Direct observation of quantum criticality in Ising spin chains

Jingfu Zhang
Supervisor: Prof. Raymond Laflamme

Institute for Quantum Computing,
and Department of Physics,
University of Waterloo,
Waterloo, ON N2L 3G1, Canada

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Outline

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I. Introduction: Simulating QPTs in quantum computers

Classical phase transitions: The most familiar examples involve temperature changes…

Quantum phase transitions (QPTs):

quantum system in its ground state, i.e.,T=0K;

non-thermal control parameter (pressure, magnetic field,…).

Quantum Computers can simulate quantum systems efficiently.

[R. P. Feynman, International Journal of Theoretical Physics, V. 21, 467 (1982)]

Introduction

Quantum computer: the information carrier is quantum bit (qubit), which can lies in a superposition of |0> and |1>, resulting to quantum parallelism, and entanglement.

In quantum computers: investigate the details of QPTs,e.g. entanglement near critical points

Ground state of the qubit system: T=0

Ground state and its evolution: quantum network

in NMR quantum computers

Observe QPTs: problems

global measurements, such as:
conductivity or susceptibility in superconductors,
total magnetization in some spin chains

cannot provide all the details and they are not suitable for closer investigations of the systems in the interesting area close to the critical points.

quantum state tomography

but this approach scales very poorly with the size of the system.

The number of experiments necessary for tomography grows exponentially with the size of the system.

Loschmidt echo (Fidelity decay)

[T. Gorin et al., Phys. Rep. 435, 33 (2006)]

Compare the evolution of systems at slightly different Hamiltonians

$$|\psi_0\rangle \xrightarrow{H_1} |\psi_1(t)\rangle = e^{-itH_1}|\psi_0\rangle$$

$$|\psi_0\rangle \xrightarrow{H_2} |\psi_2(t)\rangle = e^{-itH_2}|\psi_0\rangle$$

$$L = |\langle \psi_1 | \psi_2 \rangle|^2$$

$$(H_2 - H_1 = \delta V, |\delta| << 1.)$$

Applications in system sensitive to perturbations

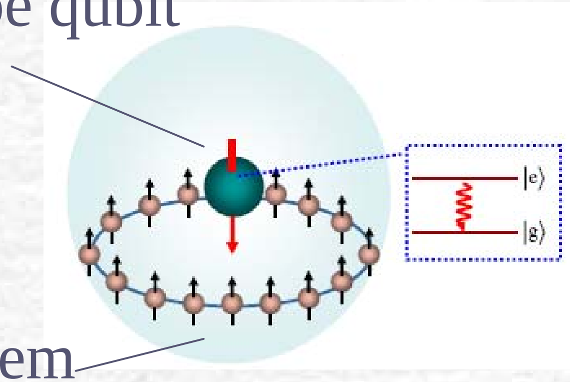
#Chaos system [C. A. Ryan *et al.*, Phys. Rev. Lett. **95**, 250502 (2005)]

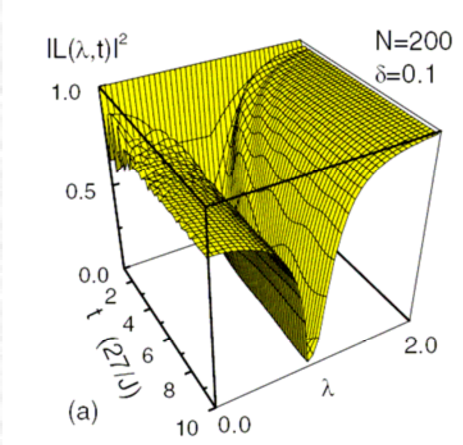
#decoherence [F. M. Cucchietti *et al.*, Phys. Rev. A **75**, 032337 (2007)]

#characterization of errors [B. Levi *et al.*, Phys. Rev. A **75**, 022314 (2007).]

