Effects of dissipation on quantum critical points with disorder

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- Introduction: disorder, dissipation, criticality
- Continuous $O(N)$ symmetry: infinite-randomness critical point
- Ising symmetry: smeared quantum phase transition

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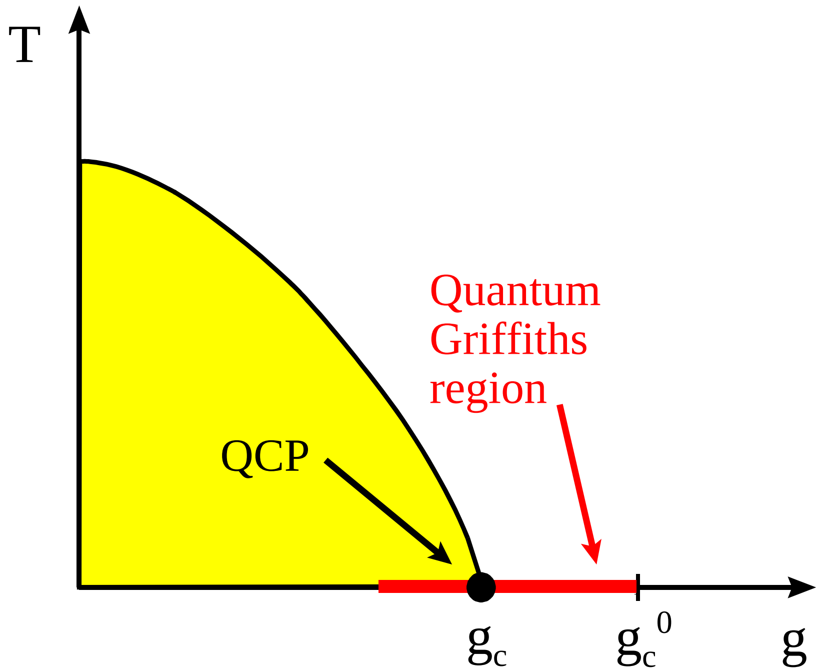
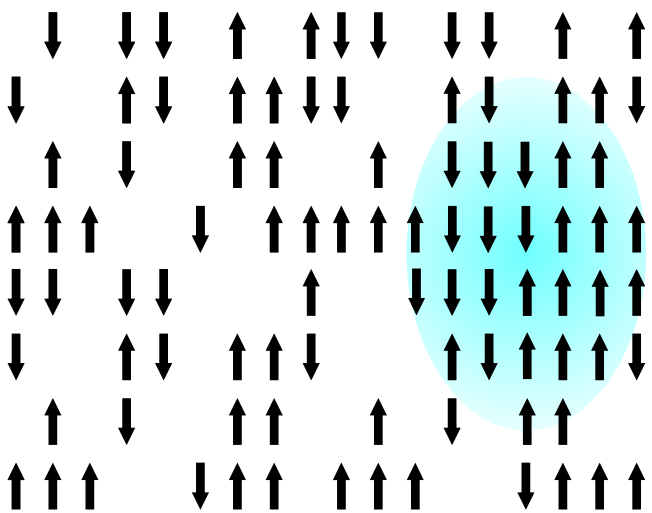
Phase transitions, disorder, and dissipation





Quenched Disorder:

- can destabilize clean critical point
- stronger effects at QPTs than at classical transitions (disorder correlations in time)
- exotic critical points with non-power-law scaling
- rare regions lead to Griffiths singularities close to actual transition

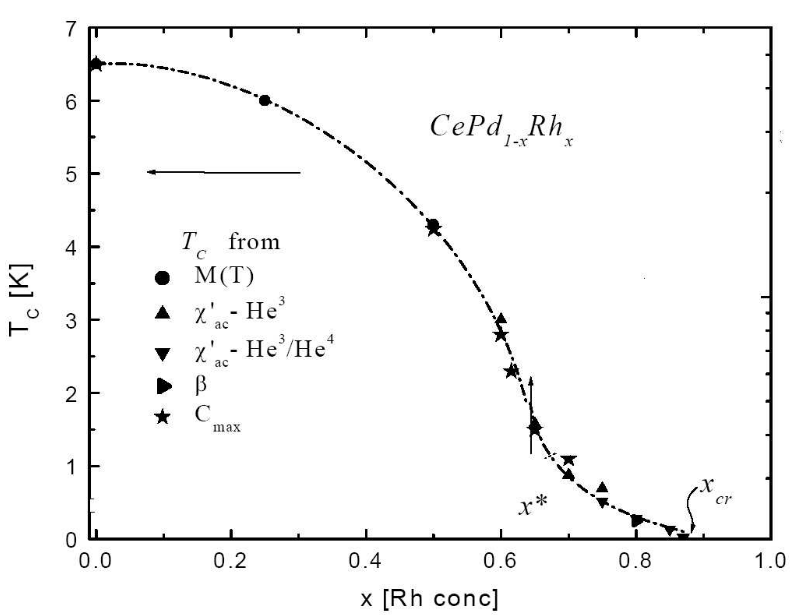
Dissipation:

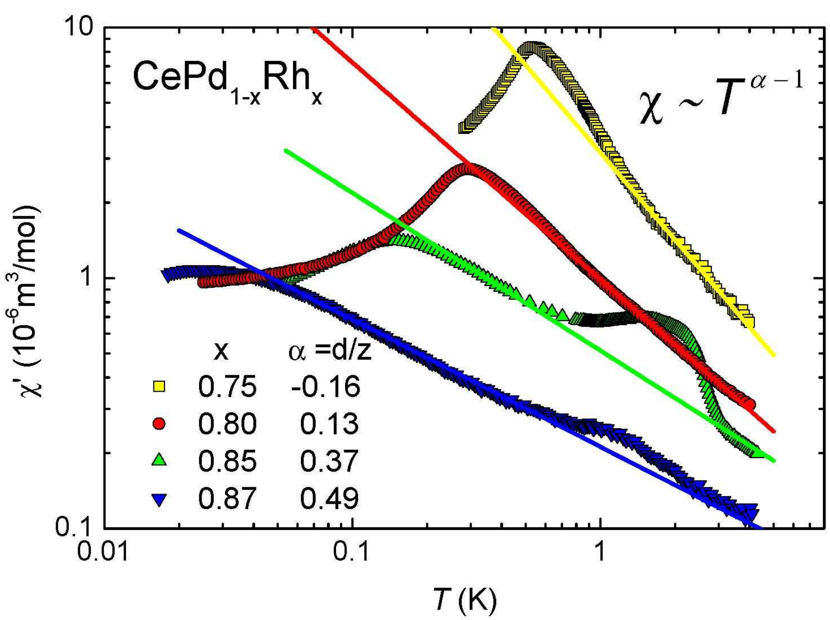
- slows down critical dynamics
- further enhances disorder effects

Question: Fate of QPT under combined influence of disorder and dissipation?

Experiment I: Itinerant quantum magnets

- quantum phase transitions between paramagnetic metal and ferromagnetic or antiferromagnetic metal
- transition often controlled by chemical composition ⇒ disorder appears naturally
- magnetic modes damped due to coupling to fermions ⇒ Ohmic dissipation
- typical example: ferromagnetic transition in $CePd_{1-x}Rh_x$



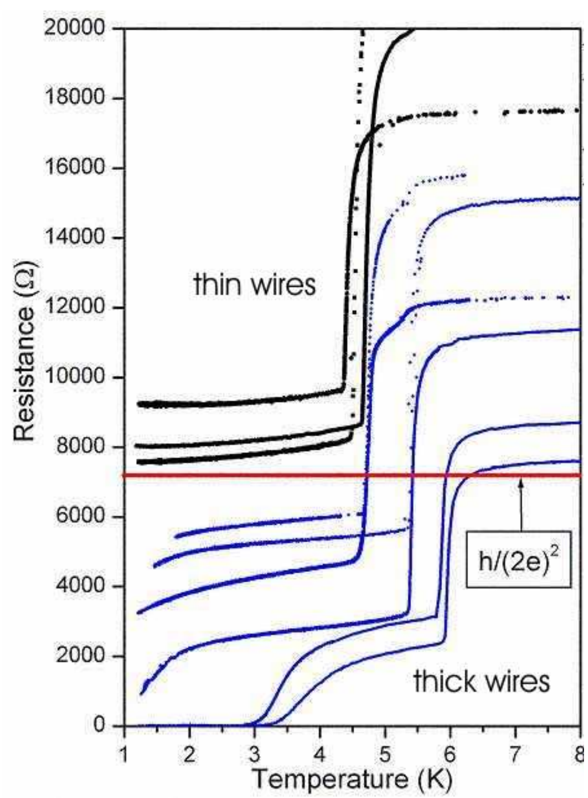


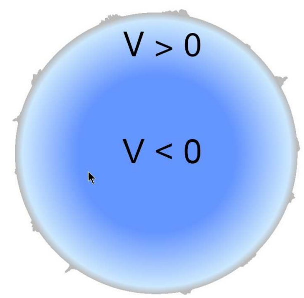
(Sereni et al., Phys. Rev. B **75** (2007) 024432 + Westerkamp, private communication)

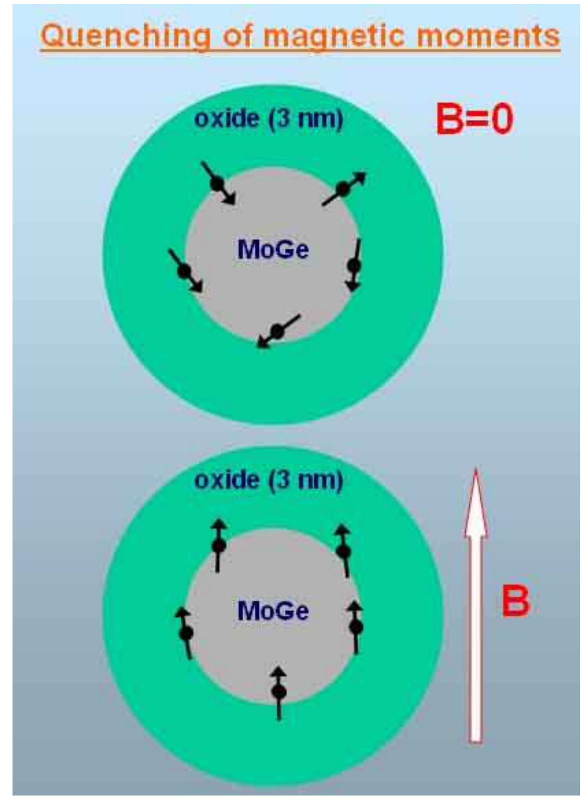
Experiment II: Superconductivity in ultrathin nanowires

