Deconfined criticality: SU(2) Dejave

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Why Dejavu?

\[ U(1) \times U(1) \quad \text{Contributing authors} \]

\[ SU(2) \quad \text{Many of the same authors} \]

Simulations of specific models

Contributing authors

Simulations of specific models

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claims of new criticality

Many of the same authors

Same type of models

problems with scaling for systems

Method of solution: flowgrams

\[ \text{generic I-order} \quad \text{Dejavu} \]
Solid = insulator with broken translational symmetry  
(integer filling excluded)

**Checkerboard solid**  
(anti-ferromagnet)

**Valence-bond solids (VBS)**

- **Columnar VBS**
- **Plaquette VBS**

**Density-order**
\[ \langle n_i n_j \rangle \]

**Current (or bond) order**
\[ \langle J_i^2 J_j^2 \rangle \]
Superfluid and Solid orders and transitions:

\[ \Psi, M, B_x, B_y, D_x, D_y, \ldots \]

order parameters:

"Naïve" Ginzburg-Landau (expansion from the generic quantum disordered phase)
\[ \vec{S} = (S_1, S_2, S_3, S_4, S_5, \ldots) \]

- multi-component order parameter; the symmetry of \( S \) is always broken in the ground state \( |\vec{S}| \neq 0 \)

\( \Psi \):

O(3)-case:

- \( \vec{S} \)-sphere

Heisenberg point = exact O(3)-symmetry

Generic case; I-order SF-S transition
SF and VBS orders are deeply related … May be even self-dual at the critical point!


*Deconfined criticality, does not fit the standard Landau-Ginsburg-Wilson paradigm.*

Deconfined spinons = vortices in the VBS order

**DCP action:**

\[
S = \int d^2 r d\tau \sum_{a=1,2} \left| \left( \partial_\mu - i A_\mu \right) z_a \right|^2 + s|z|^2 + u\left( |z|^2 \right)^2 + v|z_1|^2 |z_2|^2 + \kappa \left( \epsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2
\]
Ordered SF state for matter fields = (duality mapping) “quantum disordered” for vortex fields

Do LG expansion for vortex fields!


Filling factor 1/2 $\rightarrow$ dual magnetic flux/plaquette 1/2 $\rightarrow$ two species of vortexes + gauge field coupling

at the SF-VBS critical point

DCP action

$$S_\psi = \int d^2 r d\tau \sum_{a=1,2} \left( \partial_\mu - i A_\mu \right) \psi_a \left| \psi_a \right|^2 + s \left| \psi \right|^2 + u \left( \left| \psi \right|^2 \right)^2 + v \left| \psi_1 \right|^2 \left| \psi_2 \right|^2 + \kappa \left( \varepsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2$$

$$g = [e^2 = 1/\kappa] \rightarrow g(L) = gL \quad \text{Run-away flow to strong coupling}$$
DCP action:

\[ S_\psi = \sum_i \sum_{a=1,2} \left| \left( \partial_\mu - i A_\mu \right) \psi_a \right|^2 + s |\psi|^2 + u \left( |\psi|^2 \right)^2 + v |\psi_1|^2 |\psi_2|^2 + \kappa \left( \epsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2 \]

\[ S_{XY} = -J \sum_{r,\mu} \left[ \cos \left( \Delta_\mu \varphi_1 - A_\mu \right) + \cos \left( \Delta_\mu \varphi_2 - A_\mu \right) \right] + \kappa \sum \left( \nabla \times A \right)^2 \]

\[ S_{CP} = \sum_{i} \sum_{a=1,2} \left| \left( \partial_\mu - i A_\mu \right) \psi_a \right|^2 + \kappa \left( \epsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2 \text{ with } |\psi_1|^2 + |\psi_2|^2 = 1 \]

\[ S_J = U \sum_r F[j_{a\mu}(r)] + g \sum_{r,rr',\mu} Q(r-r') \left( j_{1\mu}(r) + j_{2\mu}(r) \right) \cdot \left( j_{1\mu}(r') + j_{2\mu}(r') \right) \]

\[ Q^1(q) = \sum_\mu \sin^2(q\mu/2) \rightarrow Q(r) \sim 1/r \]
Mappings:

“High-T” expansion (expansion in kinetic energy)

\[
S = \sum_r U|\psi_r|^4 - \mu|\psi_r|^2 - t \sum_{rr'}(\psi^*_r \psi_{r'}, e^{iA_{rr'}} + c.c.) + \kappa \sum [\nabla \times A]^2
\]

\[
Z = \int \int DA \prod_r d\psi_r e^S
\]

\[
= \int \int DA \prod_r \int d\psi_r e^{S_r} \prod_{b=rr'} \sum_{n_b=0}^{\infty} \frac{(-t)^{n_b}}{n_b!} (\psi^*_r \psi_{r'})^{n_b} \sum_{m_b=0}^{\infty} \frac{(-t)^{m_b}}{m_b!} (\psi^*_r \psi_{r'})^{m_b} e^{iA_b (n_b - m_b)} e^{S_A}
\]

\[
= \sum_{n_b,m_b \text{ with } \{j=n_b-m_b\} \text{ loops}} \exp\{-g \sum_{rr'} Q(r-r') j_r j_{r'}\} W(\{j_b\}) = \sum_{n_b,m_b \text{ with } \{j=n_b-m_b\} \text{ loops}} e^{S_j}
\]

\[
\text{loop configurations of oriented currents}
\]

\[
\mathbf{j}_r = \mathbf{j}_{1r} + \mathbf{j}_{2r} \quad \text{(DCP)}
\]
Winding numbers = MC “blessing”  

Superfluid stiffness: \[ \Lambda_s = T L^{d-2} \langle W^2 \rangle \rightarrow \langle W^2 \rangle / L \]

j-current through any cross-section

Probe system properties at the largest scales; ideal for studies of critical phenomena

(Pollock, Ceperley ’87)
Continuous critical point: the entire distribution $P(W)$ is universal

$$\langle W^2 \rangle \sim 1$$

Since

$$\sum_{W \neq 0} \frac{P(W)}{P(0)} = \begin{cases} \infty & \text{in SF} \\ 0 & \text{in I} \end{cases}$$

I-order SF-S transition:

$$\langle W^2 \rangle_{SF} \sim L$$

Transition point Def:

$$\sum_{W \neq 0} \frac{P(W)}{P(0)} = \#$$

Quantity to study at the transition point:

$$\langle W^2 \rangle = f(L) \rightarrow \begin{cases} \infty & \text{I-order} \\ \text{const} & \text{continuous (scale invariance)} \end{cases}$$
Phase diagram of the U(1)×U(1) DCP action

\[ gQ(r-r') \rightarrow V \delta(r-r') \]

short-range version

\[ g \sim 0.5 - \text{weak I-order} \]

small \(g\) maps to \(g > 0.5\)

RG flow: \(g_{\text{eff}}(L) \propto gL\)
Flowgrams of $\left\langle W^2 \right\rangle_L$. What to expect across the tricritical point.
Flowgrams of $\langle W^2 \rangle_L$ for the run-away flow to strong coupling and I-order.

- Criticality at $g = e^2 = 0$
- Large $g$
- I-order divergence

$$P(S)$$
Flowgram collapse! \[ \left\langle W^2 \right\rangle \]

\[ \left\langle W^2 \right\rangle \]

Criticality at \( g = e^2 = 0 \)

Different \( g \) are identical on length scales \( L \geq \xi(g) >> 1 \)

\[ \ln(L / \xi(g)) \]

\[ \ln(L) \]

\[ P(S) \]

\[ S \]
$U(1) \times U(1)$ case

\[
\langle W^2 \rangle = \ln [(g + g^2) L]
\]
Proof of first-order transition at $g=0.58$

Probability distributions at the critical point for $g=0.58$

Scaling of peak positions with system size $L$
\textit{SU(2) case}

\[ H = J \sum_{x,i} \tilde{S}_x \cdot \tilde{S}_{x+i} - Q \sum_x \left[ (\tilde{S}_x \cdot \tilde{S}_{x+\hat{1}} - \frac{1}{4})(\tilde{S}_{x+\hat{2}} \cdot \tilde{S}_{x+\hat{1}+\hat{2}} - \frac{1}{4}) \right. \]

\[ + (\tilde{S}_x \cdot \tilde{S}_{x+\hat{2}} - \frac{1}{4})(\tilde{S}_{x+\hat{1}} \cdot \tilde{S}_{x+\hat{1}+\hat{2}} - \frac{1}{4}) \right] \]

A. W. Sandvik '07
R. G. Melko and R. K. Kaul '07

F.-J. Jiang\textsuperscript{a}, M. Nyfeler\textsuperscript{a}, S. Chandrasekharan\textsuperscript{b}, and U.-J. Wiese\textsuperscript{a} '07
$SU(2)$ case, $CP^1$-model
“Smoking gun” of the first-order transition
$SU(2)$ case
Conclusions:

1. So far, all 2+1 dim. models of the SF-S transitions observe either I-order, or problems with scaling in agreement with both (!!)

\[ \tilde{S} = (S_1, S_2, S_3, S_4, S_5 \ldots) \neq 0 \]

GWL using \( \Psi \ S_{CB} \ B_x, B_y \)

DCP action = a continuous theory of weak I-order scenario

2. Strong renormalized pairing of vortices on large scales may be the mechanism preventing deconfined criticality from happening