Loschmidt echo: indicator for quantum criticality (theory)

coupling a probe qubit to the QPT system





$$H(\lambda, \delta) = -J\sum_j(\sigma_j^z\sigma_{j+1}^z + \lambda\sigma_j^x + \delta|e\rangle\langle e|\sigma_j^x),$$

[H. T. Quan et al., Phys. Rev. Lett. **96,** 140604 (2006)]

System "see" two Hamiltonians: H_1 when probe in $|0\rangle$

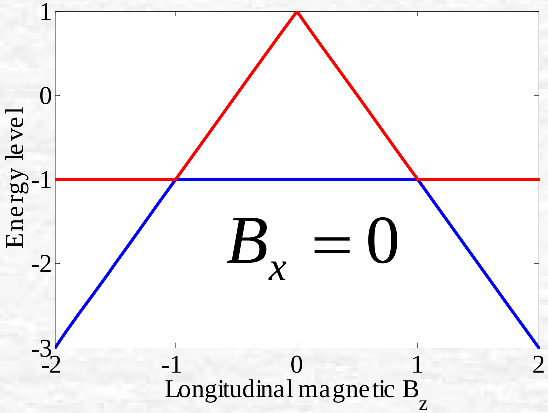
H_2, $|1\rangle$

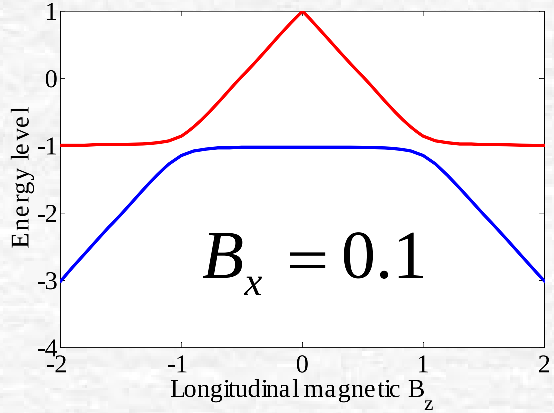
When probe in $|0\rangle+|1\rangle$, $\rho_{12} = \langle\psi_0|e^{itH_2}e^{-itH_1}|\psi_0\rangle$

Loschmidt echo: indicator for quantum criticality (experiment)

System for QPT:

$$H = \sigma_z^1\sigma_z^2 + B_z(\sigma_z^1 + \sigma_z^2) + B_x(\sigma_x^1 + \sigma_x^2)$$

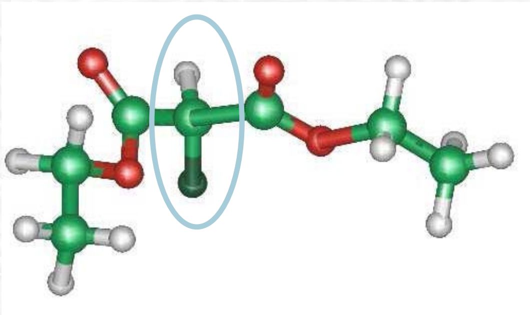




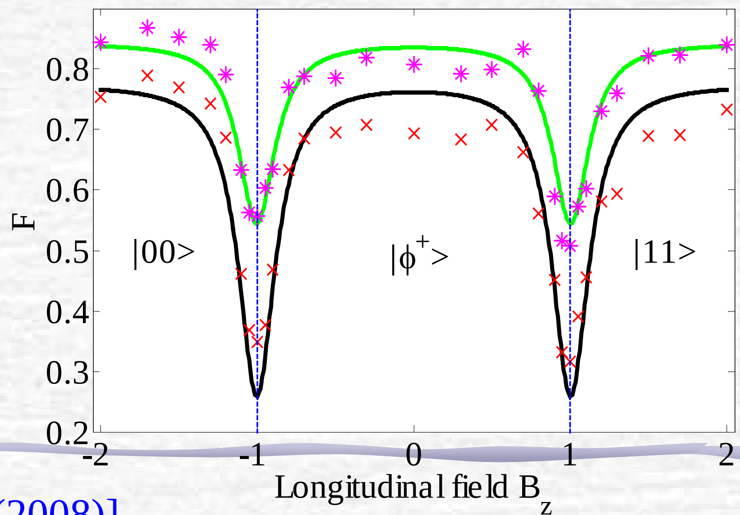
The gap $\approx 2\sqrt{2}B_x$

A coupled probe measures system

$$H_{tot} = H + \varepsilon\sigma_z^0(\sigma_z^1 + \sigma_z^2)$$



Diethylfluoromalonate.



t=1.6

*: $\varepsilon = 0.2$

X: $\varepsilon = 0.3$

[J. Zhang, Phys. Rev. Lett. **100,** 100501 (2008)]

Generality and Motivation for new scheme

In implementation, coupling one qubit to all the qubits is not always practical.

