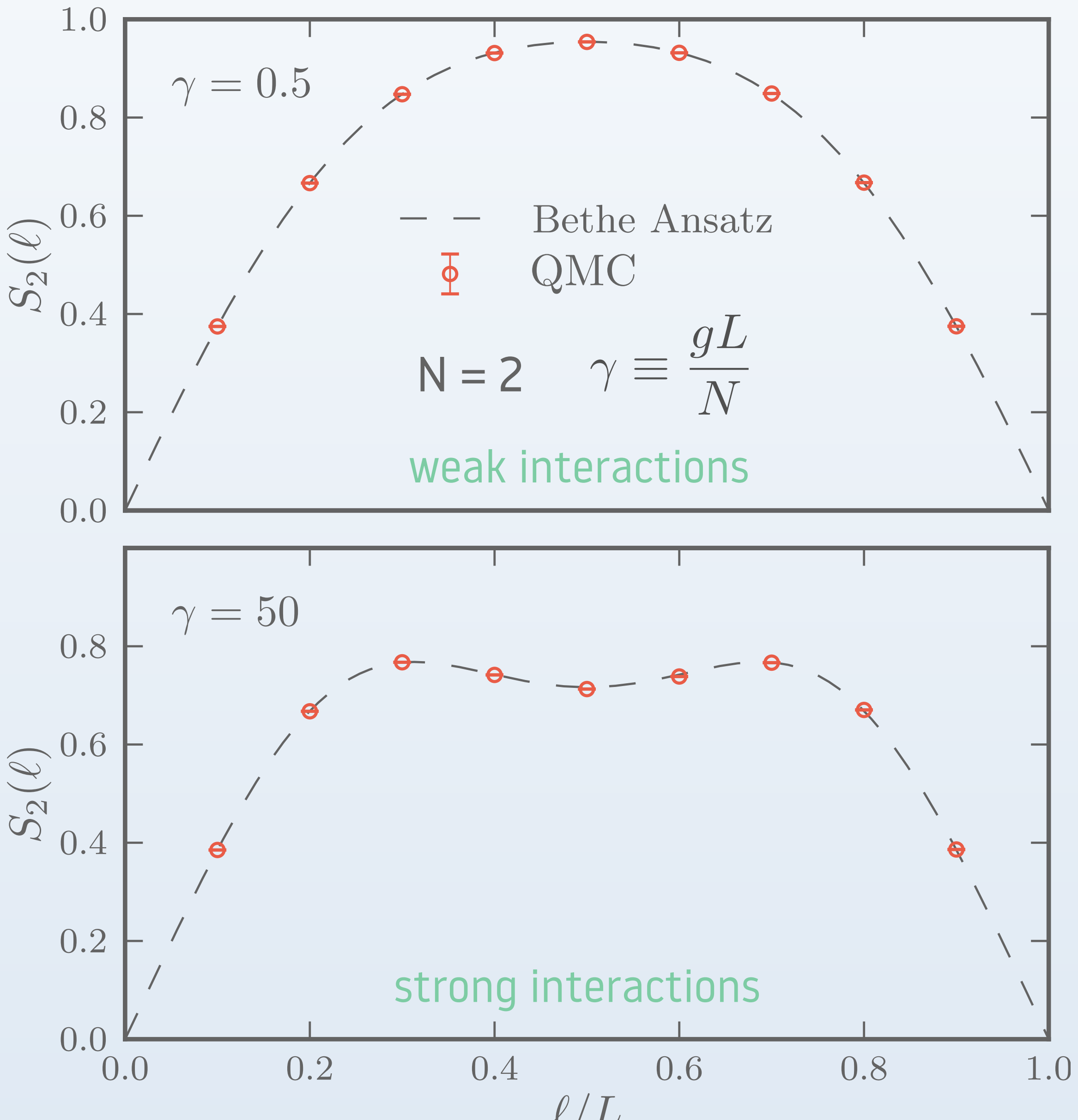
at physical slice ψ_0

Benchmarking on a Solvable Model

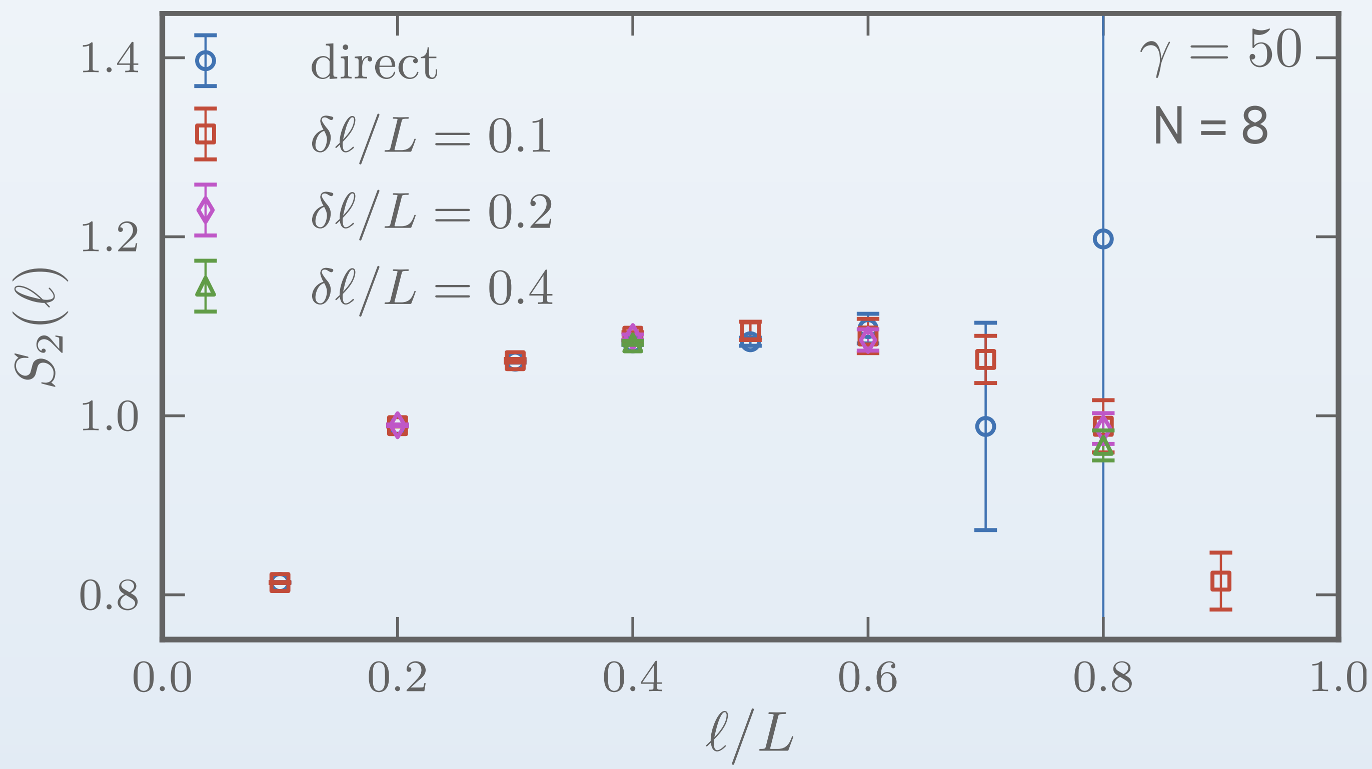
Lieb-Liniger model of δ-function interacting bosons on a ring

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + g\sum_{i<j}\delta(x_i - x_j)$$





ratio trick



