Quantum limited measurement of quantum dots.

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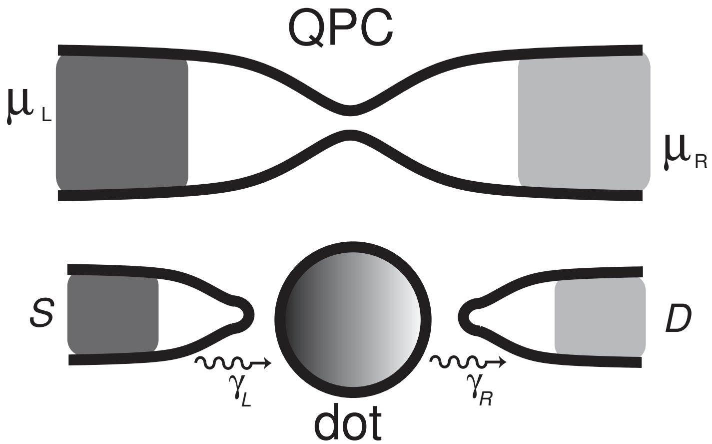
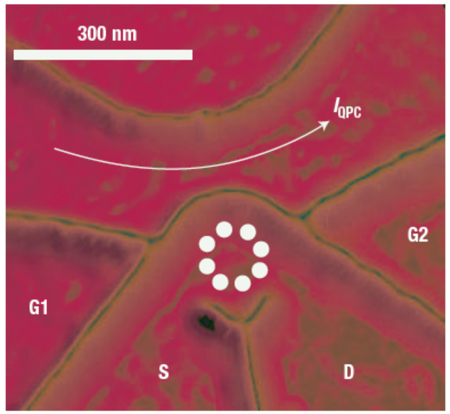
Quantum dots.

Surface gate on 2DEG structures (GaAs/AlGaAs)

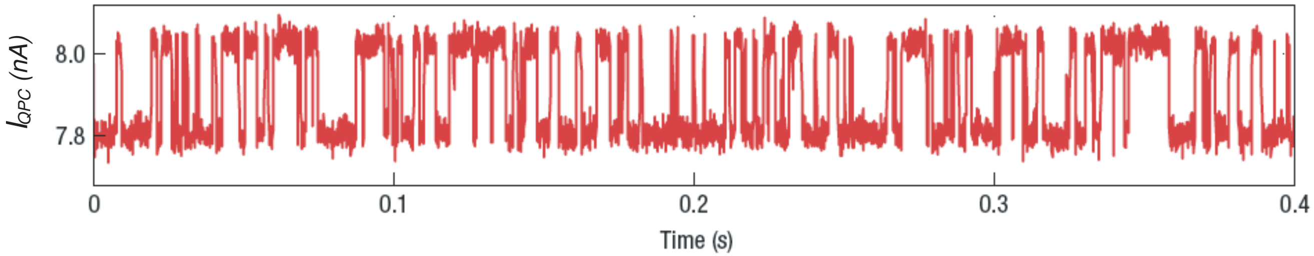




E.V.Sukhorukov, et al. , Nature Physics, **3**, 243 (2007).

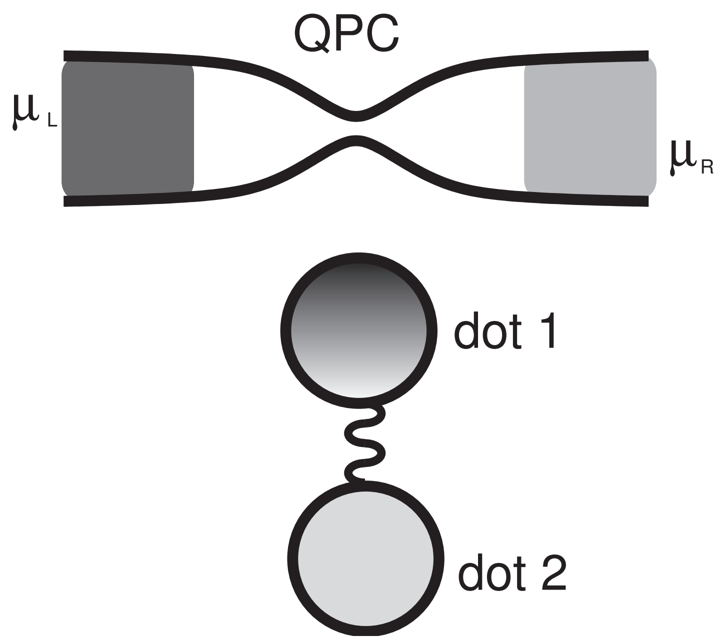
Continuous measurement.

Current through the QPC, a *measurement record.*



E.V.Sukhorukov, et al. , Nature Physics, **3**, 243 (2007).

Measurement of quantum tunnelling.

