Infinite randomness at the superconductor-metal quantum phase transition

Phys. Rev. Lett. (2008)

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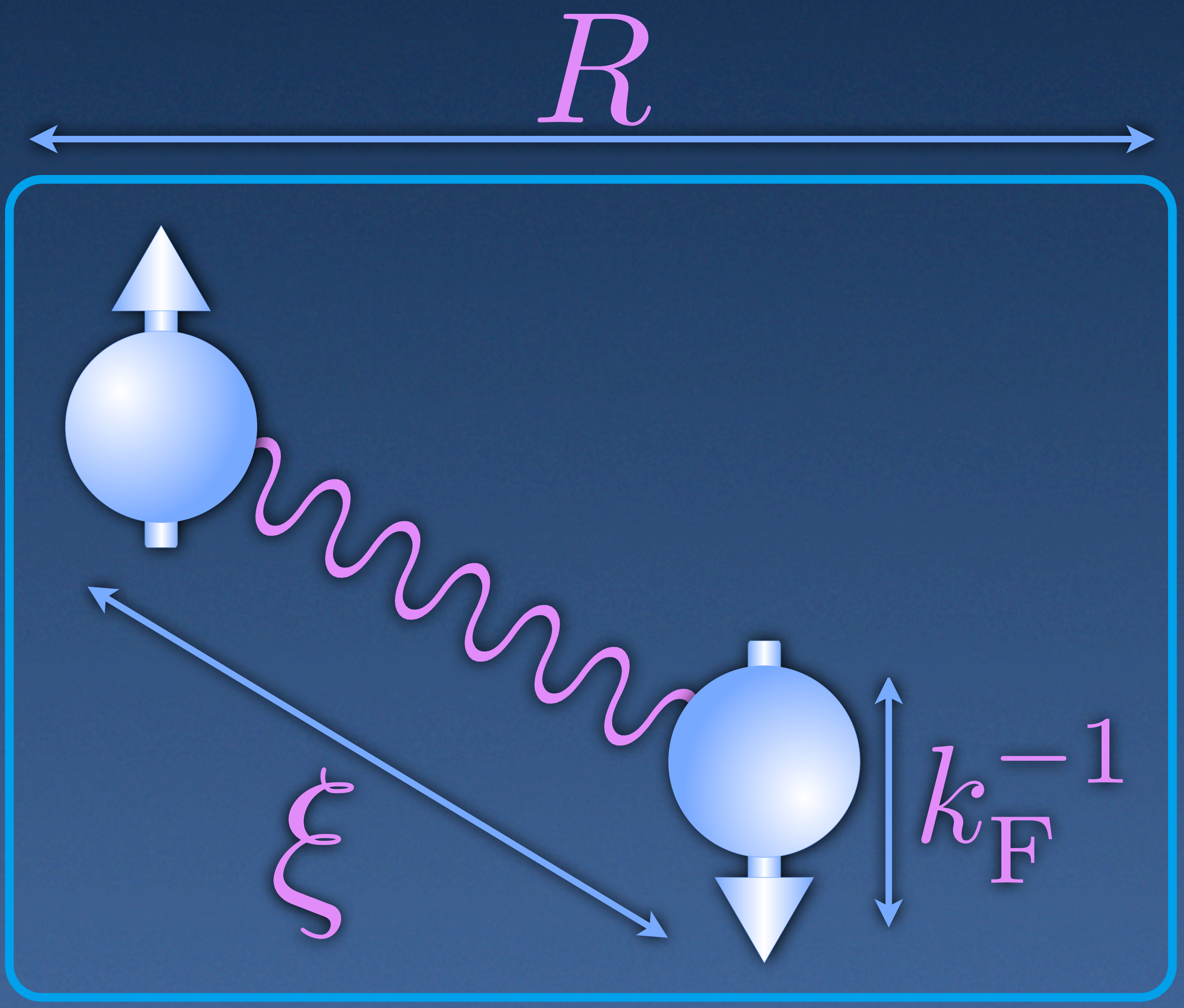
Diminutive superconductivity

- A macroscopic quantum wave function can be drastically altered by finite size effects



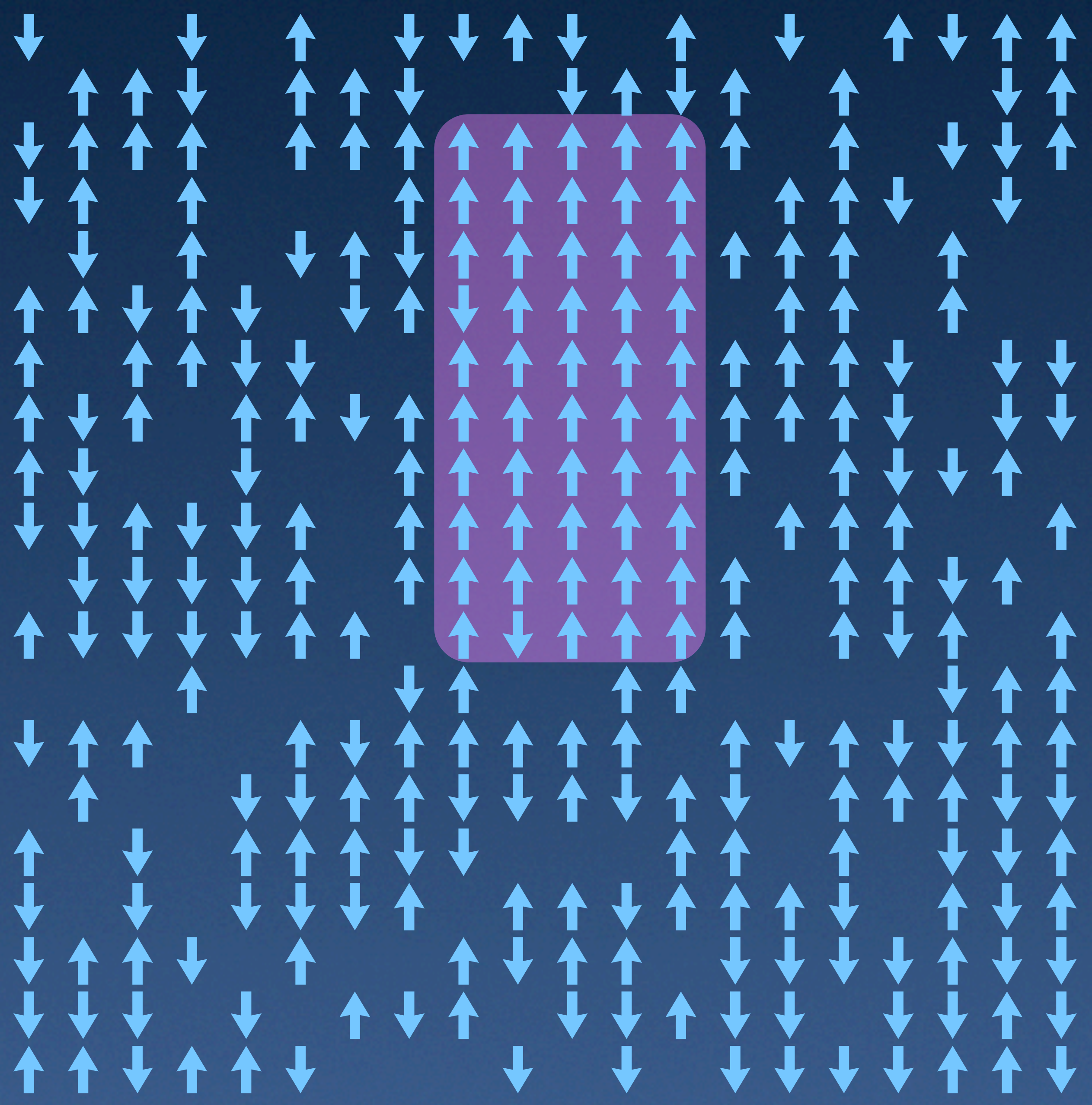
- Interesting and non-trivial new behavior when