E. H. Lieb and W. Liniger, PR **130**, 1605 (1963)
C. M. Herdman, P. N. Roy, R. G. Melko, and A.D., PRB B **94**, 064524 (2016)

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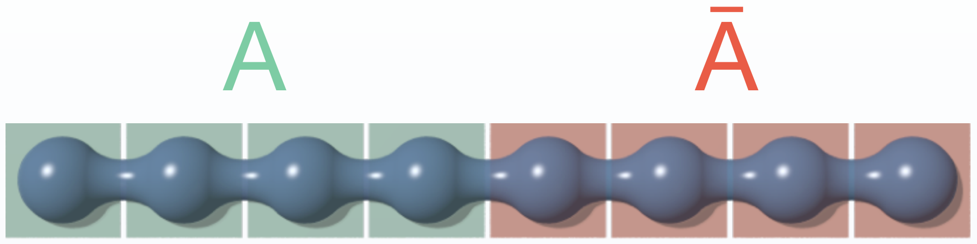
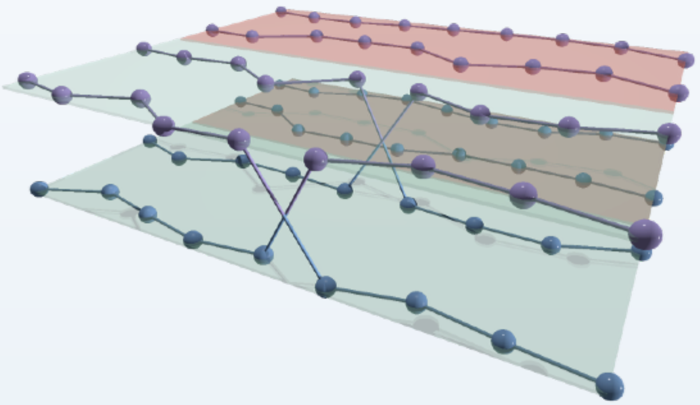
quantifying uncertainty in many-body systems