Completely non-local phase transitions, a single coupling between the probe qubit and one of the system qubits is sufficient [Phys. Rev. A 75, 032333 (2007)]

Otherwise, one qibut is not enough. However, the number of probe qubits (or operations) only scales linearly with the size of the system.

The coupled probe increases the difficulty in implementation.

New scheme: direct observation:

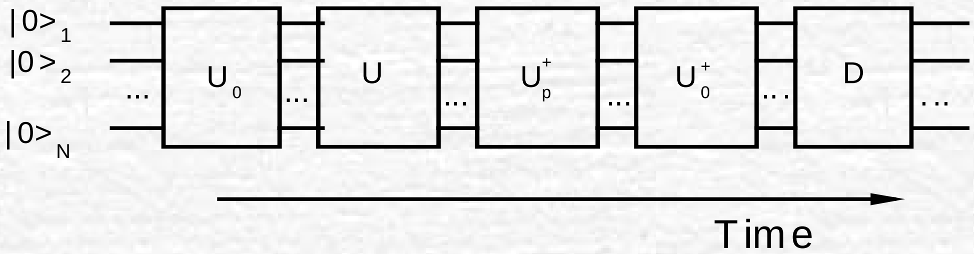
The additional probe is avoided;

Measuring one qubit of the system is sufficient for detecting the critical points in general QPTs.

II. Experimental scheme

$$L(t) = |\langle \psi(0) | U_p^{+}(t) U(t) | \psi(0) \rangle|^2$$

$$= |\langle 00...0 | U_0^{+} U_p^{+}(t) U(t) U_0 | 00...0 \rangle|^2$$



$$U_0 |00...0\rangle = |\psi(0)\rangle, \qquad U = e^{-itH},$$

$$U_p = e^{-it(H+V)}$$

D Kills non-diagonal
terms of the density matrix

$$|00...0\rangle = \begin{bmatrix} 1 \\ 0 \\ : \\ 0 \end{bmatrix}$$

$$|\psi\rangle = U_0^{+} U_p^{+} U U_0 |00...0\rangle$$

$$= \begin{bmatrix} \sqrt{L} \\ x_2 \\ : \\ x_{2^N - 1} \end{bmatrix}$$

$$\xrightarrow{[\pi/2]_i} L_e = L - \rho_{nn}$$

System for QPTs

$$H = \sum_{i=1}^{N-1} \sigma_z^i \sigma_z^{i+1} + B_z \sum_{i=1}^{N} \sigma_z^i + B_x \sum_{i=1}^{N} \sigma_x^i$$

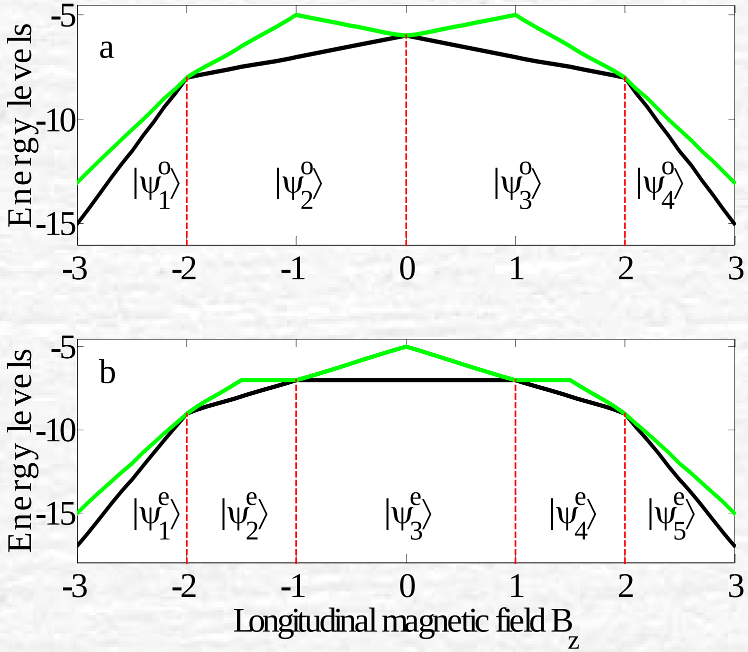
$B_x = 0$

N=odd

$$|\psi_g^o(B_z)\rangle = \begin{cases} |\underbrace{00..0}_{N}\rangle & (B_z \le -2) \\ |\underbrace{(01)(01)...(01)}_{(N-1)/2} 0\rangle & (-2 \le B_z \le 0) \\ |\underbrace{(10)(10)...(10)}_{(N-1)/2} 1\rangle & (0 \le B_z \le 2) \\ |\underbrace{11...1}_{N}\rangle & (B_z \ge 2), \end{cases}$$