$$R \sim \xi$$

Disorder in quantum systems

- Disorder has long range correlations in the imaginary time direction

$$\ln \tau \sim \xi^{\psi}$$

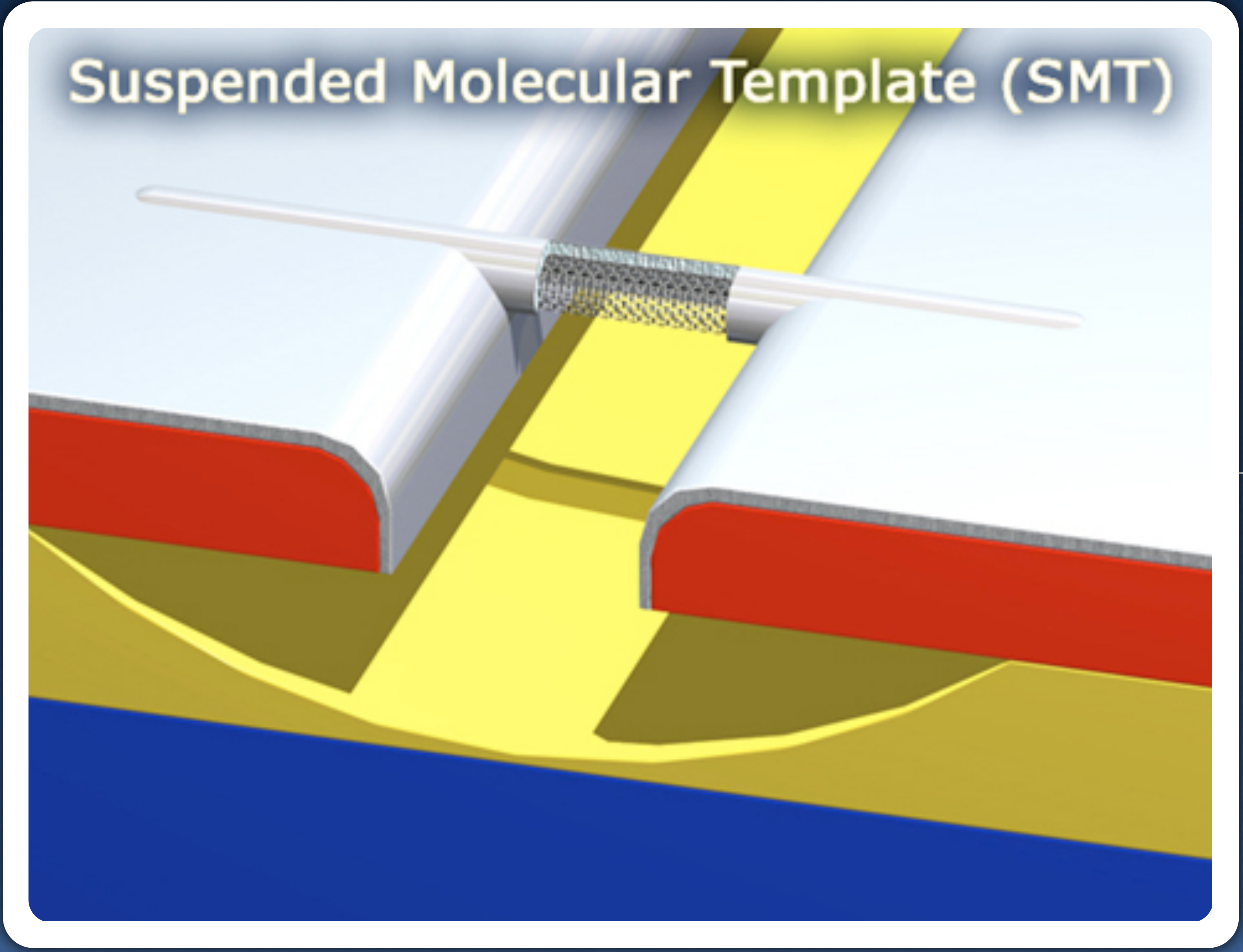


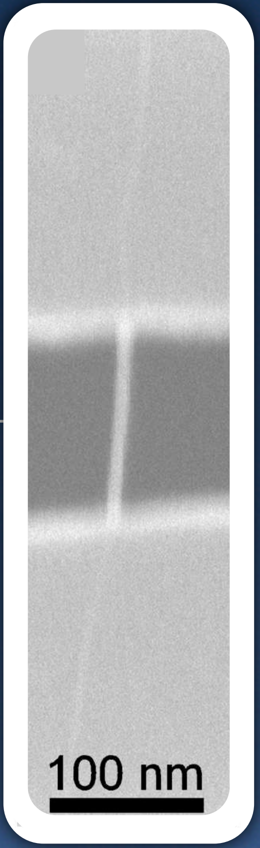
What role does disorder play near the superconductor-metal transition in one dimension?

Ultra-Narrow Wires

Fabrication of ultra-narrow wires

- Novel fabrication techniques have allowed for the study of wires with d < 10 nm

