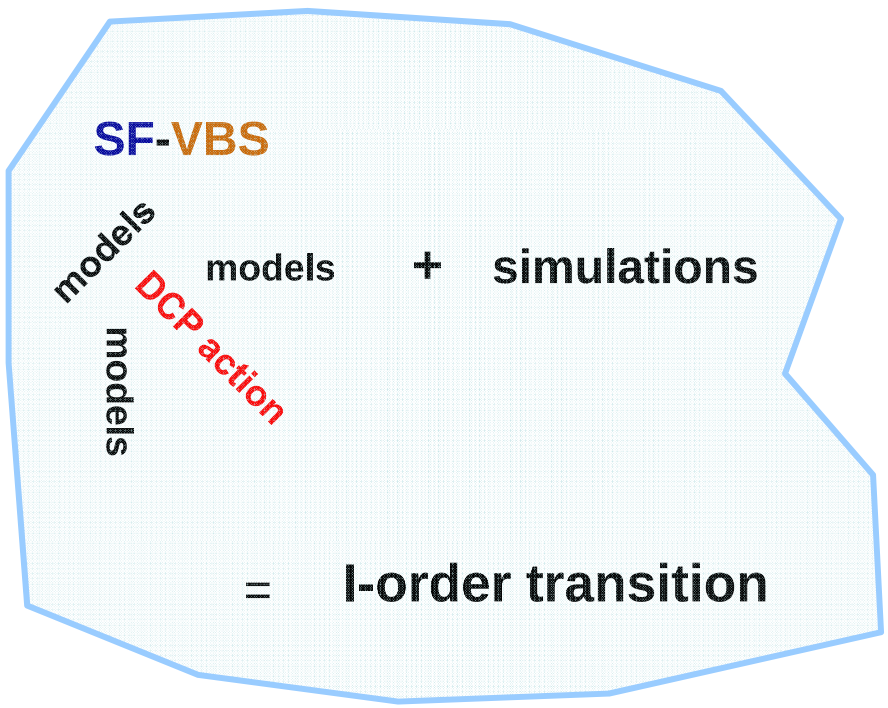
Deconfined criticality: SU(2) Dejavu



Anatoly Kuklov (CSI, CUNY)
Nikolay Prokof'ev (UMASS, Amherst)
Boris Svistunov (UMASS, Amherst)
Matthias Troyer (ETH)
Munehisa Matsumoto (UC Davis)

PRL 101, 050405 (2008)
Annals of Physics 321, 1601 (2006)
Prog. Theor. Phys. Jap. 160 (2005)
PRL 93, 230402 (2004)

National Science Foundation
WHERE DISCOVERIES BEGIN

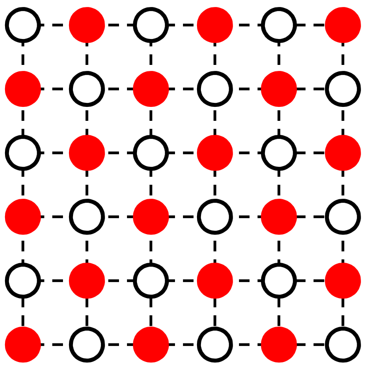
"Quantum critical phenomena" Totonto, September 25-27, 2008

Why Dejavu?

$U(1)\times U(1)$	$SU(2)$
Contributing authors	**Many of the same authors**
Simulations of specific models	**Same type of models**
claims of new criticality	**claims of new criticality**
Problems with scaling for systems	**claims of new criticality**
Method of solution: flowgrams $\rightarrow$ **generic I-order**	**Dejavu**

Solid = insulator with broken translational symmetry (integer filling excluded)

Checkerboard solid (anti-ferromagnet)

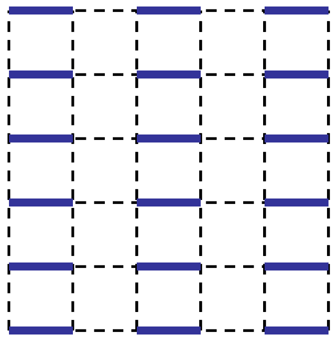


Density-order

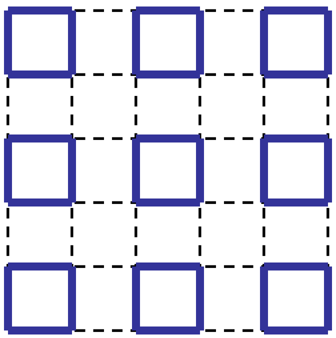
$$\langle n_i n_j \rangle$$

Valence-bond solids (VBS)

Columnar VBS



Plaquette VBS



Current (or bond) order

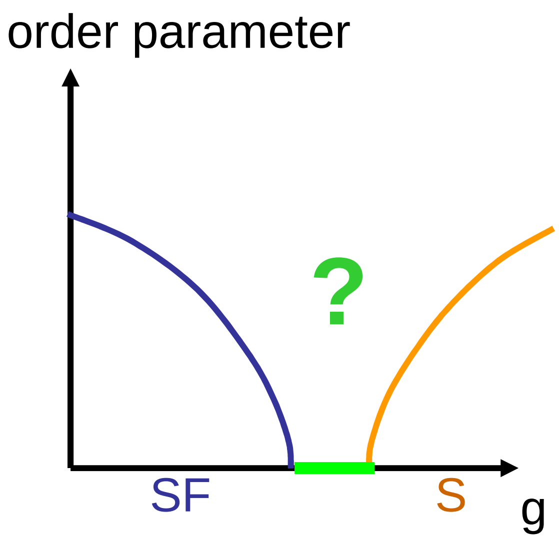
$$\langle J_i^2 J_j^2 \rangle$$

Superfluid and Solid orders and transitions:

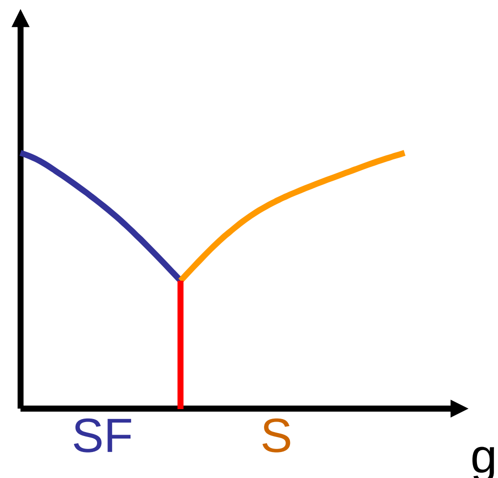
$$\Psi \qquad M \qquad B_x, B_y \qquad D_x, D_y \qquad \ldots$$

order parameters:

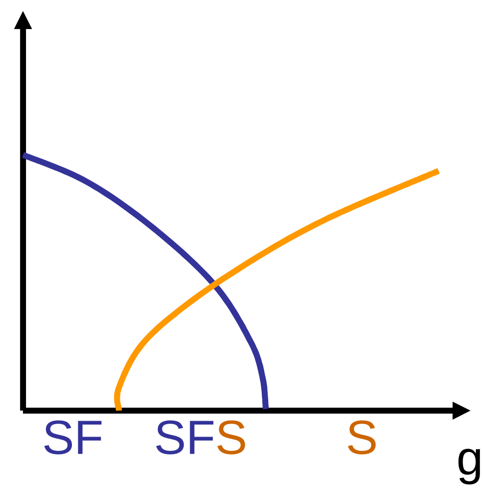
"Naïve" Ginzburg-Landau (expansion from the generic quantum disordered phase)



Quantum disordered



First-order



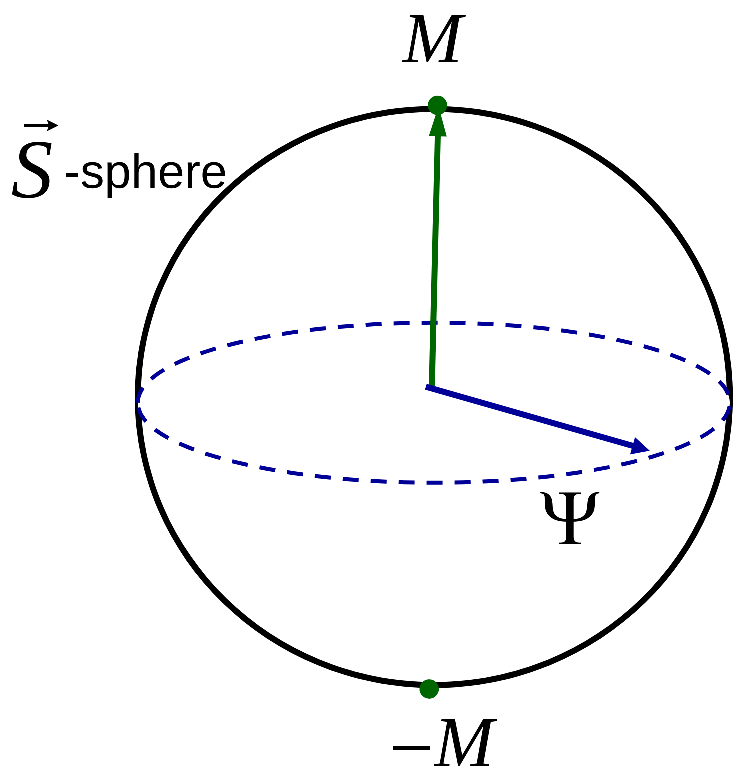
Supersolid

$$\vec{S} = (\underbrace{S_1, S_2}_{\Psi}, \underbrace{S_3}_{M}, \underbrace{S_4, S_5}_{B_x, B_y} \ldots)$$