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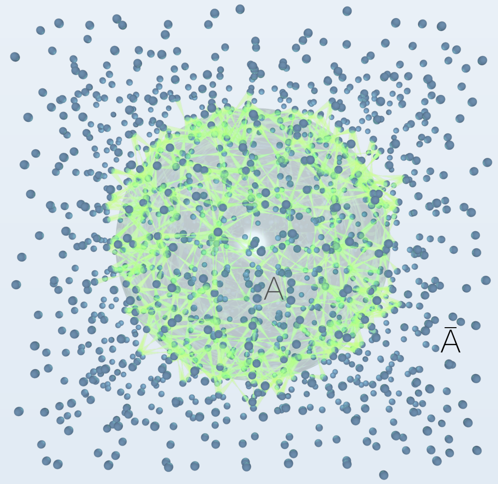
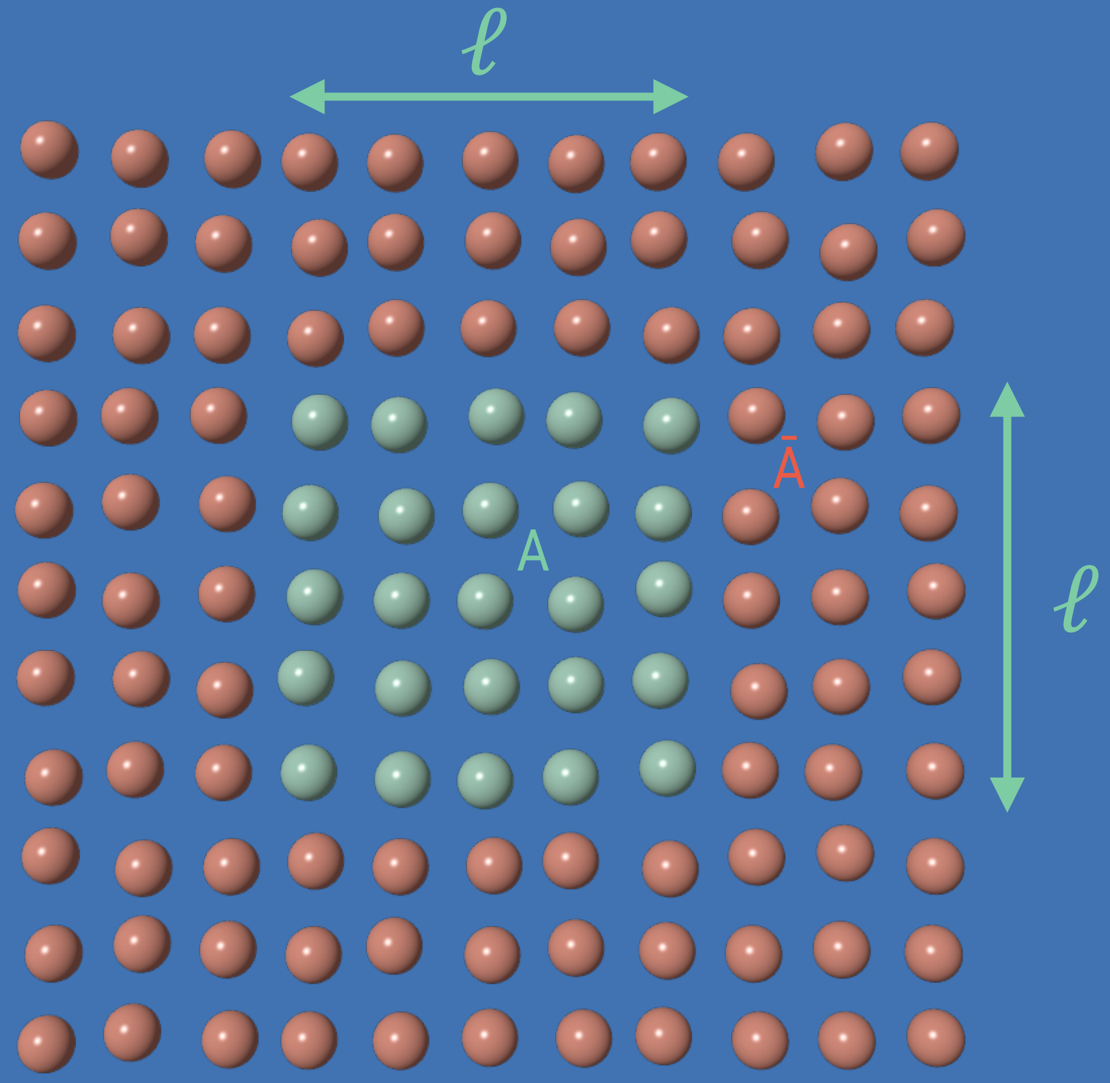
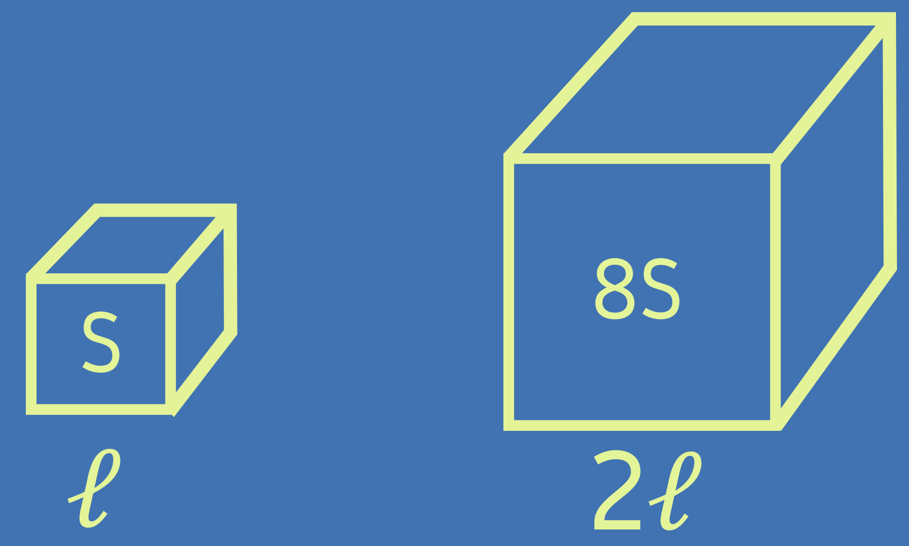


How does entanglement scale with the size of the subregion?