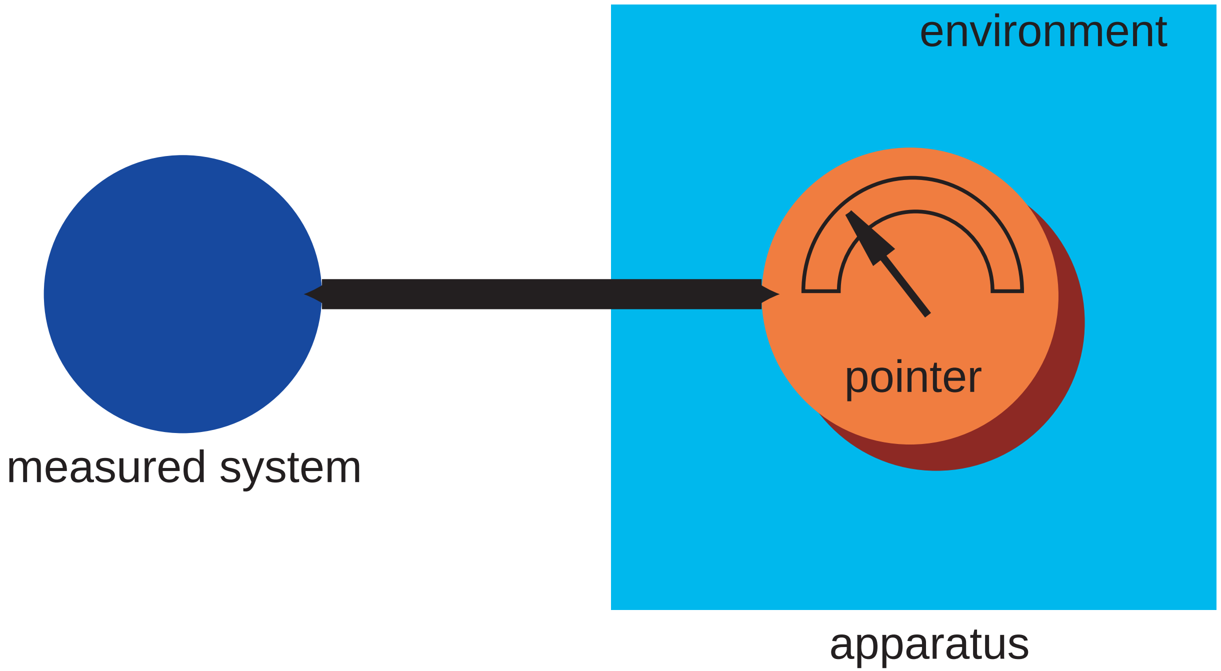


Quantum measurement.

A measured system is an open system.

Reversible interaction: measured system + pointer.

Irreversible interaction: pointer + environment.

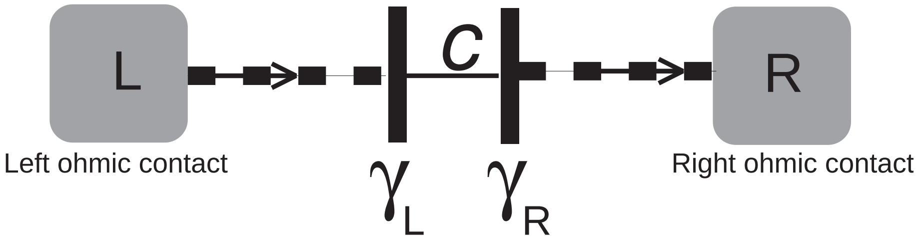


Quantum measurement.

Two questions:

- What is the measured system state *given* a record of measured results: *conditional dynamics*.
- What is the measured system state *averaged* over all measured results: *unconditional dynamics*.

Master equation for a single dot.



$$
\begin{aligned}
\frac{d\rho}{dt} &= \frac{\gamma_L}{2}\left(2c^\dagger\rho c - cc^\dagger\rho - \rho cc^\dagger\right) + \frac{\gamma_R}{2}\left(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c\right) \\
&= \frac{\gamma_L}{2}\mathcal{D}[c^\dagger]\rho + \frac{\gamma_R}{2}\mathcal{D}[c]\rho
\end{aligned}
$$

$$
\mathcal{D}[c^\dagger]\rho = \mathcal{J}[c^\dagger]\rho - \mathcal{A}[c^\dagger]\rho,
$$

$$
\begin{aligned}
\mathcal{J}[c^\dagger]\rho &= c^\dagger\rho c, \qquad \text{jump super operator} \\
\mathcal{A}[c^\dagger]\rho &= (cc^\dagger\rho + \rho cc^\dagger)/2.
\end{aligned}
$$

GJM Aust.J.Phys. **53**, (2000)

Stochastic processes in a single dot.

