A symmetry principle for Topological Quantum order

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Conclusions

(1) Intermediate symmetries lead to dimensional reductions

(2) Intermediate symmetries mandate robust topological order at zero and finite T

(3) Intermediate symmetries can enforce high dimensional fractionalization, unusual topological indices (& related Berry phases)

(4) Wigner-Eckart type selection rules associated with these symmetries enable construction of zero temperature states with robust topological orders.

Conclusions

5) Exact high dimensional fractionalization occurs in a pyrochlore antiferromagnet in a finite region of its phase diagram
6) General entangled systems have string (or higher dimensional “brane”) correlators which decay more slowly than the usual few particle correlators
7) Thermal effects seem to impose severe restrictions on several current suggestions for topological quantum computing

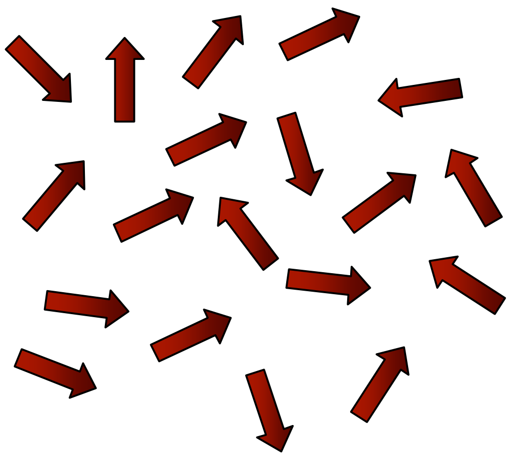
Orders and Symmetries

1. Global Symmetry Breaking Orders (Magnets)

2. "Topological Orders" (Gauge Theories, Protons in Ice, Quantum Hall, Spin Dimer States, transitions in Fermi surface topology, "string net" models)-no obvious local order parameters

Symmetry and Phase Transitions

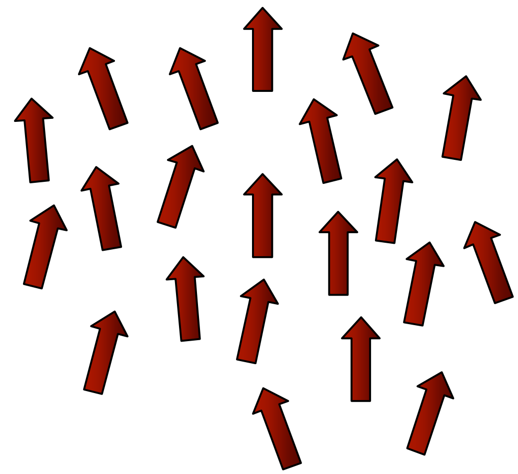
$T > T_c$



M = 0

Disordered Phase

$T < T_c$



M ≠ 0

Broken Symmetry Phase

Local order parameters

In the ferromagnet, the local expectation value is different in orthogonal ground states

$$\langle g_\alpha | M | g_\alpha \rangle \neq \langle g_\beta | M | g_\beta \rangle$$

Applying different boundary conditions can lead, at sufficiently low temperatures to spontaneous symmetry breaking

$$\langle M \rangle_\alpha \neq \langle M \rangle_\beta$$

Topological Order

For any quasi-local operator V:

T=0: $$\left\langle g_\alpha \middle| V \middle| g_\beta \right\rangle = v\delta_{\alpha\beta}$$

T>0: $$\left\langle V \right\rangle_\alpha = v$$

α boundary conditions

Here, the order is evident only in non-local "topological" quantities

Topological Order

Kitaev’s idea: use such system for quantum computing. Robustness

to local perturbations may allow greater stability to decoherence.

Topological Order

Prime examples of topological order:

Fractional Quantum Hall States

Some “spin liquids”

Kitaev’s models

Wen’s models

Gauge theories

…

Is there a common principle behind these systems?

Our central result: Intermediate symmetries & Topological Order

Any system which displays $T=0$

Topological Quantum Order (TQO)

and has interactions of finite range and strength

in which all ground states can be linked by

the discrete $d<2$ symmetries or continuous

$d<3$ symmetries, has TQO at all temperatures

$T>0$. Gapped systems with continuous $d<3$

symmetries have TQO at all temperatures.

[Idea: low dimensional symmetries can mandate the absence of

spontaneous symmetry breaking.]

Outline

- Symmetry and phase transitions (review)
- Intermediate symmetries (review/new)
- Theorem on <u>Dimensional Reduction</u> (new)
- Examples of application of the theorem (new)
- Robust topological order (new)

Outline

- Dimensional reduction and <u>fractional excitations (new)</u>

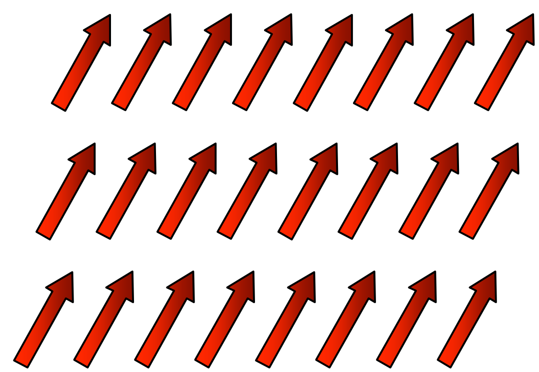
- Exact deconfinement in a pyrochlore antiferromagnet (new)

Symmetry and Phase Transitions

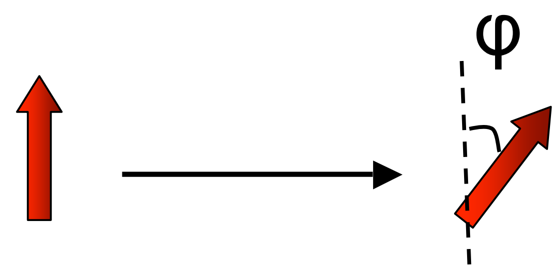
Mermin-Wagner Theorem: A continuous symmetry cannot be spontaneously broken at any finite temperature in one or two-dimensional systems with **short range interactions.**

<u>Example:</u> XY Model
(discrete symmetry)

$$H = J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$



Global U(1) symmetry:

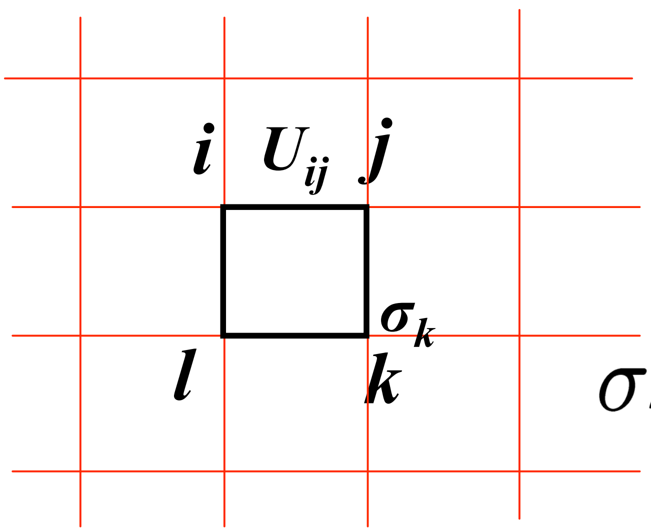


Local Symmetries and Elitzur Theorem

A local symmetry cannot be spontaneously broken at any finite temperature

<u>Example:</u> Ising lattice gauge theory.

$$H = -K \sum_{\langle ijkl \rangle} U_{ij} U_{jk} U_{kl} U_{li} - J \sum_{ij} \sigma_i U_{ij} \sigma_j$$



$$U_{ij} = \pm 1 \quad \sigma_i = \pm 1 \quad \eta_i = \pm 1$$

Local *Z_2 symmetry:*

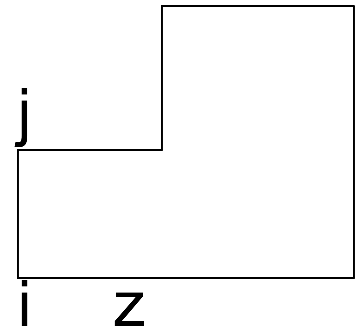
$$\sigma_i \to \eta_i \sigma_i, U_{ij} \to \eta_i U_{ij} \eta_j$$

Wilson loops

Notwithstanding the lack of rigidity with respect to any such transformation, there are topological quantities which are invariant under all local gauge transformations.