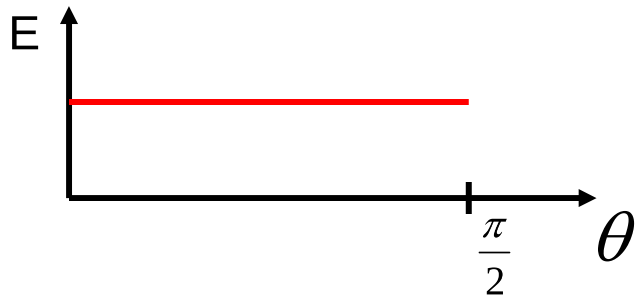
- multi-component order parameter; the symmetry of S is ALWAYS broken in the ground state

$$\left|\vec{S}\right| \neq 0$$

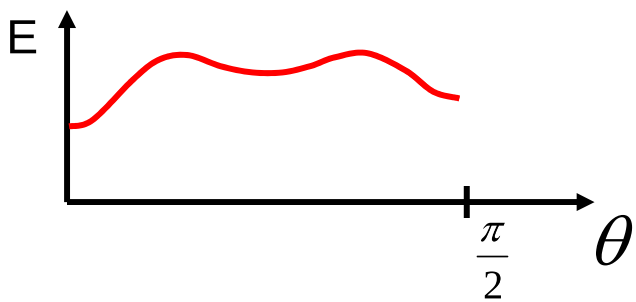
O(3)-case:



Heisenberg point = exact O(3)-symmetry



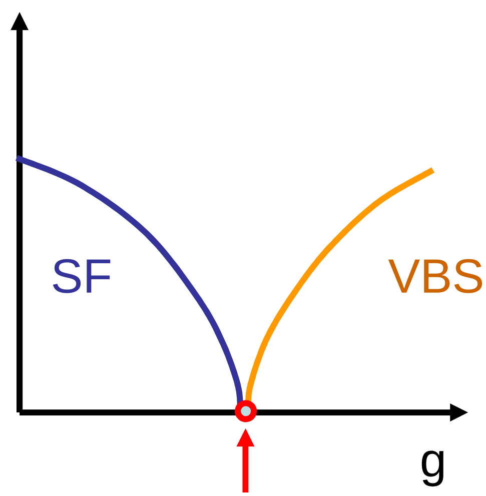
Generic case; I-order SF-S transition



SF and **VBS** orders are deeply related … May be even self-dual at the critical point!

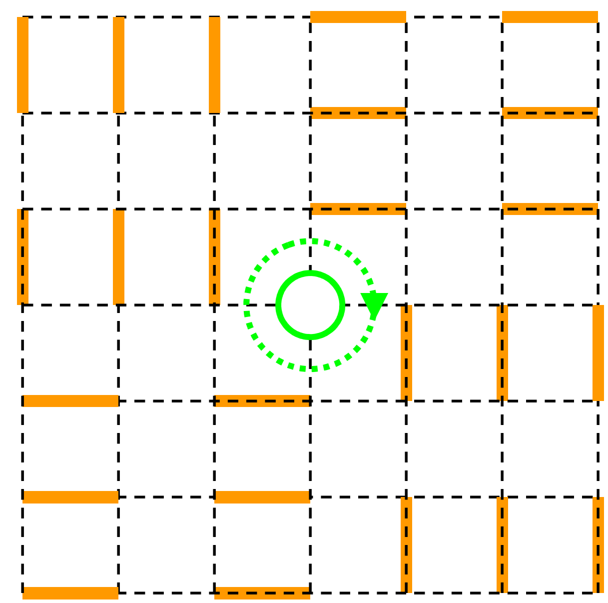
O.I. Motrunich and A. Vishwanath, Phys. Rev. 70, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M.P.A. Fisher, Science, 303, 1490 (2004):

Deconfined criticality, does not fit the standard Landau-Ginsburg-Wilson paradigm.



Deconfined spinons
= vortices in the VBS order

$B_x + iB_y$



DCP action:

$$S = \int d^2r\, d\tau \sum_{a=1,2} \left| \left(\partial_\mu - iA_\mu\right) z_a \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + v|z_1|^2 |z_2|^2 + \kappa\left(\varepsilon_{\mu\nu\beta}\partial_\nu A_\beta\right)^2$$

Ordered SF state for matter fields =
(duality mapping) "quantum disordered" for vortex fields

Do LG expansion for vortex fields!

L. Balents, L. Batosch, A. Burkov, S.Sachdev, K. Sengupta, PRB 71, 144508 and 144509 (2005).

Filling factor 1/2 ⟶ dual magnetic flux/plaquette 1/2 ⟶ two species of vortexes + gauge field coupling

at the SF-VBS critical point

DCP action

$$S_\psi = \int d^2 r d\tau \sum_{a=1,2} \left| \left(\partial_\mu - iA_\mu \right) \psi_a \right|^2 + s|\psi|^2 + u\left(|\psi|^2\right)^2 + v|\psi_1|^2 |\psi_2|^2 + \kappa\left(\varepsilon_{\mu\nu\beta}\partial_\nu A_\beta\right)^2$$

$$|\psi|^2 = |\psi_1|^2 + |\psi_2|^2$$

$$g = [e^2 = 1/\kappa] \rightarrow g(L) = gL$$ **Run-away flow to strong coupling**

DCP action:

SU(2)
$\nu = 0$

$$S_\psi = \sum_i \sum_{a=1,2} \left| \left(\partial_\mu - iA_\mu \right) \psi_a \right|^2 + s|\psi|^2 + u\left(|\psi|^2 \right)^2 + \nu |\psi_1|^2 |\psi_2|^2 + \kappa \left(\varepsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2$$

$$S_{XY} = -J \sum_{r,\mu} \left[\cos\left(\Delta_\mu \varphi_1 - A_\mu \right) + \cos\left(\Delta_\mu \varphi_2 - A_\mu \right) \right] + \kappa \sum_{\square} (\nabla \times A)^2$$

$$S_{CP^1} = \sum_i \sum_{a=1,2} \left| \left(\partial_\mu - iA_\mu \right) \psi_a \right|^2 + \kappa \left(\varepsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2 \quad \text{with} \quad |\psi_1|^2 + |\psi_2|^2 = 1$$

$$S_J = U \sum_r F[j_{a\mu}(r)] + g \sum_{rr',\mu} Q(r-r') \left(j_{1\mu}(r) + j_{2\mu}(r) \right) \cdot \left(j_{1\mu}(r') + j_{2\mu}(r') \right)$$

$$Q^{-1}(q) = \sum_\mu \sin^2(q_\mu/2) \quad \rightarrow \quad Q(r) \sim 1/r$$

Mappings:

"High-T" expansion (expansion in kinetic energy)

$$S = \sum_r U|\psi_r|^4 - \mu|\psi_r|^2 - t\sum_{<rr'>}\left(\psi_r^*\psi_{r'}\, e^{iA_{<rr'>}} + c.c.\right) + \kappa\sum_{\square}[\nabla\times A]^2$$

$$Z = \iint DAD\psi\, e^S$$

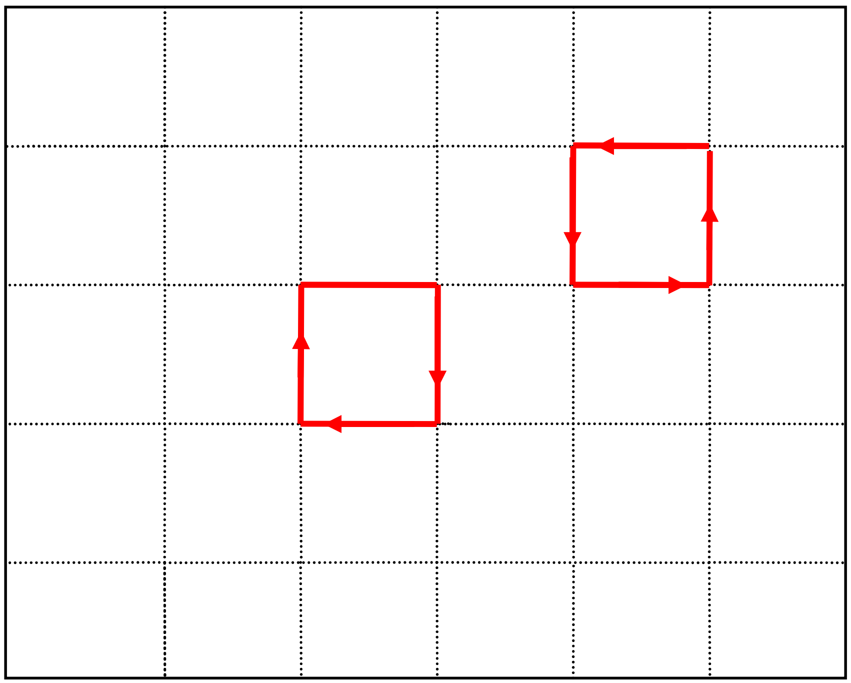
$$= \iint DA \prod_r \int d\psi_r\, e^{S_r} \prod_{b=<rr'>} \sum_{n_b=0}^{\infty} \frac{(-t)^{n_b}}{n_b\,!}(\psi_r^*\psi_{r'})^{n_b} \sum_{m_b=0}^{\infty} \frac{(-t)^{m_b}}{m_b\,!}(\psi_r\psi_{r'}^*)^{m_b}\, e^{iA_b(n_b-m_b)} e^{S_A}$$