N=even

$$|\psi_g^e(B_z)\rangle = \begin{cases} |\underbrace{00..0}_{N}\rangle & (B_z \le -2) \\ \frac{1}{\sqrt{2}}\left[|\underbrace{(01)(01)...(01)}_{(N-2)/2} 00\rangle + |00 \underbrace{(10)(10)...(10)}_{(N-2)/2}\rangle\right] & (-2 \le B_z \le -1) \\ \frac{1}{\sqrt{2}}\left[|\underbrace{(01)(01)...(01)}_{N/2}\rangle + |\underbrace{(10)(10)...(10)}_{N/2}\rangle\right] & (-1 \le B_z \le 1) \\ \frac{1}{\sqrt{2}}\left[|11 \underbrace{(01)(01)...(01)}_{(N-2)/2}\rangle + |\underbrace{(10)(10)...(10)}_{(N-2)/2} 11\rangle\right] & (1 \le B_z \le 2) \\ |\underbrace{11...1}_{N}\rangle & (B_z \ge 1) \end{cases}$$



When $N \to \infty$,

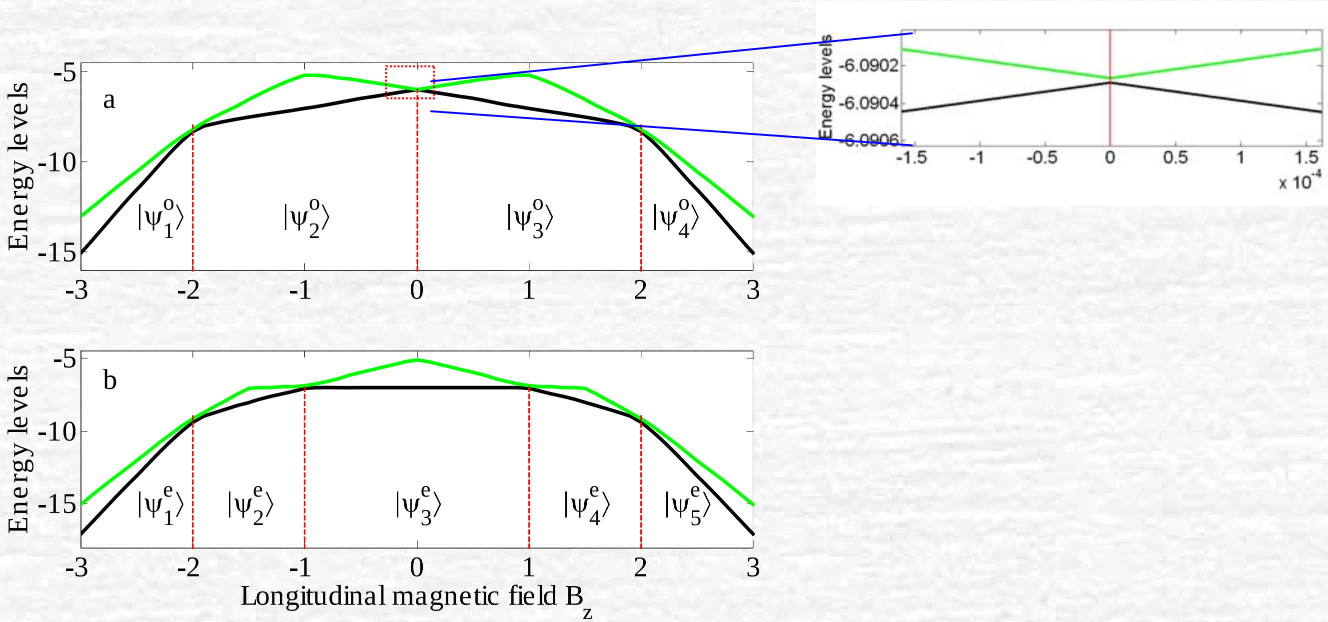
Three phases:

$|00\ ...\ 0\rangle, |0101\ ...\ 01\rangle, |11\ ...\ 1\rangle$

Critical points:2,-2

$B_x = 0.1$

Phase diagrams

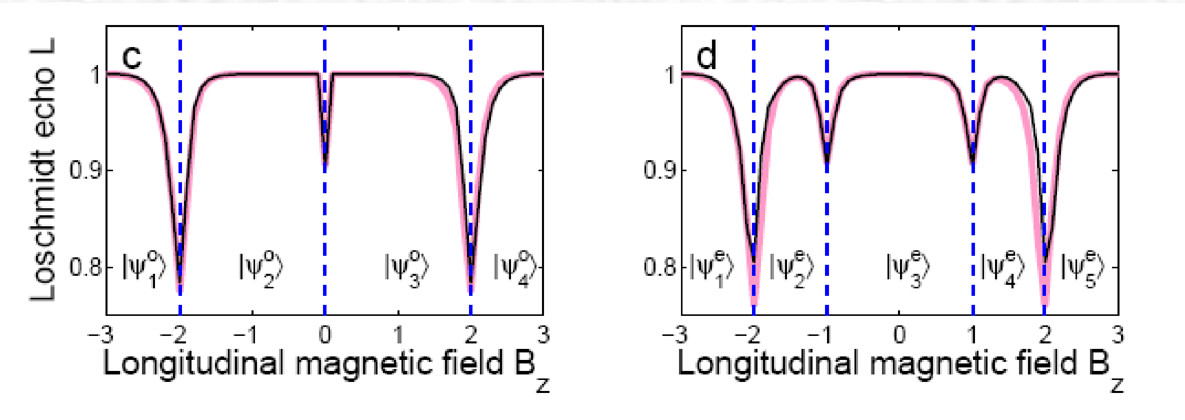


Loschmidt echo

$$L(t) \approx 1 - 2\frac{|V_{01}|^2}{\Delta^2}\varepsilon^2[1-\cos\Delta t]$$

$$\Delta = E_1 - E_0$$

$$V_{01} = \langle E_1 | V | E_0 \rangle$$



$\varepsilon = 0.1, t = \pi$

perturbation $V = -\varepsilon \sum_{i=1}^{N} \sigma_z^i,$

III. Experiment 1: QPT in odd qubit system

3 qubit system

$$H = \sum_{i=1}^{2} \sigma_z^i \sigma_z^{i+1} + B_z \sum_{i=1}^{3} \sigma_z^i + B_x \sum_{i=1}^{3} \sigma_x^i$$