Dot population:

$$\frac{d\langle n\rangle}{dt} = \gamma_L(1-\langle n\rangle) - \gamma_R\langle n\rangle$$

Define classical stochastic processes

$$\begin{aligned}
[dM_c^b(t)]^2 &= dM_c^b(t), \\
E[dM_c^L(t)] &= \gamma_L\langle cc^\dagger\rangle_c(t)dt = \gamma_L(1-\langle n\rangle_c(t))dt \\
E[dM_c^R(t)] &= \gamma_R\langle c^\dagger c\rangle_c(t)dt = \gamma_R\langle n\rangle_c(t))dt
\end{aligned}$$

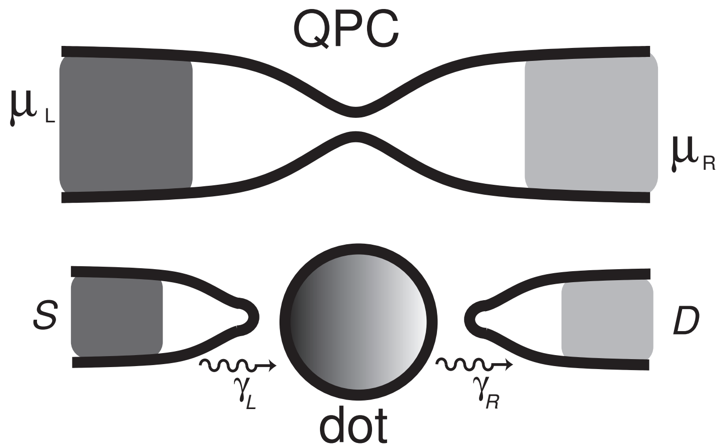
Observed current through the dot;

$$J(t)dt = \frac{e}{2}(dM_c^L(t) + dM_c^R(t))$$

In steady state:

$$J_0 = \frac{e\gamma_L\gamma_R}{\gamma_L+\gamma_R}$$

Measurement with a point contact.



If dot is charged, the QPC tunnel barrier is raised.

Readout the macroscopic current through the point contact.

Pointer variable is number of electrons tunnelling.

Environment is the Fermi reservoirs in left and right leads.

Leads in local equilibrium: chemical potentials μ_L, μ_R.

Hamiltonian.

$$\mathcal{H} = \mathcal{H}_{QD} + \mathcal{H}_{QPC} + \mathcal{H}_{coup}$$

where

$$\begin{aligned}
\mathcal{H}_{QD} &= \hbar\omega c^\dagger c \\
\mathcal{H}_{QPC} &= \hbar\sum_k \left(\omega_k^L a_{Lk}^\dagger a_{Lk} + \omega_k^R a_{Rk}^\dagger a_{Rk}\right) + \sum_{k,q}\left(T_{kq} a_{Lk}^\dagger a_{Rq} + T_{qk}^* a_{Rq}^\dagger a_{Lk}\right), \\
\mathcal{H}_{coup} &= \sum_{k,q} c^\dagger c \left(\chi_{kq} a_{Lk}^\dagger a_{Rq} + \chi_{qk}^* a_{Rq}^\dagger a_{Lk}\right).
\end{aligned}$$

$\mathcal{H}_{PC}$: tunnelling in the QPC.

$\mathcal{H}_{coup}$: QD modified tunnelling in QPC.

c_i, $c_i^\dagger$: Fermi operators in the QD system.

a_{Lk}, a_{Rk} : Fermi operators for the left and right reservoirs.

Unconditional dynamics: master equation.

H.-S. Goan and GJM, Phys. Rev. B **63***, 125326-1 , 2001*

$$
\begin{aligned}
\dot{\rho}(t) &= \gamma_L \mathcal{D}[c^\dagger]\rho + \gamma_R \mathcal{D}[c]\rho + \mathcal{D}[\mathcal{T} + \mathcal{X} n]\rho(t) \\
&\equiv \mathcal{L}\rho(t),
\end{aligned}
$$

$$
\mathcal{D}[B]\rho = \mathcal{J}[B]\rho - \mathcal{A}[B]\rho,
$$

$$
\begin{aligned}
\mathcal{J}[B]\rho &= B\rho B^\dagger, \qquad \text{jump super operator} \\
\mathcal{A}[B]\rho &= (B^\dagger B\rho + \rho B^\dagger B)/2.
\end{aligned}
$$

The measured signal in QPC.

The measured signal: current through the PC, a conditional stochastic point process.

$$I(t) = e\frac{dN(t)}{dt}$$

$$\mathcal{E}(dN(t)) = \mathrm{tr}\left(\mathcal{J}(\mathcal{T} + \chi\hat{n})\rho_c(t)\right)dt$$

$$\overline{I(t)} = e\left[D + (D' - D)\langle\hat{n}\rangle\right]$$

$D = |\mathcal{T}|^2$: tunnel rate when $n = 0$

$D' = |\mathcal{T} + \mathcal{X}|^2$: tunnel rate when $n = 1$.

Steady state current in QPC:

$$I_0 = eD\left(\frac{\gamma_R}{\gamma_L + \gamma_R}\right) + eD'\left(\frac{\gamma_L}{\gamma_L + \gamma_R}\right)$$

Conditional current in QPC

$$I_c(t) = eD + e(D' - D)\langle n \rangle_c(t)$$

Random telegraph process with two current values D, D' and transition rates γ_L, γ_R.