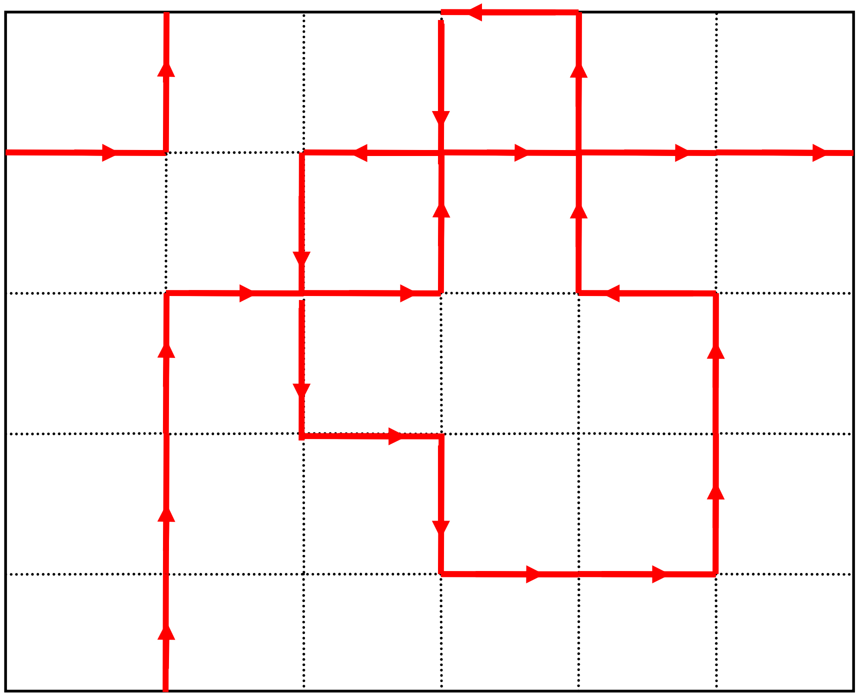
$$= \sum_{\substack{n_b, m_b \text{ with} \\ \{j=n_b-m_b\}\in loops}} \exp\{-g\sum_{rr'} Q(\mathbf{r}-\mathbf{r}')\,\mathbf{j}_r\mathbf{j}_{r'}\}\; W(\{j_b\}) = \sum_{\substack{n_b, m_b \text{ with} \\ \{j=n_b-m_b\}\in loops}} e^{S_J}$$

loop configurations of oriented currents

$\mathbf{j}_r = \mathbf{j}_{1r} + \mathbf{j}_{2r}$ (DCP)

INSULATOR (NORMAL) **SUPERFLUID**





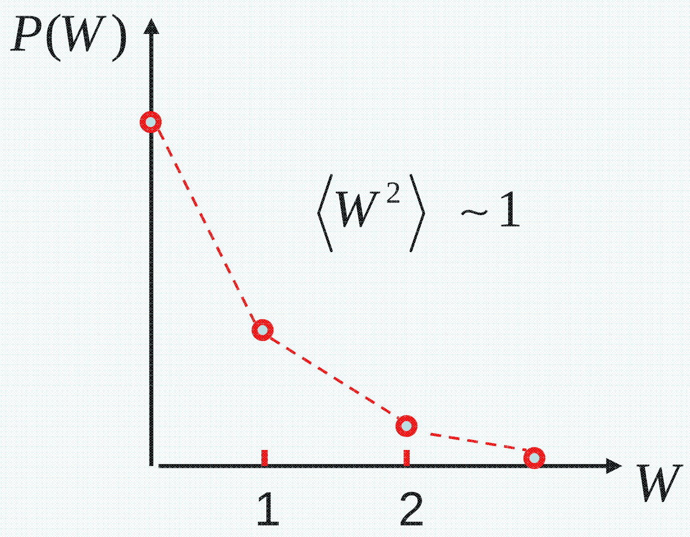
Winding numbers = MC “blessing” (Pollock, Ceperley ’87)

Superfluid stiffness: $\Lambda_s = TL^{d-2} \langle W^2 \rangle \rightarrow \langle W^2 \rangle / L$

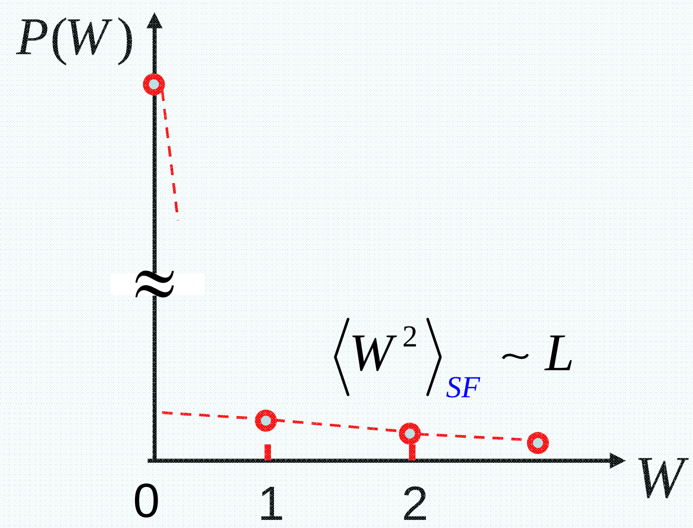
j-current through any cross-section

Probe system properties at the largest scales; ideal for studies of critical phenomena

Continuous critical point: the entire disctibution $P(W)$ is universal



I-order SF-S transition:



Since $$\frac{\sum_{W\neq 0} P(W)}{P(0)} = \begin{cases} \infty & \text{in SF} \\ 0 & \text{in I} \end{cases}$$

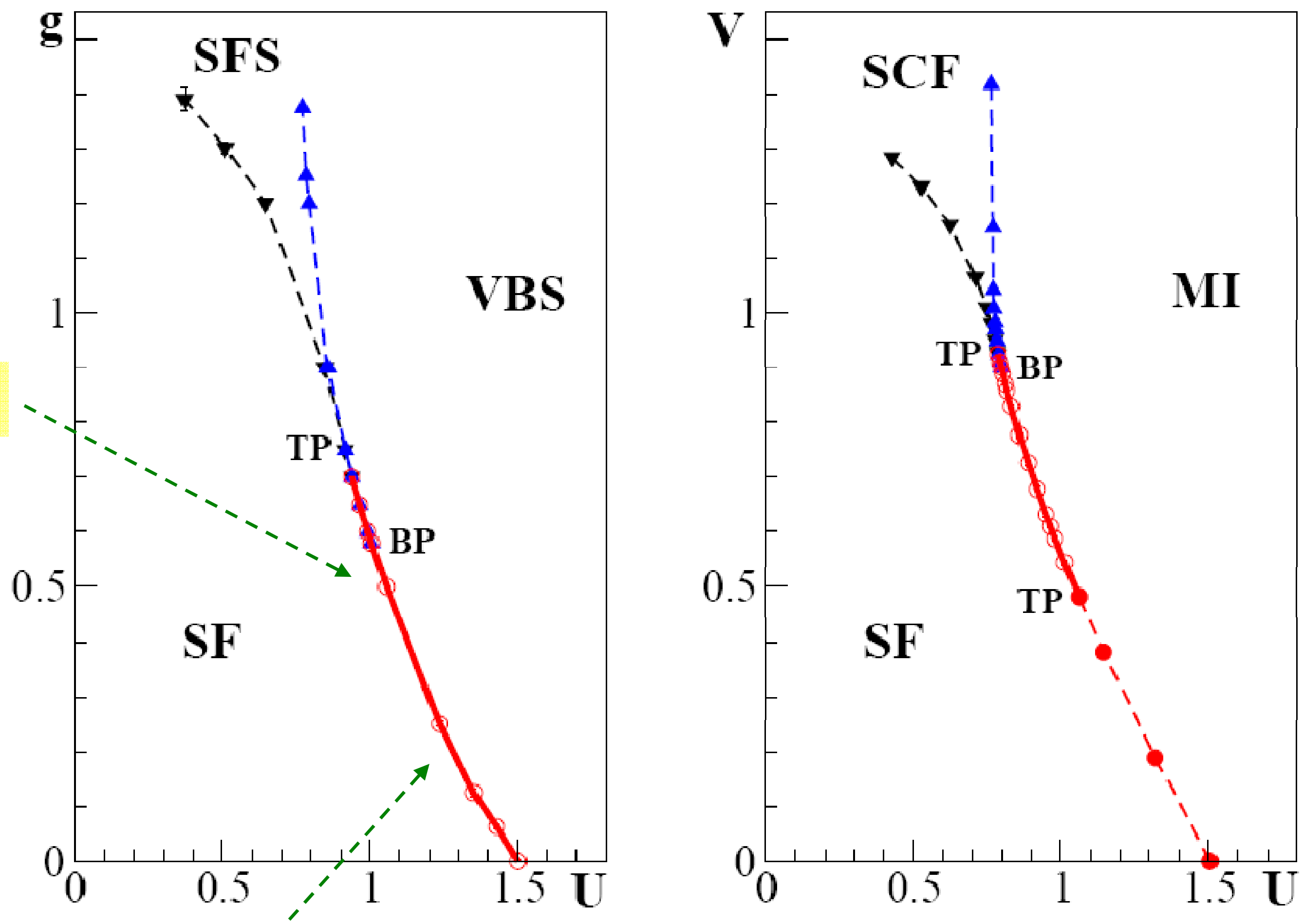
Transition point Def: $$\frac{\sum_{W\neq 0} P(W)}{P(0)} = \#$$