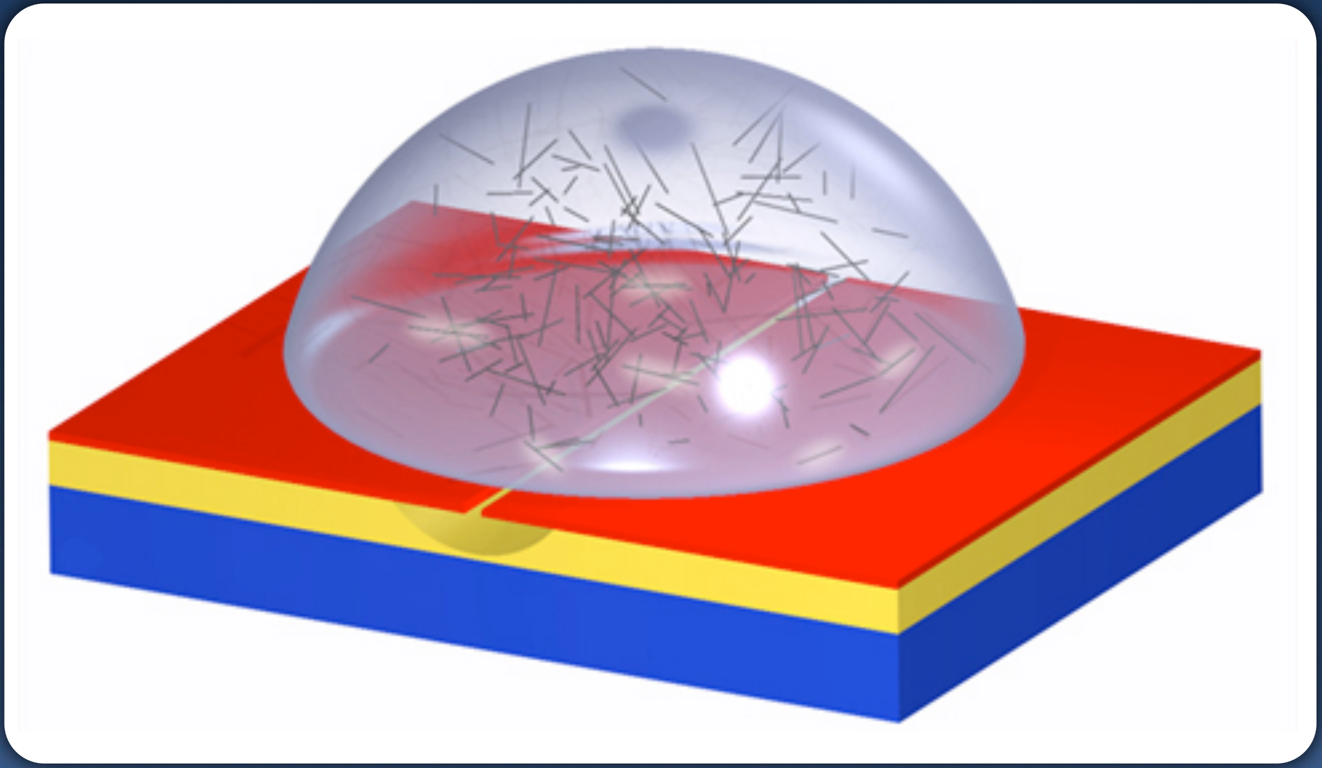




www.nanogallery.info
A. Bezryadin et al., Nature (2000)

Transport in ultra-narrow wires

- Can tune a phase transition between a metal and superconductor via wire diameter



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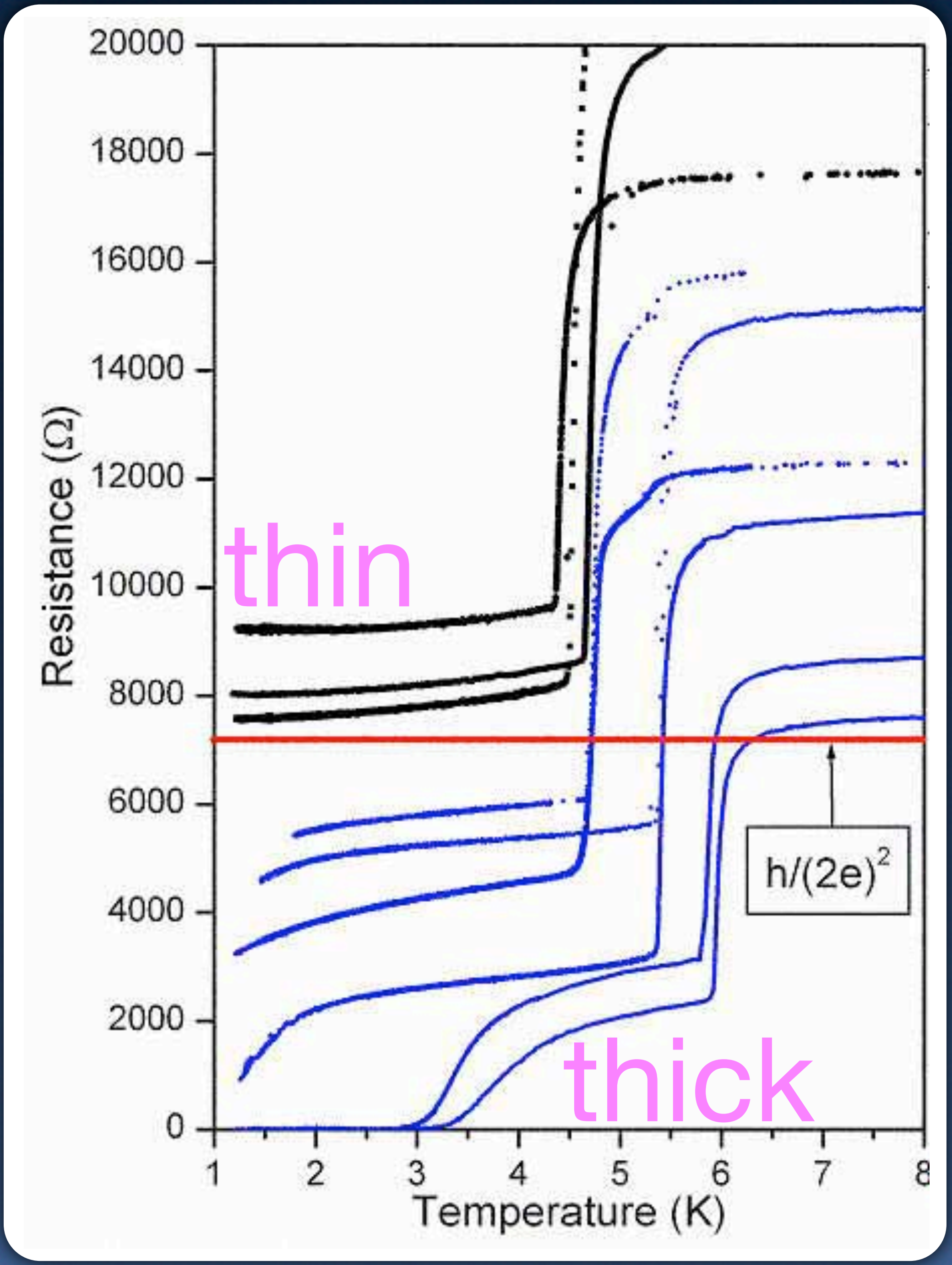
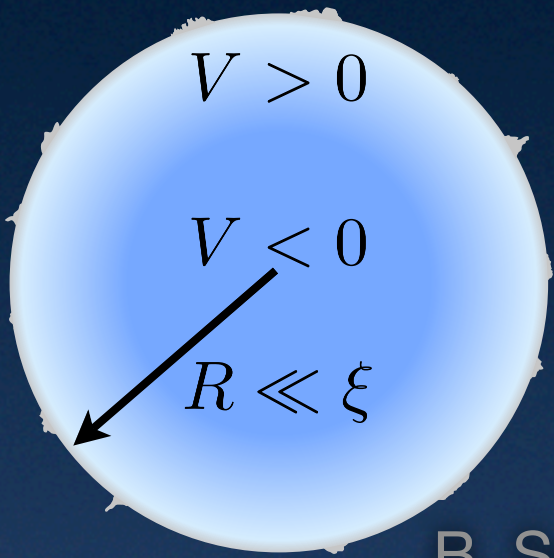


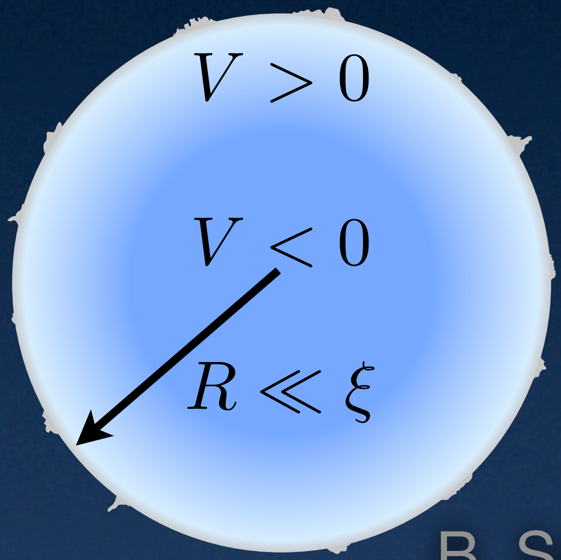
A. Bezryadin, et al., PRL (2003)

Pair-breaking in nanowires



B. Spivak et. al, PRB (2001)

Pair-breaking in nanowires



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- Evidence for magnetic moments on the wire's surface