$$E(I(t), I(s)) = e^2(D - D')^2 \frac{\gamma_L \gamma_R}{(\gamma_l + \gamma_R)^2} e^{-(\gamma_R + \gamma_L)|t-s|}$$

$$\begin{aligned} G_{I,J}(\tau) &= \frac{e^2}{2} \mathrm{Tr}\left[(\gamma_L c c^\dagger + \gamma_R c^\dagger c) e^{\mathcal{L}\tau} \mathcal{J}[\rho_\infty]\right] - I_0 J_0 \\ &= \frac{e^2 \gamma_L \gamma_R}{2(\gamma_L + \gamma_R)^2} (\gamma_L - \gamma_R)(D - D') e^{-(\gamma_L + \gamma_R)\tau} \end{aligned}$$

Noise power spectrum in QPC

Define the noise power in the QPC current as

$$P_{QPC} = \int_{-\infty}^{\infty} dt E(I(t), I(0))$$

$$P_{QPC} = \frac{2(D - D')^2 \gamma_L \gamma_R}{(\gamma_L + \gamma_R)^3}$$

Conditional dynamics of the dot

What is the dynamics of the dot given a QPC measurement record?

$$
\begin{aligned}
d\rho_c(t) \;=\; & dN_c(t)\left[\frac{\mathcal{J}[\mathcal{T}+\mathcal{X}n]}{\mathcal{P}_{1c}(t)}-1\right]\rho_c(t) \\
& + dt\left\{-\mathcal{A}[\mathcal{T}+\mathcal{X}n]\rho_c(t)+\mathcal{P}_{1c}(t)\rho_c(t)\right\} \\
& + \gamma_L\mathcal{D}[c^\dagger]\rho_c dt+\gamma_R\mathcal{D}[c]\rho_c dt,
\end{aligned}
$$

where

$$
\mathcal{P}_{1c}(t)=D+(D'-D)\langle n\rangle_c(t).
$$

(recall $I_c(t)=eD+e(D'-D)\langle n\rangle_c(t)$, conditional current in QPC)

Conditional dynamics of dot.

Diffusive limit: $\mathcal{T} >> \chi$, large QPC current.

Approximate Poisson process by diffusion process.

$$\begin{aligned}\dot{\rho}_c(t) &= \gamma_L \mathcal{D}[c^\dagger]\rho_c + \gamma_R \mathcal{D}[c]\rho_c + \mathcal{D}[\mathcal{T} + \mathcal{X} n]\rho_c(t) \\ &\quad + \xi(t)\frac{1}{|\mathcal{T}|}\left[\mathcal{T}^* \mathcal{X}\, n_1 \rho_c(t) + \mathcal{X}^* \mathcal{T} \rho_c(t) n - 2\,\mathrm{Re}(\mathcal{T}^* \mathcal{X})\langle n \rangle_c(t)\rho_c(t)\right].\end{aligned}$$

Assume $\mathcal{T}$ and χ real and opposite sign,

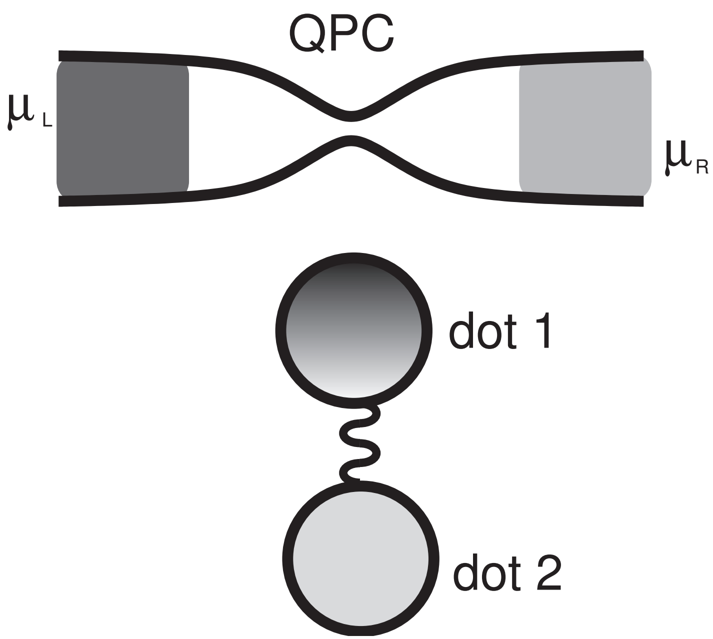
$D = |\mathcal{T}|^2$, $D' = |\mathcal{T} - \chi|^2$.

$$\frac{d\langle n \rangle_c}{dt} = \gamma_L(1 - \langle n \rangle_c) - \gamma_R \langle n \rangle_c - 2\chi(1 - \langle n \rangle_c)\langle n \rangle_c \xi(t)$$

Note: noise turns off at $\langle n \rangle_c = 1$ or 0,

called *localisation*, in other words, quantum state reduction.

Measurement of quantum tunnelling.