Quantity to study at the transition point: $$\langle W^2 \rangle = f(L) \to \begin{cases} \infty & \text{I-order} \\ \text{const} & \text{continuous} \end{cases}$$ (scale invariance)

Phase diagram of the U(1)xU(1) DCP action

short-range version

$$gQ(r-r') \to V\delta(r-r')$$

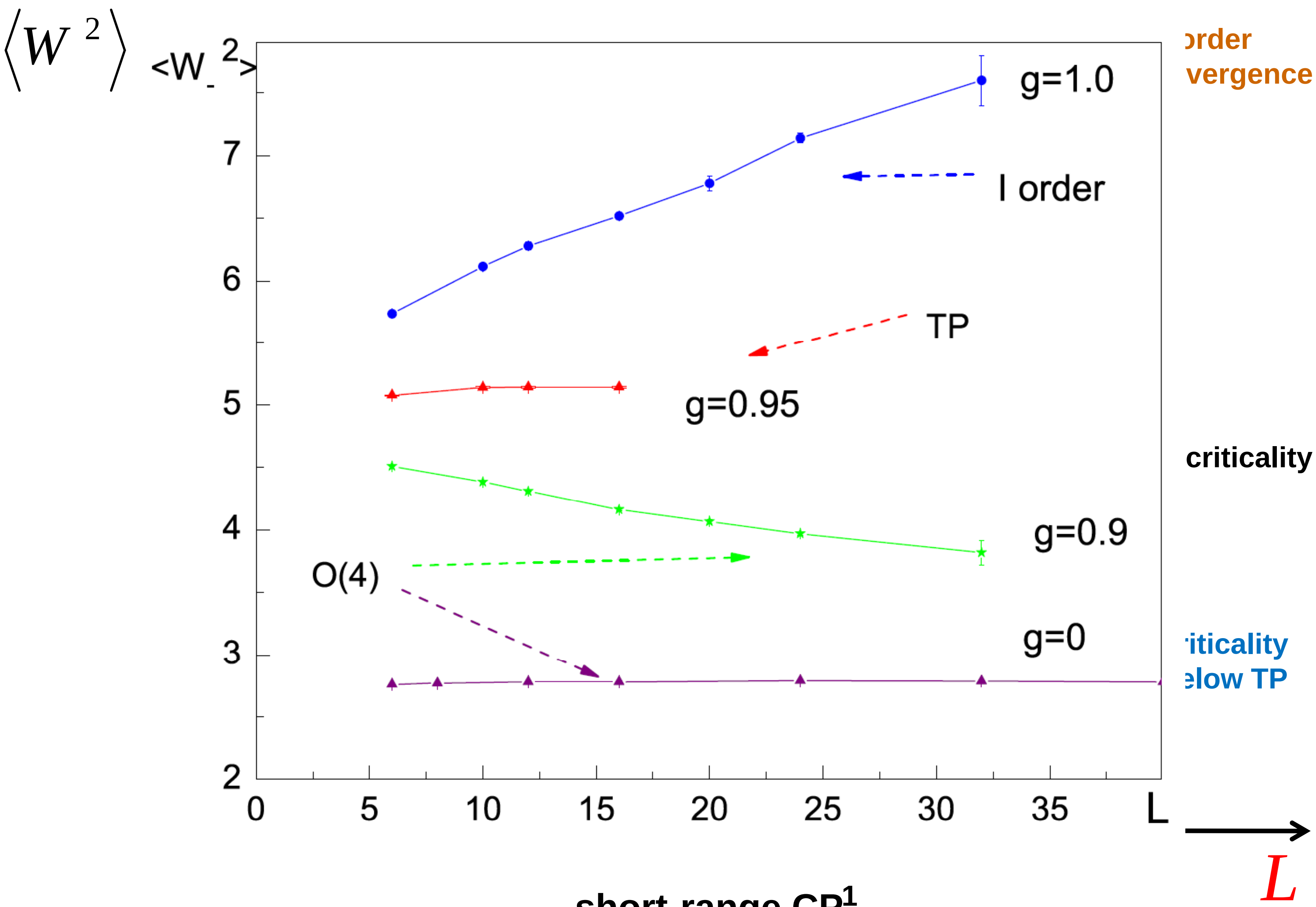


g ~ 0.5 - weak I-order

small g maps to g > 0.5

