Analytic continuation with

Ω*MaxEnt*

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$$G(i\omega_n) \text{ or } G(\tau) \Rightarrow A(\omega)?$$

$\Rightarrow$ invert

$$G(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(\omega)}{i\omega_n - \omega} \,.$$

- or

$$G(\tau) = -\int \frac{d\omega}{2\pi} \frac{e^{-\omega\tau} A(\omega)}{1 \pm e^{-\beta\omega}}$$

- Problem: discrete approximation is ill-conditioned.

$$G = KA$$

Maximum entropy

- Minimize

$$Q = \chi^2 - \alpha S$$

$$\chi^2 = \sum_{mn} (G_m - K_m A)^T C_{mn}^{-1} (G_n - K_n A)$$

$$S = -\int d\omega \, A(\omega) \ln \frac{A(\omega)}{D(\omega)}$$

$$C_{mn} = \frac{1}{N-1} \sum_{i=1}^{N} (G_m^i - \bar{G}_m)(G_n^i - \bar{G}_n)$$

$$\alpha?$$

Ω*MaxEnt*

Input data

$$G(i\omega_n) \text{ or } G(\tau)$$

- Fermionic ($A(\omega) > 0$) or bosonic ($A(\omega)/\omega > 0$)
- Diagonal or general covariance matrix