- ultrathin MoGe wires (width $\sim$ 10 nm)
- produced by molecular templating using a single carbon nanotube
  [A. Bezryadin et al., Nature 404, 971 (2000)]

superconductor-metal QPT as function of wire thickness







Pair breaking mechanism:

- magnetic impurities at the surface
- pairing interaction: repulsive at surface, attractive in core

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Dissipative $O(N)$ order parameter field theory

N-component ($N > 1$) order parameter field $\varphi(\mathbf{x}, \tau)$ in d dimensions

$$S = T \sum_{\mathbf{q},\omega_n} \left(r + \xi_0^2 \mathbf{q}^2 + \gamma\, |\omega_n|\right) |\varphi(\mathbf{q}, \omega_n)|^2 + \frac{u}{2N} \int \mathrm{d}^d x \mathrm{d}\tau\ \varphi^4(\mathbf{x}, \tau)$$

Disorder: $\left\{ \begin{array}{c} \text{distance } r \text{ from criticality} \\ \text{bare correlation length } \xi_0 \\ \text{Ohmic dissipation constant } \gamma \end{array} \right\}$ random functions of position

Applications:

- Superconductor-metal quantum phase transition in nanowires ($d = 1, N = 2$)
  $\varphi(\mathbf{x}, \tau)$ represents local Cooper pair operator (Sachdev, Werner, Troyer 2004)
- Hertz' theory of itinerant quantum Heisenberg antiferromagnets ($d = 3, N = 3$)
  $\varphi(\mathbf{x}, \tau)$ represents staggered magnetization (Hertz 1976)

Discrete large-N theory in one dimension

To apply real-space based strong-disorder renormalization group:

- discretize space by introducing "rotor" variables $\phi_j(\tau)$
- large-N limit of an infinite number of order parameter components

Resulting action:

$$S = T \sum_{i,\omega_n} (r_i + \lambda_i + \gamma_i |\omega_n|) \, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

r_i, $\gamma_i > 0$, $J_i > 0$: random functions of lattice site i

λ_i: Lagrange multiplier enforcing large-N constraint $\langle \varphi_i^2(\tau) \rangle = 1$

$\epsilon_i = r_i + \lambda_i$: renormalized (local) distance from criticality

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

in our system

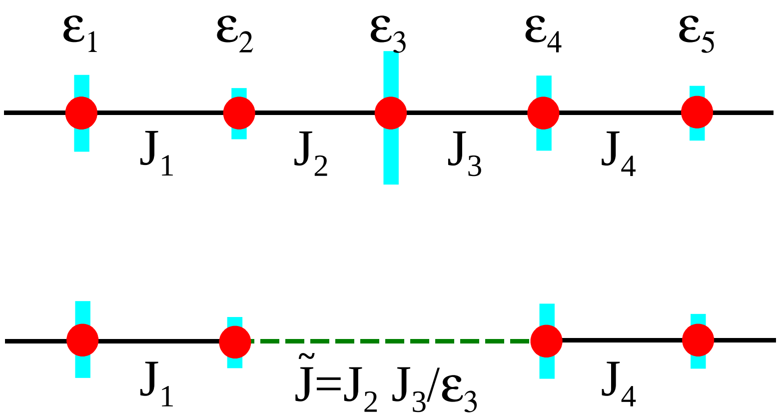
$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|)\, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i\, \phi_i(-\omega_n)\, \phi_{i+1}(\omega_n)$$

the competing local energies are:

- interactions (bonds) J_i favoring the ordered phase
- local "gaps" ϵ_i favoring the disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_i and ϵ_i

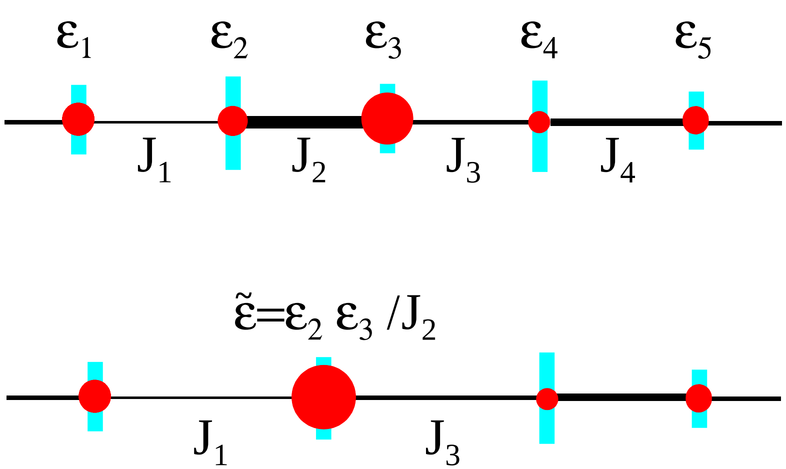
Recursion relations



if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:

- site 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3/\epsilon_3$



if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:

- rotors of sites 2 and 3 are parallel
- can be replaced by single rotor with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$

Renormalization-group flow equations

• RG step is iterated, building larger and larger clusters connected by weaker and weaker bonds (while gradually reducing maximum energy Ω)

$\Rightarrow$ **flow equations** for the probability distributions $P(J)$ and $R(\epsilon)$

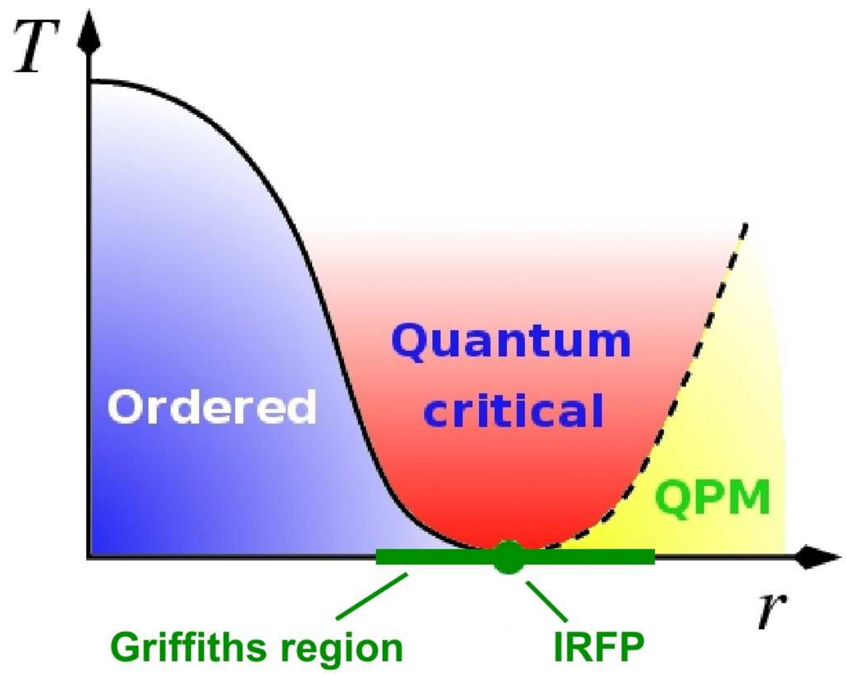
$$
\begin{aligned}
-\frac{\partial P}{\partial \Omega} &= [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right) \\
-\frac{\partial R}{\partial \Omega} &= [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1) R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right)
\end{aligned}
$$

Flow equations are identical to those of the **random transverse-field Ising chain**

$\Rightarrow$ infinite-randomness critical point
$\Rightarrow$ activated dynamical scaling

QPT of a disordered **dissipative** $O(N)$ order parameter is in the same universality class as the **dissipationless** random transverse-field Ising model.

Results: Phase diagram



finite-temperature phase boundary and crossover line take unusual form

$$T_c \sim \exp(-\text{const}\,|r|^{-\nu\psi})$$

$\Rightarrow$ **very wide quantum critical region**

Infinite-randomness critical point:

- at fixed point of the flow equations, disorder scales to infinity
- FP characterized by 3 exponents
- tunneling exponent $\psi = 1/2$ controls dynamical scaling $\ln(1/\Omega) \sim L^\psi$
- moments of surviving clusters grow like $\mu \sim \ln^\phi(1/\Omega)$ with $\phi = (1+\sqrt{5})/2$
- average correlation length diverges as $\xi \sim |r|^{-\nu}$ with $\nu = 2$

Quantum Griffiths regions:

- fixed points of the flow equations can also be found off criticality
- power-law dynamical scaling with nonuniversal exponent

Results: Observables

Thermodynamics:

to find order parameter susceptibility and specific heat at temperature T:
run RG down to energy scale $\Omega = T$ and consider remaining clusters as free

$$\chi(r,T) = \frac{1}{T}\left[\ln(1/T)\right]^{2\phi-d/\psi}\Theta_\chi\left(r^{\nu\psi}\ln(1/T)\right)$$

$$C(r,T) = \left[\ln(1/T)\right]^{-d/\psi}\Theta_C\left(r^{\nu\psi}\ln(1/T)\right)$$

at criticality: $\chi \sim \frac{1}{T}\left[\ln(1/T)\right]^{2\phi-d/\psi}$, in Griffiths phase: $\chi \sim T^{d/z'-1}$