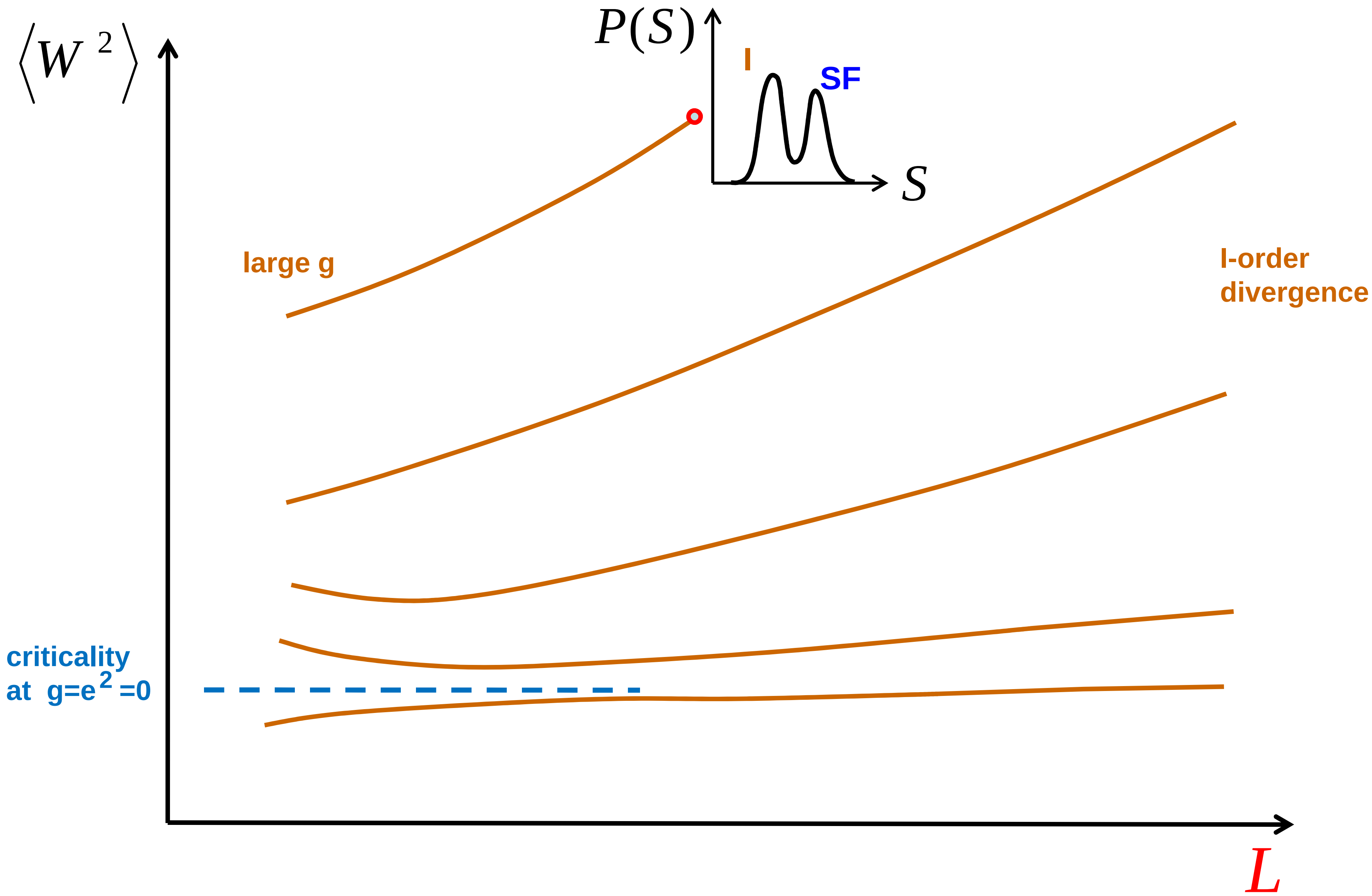
RG flow: $g_{eff}(L) \propto gL$

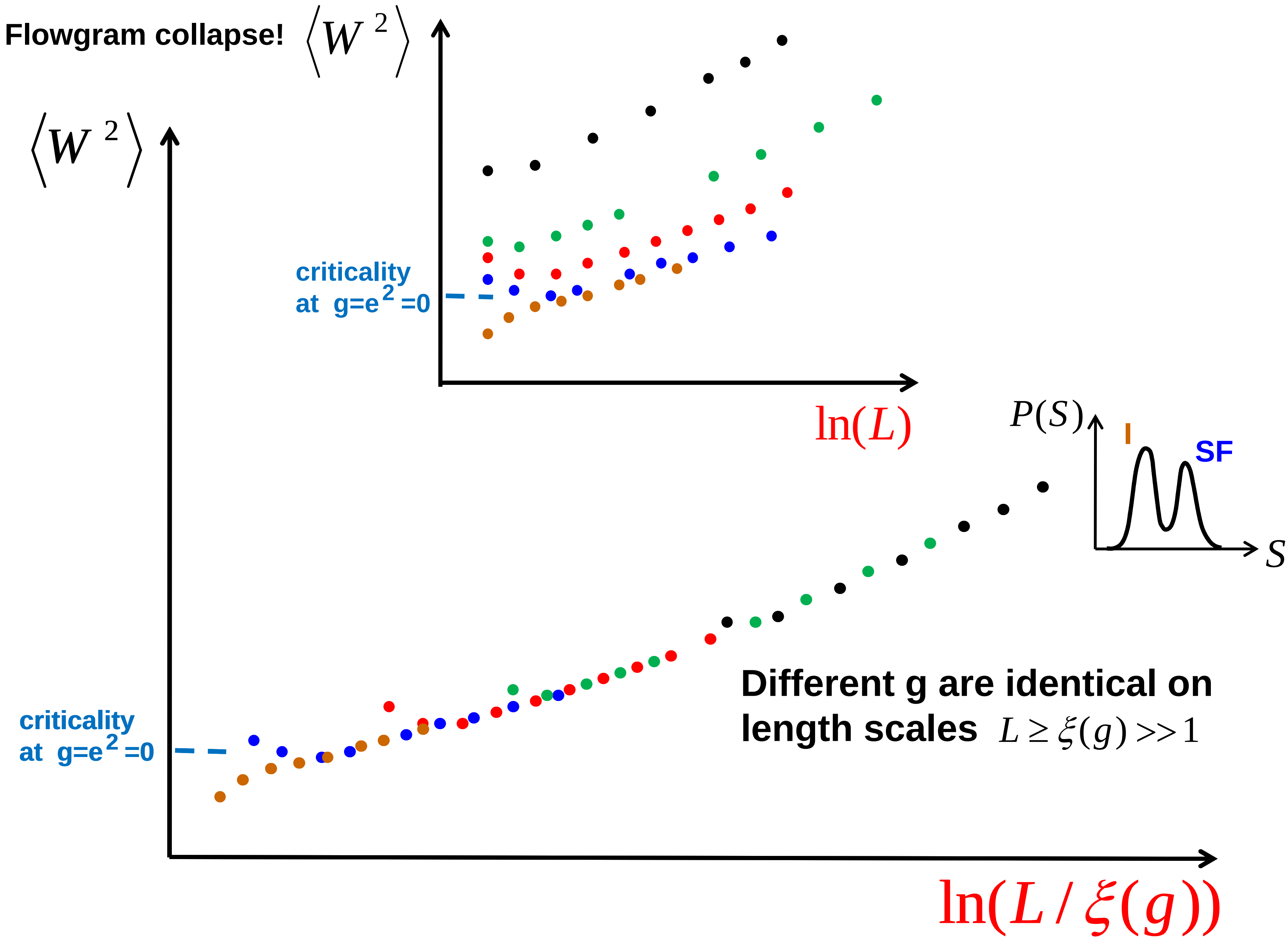
Flowgrams of $\left\langle W^2 \right\rangle_L$. What to expect across the tricitical point.



short-range CP[1]

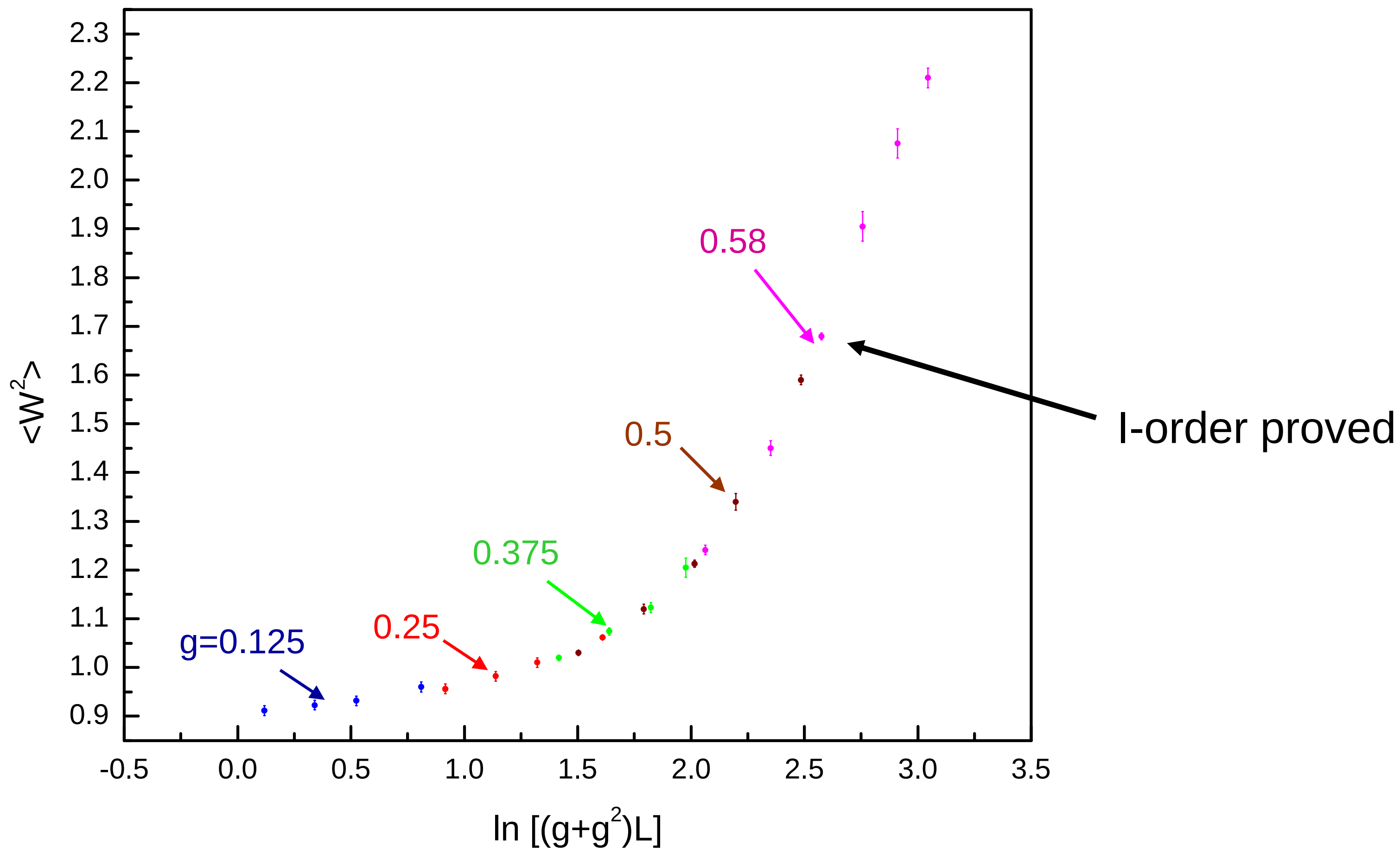
Flowgrams of $\langle W^2 \rangle_L$ for the run-away flow to strong coupling and I-order.