Ω*MaxEnt* features

- Uses $$G(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(\omega)}{i\omega_n - \omega} \ .$$
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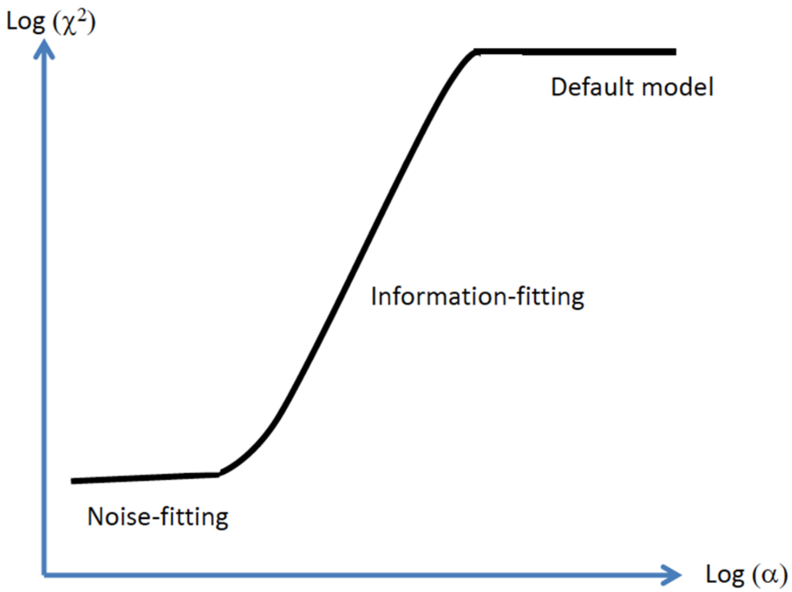
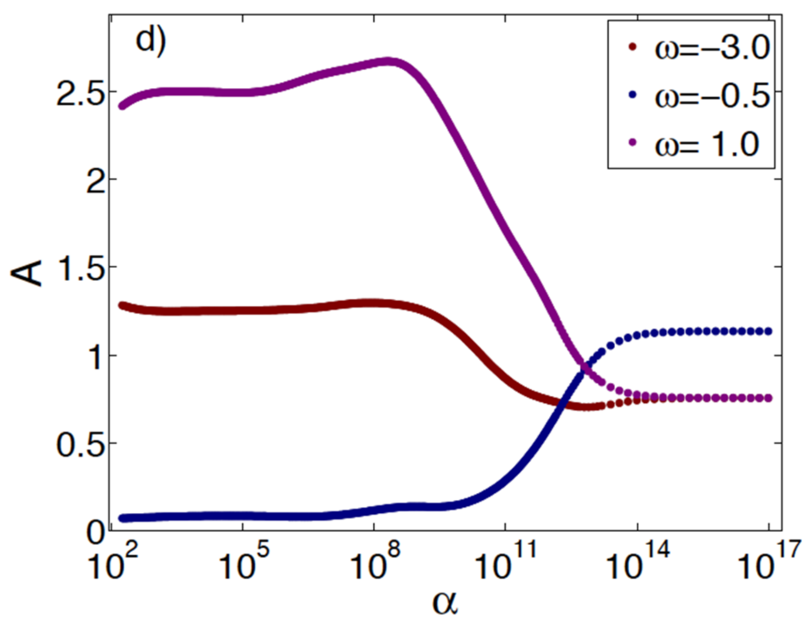
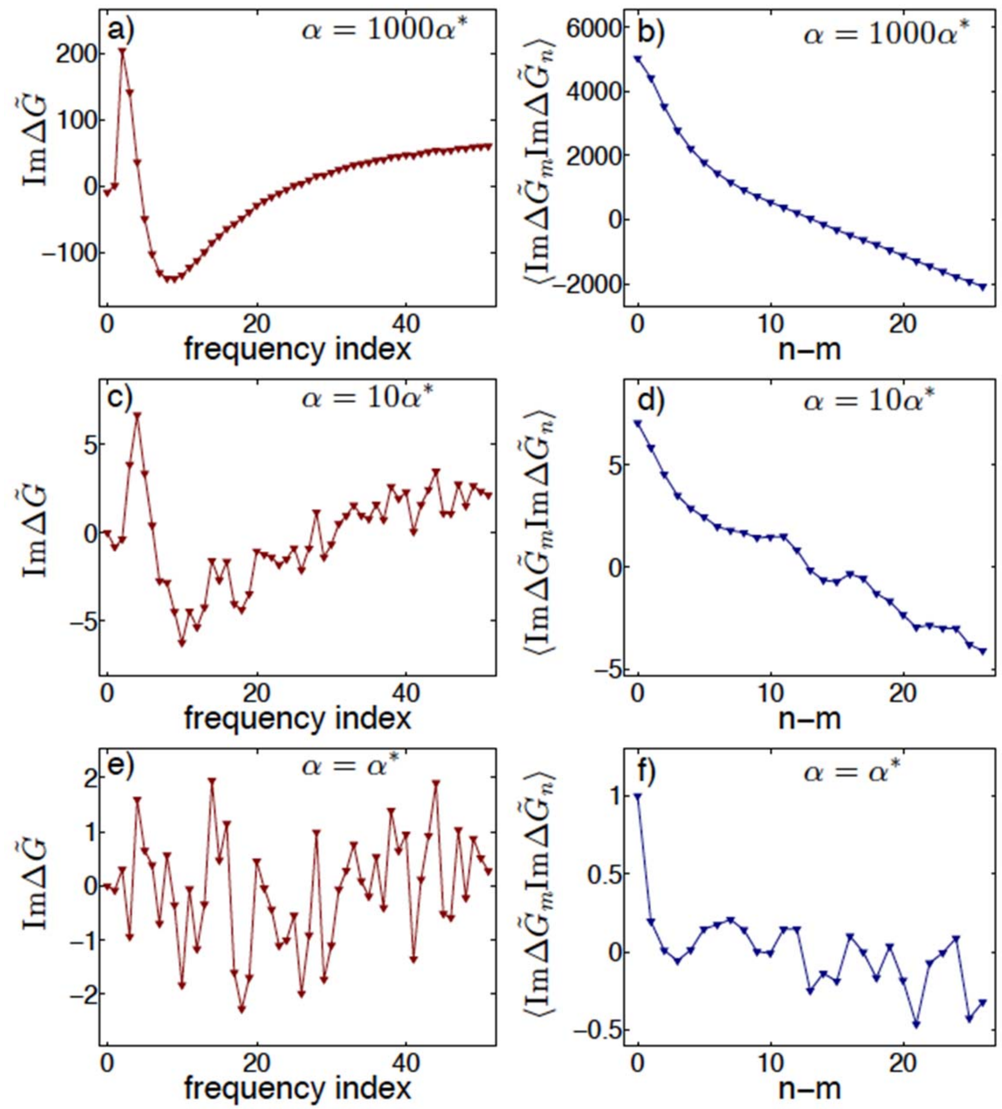
  - $\Rightarrow$ numerically more convenient.
- Adapted ω and ω_n grids
- Interactive mode: graphical diagnostic tools
- Batch mode