Wilson loop:

$$W = \left\langle U_{ij} U_{jk} U_{kp} \dots U_{zi} \right\rangle$$



In pure gauge theories, only such correlators may be finite.
The Wilson loops characterize the different phases.
Connection to percolation transitions.

Percolation transitions

Although in matter coupled lattice gauge theories, Wilson loops always exhibit a perimeter law, it can be shown that there are sharp transitions associated with percolation [a new result].

Intermediate Symmetries and Topological Orders

Intermediate Symmetries

A **d-dimensional** symmetry is a group of transformations that leave the theory invariant such that the minimum non empty set of fields that are changed under the symmetry operation occupies a d-dimensional region

<u>Example</u>: Two dimensional compass model.

$$H = J \sum_{\vec{r}} (\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y}).$$

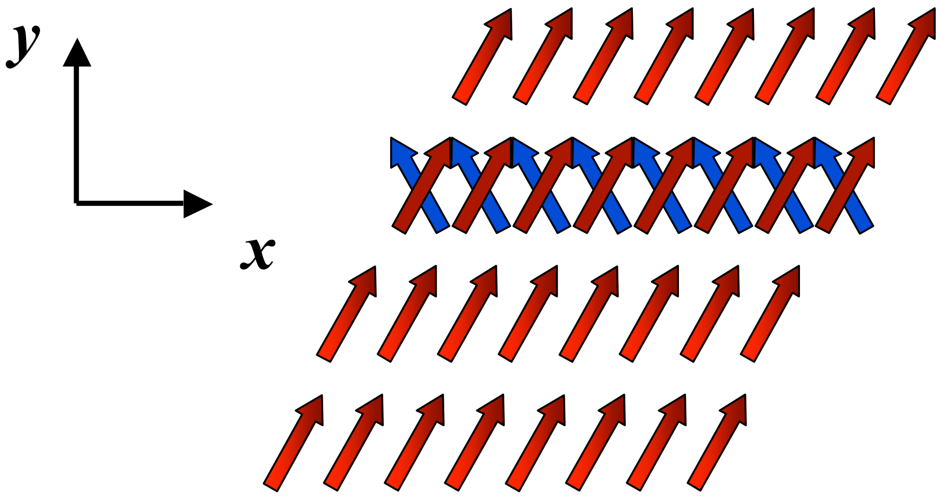
d=1 symmetry:

$$\hat{O}_{P;y} \equiv \prod_{\vec{r} \in P} \sigma^y_{\vec{r}}$$

$$\hat{O}_{P;x} \equiv \prod_{\vec{r} \in P} \sigma^x_{\vec{r}}$$

Intermediate Symmetries

$$H = J \sum_{\vec{r}} (\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y}).$$



Rotation by π
around the y-axis

(P+iP) Superconducting Arrays

A related system emulating (p+ip) superconducting grains, such as those of Sr_2RuO_4, is (C. Xu, J. Moore)

$$H = -K\sum_{\square}\sigma_i^z\sigma_j^z\sigma_k^z\sigma_l^z - h\sum_i \sigma_i^x$$

with the four spin product a product of all spins common to a given plaquette $\square$.

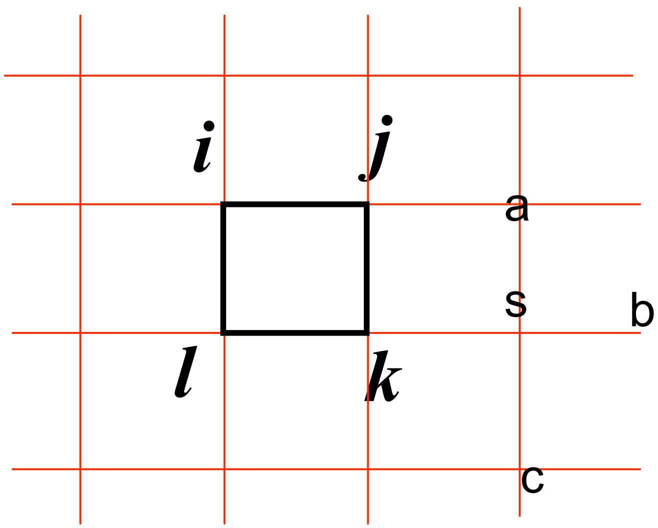
The symmetries are identical to those of the orbital compass model.

Kitaev’s toric code model

$$H = -\sum_s A_s - \sum_p B_p$$

$$A_s = \sigma^x_{sa}\sigma^x_{sb}\sigma^x_{sc}\sigma^x_{sl}$$

$$B_p = \sigma^z_{ij}\sigma^z_{jk}\sigma^z_{kl}\sigma^z_{li}$$



Here, we have two d=1 Ising symmetries

$$X_{1,2} = \prod_{C_{1,2}} \sigma^x_{i\in C_{1,2}}$$

$$Z_{1,2} = \prod_{C'_{1,2}} \sigma^z_{i\in C'_{1,2}}$$

Theorem on Dimensional Reduction

The absolute mean value of any local quantity which is not invariant under a **d-dimensional symmetry group G** of the **D-dimensional** Hamiltonian H is equal or smaller than the absolute mean value of the same quantity computed for a **d-dimensional** Hamiltonian H that preserves the range of interactions in H.

Theorem on Dimensional Reduction

Elements of the group G: $$\mathbf{U}_{lk} = \prod_{\mathbf{i} \in \mathcal{C}_l} \mathbf{g}_{\mathbf{i}k}$$

Non-invariance: $$\mathcal{C}_l \subset \Lambda \text{ and } \Lambda = \cup_l \mathcal{C}_l$$

Definition: $$\sum_k f\left[\mathbf{g}_{\mathbf{i}k}(\phi_{\mathbf{i}})\right] = 0$$

$$\langle f(\phi_{\mathbf{i}}) \rangle = lim_{h \to 0} lim_{N \to \infty} \langle f(\phi_{\mathbf{i}}) \rangle_{h,N}$$

h is a symmetry breaking field

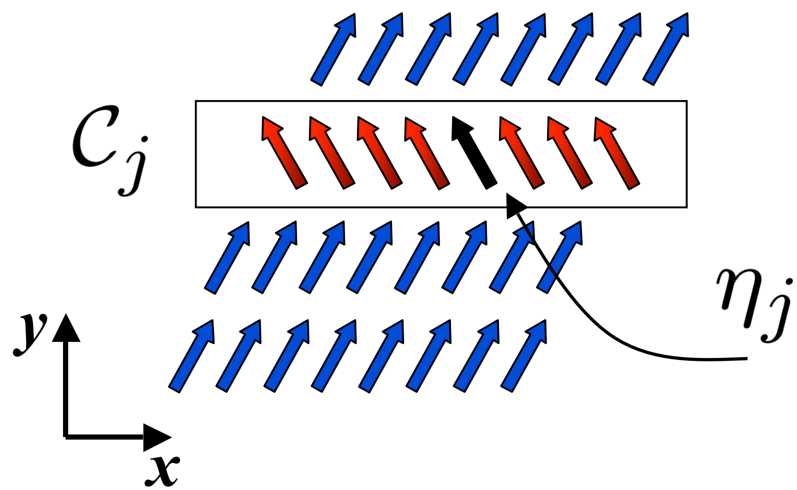
N is the system size

Theorem on Dimensional Reduction

$$\langle f(\phi_{\mathbf{j}})\rangle_{h,N} = \frac{\sum_{\{\phi_{\mathbf{i}}\}} f(\phi_{\mathbf{j}}) e^{-\beta H(\phi)} e^{-\beta h \sum_{\mathbf{i}} \phi_{\mathbf{i}}}}{\sum_{\{\phi_{\mathbf{i}}\}} e^{-\beta H(\phi) - \beta h \sum_{\mathbf{i}} \phi_{\mathbf{i}}}} =$$

