Effects of dissipation on quantum critical points with disorder

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- Introduction: disorder, dissipation, criticality
- Continuous $O(N)$ symmetry: infinite-randomness critical point
- Ising symmetry: smeared quantum phase transition

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  • Ising symmetry: smeared quantum phase transition
Phase transitions, disorder, and dissipation

Quenched Disorder:
- can destabilize clean critical point
- stronger effects at QPTs than at classical transitions (disorder correlations in time)
- exotic critical points with non-power-law scaling
- rare regions lead to Griffiths singularities close to actual transition

Dissipation:
- slows down critical dynamics
- further enhances disorder effects

Question: Fate of QPT under combined influence of disorder and dissipation?
Experiment I: Itinerant quantum magnets

- quantum phase transitions between paramagnetic metal and ferromagnetic or antiferromagnetic metal
- transition often controlled by chemical composition $\Rightarrow$ disorder appears naturally
- magnetic modes damped due to coupling to fermions $\Rightarrow$ Ohmic dissipation
- typical example: ferromagnetic transition in CePd$_{1-x}$Rh$_x$

Experiment II: Superconductivity in ultrathin nanowires

- ultrathin MoGe wires (width $\sim 10$ nm)
- produced by molecular templating using a single carbon nanotube
  [A. Bezryadin et al., Nature 404, 971 (2000)]

superconductor-metal QPT as function of wire thickness

Pair breaking mechanism:
- magnetic impurities at the surface
- pairing interaction: repulsive at surface, attractive in core
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Dissipative $O(N)$ order parameter field theory

$N$-component ($N > 1$) order parameter field $\varphi(x, \tau)$ in $d$ dimensions

$$S = T \sum_{\mathbf{q}, \omega_n} \left( r + \xi_0^2 q^2 + \gamma |\omega_n| \right) |\varphi(\mathbf{q}, \omega_n)|^2 + \frac{u}{2N} \int d^d x d\tau \varphi^4(x, \tau)$$

Disorder: \[
\left\{ \begin{array}{l}
\text{distance } r \text{ from criticality} \\
\text{bare correlation length } \xi_0 \\
\text{Ohmic dissipation constant } \gamma
\end{array} \right. \] random functions of position

Applications:

- Superconductor-metal quantum phase transition in nanowires ($d = 1, N = 2$)
  $\varphi(x, \tau)$ represents local Cooper pair operator (Sachdev, Werner, Troyer 2004)
- Hertz’ theory of itinerant quantum Heisenberg antiferromagnets ($d = 3, N = 3$)
  $\varphi(x, \tau)$ represents staggered magnetization (Hertz 1976)
Discrete large-$N$ theory in one dimension

To apply real-space based strong-disorder renormalization group:

- discretize space by introducing “rotor” variables $\phi_j(\tau)$
- large-$N$ limit of an infinite number of order parameter components

Resulting action:

$$ S = T \sum_{i,\omega_n} \left( r_i + \lambda_i + \gamma_i |\omega_n| \right) |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \phi_i(-\omega_n) \phi_{i+1}(\omega_n) $$

$r_i, \gamma_i > 0, J_i > 0$: random functions of lattice site $i$

$\lambda_i$: Lagrange multiplier enforcing large-$N$ constraint $\langle \phi_i^2(\tau) \rangle = 1$

$\epsilon_i = r_i + \lambda_i$: renormalized (local) distance from criticality
Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

in our system

\[ S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|) |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \phi_i(-\omega_n) \phi_{i+1}(\omega_n) \]

the competing local energies are:

- interactions (bonds) \( J_i \) favoring the ordered phase
- local "gaps" \( \epsilon_i \) favoring the disordered phase

\( \Rightarrow \) in each RG step, integrate out largest among all \( J_i \) and \( \epsilon_i \)
Recursion relations

if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:
- site 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3 / \epsilon_3$

if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:
- rotors of sites 2 and 3 are parallel
- can be replaced by single rotor with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$
Renormalization-group flow equations

- RG step is iterated, building larger and larger clusters connected by weaker and weaker bonds (while gradually reducing maximum energy $\Omega$)

$\Rightarrow$ flow equations for the probability distributions $P(J)$ and $R(\epsilon)$

\begin{align*}
- \frac{\partial P}{\partial \Omega} &= [P(\Omega) - R(\Omega)] P + R(\Omega) \int dJ_1 dJ_2 P(J_1)P(J_2) \delta \left( J - \frac{J_1 J_2}{\Omega} \right) \\
- \frac{\partial R}{\partial \Omega} &= [R(\Omega) - P(\Omega)] R + P(\Omega) \int d\epsilon_1 d\epsilon_2 R(\epsilon_1) R(\epsilon_2) \delta \left( \epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega} \right)
\end{align*}