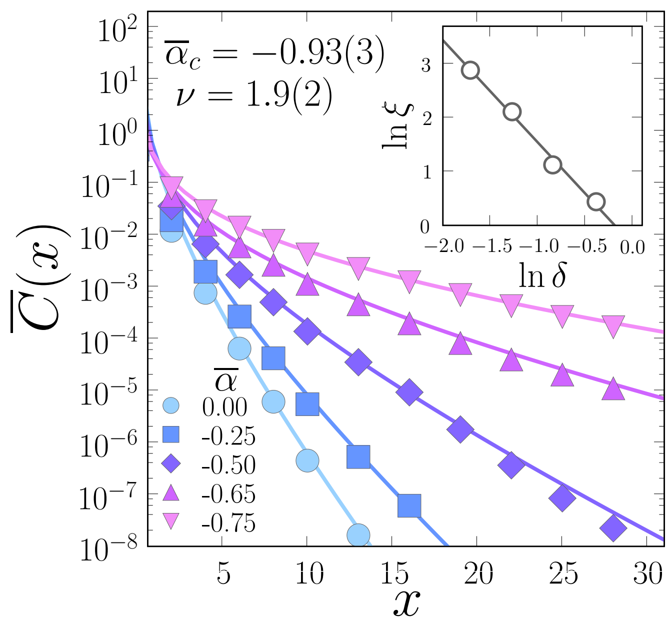
Dynamic susceptibilities at $T=0$:

found by running RG to energy scale $\Omega \approx \omega$

$$\mathrm{Im}\chi(r,\omega) \sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{\phi-d/\psi}\ X\left(r^{\nu\psi}\ln(1/\omega)\right)$$

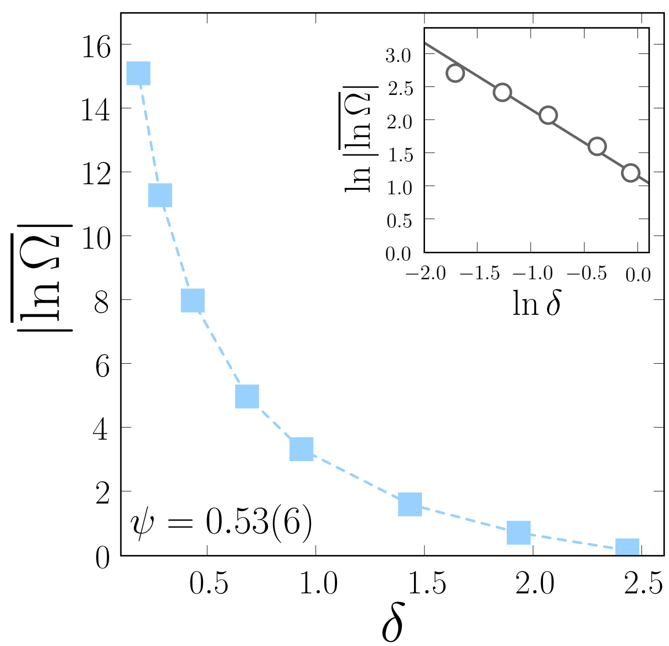
$$\mathrm{Im}\chi^{\mathrm{loc}}(r,\omega) \sim \frac{1}{\omega}\left[\ln(1/\omega)\right]^{-d/\psi}\ X^{\mathrm{loc}}\left(r^{\nu\psi}\ln(1/\omega)\right)$$

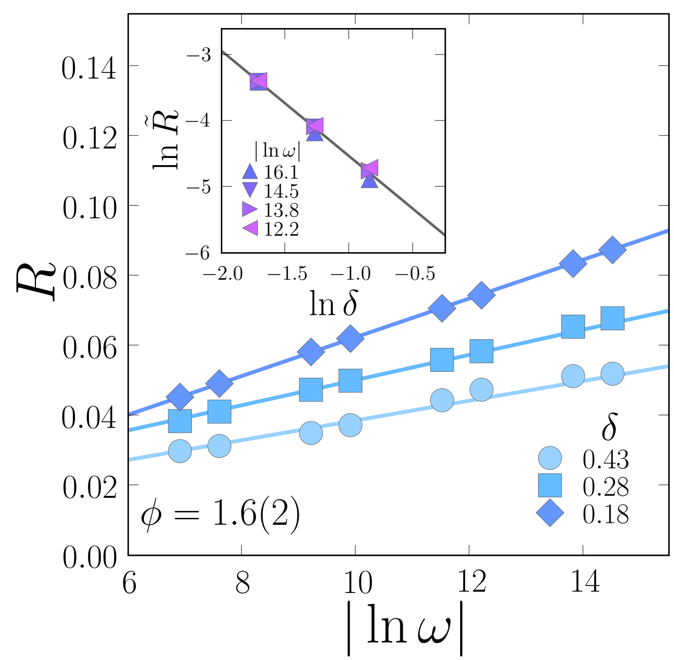
Results: Numerical confirmation



- A. Del Maestro et al. (2008) solved disordered large-N problem numerically exactly
- calculated equal time correlation function C, energy gap Ω, and ratio R of local and order parameter dynamic susceptibilities