$$\frac{\sum_{\{\psi_{\mathbf{i}}\}} z_{\{\psi\}} e^{-\beta h \sum_{\mathbf{i} \notin \mathcal{C}_j} \psi_{\mathbf{i}}} \left[\sum_{\{\eta_{\mathbf{i}}\}} \frac{f(\eta_{\mathbf{j}}) e^{-\beta H(\phi) - \beta h \sum_{\mathbf{i} \in \mathcal{C}_j} \eta_{\mathbf{i}}}}{z_{\{\psi\}}}\right]}{\sum_{\{\psi_{\mathbf{i}}\}} z_{\{\psi\}} e^{-\beta h \sum_{\mathbf{i} \notin \mathcal{C}_j} \psi_{\mathbf{i}}}}$$

$$H = J \sum_{\vec{r}} \left(\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y}\right)$$



 $\equiv \psi_i$  $\equiv \eta_i$

Theorem on Dimensional Reduction

$$z_{\{\psi\}} = \sum_{\{\eta_{\mathbf{i}}\}} e^{-\beta H(\psi,\eta) - \beta h \sum_{\mathbf{i}\in\mathcal{C}_j} \eta_{\mathbf{i}}}$$

$$|\langle f(\phi_{\mathbf{i}})\rangle_{h,N}| \leq \left| \frac{\sum_{\{\eta_{\mathbf{i}}\}} f(\eta_{\mathbf{i}}) e^{-\beta H(\bar{\psi},\eta) - \beta h \sum_{\mathbf{i}\in\mathcal{C}_j} \eta_{\mathbf{i}}}}{z_{\{\bar{\psi}\}}} \right|$$

$$\boxed{\bar{H}(\eta) = H(\bar{\psi}, \eta)}$$

Corollaries

Corollary I: **Elitzur's theorem**.

Corollary II: Any local quantity that is **not** invariant under a **d=1** symmetry group has a **vanishing mean value at any finite temperature if the interactions are short ranged**.

Corollary III: Any local quantity that is **not** invariant under a **d=2 continuous** symmetry group has a **vanishing mean value at any finite temperature if the interactions are short ranged**.

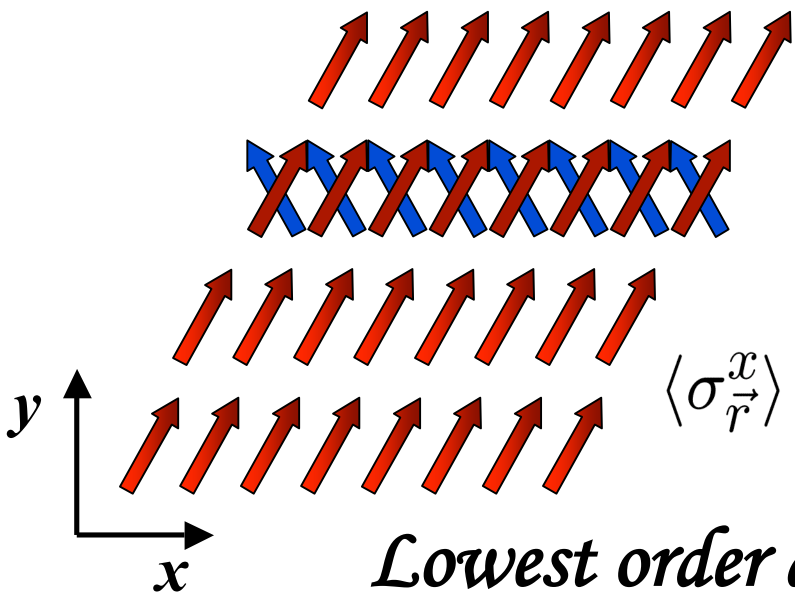
Corollaries

Corollary IV: If the energy spectrum displays a gap between the (degenerate) ground state sector and the excited states in any system displaying an emergent (low energy) continuous d=1 or 2 symmetries then the expectation value of any local quantity which is not invariant under this symmetry strictly vanishes at zero temperature

Example of application

Orbital Compass Model

$$H = J\sum_{\vec{r}}(\sigma^x_{\vec{r}}\sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}}\sigma^y_{\vec{r}+\hat{e}_y})$$



Rotation by π around the y-axis ➡

$$\langle\sigma^x_{\vec{r}}\rangle = \langle\sigma^y_{\vec{r}}\rangle = \langle\sigma^z_{\vec{r}}\rangle = 0 \; for \; T > 0$$

Lowest order allowed order parameter:

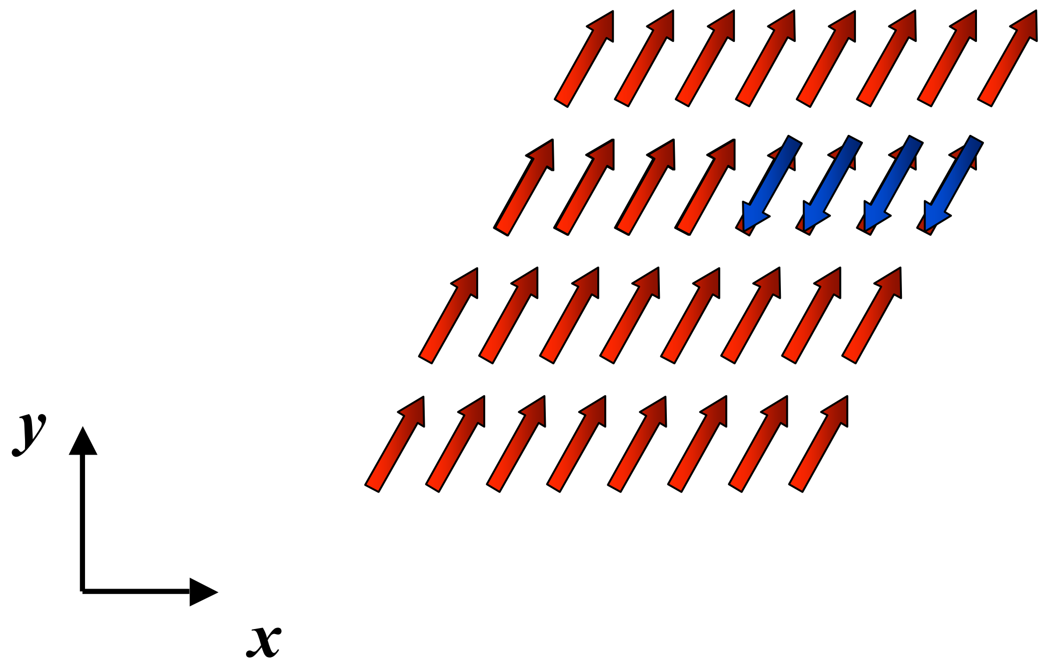
Nematic:

$$\langle\sigma^x_{\vec{r}}\sigma^x_{\vec{r}+\hat{e}_x} - \sigma^y_{\vec{r}}\sigma^y_{\vec{r}+\hat{e}_y}\rangle$$

Intuitive Physical Picture

Orbital Compass Model

$$H = J \sum_{\vec{r}} (\sigma^x_{\vec{r}} \sigma^x_{\vec{r}+\hat{e}_x} + \sigma^y_{\vec{r}} \sigma^y_{\vec{r}+\hat{e}_y})$$



A soliton has a local energy cost.