A. Rogachev et al., PRL (2006)

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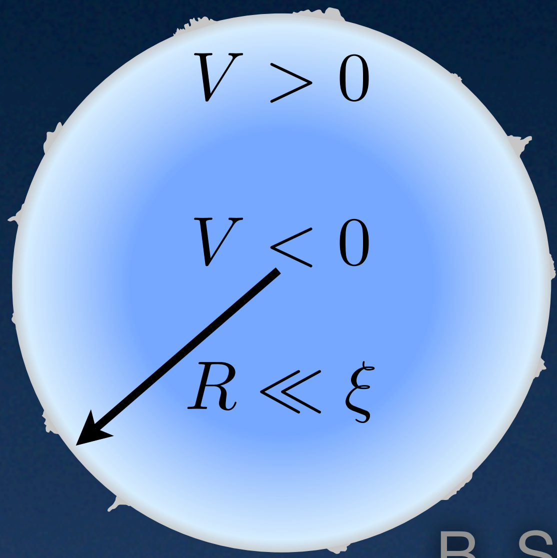

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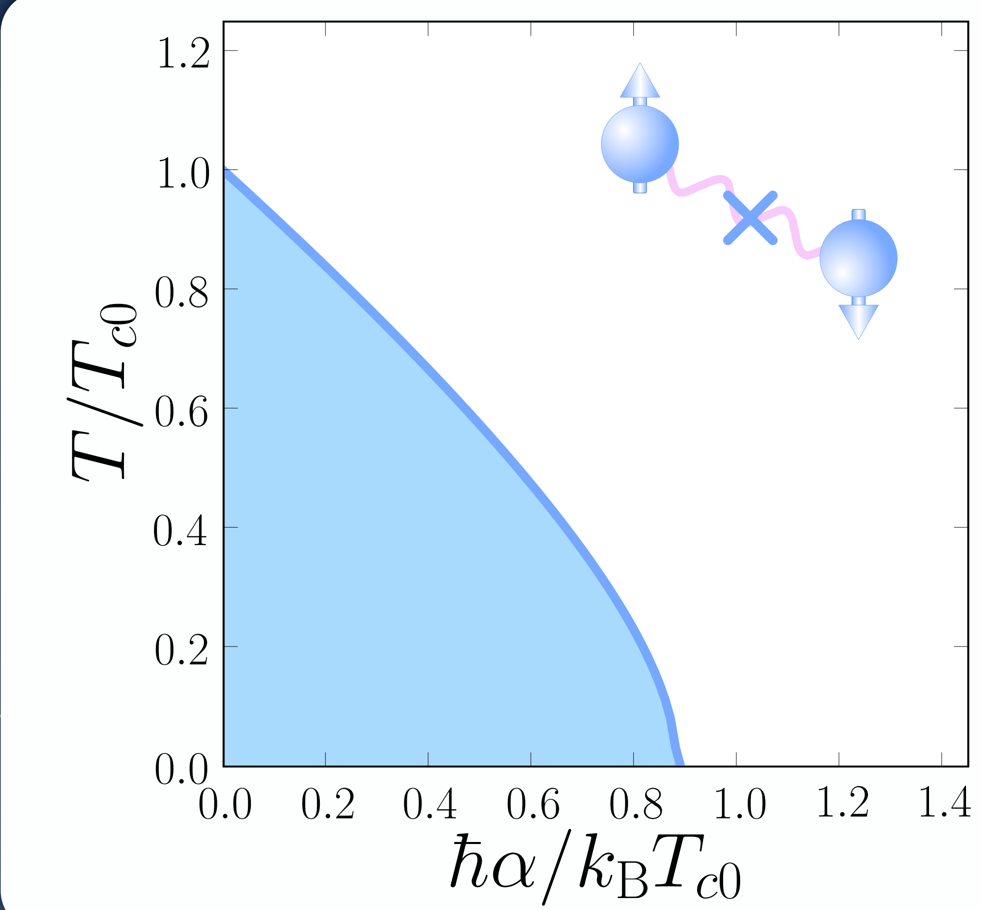
- Pair-breaking interactions break time-reversal symmetry and allow for the destruction of superconductivity at T > 0

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\hbar\alpha}{2\pi k_B T_c}\right)$$

A. Abrikosov and G. Gor'kov, JETP (1961)



Dissipative Cooperon theory

- Diffusive electrons with Ohmic dissipation coming from decay of repulsive Cooper pairs into gapless fermions

$$z = 2$$

$$\mathcal{S}_\alpha = \int_0^L dx \int_0^{\hbar\beta} d\tau \left[\widetilde{D} |\partial_x \Psi(x,\tau)|^2 + \alpha |\Psi(x,\tau)|^2 + \frac{u}{2} |\Psi(x,\tau)|^4 \right.$$
$$+ \frac{1}{\hbar\beta} \sum_{\omega_n} \int_0^L dx\, \gamma |\omega_n| |\Psi(x,\omega_n)|^2$$

Ramazashvili and Coleman; Herbut; Dalidovich and Phillips; Spivak, etc.

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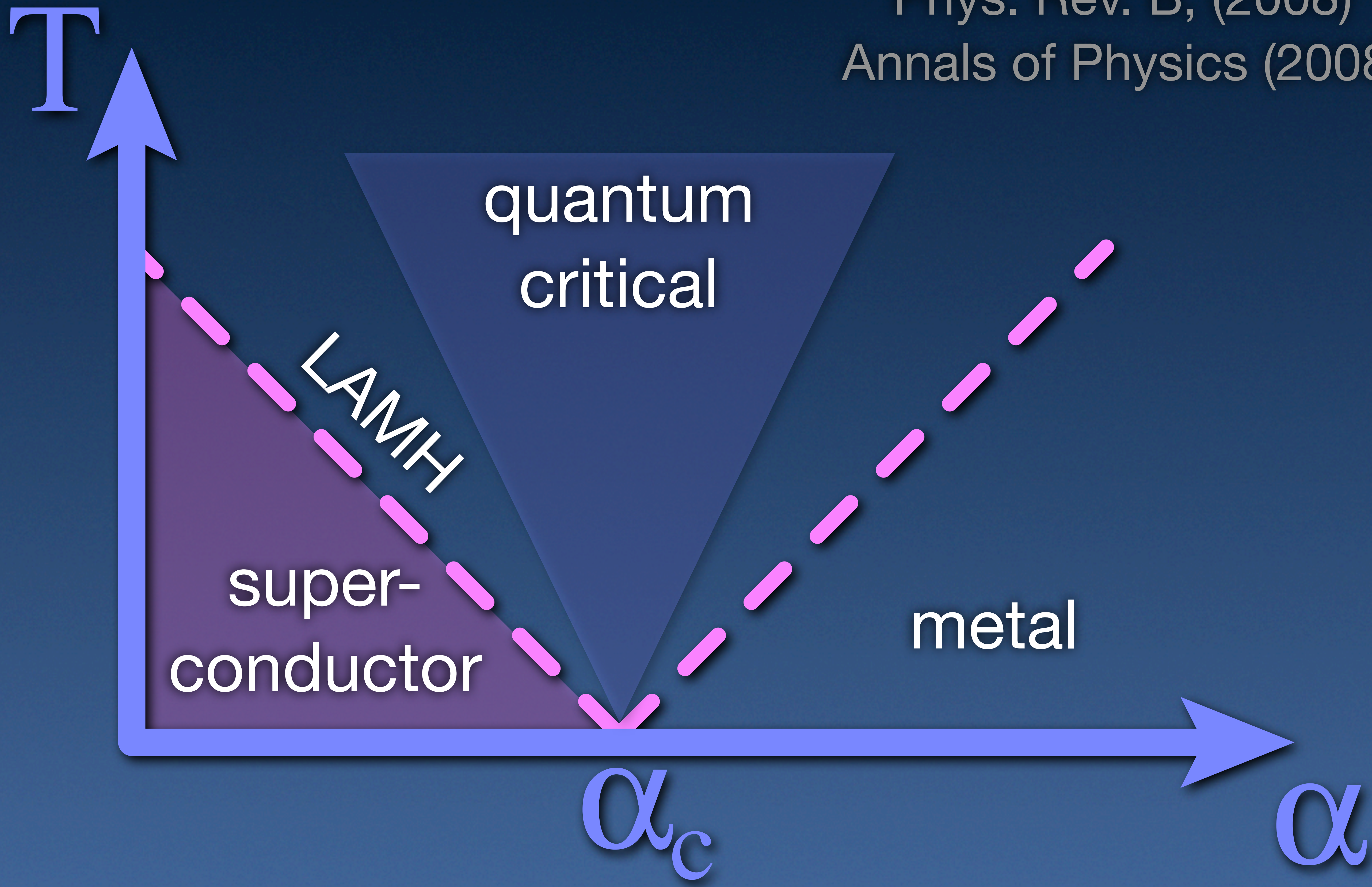
particle-hole continua