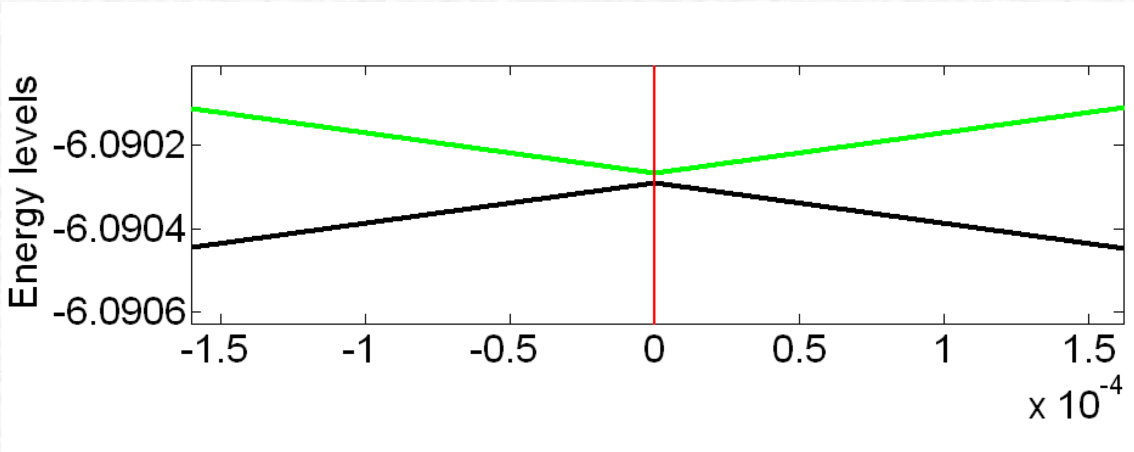
Ground state, near

$$B_c = -2, |\psi\rangle = |000\rangle \cos\varphi - |010\rangle \sin\varphi$$

$$B_c = 2, |\psi\rangle = |111\rangle \cos\varphi - |101\rangle \sin\varphi$$

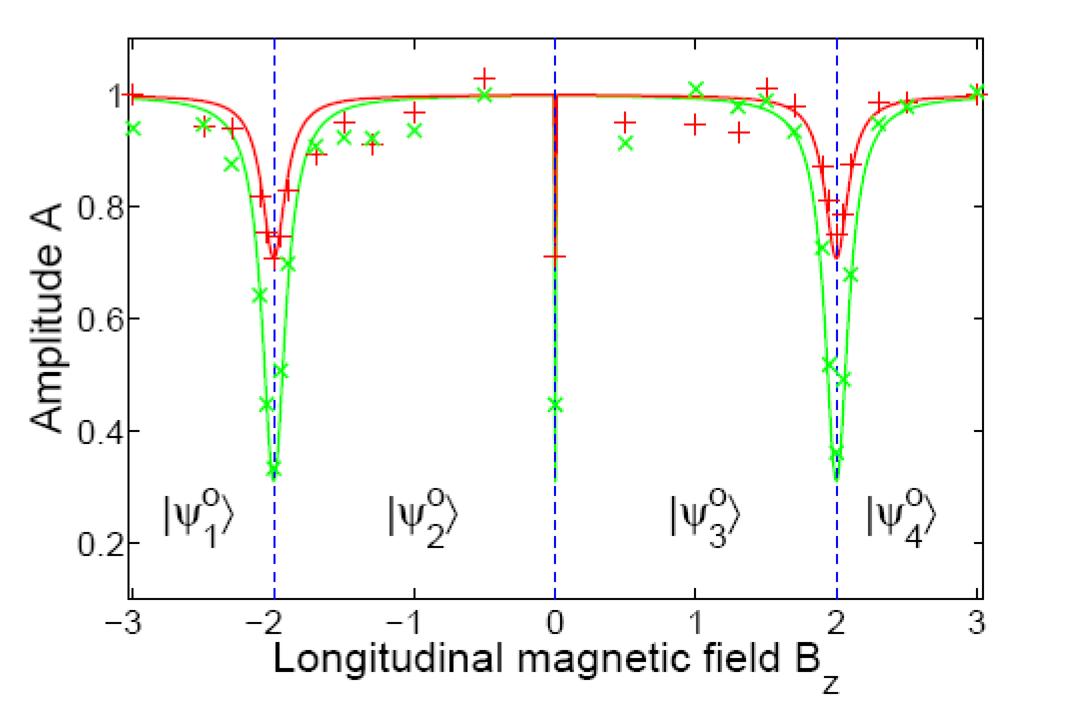
$$\tan\varphi = [(2 - |B_z|) + \sqrt{(2 - |B_z|)^2 + B_x^2}\,] / B_x$$

Near $B_c = 0$



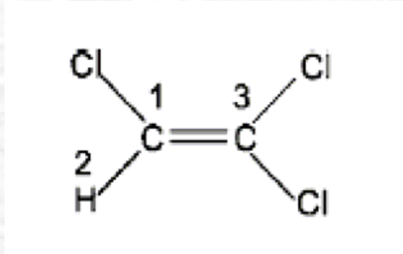
$$|010\rangle \rightarrow \quad (|010\rangle - |101\rangle)/\sqrt{2} \rightarrow \quad |101\rangle$$

Results in 3 qubit system



$\tau = \pi, \times : \varepsilon = 0.2, + : \varepsilon = 0.125$

carbon-13 labeled trichloroethylene (TCE)



IV. Experiment 2: QPT in even qubit system

4 qubit system
$$H=\sum_{i=1}^{3}\sigma_z^i\sigma_z^{i+1}+B_z\sum_{i=1}^{4}\sigma_z^i+B_x\sum_{i=1}^{4}\sigma_x^i$$

Ground state, near
$$B_c=-2, |\psi\rangle=|0000\rangle\sin\varphi-\frac{1}{\sqrt{2}}(|0010\rangle+|0100\rangle)\sin\varphi$$

$$B_c=2, |\psi\rangle=|1111\rangle\sin\varphi-\frac{1}{\sqrt{2}}(|1101\rangle+|1011\rangle)\sin\varphi$$