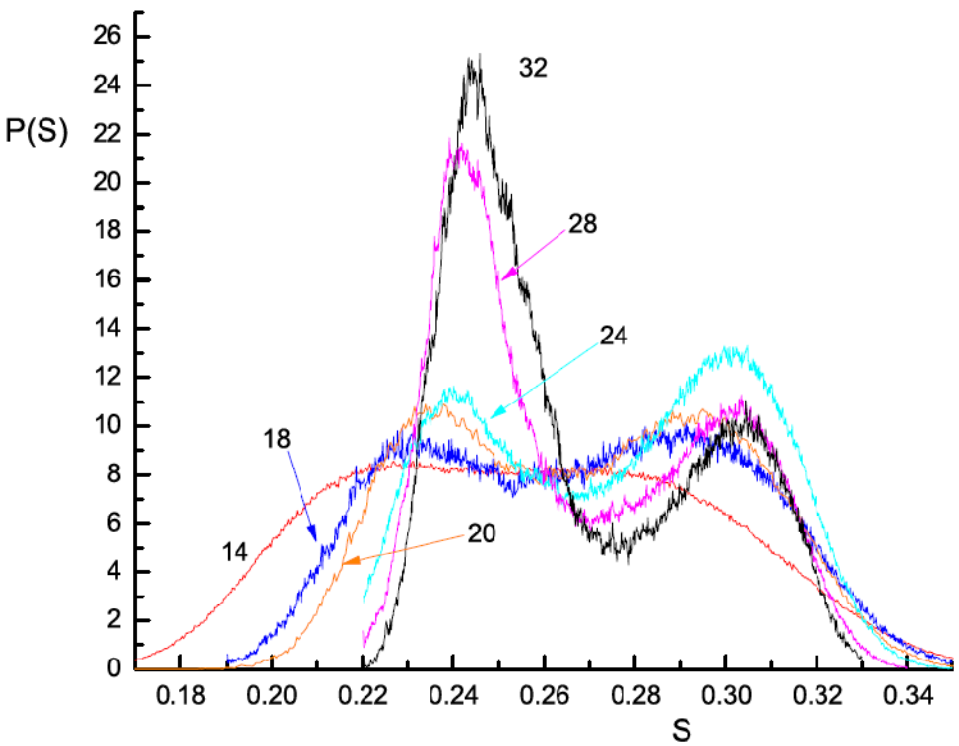


Flowgram collapse!
$\langle W^2 \rangle$
criticality
at g=e^2=0
$\ln(L)$
$P(S)$
I
SF
S
$\langle W^2 \rangle$
criticality
at g=e^2=0
Different g are identical on
length scales $L \geq \xi(g) >> 1$
$\ln(L/\xi(g))$

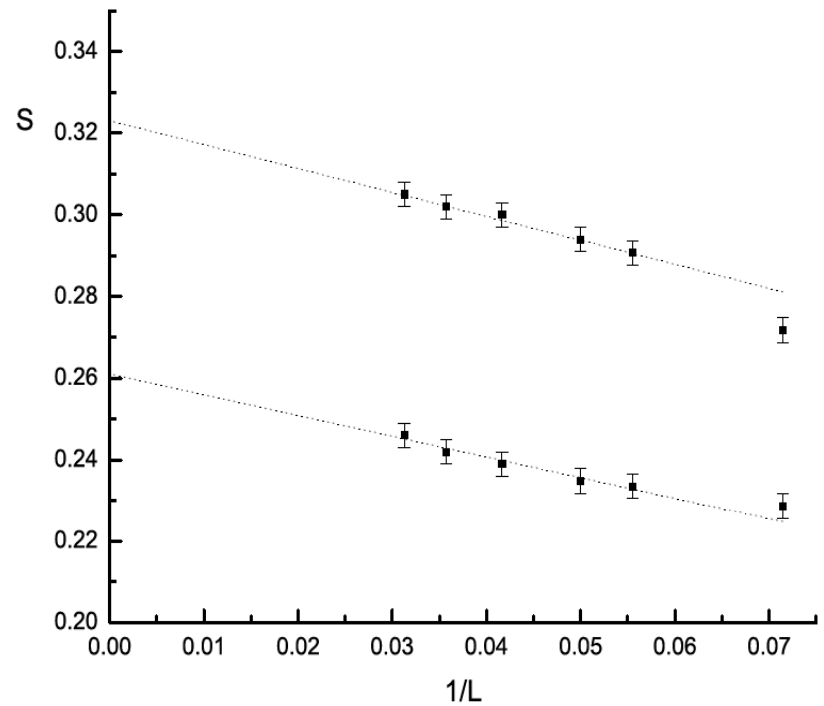
$U(1) \times U(1)$ case



Proof of first-order transition at g=0.58



Probability distributions at the critical point for g=0.58



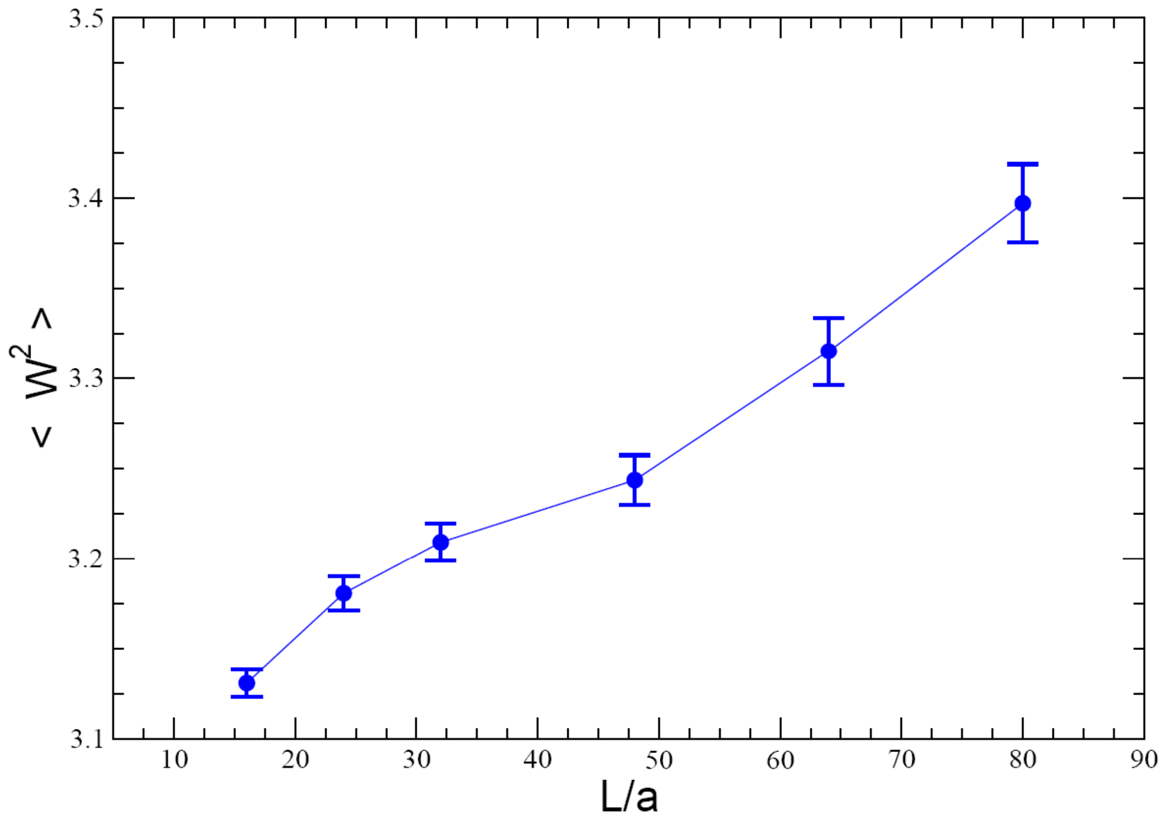
Scaling of peak positions with system size L

$SU(2)$ case

$$H = J\sum_{x,i}\vec{S}_x\cdot\vec{S}_{x+\hat{i}} - Q\sum_x\Big[(\vec{S}_x\cdot\vec{S}_{x+\hat{1}}-\frac{1}{4})(\vec{S}_{x+\hat{2}}\cdot\vec{S}_{x+\hat{1}+\hat{2}}-\frac{1}{4}) + (\vec{S}_x\cdot\vec{S}_{x+\hat{2}}-\frac{1}{4})(\vec{S}_{x+\hat{1}}\cdot\vec{S}_{x+\hat{1}+\hat{2}}-\frac{1}{4})\Big]$$