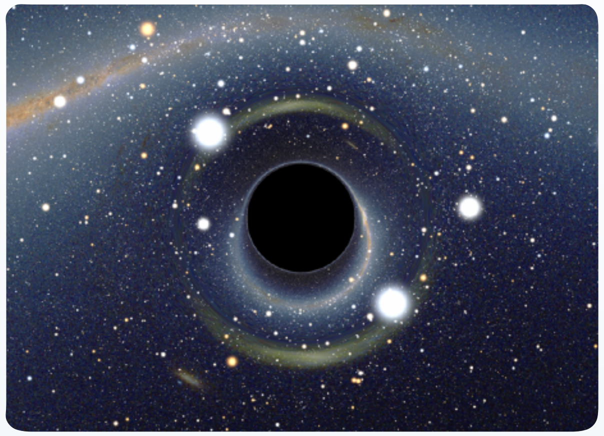
$$S(\ell) \sim \ell^{\lambda}$$

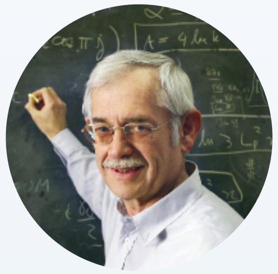
thermodynamic entropy is extensive ⇒ $\lambda = d$

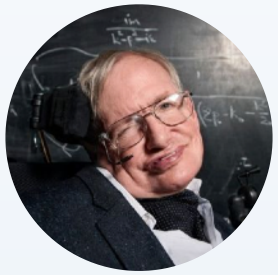


is this always the case?

Black Hole Entropy Area Law







J.D. Bekenstein, *PRD* **7**, 2333 (1973)
S.W. Hawking, *Nature* **248**, 30 (1974)

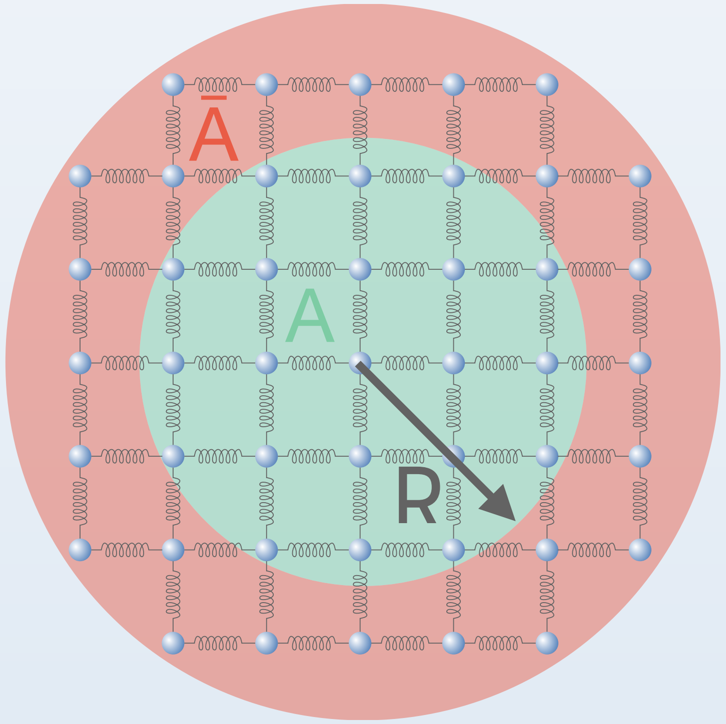
Black hole thermodynamics:

- Quantum black holes emit thermal radiation
- Area Law: entropy of a black hole is proportional to surface area, not volume!

$$S_{BH} \propto \text{area}$$

Is this due to entanglement?

- Toy model: coupled harmonic oscillators
- "Area Law": number of springs connecting A with $\bar{A}$ scales with boundary size