Ramazashvili and Coleman; Herbut; Dalidovich and Phillips; Spivak, etc.

Superconductor-metal transition

Phys. Rev. B, (2008)
Annals of Physics (2008)



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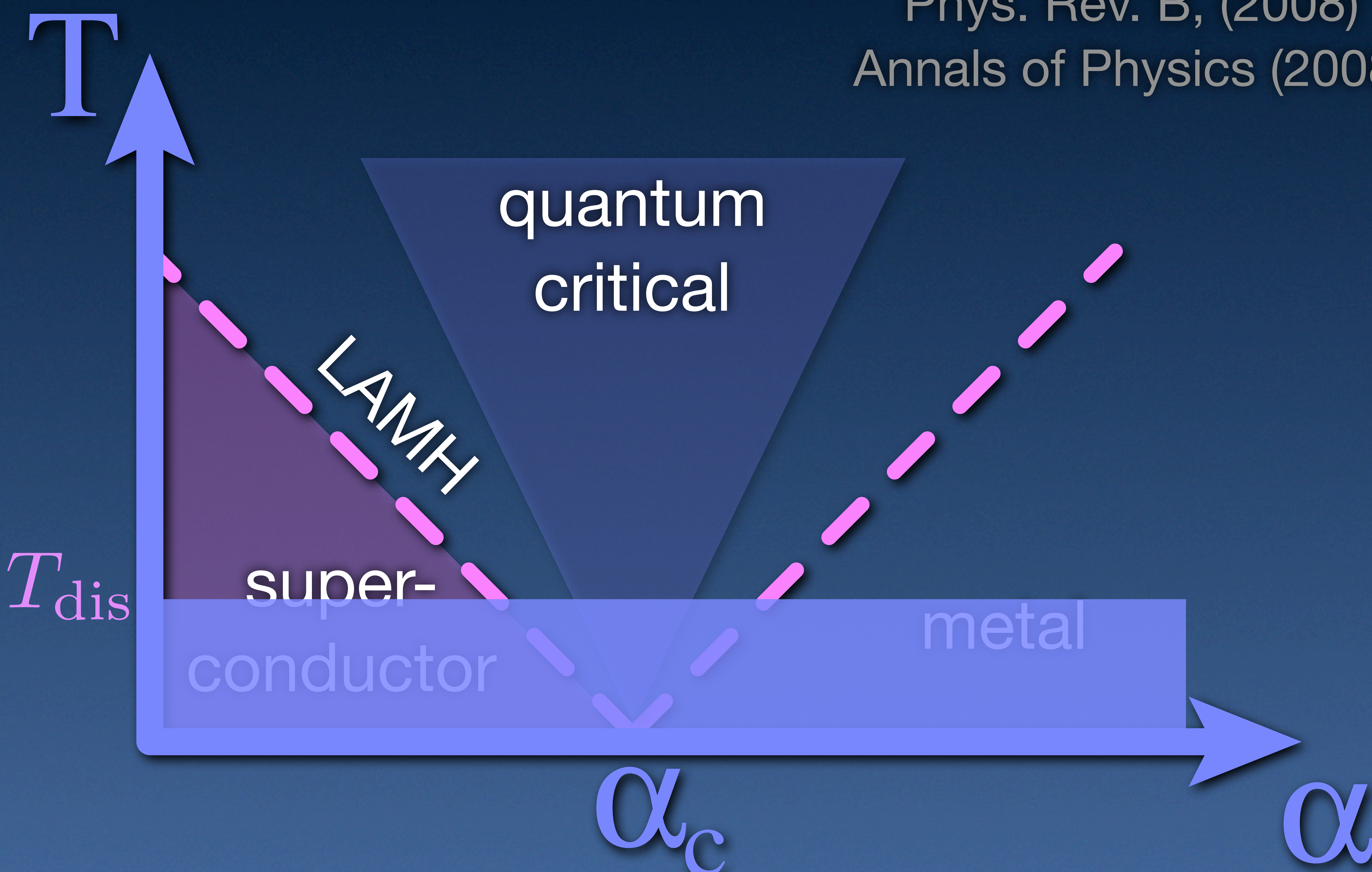


Infinite Randomness

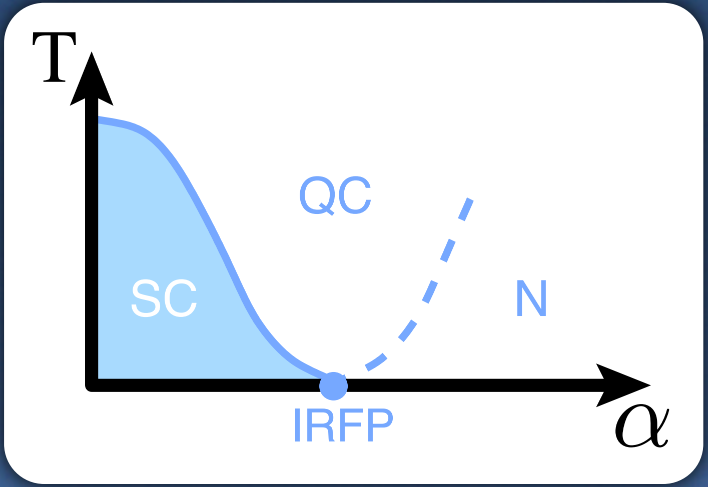
Spatially dependent random couplings

$$\mathcal{S} = \int dx \int d\tau \left[D(x)|\partial_x \Psi(x,\tau)|^2 + \alpha(x)|\Psi(x,\tau)|^2 + \frac{u(x)}{2}|\Psi(x,\tau)|^4 \right] + \int \frac{d\omega}{2\pi}\gamma(x)|\omega||\Psi(x,\omega)|^2$$

T = 0

Hoyos, Kotabage and Vojta, PRL 2007

- Real space RG predicts the flow to a strong randomness fixed point for $z = 2$



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Fisher's
RTFIM

Manifestations of strong disorder

- Under renormalization, probability distributions for observables become extremely broad

- Dynamics are highly anisotropic in space and time: activated scaling

$$\ln \xi_\tau \sim \xi^\psi$$

- Averages become dominated by rare events (spurious disorder configurations)