	ν	ψ	ϕ
SDRG	2	1/2	$(\sqrt{5}+1)/2$
Numerics	1.9(2)	0.53(6)	1.6(2)





Generalizations: $N < \infty$, $d > 1$, nonohmic damping

Order parameter symmetry

- our explicit calculations are for an infinite number of OP components, $N = \infty$
- results apply to all **continuous symmetry** cases $N > 1$
  (clusters are marginal – gaps depends exponentially on size)

Higher dimensions $d > 1$

- infinite randomness scaling scenario also appears in 2D and 3D
- critical exponent values are different, only known numerically

Nonohmic damping

- if damping term is nonohmic, $\gamma|\omega_n|^{2/z_0}$, recursion relations change
- **subohmic** case, $z_0 > 2$: quantum phase transition **destroyed by smearing**
- **superohmic** case, $z_0 < 2$: transition survives, likely with **conventional scaling**

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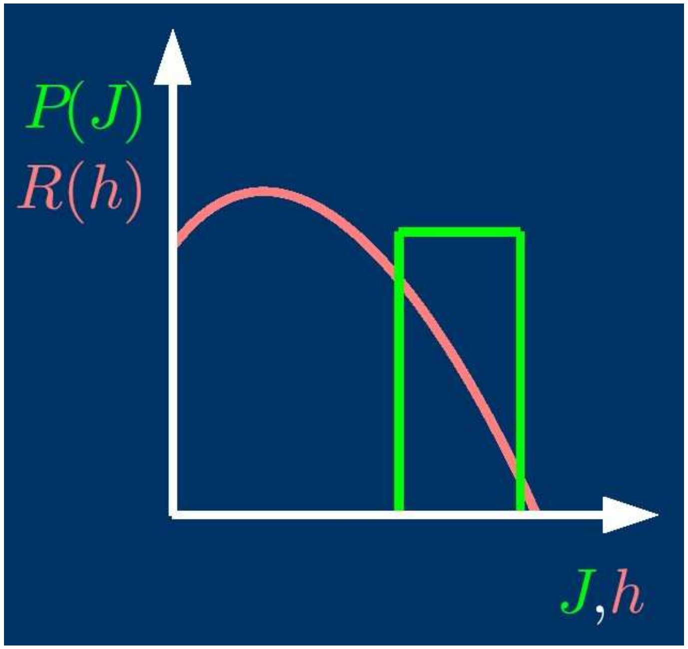
Dissipative random transverse-field Ising chain

$$H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + \sum_{i,n} \sigma_i^z \lambda_{i,n} (a_{i,n}^\dagger + a_{i,n}) + \sum_{i,n} \nu_{i,n} a_{i,n}^\dagger a_{i,n}$$

J_i: exchange interaction between z-components of spin σ_i
h_i: transverse magnetic field, acting on x-component of spin σ_i
$a_{i,n}^\dagger, a_{i,n}$: harmonic oscillator bath coupling to z-component of spin σ_i



Bath spectral function

$$\mathcal{E}(\omega) = \pi \sum_n \lambda_{i,n}^2 \delta(\omega - \nu_{i,n}) = 2\pi\alpha\omega e^{-\omega/\omega_c}$$

α: dimensionless dissipation strength
ω_c: oscillator energy cutoff

Linear low freq. spectrum: Ohmic dissipation

Strong-disorder renormalization group

Integrate out local high energy modes: $\Omega = \max(J_i, h_i, \omega_c/p)$

To reduce maximum energy from Ω to $\Omega - d\Omega$:

1. Integrate out all oscillators with frequencies $\nu \in [p(\Omega - d\Omega), p\Omega]$

$$\tilde{h}_i = h_i \exp\left(-\alpha_i \int_{p(\Omega-\mathrm{d}\Omega)}^{p\Omega} \frac{\mathrm{d}\omega}{\omega}\right) = h_i \left(1 - \alpha\mu_i \frac{\mathrm{d}\Omega}{\Omega}\right)$$

2. Decimate all transverse fields $h_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{J} = J_{i-1} J_i / h_i$$

3. Decimate all interaction energies $J_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{h} = h_i h_{i+1} / J_i, \quad \tilde{\mu} = \mu_i + \mu_{i+1}$$