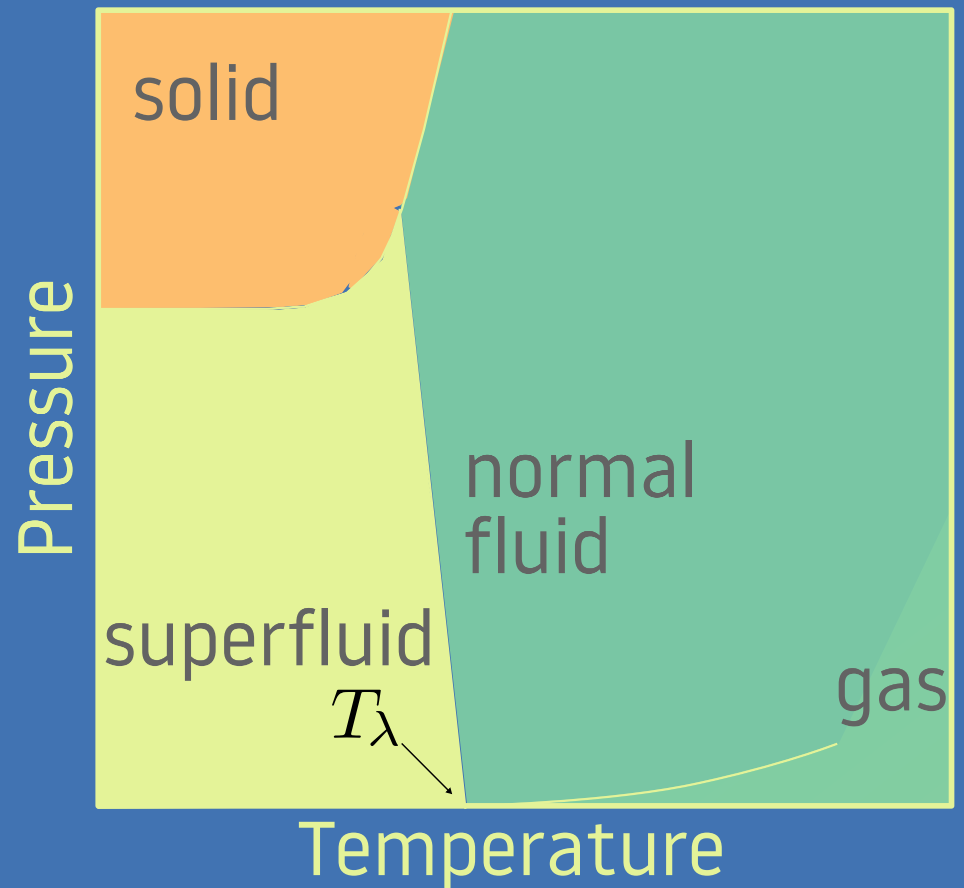
L. Bombelli, et al., PRD **34**, 373 (1986)
M. Srednicki PRL **71**, 666 (1993)

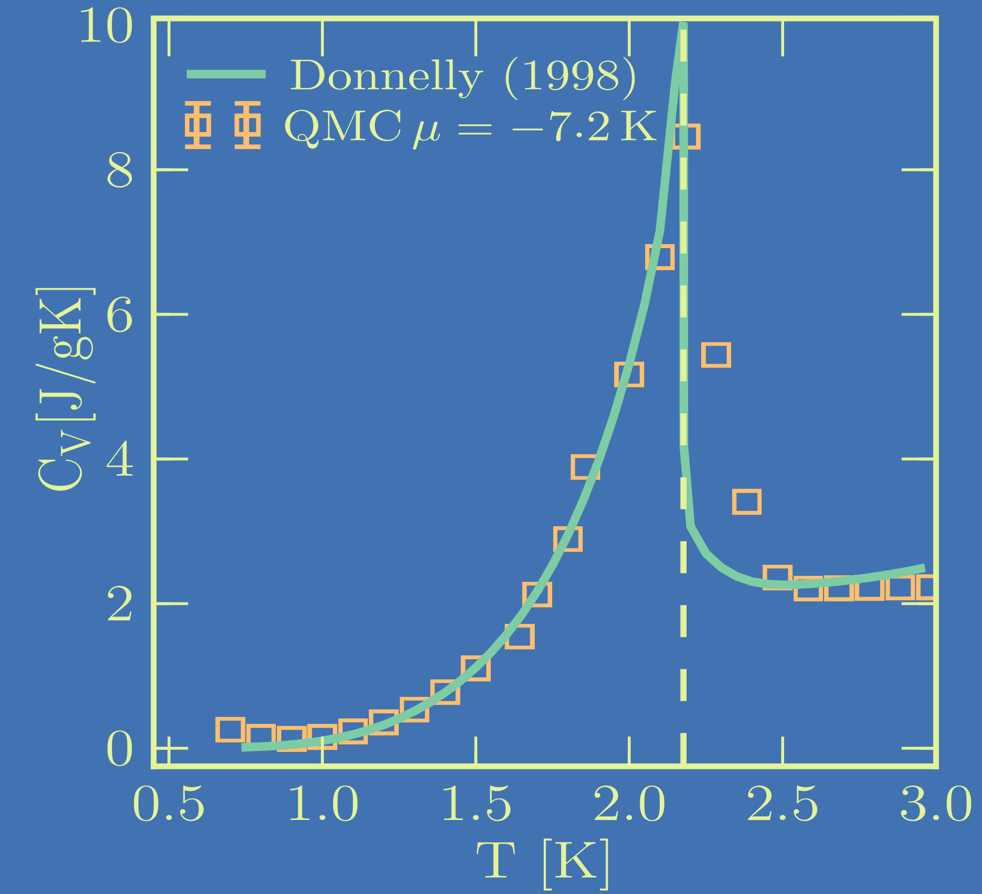
Proof for gapped ground states in d = 1

M.B. Hastings, J. Stat. Mech., P08024 (2007)