Exact predictions from the RTFIM

D. Fisher, PRL (1992); PRB (1995)

$$\mathcal{H} = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \, \sigma_i^x$$

- For a finite size system

$$\ln(1/\Omega) \sim L^{\psi}$$

$$\mu \sim \ln(1/\Omega)^{\phi}$$

- In the disordered phase

$$\xi \sim |\delta|^{-\nu}$$

$$\overline{C}(x) \sim \frac{\exp\left[-(x/\xi) - (27\pi^2/4)^{1/3}(x/\xi)^{1/3}\right]}{(x/\xi)^{5/6}}$$

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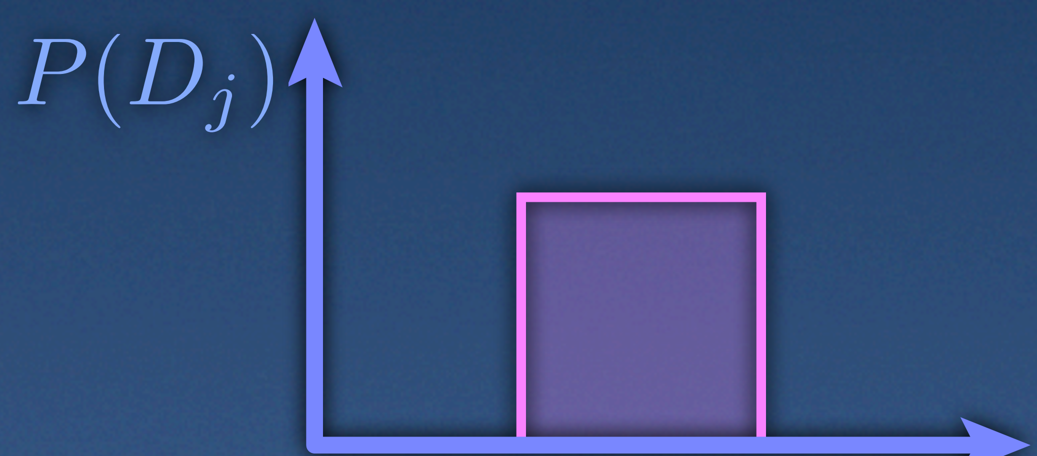
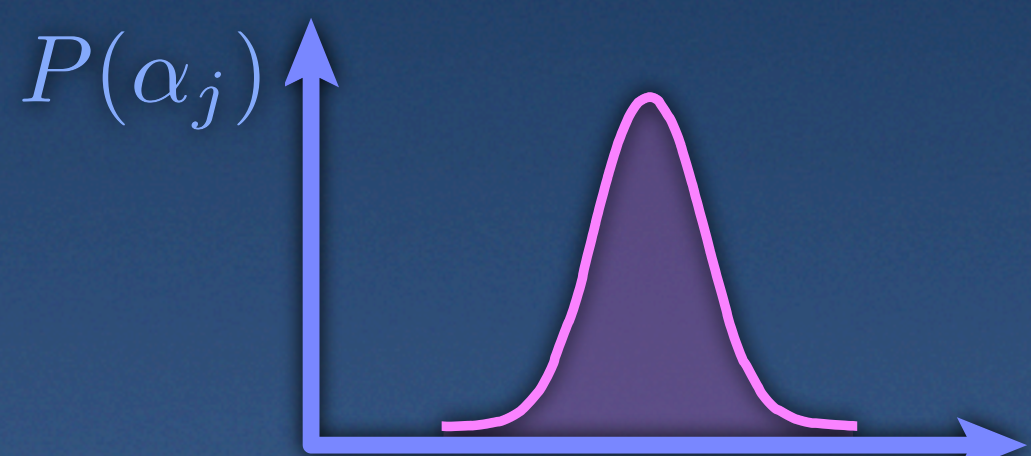
critical exponents
$\psi = 1/2$
$\nu = 2$
$\phi = (1+\sqrt{5})/2$

Discretize to a lattice of L sites

$$\mathcal{S} = \int d\tau \left\{ \sum_{j=1}^{L-1} D_j |\Psi_j(\tau) - \Psi_{j+1}(\tau)|^2 + \sum_{j=1}^{L} \left[\alpha_j |\Psi_j(\tau)|^2 + \frac{u_j}{2} |\Psi_j(\tau)|^4 \right. \right.$$
$$\left. + c_l |\Psi_1(\tau)|^2 + c_r |\Psi_L(\tau)|^2 \right\} + \int \frac{d\omega}{2\pi} \sum_{j=1}^{L} \gamma_j |\omega| |\Psi_j(\omega)|^2$$





- Measure all quantities with respect to γ^2 and set $u_j = 1$

Take the large-N limit

- Enforce a large-N constraint by solving the self-consistent saddle point equations numerically

$$\mathcal{S}_N = \int \frac{d\omega}{2\pi} \sum_{i,j=1}^{L} \Psi_i^*(\omega) \left[|\omega|\delta_{ij} + \mathsf{M}_{ij} \right] \Psi_j(\omega)$$

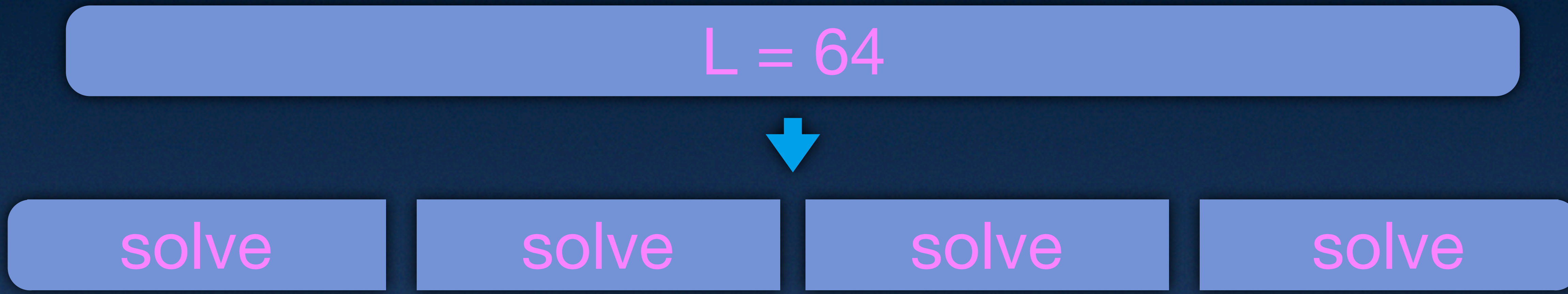
$$r_j = \alpha_j + \langle |\Psi_j(\tau)|^2 \rangle_{\mathcal{S}_N}$$