Fractionalization

Any combination of non-symmetry invariant correlators

$$\left\langle \prod_{i \in \Omega_j} \phi_i \right\rangle$$

with $\Omega_j \subset C_j$ is bounded by the expectation value of the same combination of fields in a lower dimensional system defined by C_j in the presence of transverse non-symmetry breaking external fields. In some instances, this is strongly suggestive of fractionalization. Quasi-particle (qp) weights in high dimension bounded by those in the lower dimensional system (which vanish): no resonant (qp) contributions.

Theorem linking topological order and symmetries

Any system which displays T=0 Topological Quantum Order (TQO) and has interactions of finite range and strength in which all ground states can be linked by discrete d <2 symmetries or continuous d <3 symmetries, has TQO at all temperatures T>0. Gapped systems with continuous d<3 symmetries have TQO at all temperatures

Theorem linking topological order and symmetries

Proof: For any symmetry $U \in G_d$, we separate $V = V_{G,0} + V_{G,\perp}$ where

$$[V_{G,0}, U] = 0$$

and

$$\int dU U^{\dagger} V_{G,\perp} U = 0$$

Theorem linking topological order and symmetries

Singlet component:

$$[V_{G,0}, U] = 0$$

$$\langle V_{G,0} \rangle_\alpha = \frac{\sum_a \langle a | V_{G,0} | a \rangle e^{-\beta(E_a + \phi_\alpha^a)}}{\sum_a e^{-\beta(E_a + \phi_\alpha^a)}}$$

$$= \frac{\sum_a \langle a | U^\dagger V_{G,0} U | a \rangle e^{-\beta(E_a + \phi_{U^\dagger\beta}^a)}}{\sum_a e^{-\beta(E_a + \phi_{U^\dagger\beta}^\alpha)}}$$

$$= \frac{\sum_b \langle b | V_{G,0} | b \rangle e^{-\beta(E_b + \phi_\beta^b)}}{\sum_b e^{-\beta(E_b + \phi_\beta^b)}} = \langle V_{G,0} \rangle_\beta$$

ϕ_α^a monitors effect of boundary conditions favoring state α

Theorem linking topological order and symmetries

Non symmetry invariant component: $[V_{G,\perp}, U] \neq 0$

By our generalized Elitzur theorem:

$$\left\langle V_{G,\perp} \right\rangle_{\alpha} = 0$$

Putting all of the pieces together:

$$\left\langle V \right\rangle_{\alpha} = \left\langle V \right\rangle_{\beta}$$

for any local operator V

We thus have TQO.

Theorem linking topological order and symmetries

In a gapped system with d<3 continuous symmetries for all non symmetry invariant component:

$$[V_{G,\perp}, U] \neq 0$$

and $$\left\langle V_{G,\perp} \right\rangle_{\alpha} = 0$$ also at T=0

Putting all of the pieces together: $$\left\langle V \right\rangle_{\alpha} = \left\langle V \right\rangle_{\beta}$$

for any local operator V.

We thus have T=0 TQO (and thus also T>0 TQO)

Intermediate symmetries & Topological Order

We can use selection rules associated with the d dimensional symmetries to ensure TQO in a set of states.

d - dimensional SU(2): For a quasi-local operator

$$\left\langle jm \,|\, V \,|\, j'm' \right\rangle = 0$$

if

$$|\, j - j' \,| > r \quad \text{or} \quad |\, m - m' \,| > r$$

with r the range (the number of local operators appearing in the product(s) of local operators which make V). General matrix elements of V between different eigenstates of d>0 dimensional symmetries are exponentially small in L^d

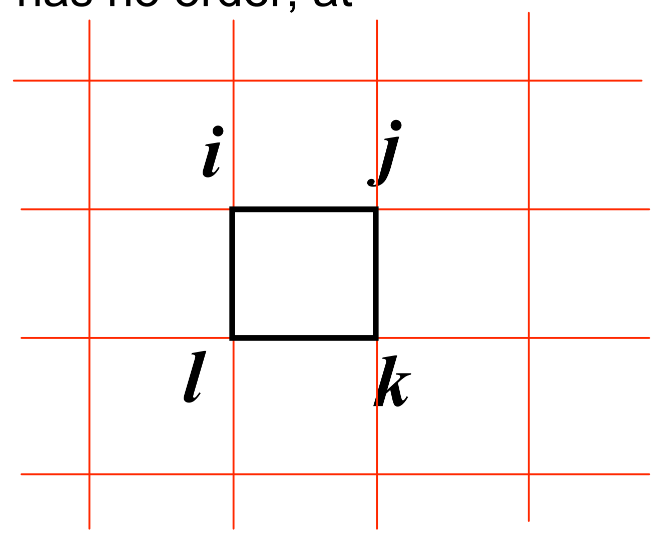
Thermal fragility

TQO is encoded in the eigenstates in a particular representation- not in the energy spectrum.

By a relabelling of commuting variables, Kitaev's model = two decoupled Ising chains. As the 1D Ising chain has no order, at any T>0, the non-local toric code operators

$$X_{1,2} = \prod_{C_{1,2}} \sigma^{x}_{i\in C_{1,2}}$$

$$Z_{1,2} = \prod_{C'_{1,2}} \sigma^{z}_{i\in C'_{1,2}}$$



are not indefinitely stable.

Non-local ("string") correlators

The spin S=1 AKLT chain

$$H = \sum_i [\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}(\vec{S}_i \cdot \vec{S}_{i+1})^2]$$

Has a uniform "string" correlator (irrespective of its length):

$$\left\langle S_i^z \exp[i\pi \sum_{j=i+1}^{k-1} S_j^z] S_k^z \right\rangle = \frac{4}{9}$$

It turns out that all systems with entangled ground states have such correlators. An algorithm for their construction (even when the ground states cannot be determined exactly) exists for general gapped systems.

Non-local ("string") correlators

The spin S=1 AKLT chain is not topologically ordered. Different ground states can be differentiated by local measurements. E.g. the measurement of an edge spin

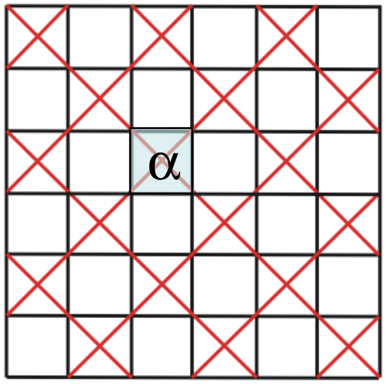
$$\frac{2}{3} = \left\langle g_1 \middle| S_1^z \middle| g_1 \right\rangle \neq \left\langle g_2 \middle| S_1^z \middle| g_2 \right\rangle = -\frac{2}{3}$$

General string (or higher dimensional "brane" correlators appear in general entangled systems- they do not mandate TQO.