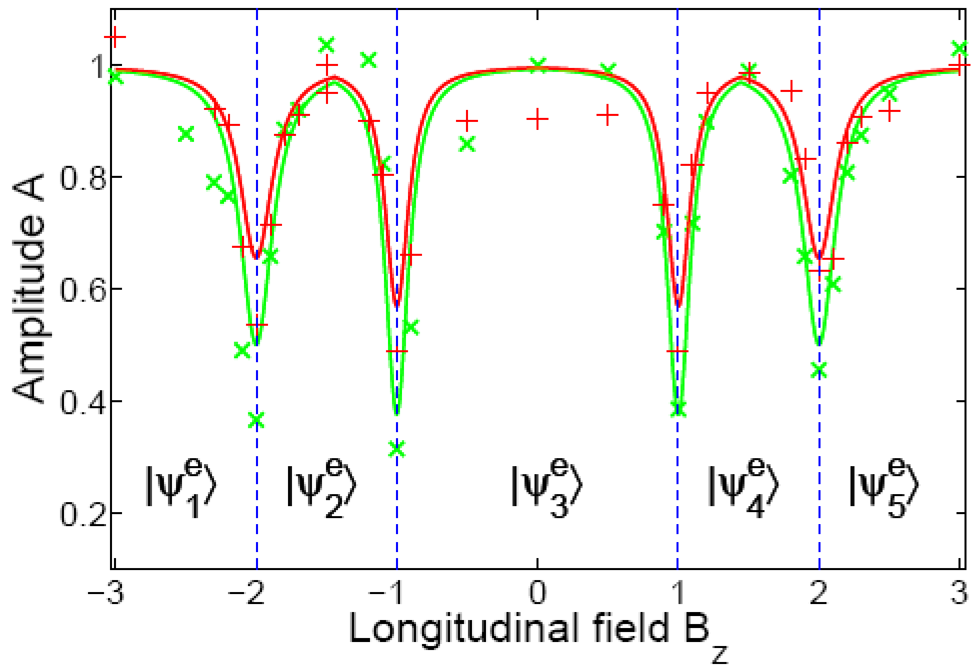
$$\tan\varphi=[(2-|B_z|)+\sqrt{(2-|B_z|)^2+2B_x^2}]/(\sqrt{2}B_x)$$

$$B_c=-1, |\psi\rangle=\frac{1}{\sqrt{2}}(|0010\rangle+|0100\rangle)\cos\varphi-\frac{1}{\sqrt{2}}(|0101\rangle+|1010\rangle)\sin\varphi$$

$$B_c=1, |\psi\rangle=\frac{1}{\sqrt{2}}(|1101\rangle+|1011\rangle)\cos\varphi-\frac{1}{\sqrt{2}}(|0101\rangle+|1010\rangle)\sin\varphi$$

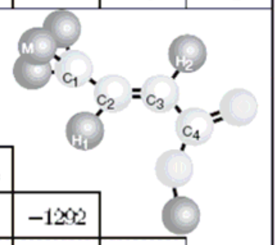
$$\tan\varphi=[(1-|B_z|)+\sqrt{(1-|B_z|)^2+2B_x^2}]/B_x$$

Experiment result: QPTs in 4 qubit system



Trans-crotonic acid

	C_1	C_2	C_3	C_4	M	H_1	H_2
C_1	−2972						
C_2	40.5	−25518					
C_3	1.5	69.9	−21583				
C_4	7.1	1.4	72.4	−29544			
M	127.5	−7.1	6.6	−0.9	−1292		
H_1	3.8	156.0	−1.8	6.5	6.9	−4855	
H_2	6.2	−0.7	162.9	3.3	−1.7	15.5	−4066



$$\tau = \pi/2, \times : \varepsilon = 0.5, + : \varepsilon = 0.4$$

V. Conclusions and outlook

Loschmidt echo is susceptible to the critical points

Only one qubit is measured, independent of the size of the quantum system. Hence this method scales very favorably with the size of the system.

Gapless (degenerated ground state), e.g. Spin glass states : state- independent indicator

[X. Wang et al., arXiv:0803.2940v2]

Possible work

different types of phase transition, such as quantum chaos; Sudden death of entanglement; Critical exponent, correlation length…

(arXiv:0808.1536[quant-ph])