Extra downward renormalization of the transverse fields due to dissipation

Renormalization-group flow equations

Flow equations for the probability distributions $P(J)$ and $R(h, \mu)$

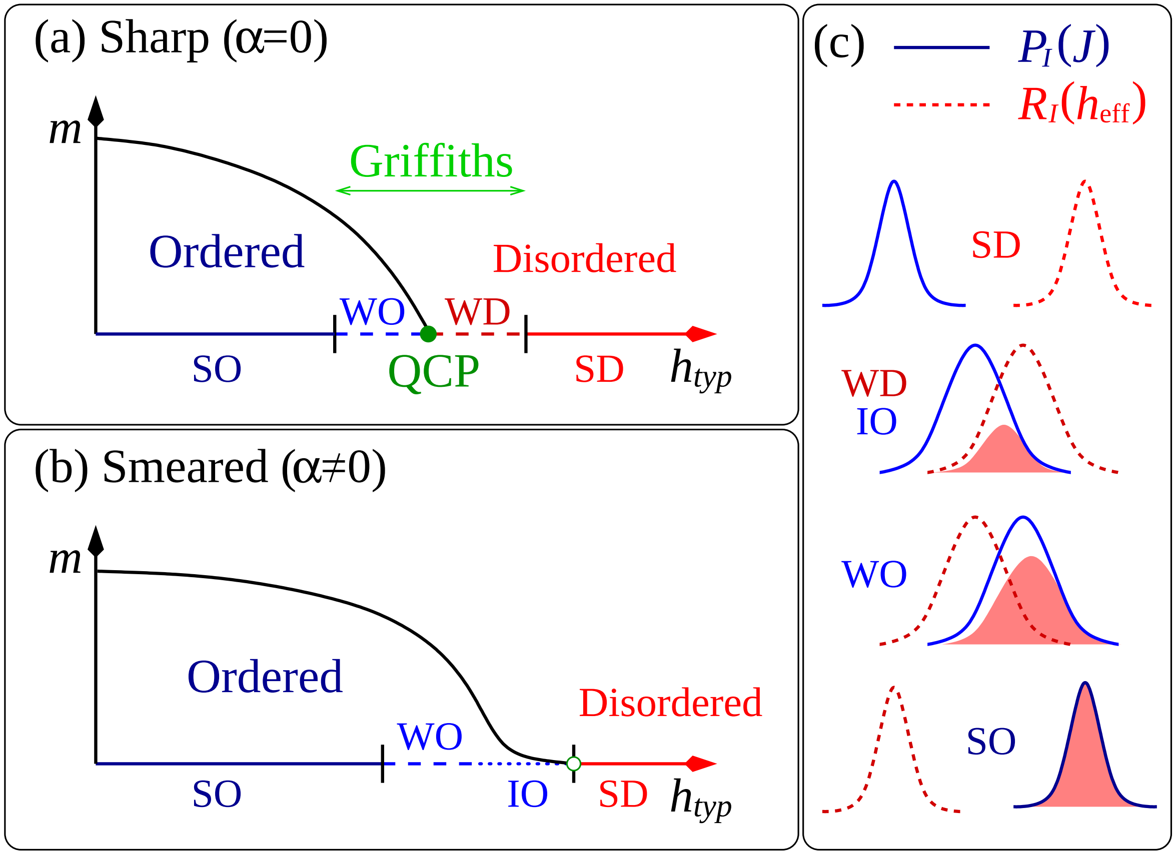
$$
\begin{aligned}
-\frac{\partial P}{\partial \Omega} &= \left[P(\Omega) - (1-\alpha\bar{\mu})R_h(\Omega)\right] P + (1-\alpha\bar{\mu})R(\Omega)\int dJ_1 dJ_2\, P(J_1)P(J_2)\,\delta\left[J - \frac{J_1 J_2}{\Omega}\right] \\
-\frac{\partial R}{\partial \Omega} &= \left[(1-\alpha\bar{\mu})R_h(\Omega) - P(\Omega)\right] R + \frac{\alpha\mu}{\Omega}\left[R + h\frac{\partial R}{\partial h}\right] + \\
&\quad + P(\Omega)\int dh_1 dh_2 d\mu_1 d\mu_2\, R(h_1,\mu_1)R(h_2,\mu_2)\,\delta\left[h - \frac{h_1 h_2}{\Omega}\right]\delta[\mu - \mu_1 - \mu_2]
\end{aligned}
$$

$(1-\alpha\bar{\mu})$: probability for decimating field vanishes for $\mu > 1/\alpha$
$\Rightarrow$ important finite "volume" scale $1/\alpha$

- clusters act as Ohmic spin-boson problem with effective damping constant $\alpha\mu$
- if $\alpha\mu > 1$, they undergo **localization transition** (Caldeira, Leggett, Weiss)

Large clusters freeze independently $\Rightarrow$ quantum phase transition is **smeared**