An exact solution: Topological invariants at a deconfined quantum critical point of a pyrochlore antiferromagnet

Klein models on Checkerboard and Pyrochlore Lattices

$$H = J \sum_{\langle ij \rangle, \alpha} H_{ij}^{\alpha} + K \sum_{\alpha} (H_{ij}^{\alpha} H_{kl}^{\alpha} + H_{il}^{\alpha} H_{jk}^{\alpha} + H_{ik}^{\alpha} H_{jl}^{\alpha}),$$



where $H_{ij}^{\alpha} = \vec{S}_i^{\alpha} \cdot \vec{S}_j^{\alpha}$. We can rewrite H as:

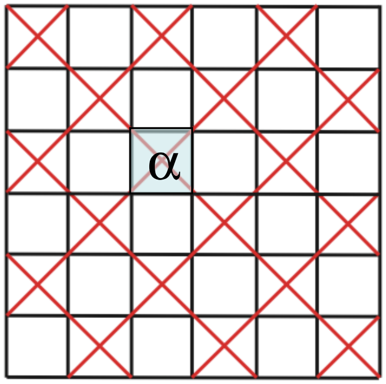
$$H = \frac{J_1}{2} \sum_{\square} \vec{S}_{\square}^2 + \frac{J_2}{4} \sum_{\square} \vec{S}_{\square}^4,$$

For $K = K_c = 4J/5$, H becomes a (semi-positive definite) Klein Hamiltonian:

$$H_K = \frac{12}{5} J \sum_{\square} P^{S_T = 2}$$

Klein models on Checkerboard and Pyrochlore Lattices

$$H = J \sum_{\langle ij \rangle, \alpha} H_{ij}^{\alpha} + K \sum_{\alpha} (H_{ij}^{\alpha} H_{kl}^{\alpha} + H_{il}^{\alpha} H_{jk}^{\alpha} + H_{ik}^{\alpha} H_{jl}^{\alpha}),$$



where $H_{ij}^{\alpha} = \vec{S}_i^{\alpha} \cdot \vec{S}_j^{\alpha}$. We can rewrite H as:

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$$H_K = \frac{12}{5} J \sum_{\square} P^{S_T = 2}$$

Hubbard model at half-filling

$$H_{Hubbard} = -t \sum_{\langle ij \rangle,\sigma} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$$H_{Hubbard} = H + J_3 \sum_{<<ij>>} \vec{S}_i \cdot \vec{S}_j$$

$$J_1 = \frac{4t^2}{U} - \frac{160t^4}{U^3} + O\left(\frac{t^6}{U^5}\right)$$

$$J_2 = \frac{40t^4}{U^3} + O\left(\frac{t^6}{U^5}\right)$$

$$J_3 = \frac{4t^4}{U^3} + O\left(\frac{t^6}{U^5}\right)$$

Rules for constructing all ground states

Spin of single site

$$1/2 \otimes 1/2 \otimes 1/2 \otimes 1/2$$

$$= 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 2$$

Total spin of plaquette

As for any two spins: $1/2 \otimes 1/2 = 0 \oplus 1$

If, in any plaquette α, at least two spins bind into a singlet state then in each of these plaquettes the total spin

$S_{\alpha}^{T} < 2$ and the total energy of the system is zero

Rules for constructing all ground states

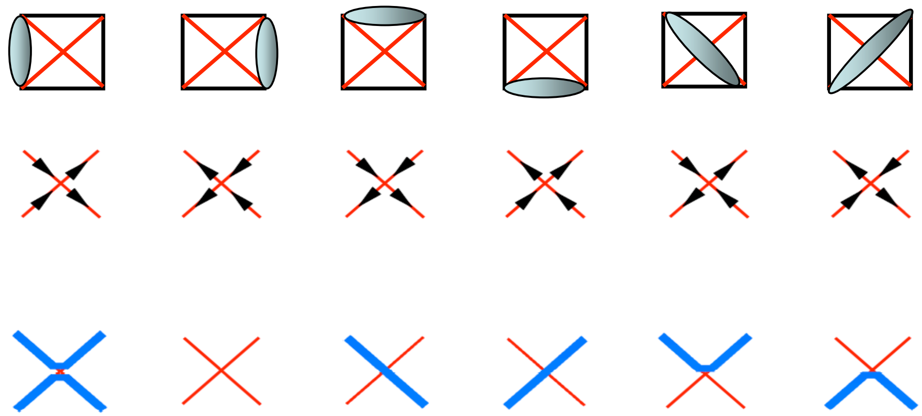
What do such states look like?

A general superposition of all singlet dimer coverings
with (at least) one dimer per plaquette.

It can be proven that these dimer coverings exhaust
all of the Klein model ground states.
Z. Nussinov, cond-mat/0606075

A spectral gap between these and all other
excited states can be proven on decorated lattices
K. Raman et al., PRB 72, 64413 (2005)

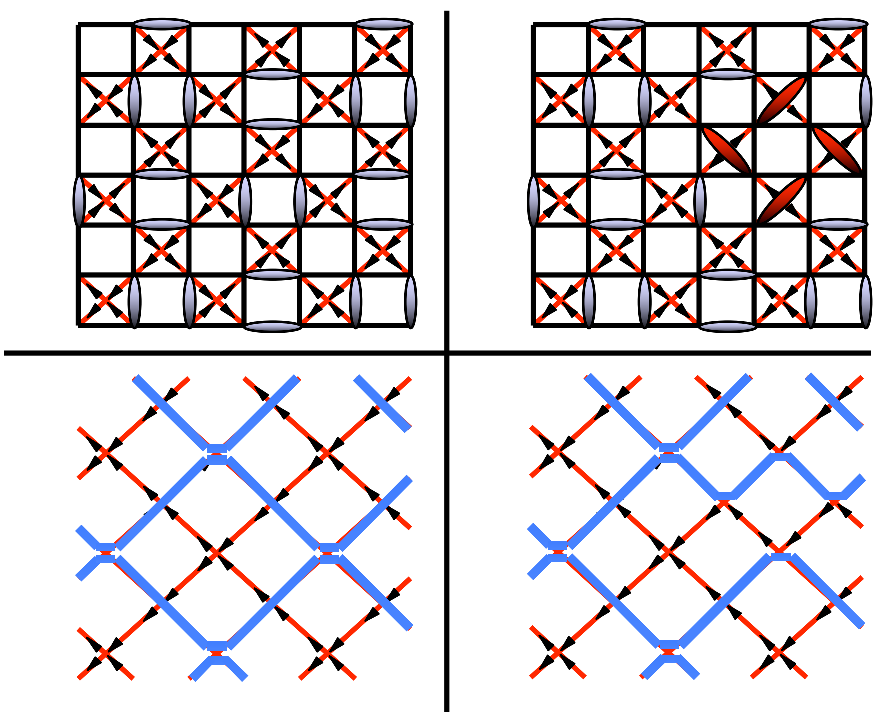
Six Vertex Representation



Vertex Representation

Line Representation

Ice rule leads to six zero-divergence configurations



The six vertex model is exactly solvable! [R. J. Baxter, *Exactly Solved Models in Statistical Mechanics*, (Academic Press, London, 1982).]

Emergent local symmetry

Number of “lines” = topological invariant.

Extensive configurational entropy

The number of dimer coverings is exponential in the volume (a consequence of local [gauge] symmetry):

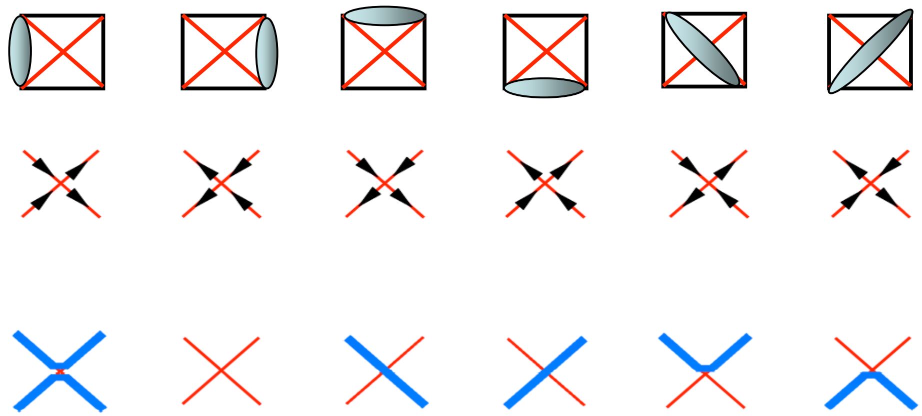
Checkerboard lattice:

$$S = \frac{3N}{4} \ln\left[\frac{4}{3}\right]$$

Pyrochlore lattice:

$$S > \frac{N}{2} \ln\left[\frac{3}{2}\right]$$

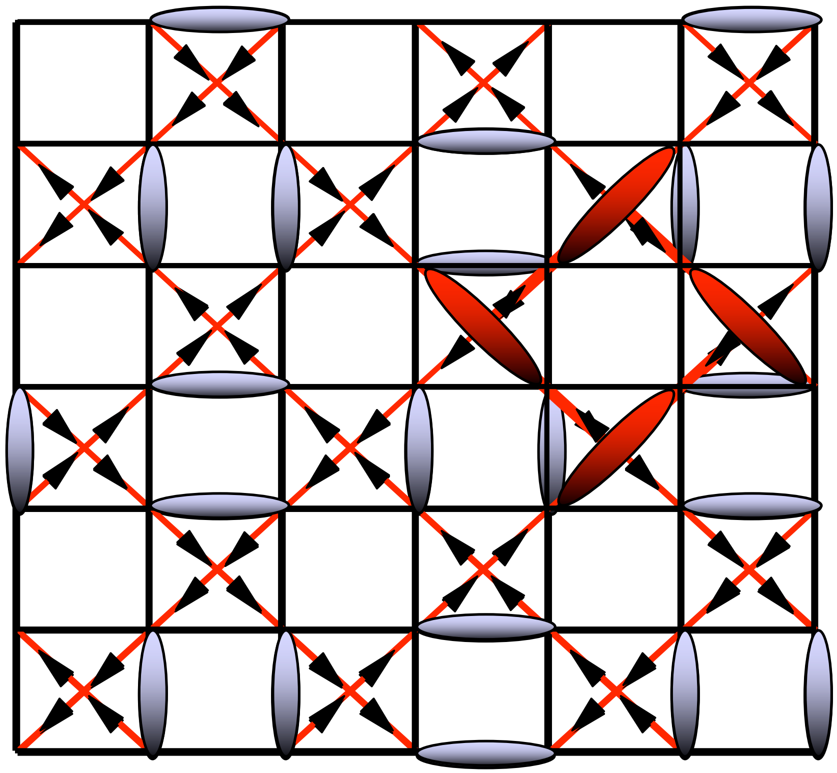
Critical correlations



“Polarization” P = (1,0,0) (-1,0,0) (0,1,0) (0,-1,0) (0,0,1) (0,0,-1)

$$\left\langle P_i(0).P_j(r)\right\rangle \sim |\, r\,|^{-3}$$

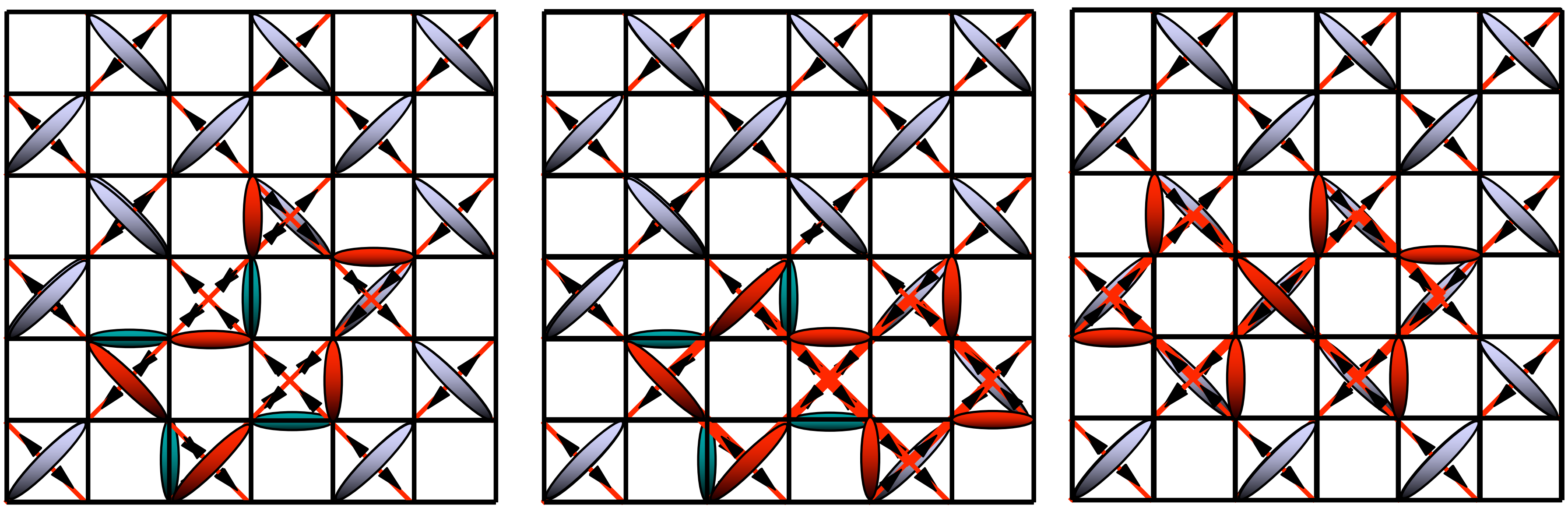
Perturbations and Dimer Models



This Rokhsar-Kivelson (RK) term is the minimal process that connects two different dimer coverings. The low energy theory in the presence of a perturbation that moves the system away from the Klein point is a dimer model on a non-orthogonal dimer basis.

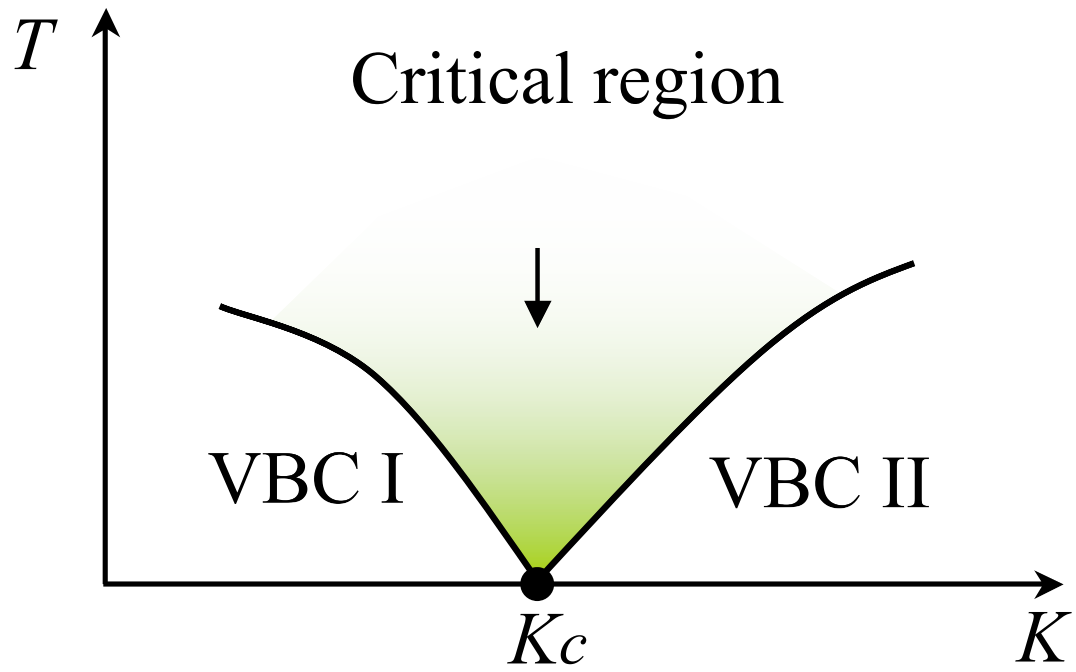
Perturbations and Dimer Models

The RK process cancels out exactly for the most natural perturbation: $K \neq K_c = 4J/5$. In this case, the next minimal processes that contributes with a non-zero matrix element are:



The RK process similarly cancels on of the pyrochlore lattice- topological origin for the cancellation of simplest loops