Flow equations are identical to those of the random transverse-field Ising chain

$\Rightarrow$ infinite-randomness critical point

$\Rightarrow$ activated dynamical scaling

QPT of a disordered dissipative $O(N)$ order parameter is in the same universality class as the dissipationless random transverse-field Ising model.
Results: Phase diagram

finite-temperature phase boundary and crossover line take unusual form

\[ T_c \sim \exp(-\text{const} \, |r|^{-\nu \psi}) \]

\( \Rightarrow \text{very wide quantum critical region} \)

Infinite-randomness critical point:
- at fixed point of the flow equations, disorder scales to infinity
- FP characterized by 3 exponents
- tunneling exponent \( \psi = 1/2 \) controls dynamical scaling \( \ln(1/\Omega) \sim L^\psi \)
- moments of surviving clusters grow like \( \mu \sim \ln^\phi (1/\Omega) \) with \( \phi = (1 + \sqrt{5})/2 \)
- average correlation length diverges as \( \xi \sim |r|^{-\nu} \) with \( \nu = 2 \)

Quantum Griffiths regions:
- fixed points of the flow equations can also be found off criticality
- power-law dynamical scaling with nonuniversal exponent
Results: Observables

**Thermodynamics:**

to find order parameter susceptibility and specific heat at temperature $T$:
run RG down to energy scale $\Omega = T$ and consider remaining clusters as free

\[
\chi(r, T) = \frac{1}{T} [\ln(1/T)]^{2\phi-d/\psi} \Theta_\chi (r^{\nu\psi} \ln(1/T))
\]

\[
C(r, T) = [\ln(1/T)]^{-d/\psi} \Theta_C (r^{\nu\psi} \ln(1/T))
\]

at criticality: $\chi \sim \frac{1}{T} [\ln(1/T)]^{2\phi-d/\psi}$, in Griffiths phase: $\chi \sim T^{d/z'-1}$

**Dynamic susceptibilities at $T = 0$:**

found by running RG to energy scale $\Omega \approx \omega$

\[
\text{Im}\chi(r, \omega) \sim \frac{1}{\omega} [\ln(1/\omega)]^{\phi-d/\psi} X (r^{\nu\psi} \ln(1/\omega))
\]

\[
\text{Im}\chi^\text{loc}(r, \omega) \sim \frac{1}{\omega} [\ln(1/\omega)]^{-d/\psi} X^\text{loc} (r^{\nu\psi} \ln(1/\omega))
\]
• A. Del Maestro et al. (2008) solved disordered large-$N$ problem numerically exactly
• calculated equal time correlation function $C$, energy gap $\Omega$, and ratio $R$ of local and order parameter dynamic susceptibilities

SDRG $\nu$ $\psi$ $\phi$
Numerics 1.9(2) 0.53(6) 1.6(2)
Generalizations: \( N < \infty, \ d > 1, \) nonohmic damping

Order parameter symmetry

- our explicit calculations are for an infinite number of OP components, \( N = \infty \)
- results apply to all continuous symmetry cases \( N > 1 \)
  (clusters are marginal – gaps depends exponentially on size)

Higher dimensions \( d > 1 \)

- infinite randomness scaling scenario also appears in 2D and 3D
- critical exponent values are different, only known numerically

Nonohmic damping

- if damping term is nonohmic, \( \gamma|\omega_n|^{2/z_0} \), recursion relations change
- subohmic case, \( z_0 > 2 \): quantum phase transition destroyed by smearing
- superohmic case, \( z_0 < 2 \): transition survives, likely with conventional scaling
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Dissipative random transverse-field Ising chain

\[ H = - \sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + \sum_{i,n} \sigma_i^z \lambda_{i,n} (a_{i,n}^\dagger + a_{i,n}) + \sum_{i,n} \nu_{i,n} a_{i,n}^\dagger a_{i,n} \]

- \( J_i \): exchange interaction between \( z \)-components of spin \( \sigma_i \)
- \( h_i \): transverse magnetic field, acting on \( x \)-component of spin \( \sigma_i \)
- \( a_{i,n}^\dagger, a_{i,n} \): harmonic oscillator bath coupling to \( z \)-component of spin \( \sigma_i \)

**Bath spectral function**

\[ E(\omega) = \pi \sum_n \lambda_{i,n}^2 \delta(\omega - \nu_{i,n}) = 2\pi \alpha \omega e^{-\omega/\omega_c} \]

- \( \alpha \): dimensionless dissipation strength
- \( \omega_c \): oscillator energy cutoff