Y. Tu and P. Weichman, PRL (1994); J. Hartman and Weichman, PRL (1995)

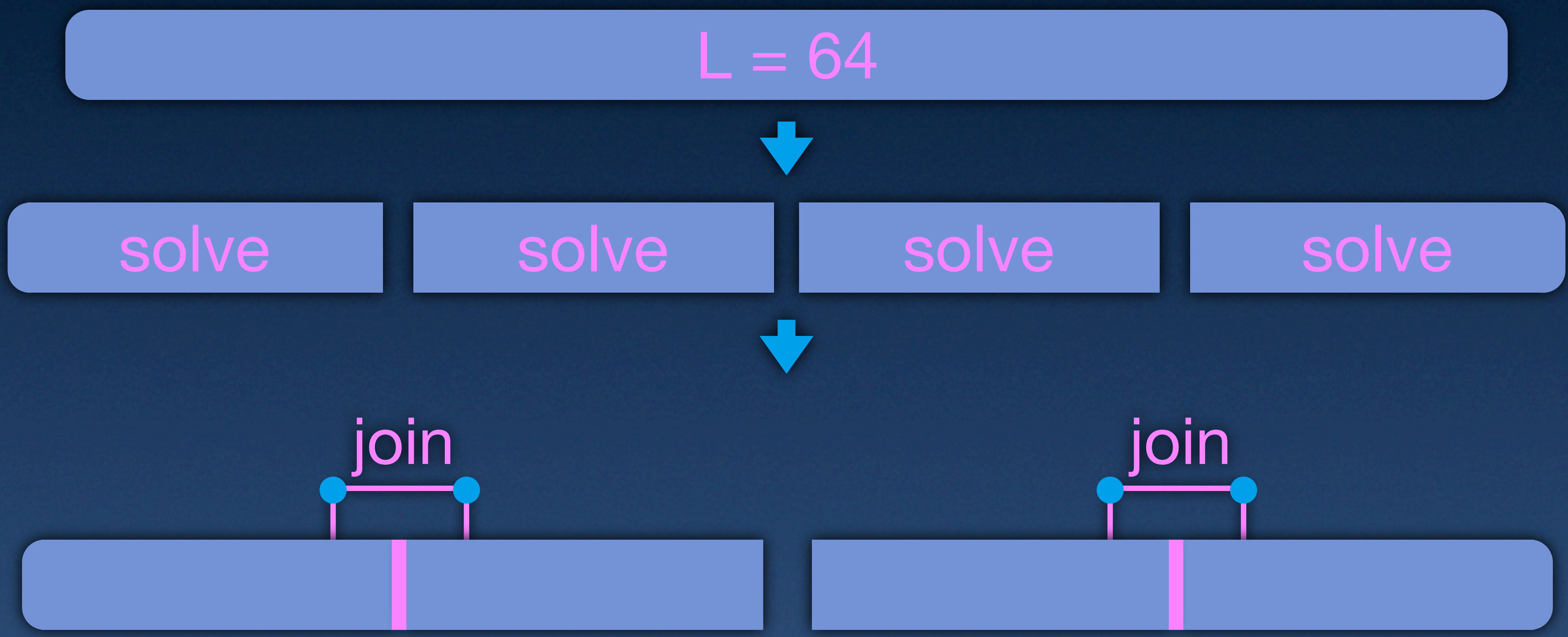
Solve-Join-Patch method

L = 64

Solve-Join-Patch method



Solve-Join-Patch method



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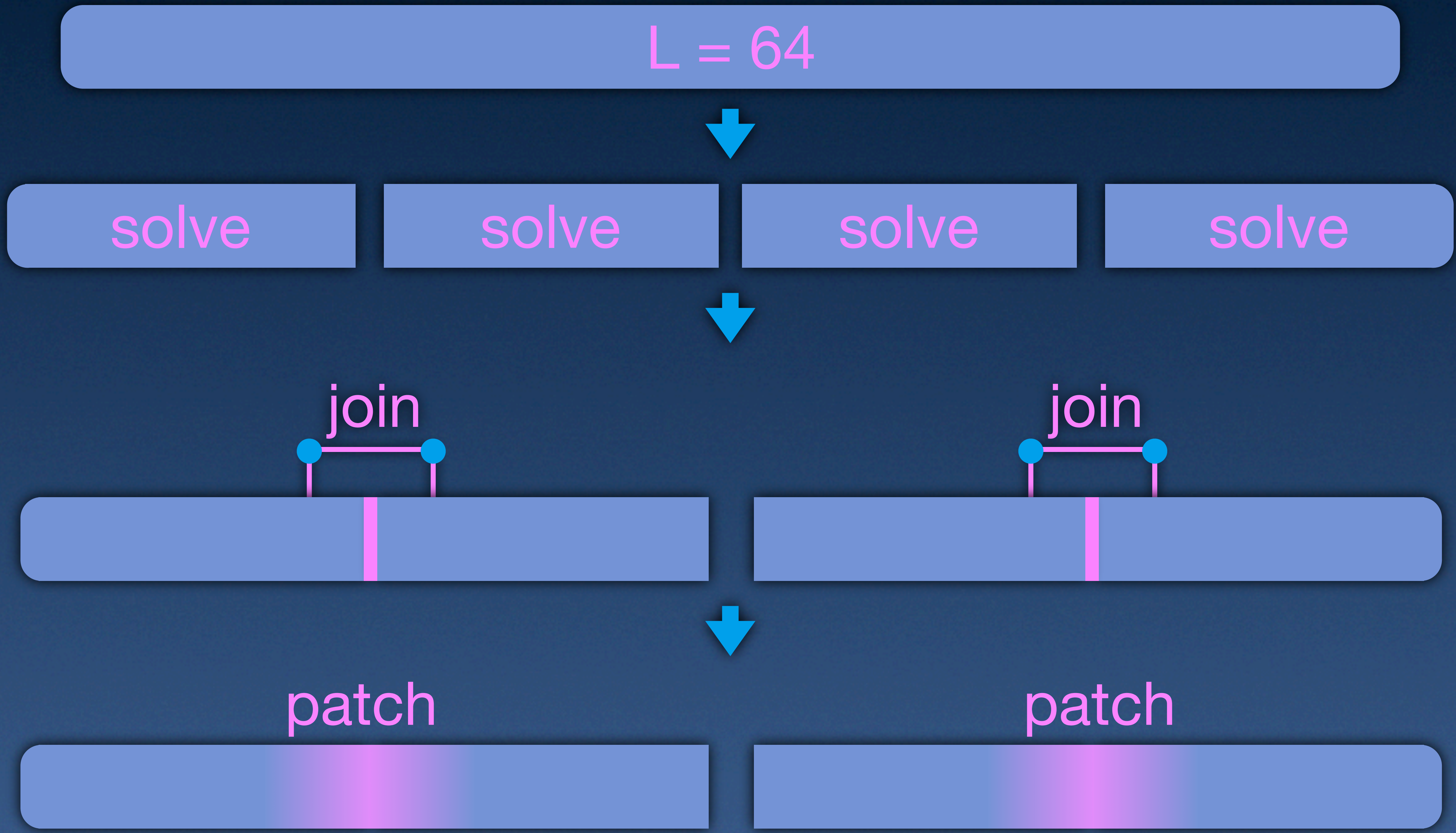