What about our real quantum phase of matter?

helium-4

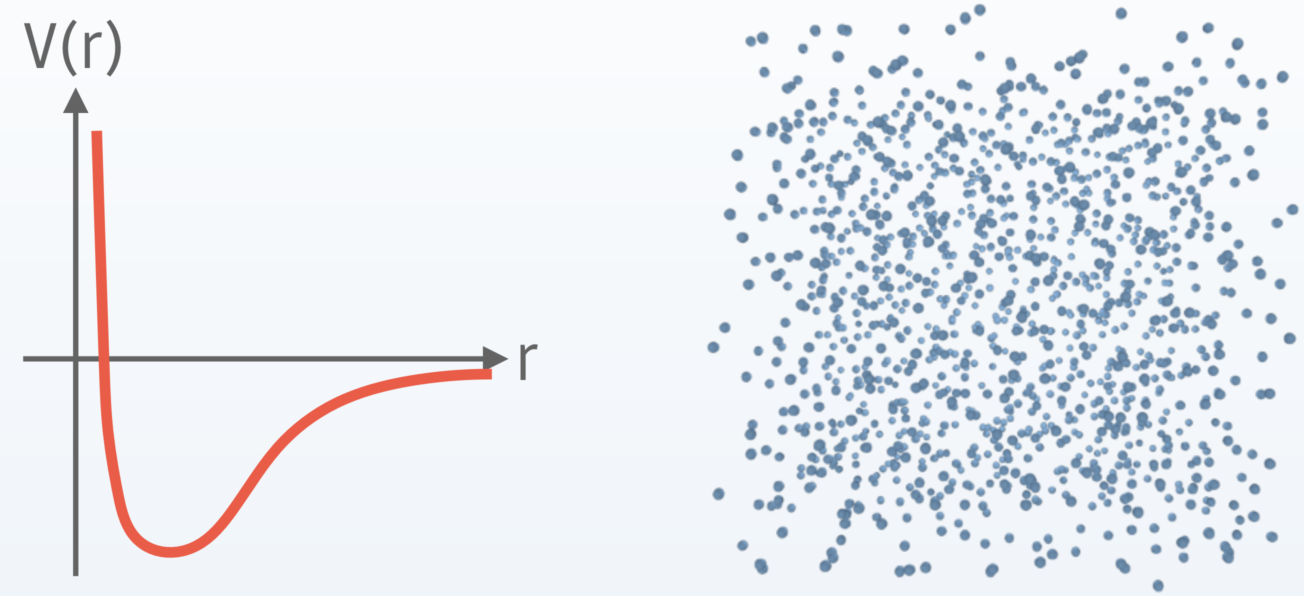




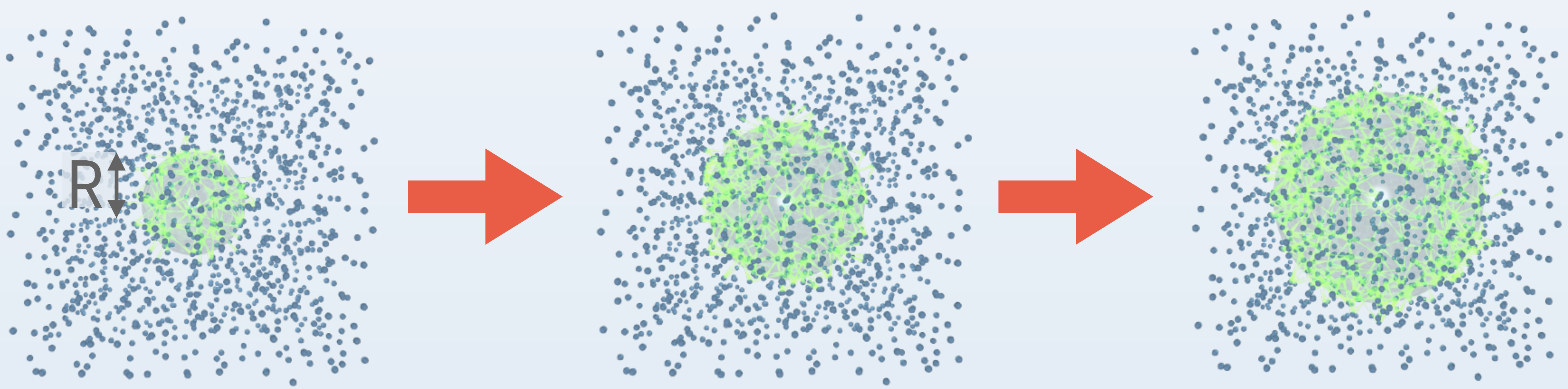
Entanglement in Superfluid ^{4}He

3d box at $T = 0$ with periodic boundary conditions at SVP

$$H = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + U_i\right) + \sum_{i<j} V_{ij}$$