Hamiltonian.

$$\mathcal{H} = \mathcal{H}_{CQD} + \mathcal{H}_{PC} + \mathcal{H}_{coup}$$

$$\begin{aligned}
\mathcal{H}_{CQD} &= \hbar\left[\omega_1 c_1^\dagger c_1 + \omega_2 c_2^\dagger c_2 + \Omega(c_1^\dagger c_2 + c_2^\dagger c_1)\right], \\
\mathcal{H}_{PC} &= \hbar \sum_k \left(\omega_k^L a_{Lk}^\dagger a_{Lk} + \omega_k^R a_{Rk}^\dagger a_{Rk}\right) + \sum_{k,q}\left(T_{kq} a_{Lk}^\dagger a_{Rq} + T_{qk}^* a_{Rq}^\dagger a_{Lk}\right), \\
\mathcal{H}_{coup} &= \sum_{k,q} c_1^\dagger c_1 \left(\chi_{kq} a_{Lk}^\dagger a_{Rq} + \chi_{qk}^* a_{Rq}^\dagger a_{Lk}\right).
\end{aligned}$$

$\mathcal{H}_{CQD}$: tunnelling in QD system.

$\mathcal{H}_{PC}$: tunnelling in the PC.

$\mathcal{H}_{coup}$: QD modified tunnelling in PC.

c_i, $c_i^\dagger$: Fermi operators in the QD system.

a_{Lk}, a_{Rk} : Fermi operators for the left and right reservoirs.

Unconditional dynamics: matrix elements.

Measurement of number $\longrightarrow$ decoherence in number basis.

$$\begin{aligned}
\dot\rho_{11}(t) &= i\Omega[\rho_{12}(t) - \rho_{21}(t)] \\
\dot\rho_{12}(t) &= i\mathcal{E}\rho_{12}(t) + i\Omega[\rho_{11}(t) - \rho_{22}(t)] - (|\mathcal{X}|^2/2)\rho_{12}(t) \\
&\quad + i\,\mathrm{Im}(\mathcal{T}^*\mathcal{X})\rho_{12}(t),
\end{aligned}$$

$\hbar\mathcal{E} = \hbar(\omega_2 - \omega_1)$: energy mismatch between the dots,

$\Gamma_d = |\mathcal{X}|^2/2$: the *decoherence rate*,

Conditional dynamics.

Current through the PC: $i(t) = edN(t)/dt$.

Conditional point process:

$$E[dN_c(t)] = \mathrm{Tr}[\tilde{\rho}_{1c}(t+dt)] = [D + (D' - D)\langle n_1 \rangle_c(t)]dt.$$

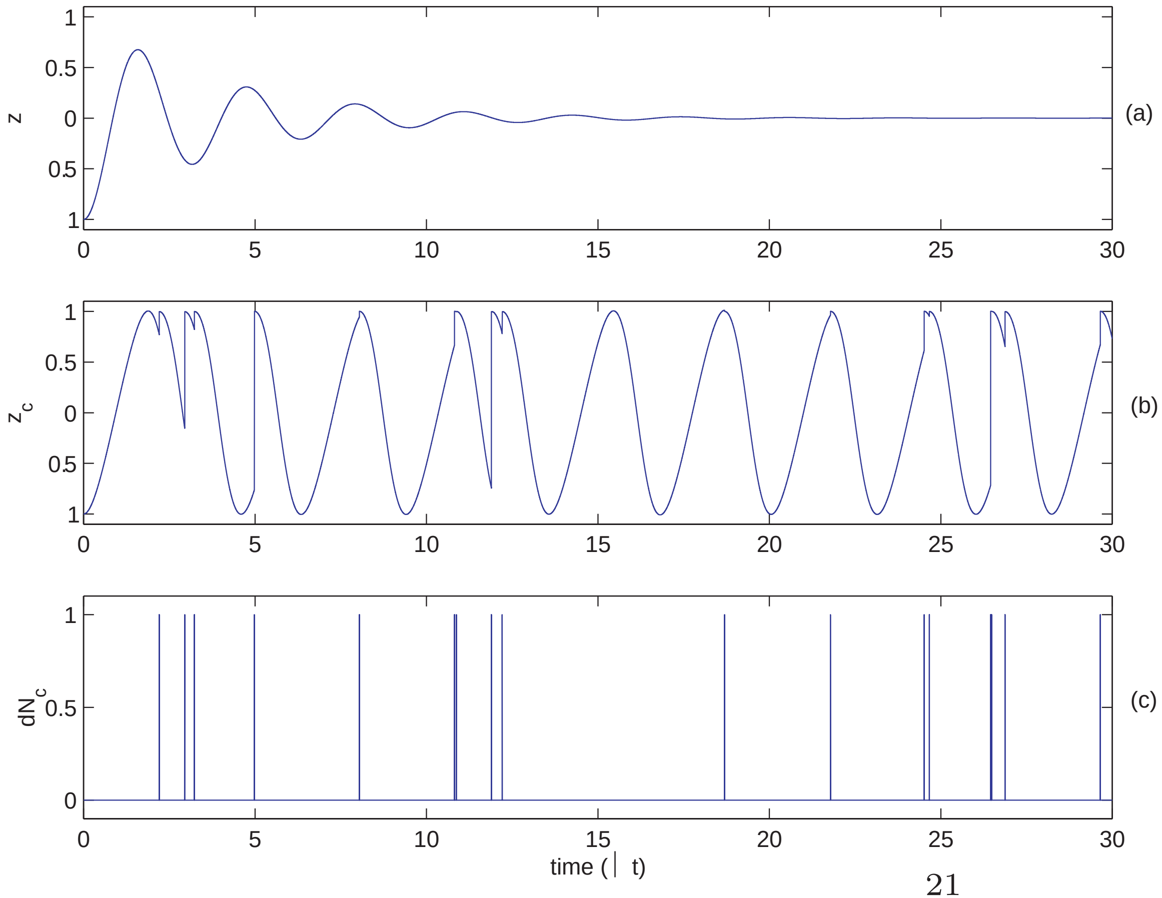
Conditional master equation:

$$\begin{aligned} d\rho_c(t) &= dN_c(t)\left[\frac{\mathcal{J}[\mathcal{T} + \mathcal{X} n_1]}{\mathcal{P}_{1c}(t)} - 1\right]\rho_c(t) \\ &\quad + dt\,\{no\ jump\}\,, \end{aligned}$$

Conditional dynamics.

Quantum jump limit: $D' = |\mathcal{T} + \chi|^2 = 0$.
Can resolve individual tunnelling events.

Define: $z_c(t) = P_c(2,t) - P_c(1,t)$.



ensemble average

conditional

jump record

Conditional dynamics.

Localisation rate: average rate for conditional state to approach an $\hat{n}_1$ eigenstate.
i.e. for $E(z_c(t)^2) = 1$.

$$\gamma_{loc}^{jump} = \frac{(D' - D)^2}{(D' + D)}$$

Good measurement: $D' = 0: \quad \gamma_{loc}^{jump} = \Gamma_d.$

Localisation rate = decoherence rate.

Good measurement: the conditional state approaches an eigenstate of the measured operator at the decoherence rate.

Conditional dynamics.

Diffusive limit: $|\mathcal{T}| >> |\chi|$, individual events cannot be resolved.

$$
\begin{aligned}
\dot{\rho}_c(t) &= -\frac{i}{\hbar}[\mathcal{H}_{CQD}, \rho_c(t)] + \mathcal{D}[\mathcal{T} + \mathcal{X} n_1]\rho_c(t) \\
&\quad + \xi(t)\frac{1}{|\mathcal{T}|}\left[\mathcal{T}^*\mathcal{X}\, n_1\rho_c(t) + \mathcal{X}^*\mathcal{T}\rho_c(t) n_1 - 2\,\mathrm{Re}(\mathcal{T}^*\mathcal{X})\langle n_1\rangle_c(t)\rho_c(t)\right].
\end{aligned}
$$

Define:

$$\rho(t) = [I + x(t)\sigma_x + y(t)\sigma_y + z(t)\sigma_z]/2,$$

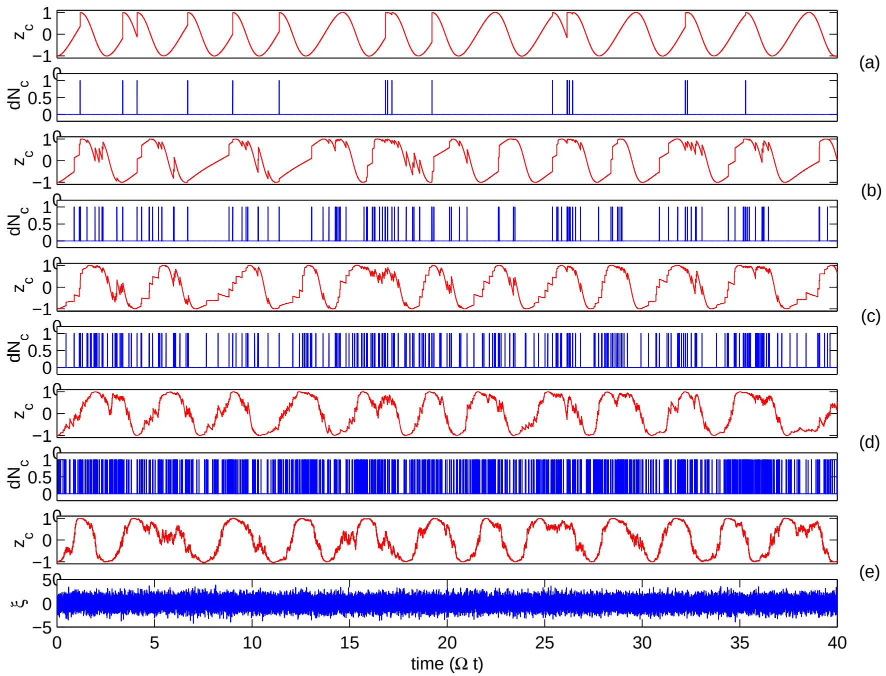
σ_i; Pauli matrices.

Conditional dynamics.