Thermally Driven Deconfinement



T=0 Critical Point

Dualities

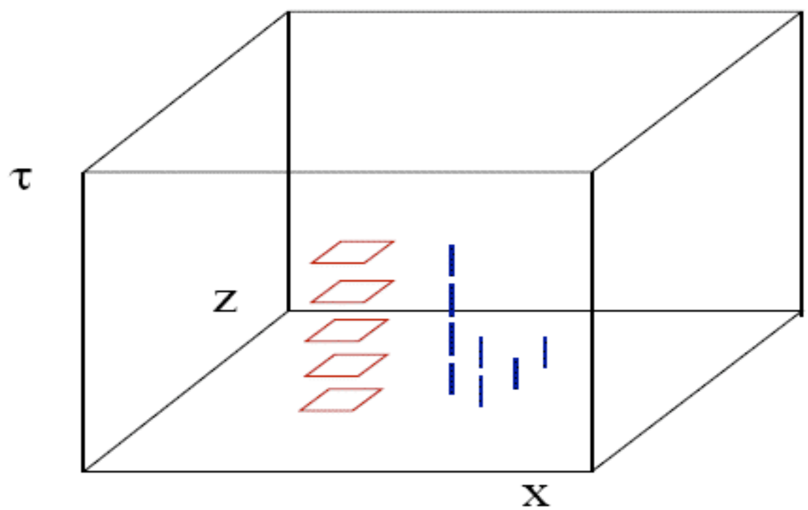
Several of the studied intermediate symmetry systems are dual to each other:

(p+ip) lattice ←→ orbital compass model

In the path integral formulation, these and many other dualities correspond to Z_2 geometrical reflections in space-time.

Dualities

Space-time action schematic for the orbital compass model and p+ip arrays



$$S_{p+ip} = -K\sum_{xz\square}\sigma_i\sigma_j\sigma_k\sigma_l - J\sum_{i}\sigma_i\sigma_{i+e_\tau}$$

$$S_{compass} = -K\sum_{x\tau\square}\sigma_i\sigma_j\sigma_k\sigma_l - J\sum_{z}\sigma_i\sigma_{i+e_z}$$

Conclusions

- Effective dimensional reduction (insofar as symmetry breaking is concerned) occurs in systems with intermediate symmetries. Such systems include liquid crystalline phases of Quantum Hall systems, orbital systems, geometrically frustrated spin systems, ring exchange systems and Bose metals, and models of (p+ip) superconducting arrays. Some of these systems exhibit fractionalization and topological order.

Conclusions (continued)

- Several of these well studied systems are dual to each other. By considering spatial reflection symmetries and discrete space-time rotations, we obtain, in a new transparent unified geometrical way, several new strong coupling- weak coupling dualities.

- Entropic “order our disorder” effects may stabilize symmetry allowed orders. In particular, orbital models rigorously display order even without the incorporation of zero point quantum fluctuations.

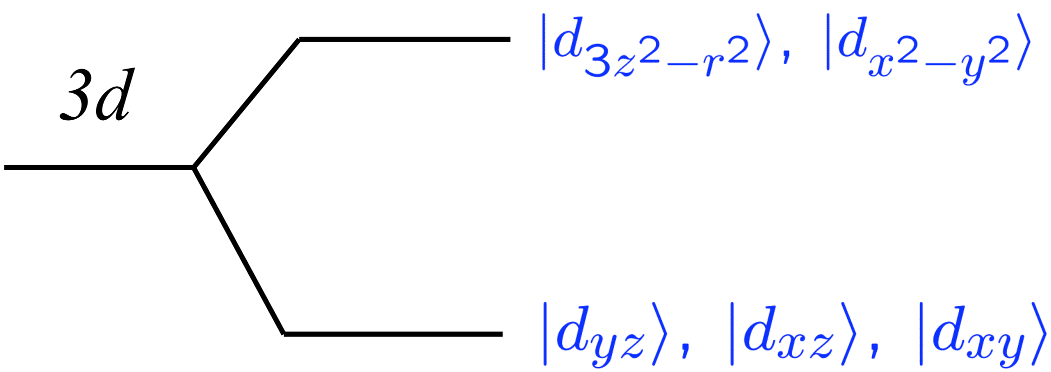
Super-exchange in 3d orbital systems

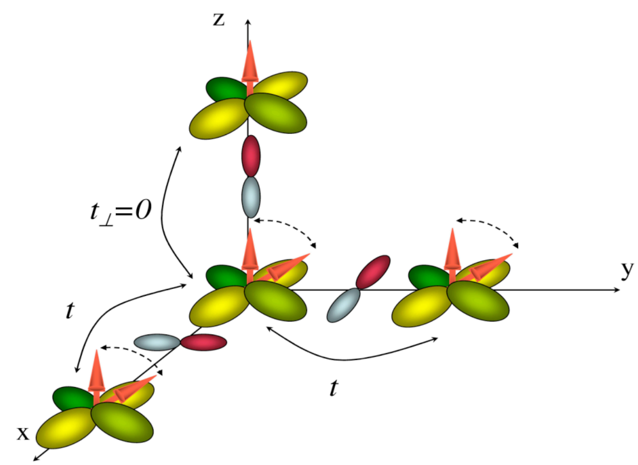
Kugel-Khomskii Model

$$H = H_X + H_Y + H_Z$$

$$H_\alpha = J \sum_{\langle ij \rangle \in \alpha} \sum_{\beta,\gamma \neq \alpha} \sum_{\sigma\eta} c^\dagger_{i,\beta,\sigma} c_{i,\gamma,\eta} c^\dagger_{j,\gamma,\eta} c_{j,\beta,\sigma}.$$

Cubic crystal field





(A. B. Harris et al., PRL 91, 87206 (2003))

Our Experimental Prediction

If the KK Hamiltonian captures much of the physics then the d=2 SU(2) symmetry invariant nematic spin order should be far more robust than the non symmetry invariant magnetization.

Nematic order persists to high temperatures

In the presence of orbital ordering in the $|\alpha\rangle$ state, superpositions of scalar products along $\eta = x, y, z$ where $\eta \neq \alpha$ need not vanish.

The “120 degree orbital Hamiltonian”

JT contributions/orbital component of superexchange lead to an effective XY Hamiltonian in the two dimensional (S=1/2) isospin basis spanned by the two e_g states

$$\left|d_{3z^2-r^2}\right\rangle, \left|d_{x^2-y^2}\right\rangle$$

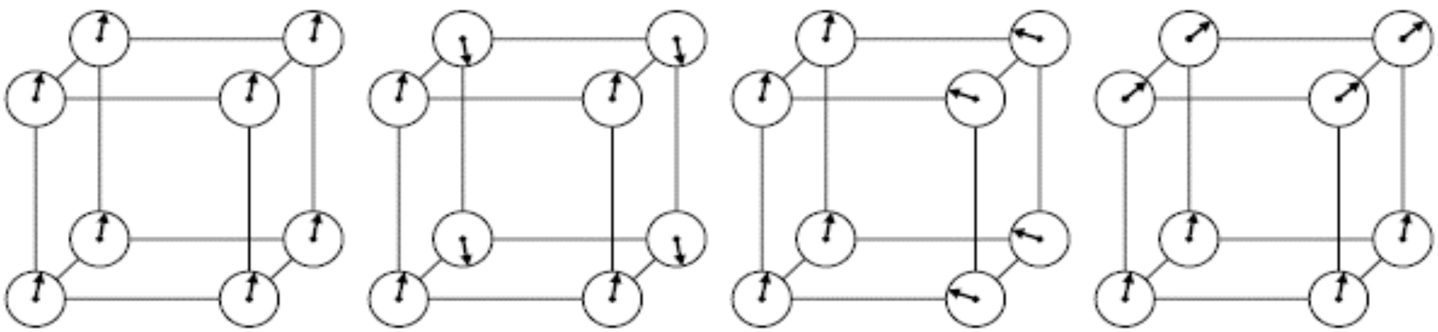
$$\hat{a} = (1,0), \hat{b} = (-\frac{1}{2}, \frac{\sqrt{3}}{2}), \hat{c} = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

$$S_r^{(\alpha=a,b,c)} = \vec{S}_r \bullet \hat{\alpha}$$

$$H = -J\sum_r (S_r^{(a)} S_{r+\hat{e}_x}^{(a)} + S_r^{(b)} S_{r+\hat{e}_y}^{(b)} + S_r^{(c)} S_{r+\hat{e}_z}^{(c)})$$

Symmetries of the 120 degree orbital Hamiltonian

Discrete symmetries of the 120 degree model on a single cube with one spin fixed.