ΩMaxEnt diagnostic tools







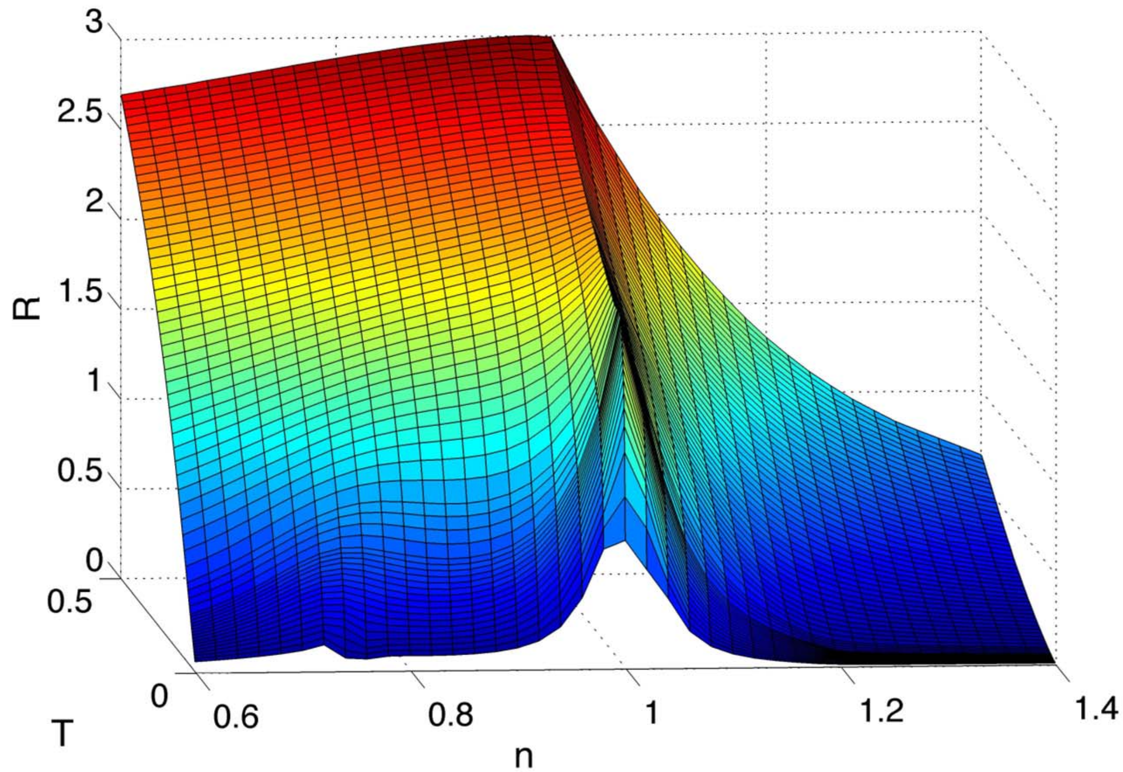
- http://www.physique.usherbrooke.ca/MaxEnt

- Algorithms for optimized maximum entropy and
- diagnostic tools for analytic continuation. D.Bergeron, A.-M.S. Tremblay.
- http://arxiv.org/abs/1507.01012

- Maximum entropy analytic continuation for spectral functions
- with nonpositive spectral weight.
- A. Reymbaut, D. Bergeron, and A.-M.S. Tremblay.
- Phys. Rev. B 92, 060509, 2015

Thanks!

- A.-M. Tremblay,
- Alexis Reymbault,
- Reza Nourafkan,
- Shaheen Acheche,
- Michel Barrette
- Patrick Vachon



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Conditioning Problem

- If we discretize ω $G = KA \quad \Rightarrow \quad A = K^{-1}G$
- error on *A*:

$$\frac{1}{\|K\|\|K^{-1}\|}\frac{\|\delta G\|}{\|G\|} \leq \frac{\|\delta A\|}{\|A\|} \leq \|K\|\|K^{-1}\|\frac{\|\delta G\|}{\|G\|}$$

- $\|K\|\|K^{-1}\|$ is large => $\frac{\|\delta A\|}{\|A\|}$ not bounded
- Analytic continuation unique in principle
- need constraints on

$\Rightarrow$ $A(\omega)$