Conditional number difference between the dots: $z_c(t)$,

$$\frac{dz_c(t)}{dt} \;=\; 2\Omega\, y_c(t) - \sqrt{2\Gamma_d}\left[1 - z_c^2(t)\right]\xi(t).$$

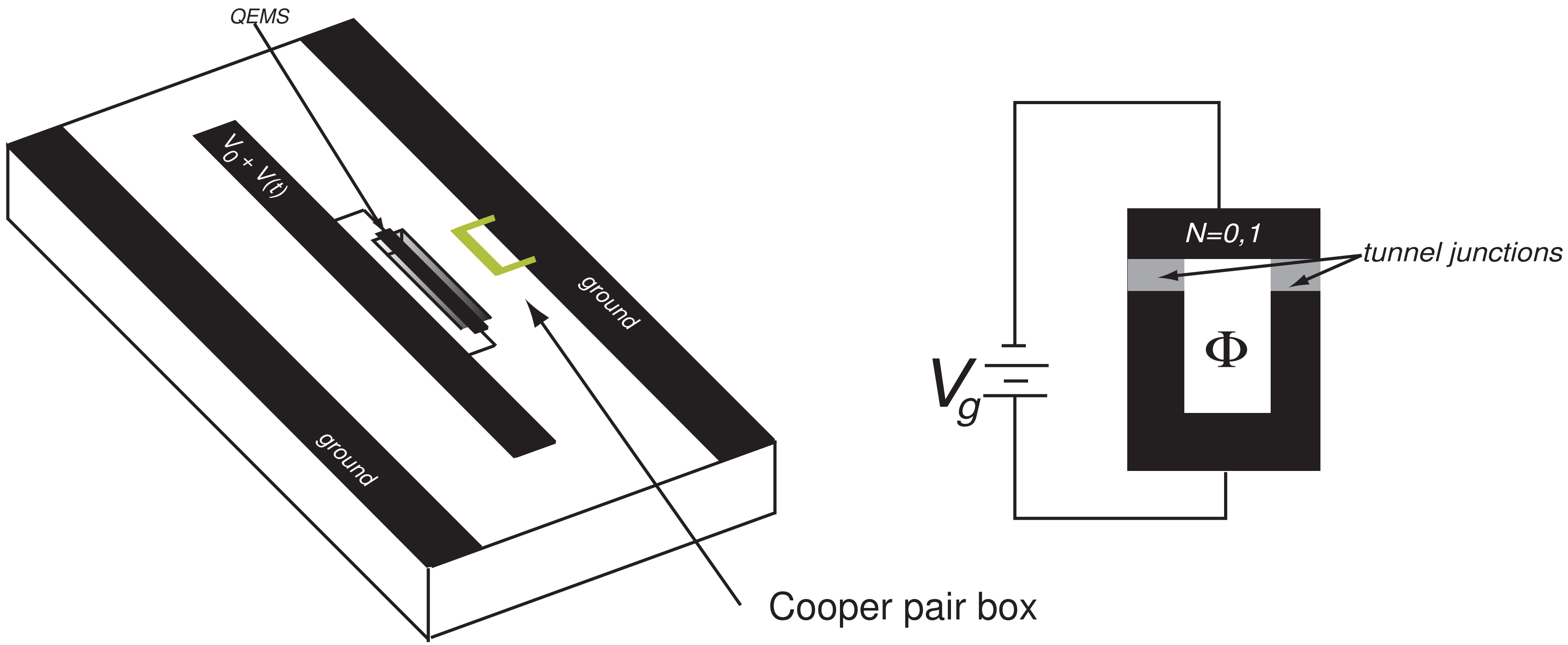
Conditional dynamics.



Initial state is $|1\rangle$.

$\zeta = 1$, $\mathcal{E} = 0$, $\theta = \pi$, $|\mathcal{X}|^2 = \Omega$, time in units of Ω^{-1}. $|\mathcal{T}|/|\mathcal{X}| =$ (a) 1, (b) 2, (c) 3, (d) 5.

Creation of entanglement by measurement.



Creation of entanglement by measurement.

Sun et al. quant-ph/0504056

Diagonalise H_{CPB} at $n_g^{(0)} = 1/2$

$$H = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_0}{2}\sigma_z - \hbar g(a\sigma_+ + a^\dagger\sigma_-)$$

$\hbar\omega_c a^\dagger a$: cavity field

σ_z, diagonal in charge basis, $\{|0\rangle, |1\rangle\}$.

$\sigma_+ = \sigma_-^\dagger = |1\rangle\langle 0|$

$$\hbar g = e\frac{C_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_r}{Lc}}$$

Dispersive limit.

Walraff et al. Nature, (2004)

Take dispersive limit: $\delta = (\omega_c - \omega_0)$ is large,

$$H_I \approx \hbar\chi a^\dagger a\sigma_z$$

where $\chi = \frac{g^2}{\delta}$

CQED as a qubit bus mode.

Sarovar, Goan, Spiller , GJM, Phys. Rev. A, 72, 062327 (2005)

Two CPB qubits, dispersive limit.