Linear low freq. spectrum: Ohmic dissipation
Integrate out local high energy modes: $\Omega = \max(J_i, h_i, \omega_c/p)$

To reduce maximum energy from $\Omega$ to $\Omega - d\Omega$:

1. Integrate out all oscillators with frequencies $\nu \in [p(\Omega - d\Omega), p\Omega]$

$$\tilde{h}_i = h_i \exp \left( -\alpha_i \int_{p(\Omega-d\Omega)}^{p\Omega} \frac{d\omega}{\omega} \right) = h_i \left( 1 - \alpha \mu_i \frac{d\Omega}{\Omega} \right)$$

2. Decimate all transverse fields $h_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{J} = J_{i-1} J_i / h_i$$

3. Decimate all interaction energies $J_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{\mu} = \mu_i + \mu_{i+1}$$

Extra downward renormalization of the transverse fields due to dissipation
Renormalization-group flow equations

**Flow equations** for the probability distributions $P(J)$ and $R(h, \mu)$

\[
\begin{align*}
\frac{-\partial P}{\partial \Omega} &= [P(\Omega) - (1 - \alpha \bar{\mu})R_h(\Omega)] P + (1 - \alpha \bar{\mu})R(\Omega) \int dJ_1 dJ_2 P(J_1)P(J_2) \delta \left[J - \frac{J_1 J_2}{\Omega}\right] \\
\frac{-\partial R}{\partial \Omega} &= [(1 - \alpha \bar{\mu})R_h(\Omega) - P(\Omega)] R + \frac{\alpha \mu}{\Omega} \left[R + h \frac{\partial R}{\partial h}\right] + \\
&\quad + P(\Omega) \int dh_1 dh_2 d\mu_1 d\mu_2 R(h, \mu_1)R(h, \mu_2) \delta \left[h - \frac{h_1 h_2}{\Omega}\right] \delta[\mu - \mu_1 - \mu_2]
\end{align*}
\]

$(1 - \alpha \bar{\mu})$: probability for decimating field vanishes for $\mu > 1/\alpha$

$\Rightarrow$ important finite “volume” scale $1/\alpha$

- clusters act as Ohmic spin-boson problem with effective damping constant $\alpha \mu$
- if $\alpha \mu > 1$, they undergo **localization transition** (Caldeira, Leggett, Weiss)

Large clusters freeze independently $\Rightarrow$ quantum phase transition is smeared
Smeared quantum phase transition

Low temperature thermodynamics: dominated by large frozen clusters

Example: uniform susceptibility $\chi \sim T^{-1-1/z}$
Rare region effects in CePd$_{1-x}$Rh$_x$?

- ferromagnetic phase shows pronounced tail $\Rightarrow$ evidence for smeared transition
- evidence for spin-glass like behavior in tail
- above tail: nonuniversal power-laws characteristic of quantum Griffiths effects

Classification of weakly disordered phase transitions according to importance of rare regions


<table>
<thead>
<tr>
<th>Dimensionality of rare regions</th>
<th>Griffiths effects</th>
<th>Dirty critical point</th>
<th>Examples</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(classical PT, QPT, non-eq. PT)</td>
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<tr>
<td>$d_{RR} &lt; d_c^{-}$</td>
<td>weak exponential</td>
<td>conv. finite disorder</td>
<td>class. magnet with point defects</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>dilute bilayer Heisenberg model</td>
</tr>
<tr>
<td>$d_{RR} = d_c^{-}$</td>
<td>strong power-law</td>
<td>infinite randomness</td>
<td>Ising model with linear defects</td>
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<tr>
<td></td>
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<td>random quantum Ising model</td>
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<tr>
<td></td>
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<td>disordered directed percolation (DP)</td>
</tr>
<tr>
<td>$d_{RR} &gt; d_c^{-}$</td>
<td>RR become static</td>
<td>smeared transition</td>
<td>Ising model with planar defects</td>
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<td></td>
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<td>itinerant quantum Ising magnet</td>
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<td>DP with extended defects</td>
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Conclusions

• We have studied quantum phase transitions in the presence of both disorder and Ohmic dissipation using the strong-disorder renormalization group.

• For continuous symmetry order parameters, the RG recursion relations for the local gaps and interactions are multiplicative.

⇒ infinite-randomness critical point in the universality class of the random transverse field Ising model.

• For Ising symmetry, the dissipation introduces a finite length scale beyond which the clusters freeze.

⇒ quantum phase transition is smeared.

Exotic QPTs due to interplay between disorder and dissipation: disorder creates locally ordered rare regions, dissipation makes their dynamics ultraslow.