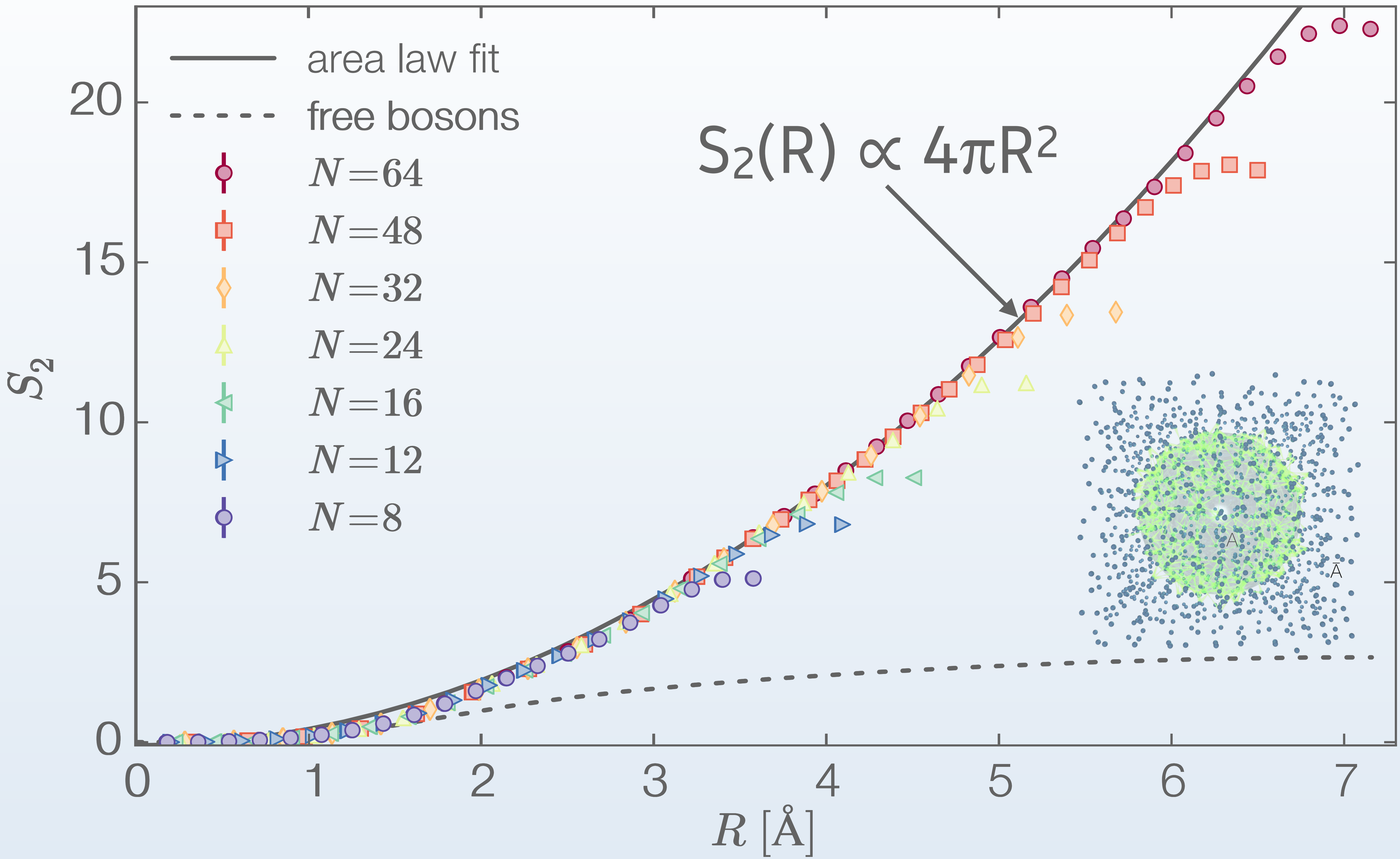
Measure entanglement $S_2(R)$ between spherical region of radius R and the rest of the box



Investigate scaling by changing the radius of the sphere

C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys 13, 556 (2017)