Smeared quantum phase transition

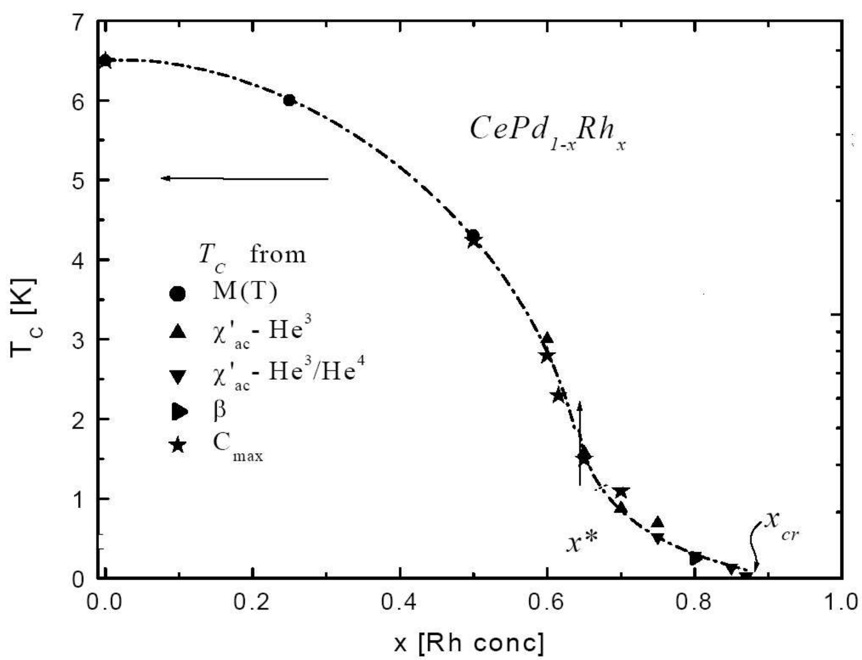


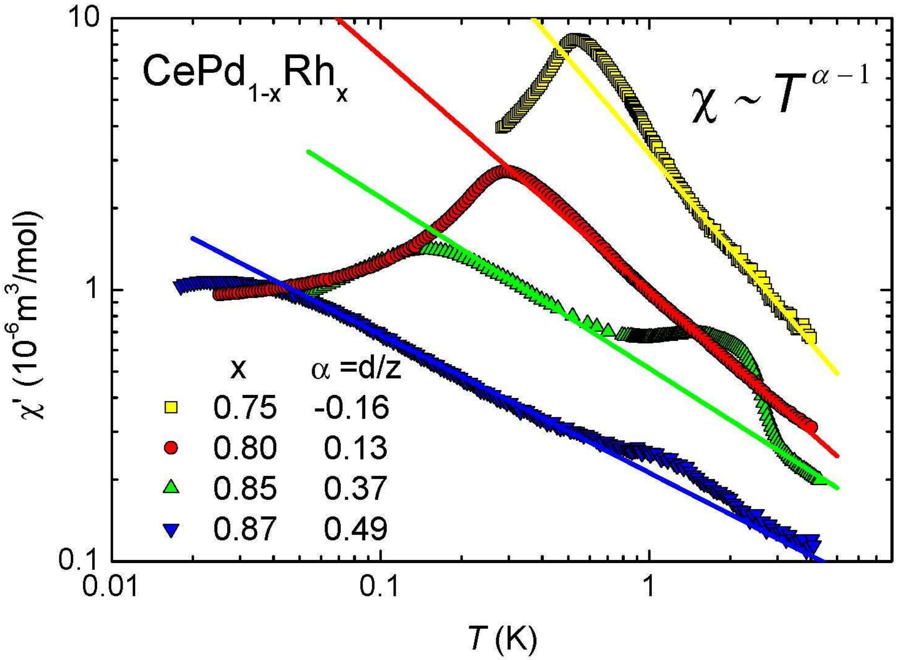
- quantum critical point and disordered Griffiths phase destroyed
- replaced by **inhomogeneously ordered** region in the tail of the ordered phase

Low temperature thermodynamics: dominated by large frozen clusters

Example: uniform susceptibility $\chi \sim T^{-1-1/z}$

Rare region effects in $CePd_{1-x}Rh_x$?





- ferromagnetic phase shows pronounced tail ⇒ evidence for smeared transition
- evidence for spin-glass like behavior in tail
- above tail: nonuniversal power-laws characteristic of quantum Griffiths effects

(Sereni et al., Phys. Rev. B **75** (2007) 024432 + Westerkamp, private communication)

Classification of weakly disordered phase transitions according to importance of rare regions

T. Vojta, J. Phys. A **39**, R143–R205 (2006)

Dimensionality of rare regions	Griffiths effects	Dirty critical point	Examples (classical PT, QPT, non-eq. PT)
$d_{RR} < d_c^-$	weak exponential	conv. finite disorder	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	strong power-law	infinite randomness	Ising model with linear defects random quantum Ising model disordered directed percolation (DP)
$d_{RR} > d_c^-$	RR become static	smeared transition	Ising model with planar defects itinerant quantum Ising magnet DP with extended defects

Conclusions

- We have studied quantum phase transitions in the presence of both **disorder** and **Ohmic dissipation** using the strong-disorder renormalization group

- For **continuous symmetry** order parameters, the RG recursion relations for the local gaps and interactions are **multiplicative**

⇒ **infinite-randomness** critical point in the universality class of the random transverse field Ising model

- For **Ising symmetry**, the dissipation introduces a finite length scale beyond which the clusters freeze.

⇒ quantum phase transition is **smeared**

Exotic QPTs due to interplay between disorder and dissipation: disorder creates locally ordered rare regions, dissipation makes their dynamics ultraslow

For details see: J. A. Hoyos, C. Kotabage, T. Vojta, Phys. Rev. Lett. **99**, 230601 (2007)
J. A. Hoyos and T. Vojta, Phys. Rev. Lett. **100**, 240601 (2008)