Formal classification of the symmetries of the 120 degree orbital Hamiltonian

The symmetries have the form

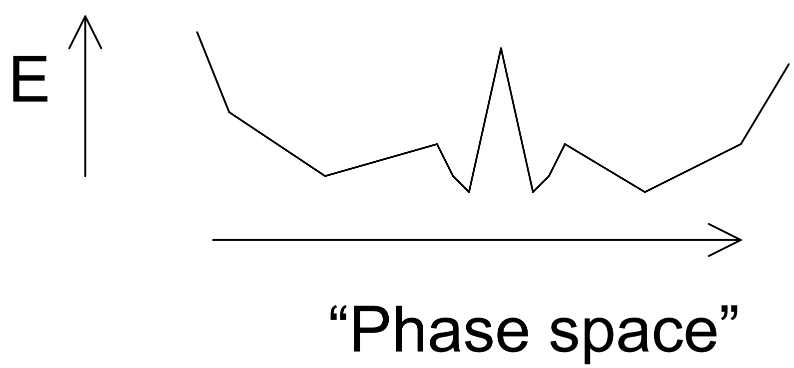
$$O_x = \prod_{r \in P_x} \sigma_r^a, O_y = \prod_{r \in P_y} \sigma_r^b, O_z = \prod_{r \in P_z} \sigma_r^c$$

Here, P_x denotes a yz plane (any plane orthogonal to the x-axis).

The planar symmetries that we found are discrete d=2 dimensional symmetries.

Discrete Symmetry breaking in the 120 degree model

As the symmetries are discrete Ising like d=2 symmetries, similar to the two dimensional Ising model, symmetry breaking can occur. We indeed proved that this is the case for the classical (infinite S) variant of this model. The physical engine behind this order in this highly degenerate system is an “order by disorder” mechanism.



As there is more phase space Volume for fluctuations about some ground states relative to others, those states may be stabilized by entropic fluctuations

Subtle facets of "Order by Disorder" in the 120 degree model

Uniform polarization on all sites i :

$$\vec{S}_i = \pm S\hat{a}, \vec{S}_i = \pm S\hat{b}, \vec{S}_i = \pm S\hat{c}$$

New results: no zero point quantum (1/S) corrections are mandatory to lift the degeneracy, temperature independent surface tension

Experimentally: orbital order may persist to T=O(100K).

Low Energy Penalties and Fractionalization

Deconfinement may occur due to low energy penalties.

Consider, in the basis of x direction of spin polarization,

$$\begin{array}{cccccccc} \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow \\ \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \end{array} \qquad \begin{array}{cccccccc} \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ \downarrow & \uparrow & \downarrow & \uparrow & \mathrm{O} & \uparrow & \downarrow & \uparrow \\ \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \end{array}$$

$$H = -t \sum_{<ij>,\sigma} (c_{i\sigma}^{+} c_{j\sigma} + h.c.) + J \sum_{i} (S_i^z S_{i+e_z}^z + S_i^x S_{i+e_x}^x)$$

Low Energy Penalties and Fractionalization (continued)

After many hops along the x axis, no energy penalties are compounded (no “string” confining potentials)

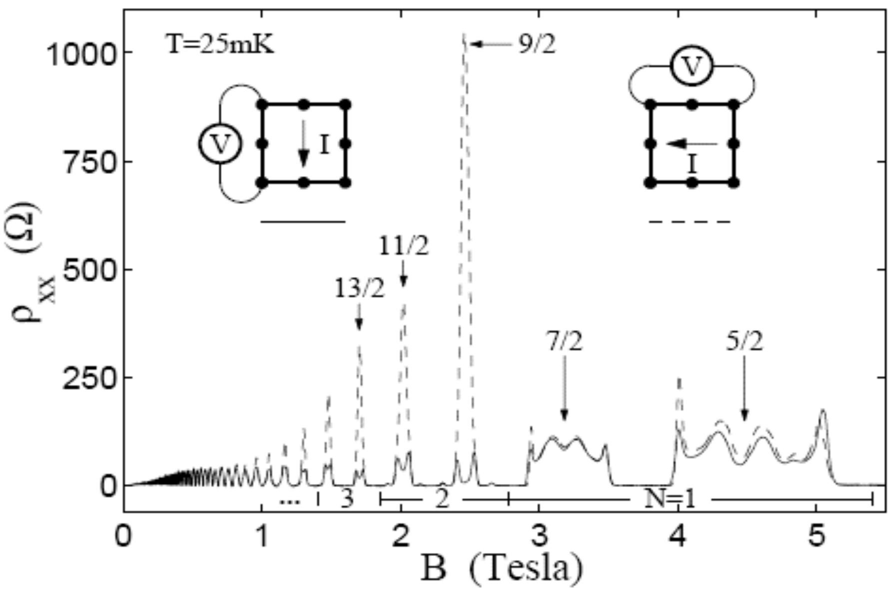
$\uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \; \downarrow$

$\downarrow \; \uparrow \; \downarrow \; \uparrow \; \uparrow \; \downarrow \; \uparrow \; \mathrm{O}$

$\uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \; \downarrow$

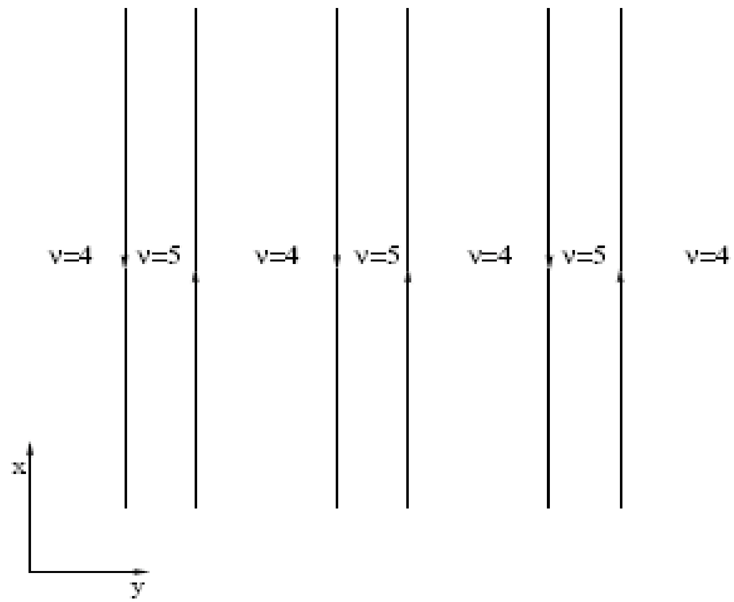
Due to the nature of the interaction there are no energy penalties from neighboring rows.

Quantum Hall Smectics I



The anisotropy of ρ_{xx} in a sample along different directions. From M. P. Lilly et al. (cond-mat/9808227, PRL)

Quantum Hall Smectics II



E. Fradkin and S. Kivelson (cond-mat/9810151, PRL)

Charge density variations $\phi(x, y) \rightarrow \phi(x, y) + f(x)$

lead to no change in energy

Frustrated Magnets

Many frustrated magnets (Checkerboard and Pyrochlore) exhibit d=1 gauge like symmetries