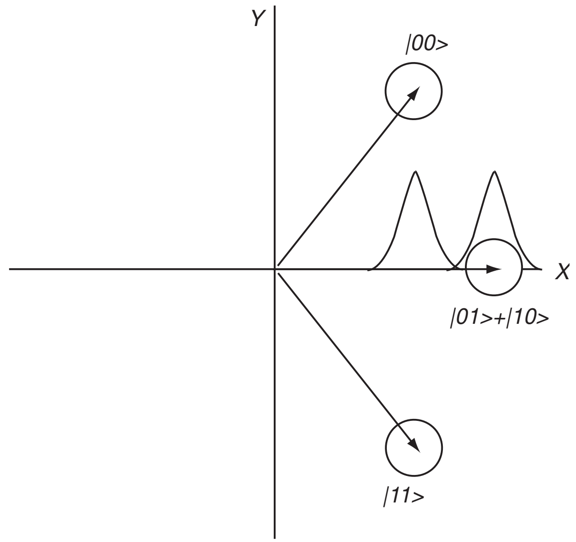
$$H_I = 2\chi J_z a^\dagger a + \chi(\sigma_1^+\sigma_2^- + \sigma_2^-\sigma_1^+)$$

where $J_z = \sigma_{z1} + \sigma_{z2}$.

$$\begin{aligned} & e^{-i\theta J_z a^\dagger a}(|00> + |01> + |10> + |11>)|\alpha\rangle \\ = \ & |00\rangle|\alpha e^{i\theta}\rangle + |11\rangle|\alpha e^{-i\theta}\rangle + (|10\rangle + |01\rangle)|\alpha\rangle \end{aligned}$$

Measure phase of field by homodyne detection.

CQED as a qubit bus mode.



Nemoto & Munro. PRL 2004.

Feedback creation of entanglement.

Feedback homodyne current from SET to change bias conditions of the CPB.

Process signal by low-pass filter:

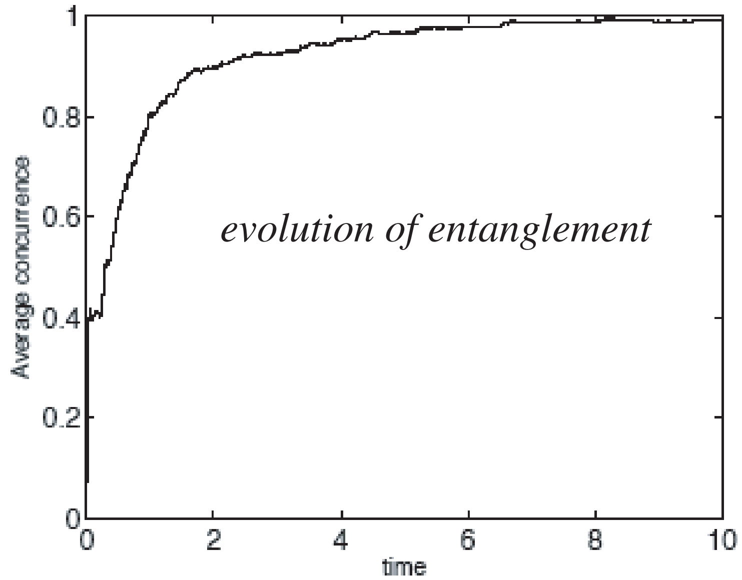
$$R(t) = \frac{1}{N}\int_{t-T}^{t} e^{-\gamma(t-t')} dI(t')$$

Add control Hamiltonian

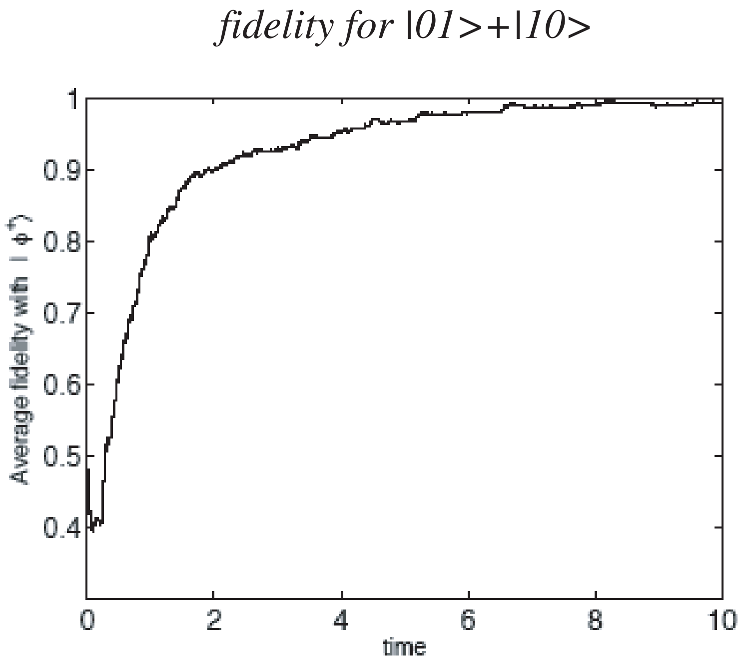
$$H_{FB} = \lambda R(t)^3(\sigma_{x1} + \sigma_{x2})$$

Feedback creation of entanglement.

$$\begin{aligned} d\,|\psi(t)\rangle &= [-iH - \frac{\kappa}{2}a^\dagger a]\,|\psi(t)\rangle\,dt + dI(t)a\,|\psi(t)\rangle \\ &- i\lambda R(t)^P J_x\,|\psi(t)\rangle\,dt \end{aligned}$$



Feedback creation of entanglement.



Sarovar et al., Phys. Rev. A 72, 062327 (2005)