Scaling of the Entanglement



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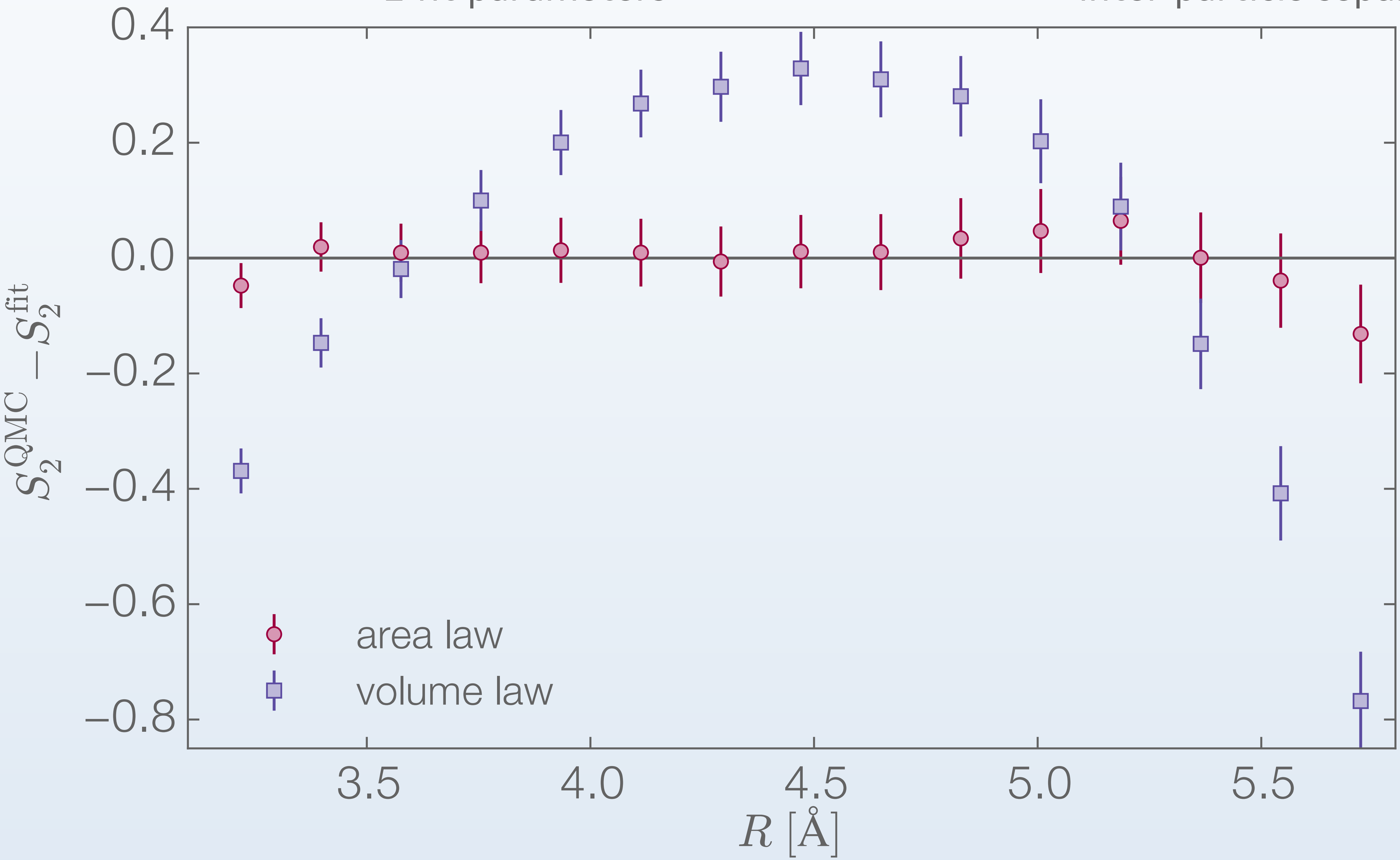
Area Law vs. Volume Law

Area: $S_2^{fit} = 4\pi a \left(\frac{R}{r_0}\right)^2 + c$

Volume: $S_2^{fit} = \frac{4\pi}{3} a \left(\frac{R}{r_0}\right)^3 + c$

2 fit parameters

inter-particle separation



C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys 13, 556 (2017)

Discovery of an area law in a real quantum liquid

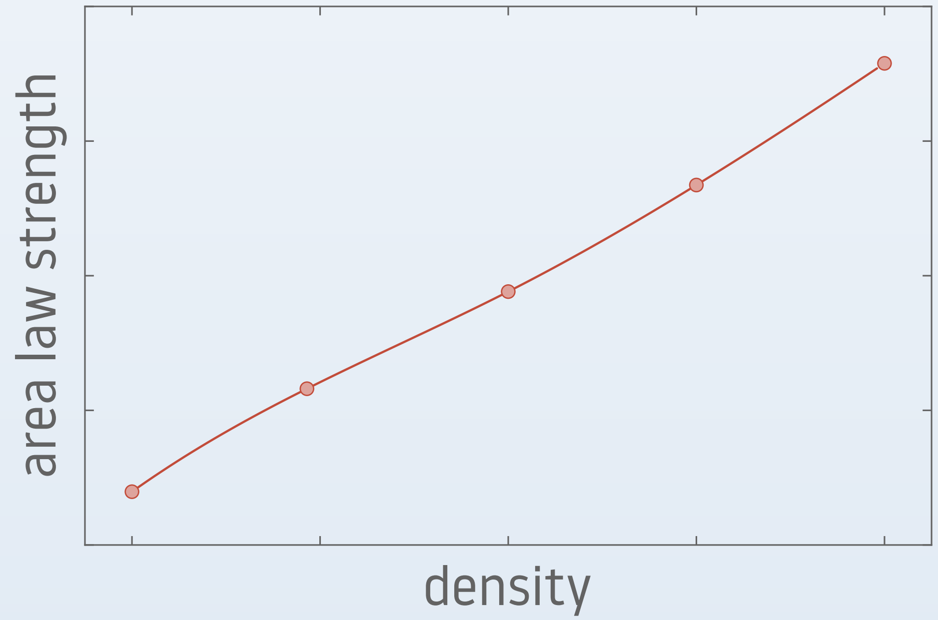
Quantum entanglement scales with the surface area and not volume in superfluid ^{4}He

$$S_2 \propto R^2$$

Analogous to Bekenstein-Hawking black hole entropy

Prospects for measurement and manipulation?

Can numerically determine an *entanglement equation of state*



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