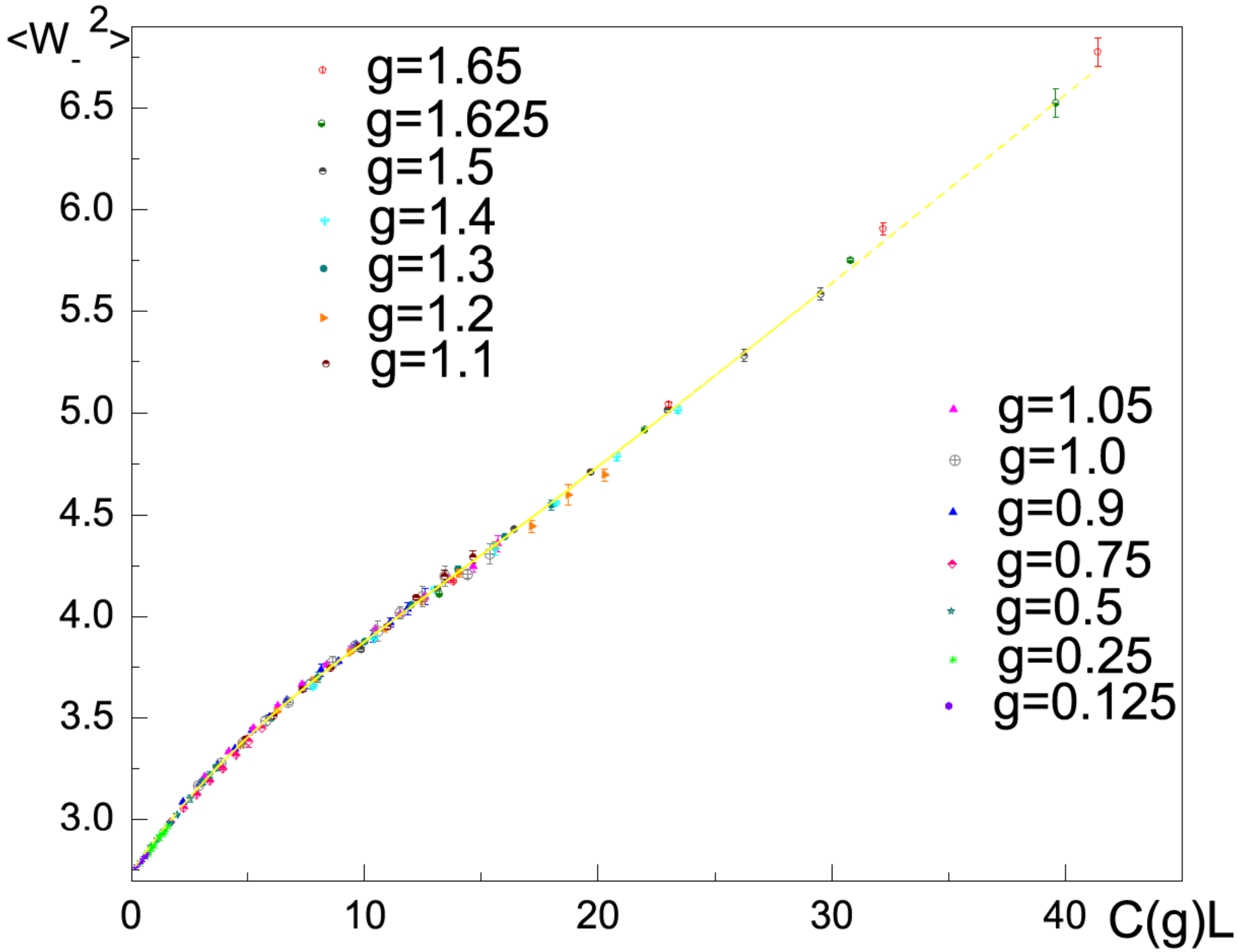
A. W. Sandvik '07

R. G. Melko and R. K. Kaul '07

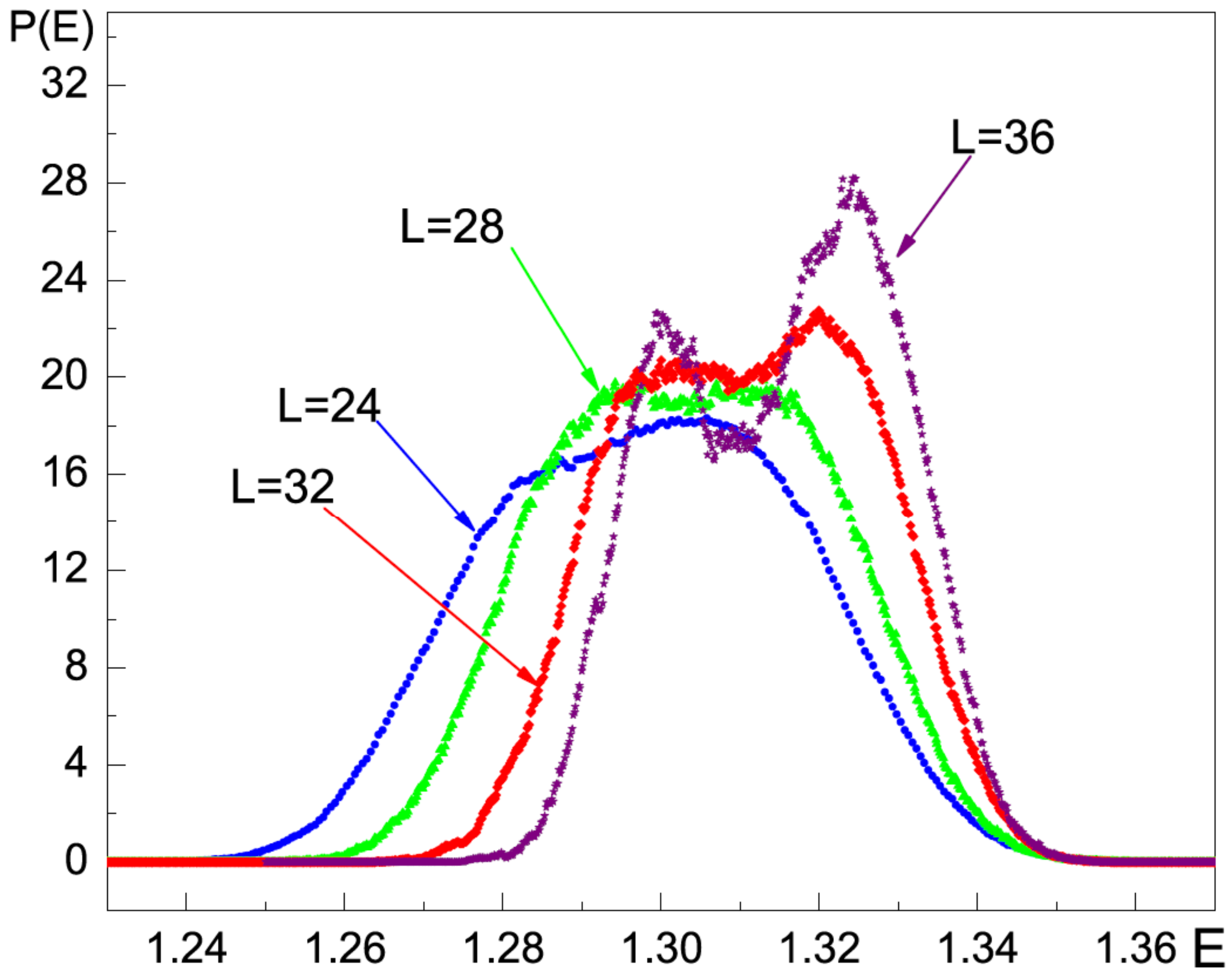


F.-J. Jiang[a], M. Nyfeler[a], S. Chandrasekharan[b], and U.-J. Wiese[a] '07

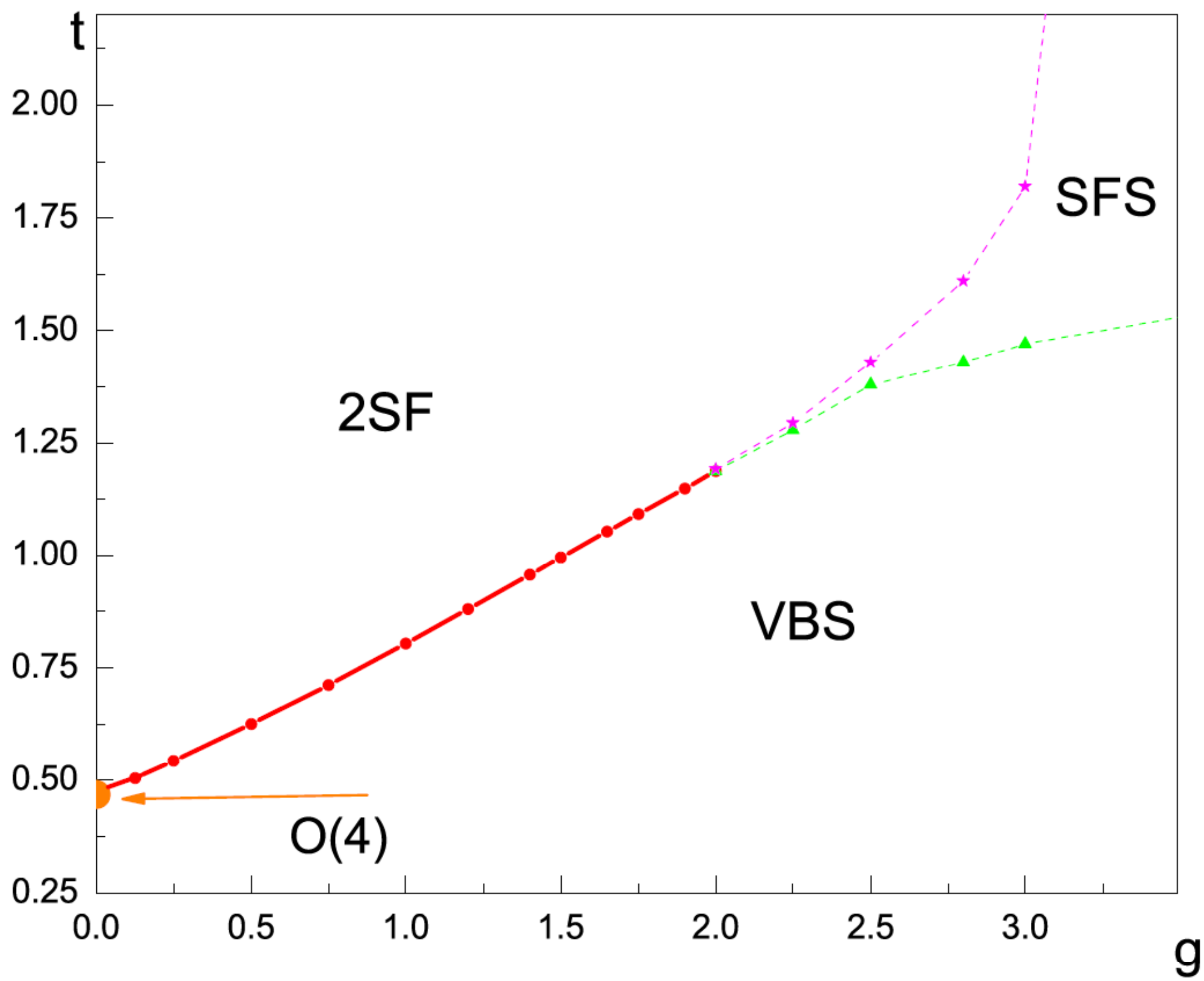
$SU(2)$ case, CP^1- model



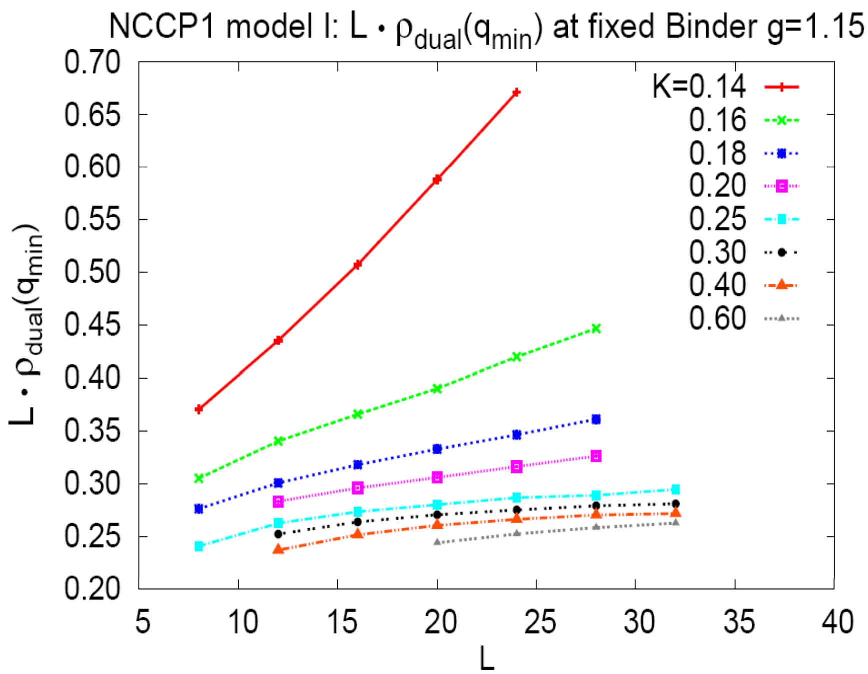
“Smoking gun” of the first-order transition

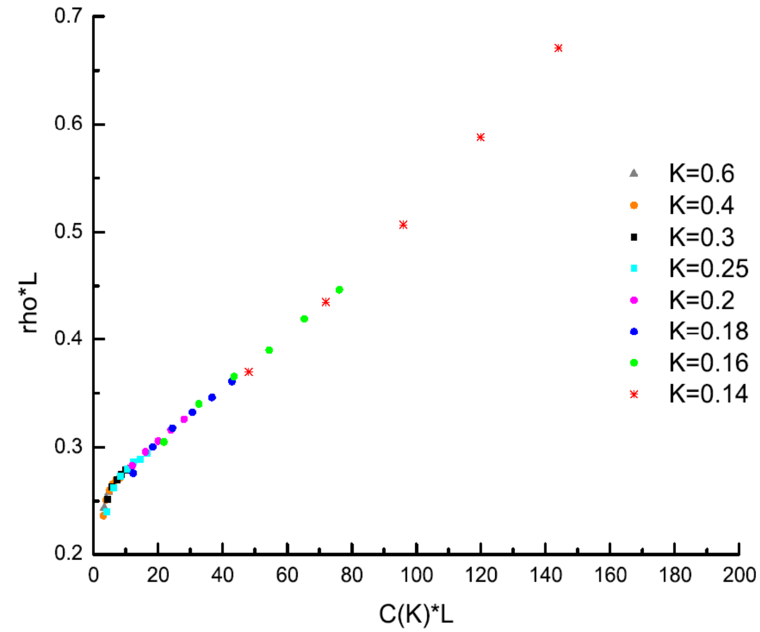


$SU(2)$ case

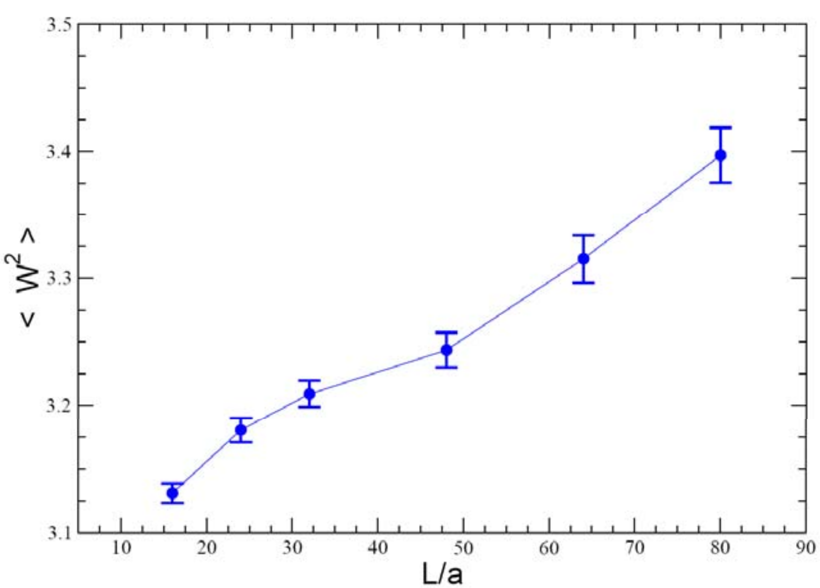


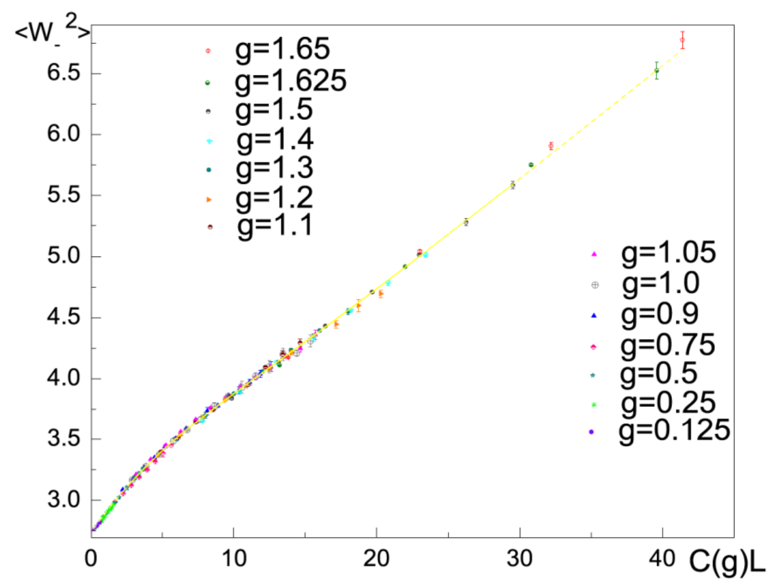
t
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
0.5
1.0
1.5
2.0
2.5
3.0
g
2SF
SFS
VBS
O(4)





O.I. Motrunich and A. Vishwanath ‘08





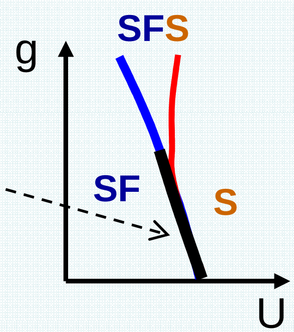
Conclusions:

1. So far, all 2+1 dim. models of the SF-S transitions observe either I-order, or problems with scaling

in agreement with both (!!)

GWL using $$\left|\vec{S} = (\underbrace{S_1, S_2}_{\Psi}, \underbrace{S_3}_{S_{CB}}, \underbrace{S_4, S_5}_{B_x, B_y} \ldots)\right| \neq 0$$

DCP action = a continuous theory of weak I-order scenario



2. Strong renormalized pairing of vortices on large scales may be the mechanism preventing deconfined criticality from happening