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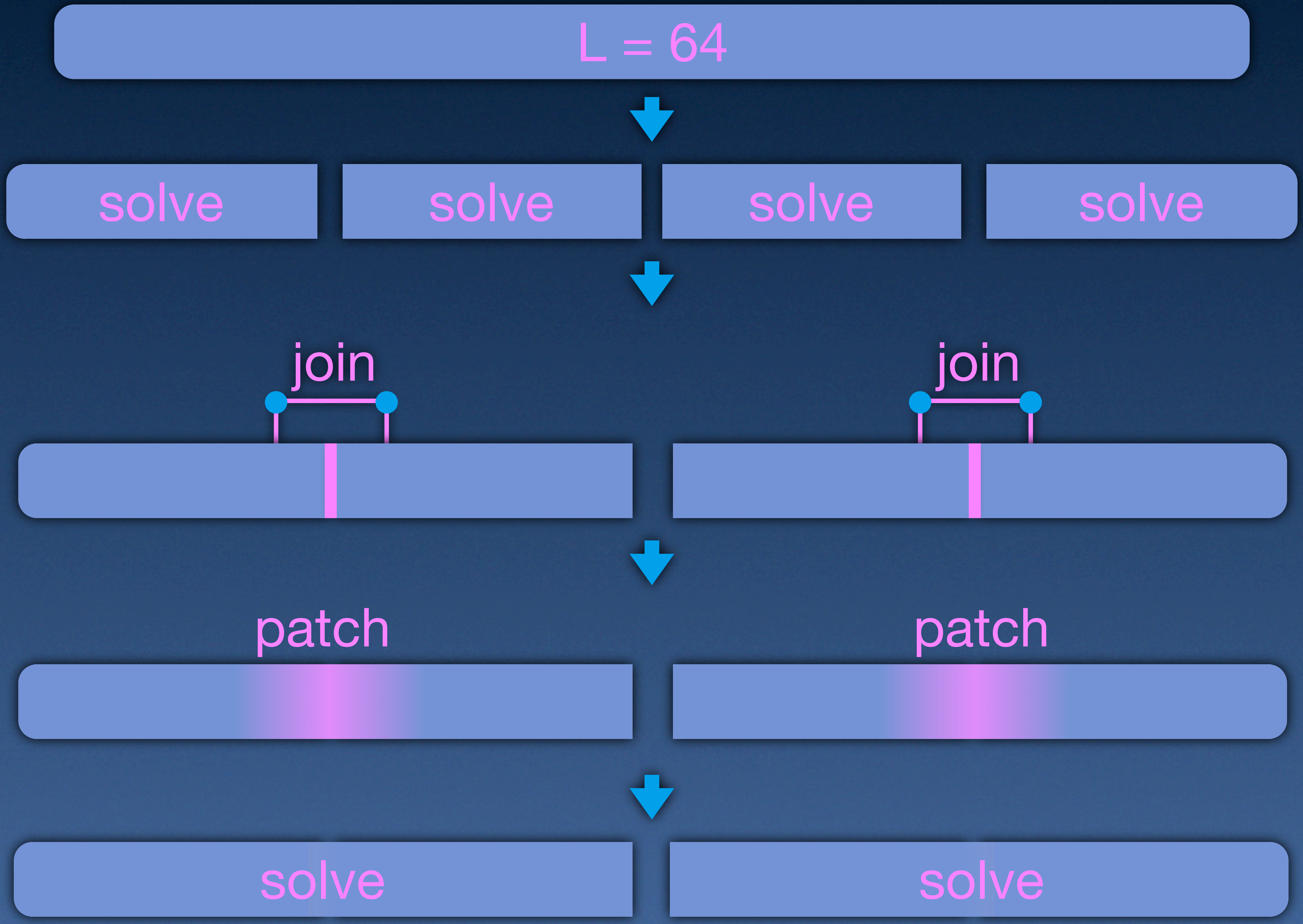


Numerical investigation of observables

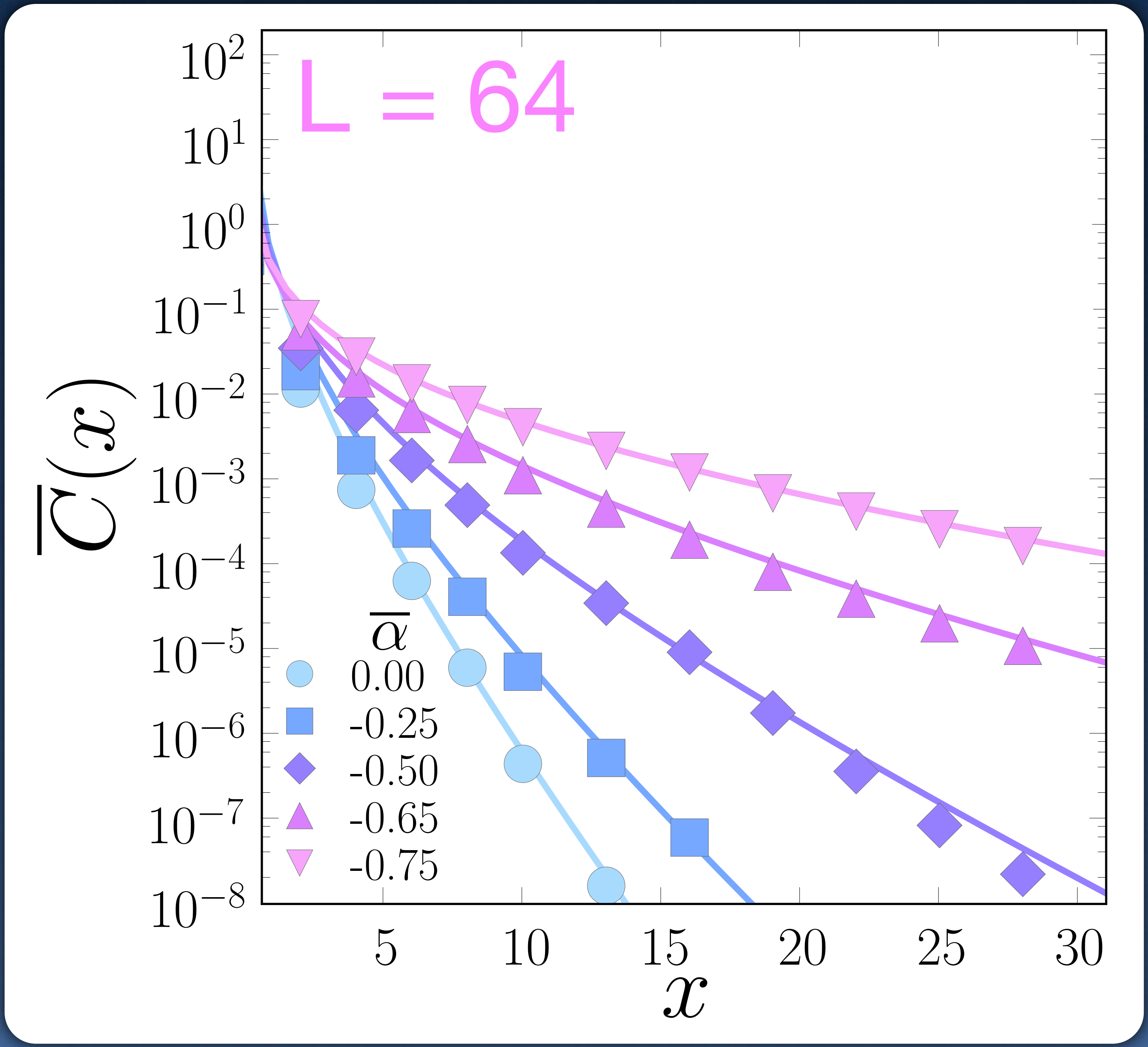
- Measure observables for L = 16, 32, 64, 128 averaged over 3000 realizations of disorder
- Tune the transition by shifting the mean of the α_j distribution, $\delta \sim \overline{\alpha} - \overline{\alpha}_c$
- Equal time correlators are easily determined from the self-consistency condition

$$\overline{C}(x) = \overline{\langle \Psi_x^*(\tau) \Psi_0(\tau) \rangle}_{\mathcal{S}_N}$$

Equal time correlation functions

- Fisher's asymptotic scaling form

$$\overline{C}(x) \sim \frac{\exp\left[-(x/\xi) - (27\pi^2/4)^{1/3}(x/\xi)^{1/3}\right]}{(x/\xi)^{5/6}}$$



$$\xi \sim \delta^{-\nu}$$

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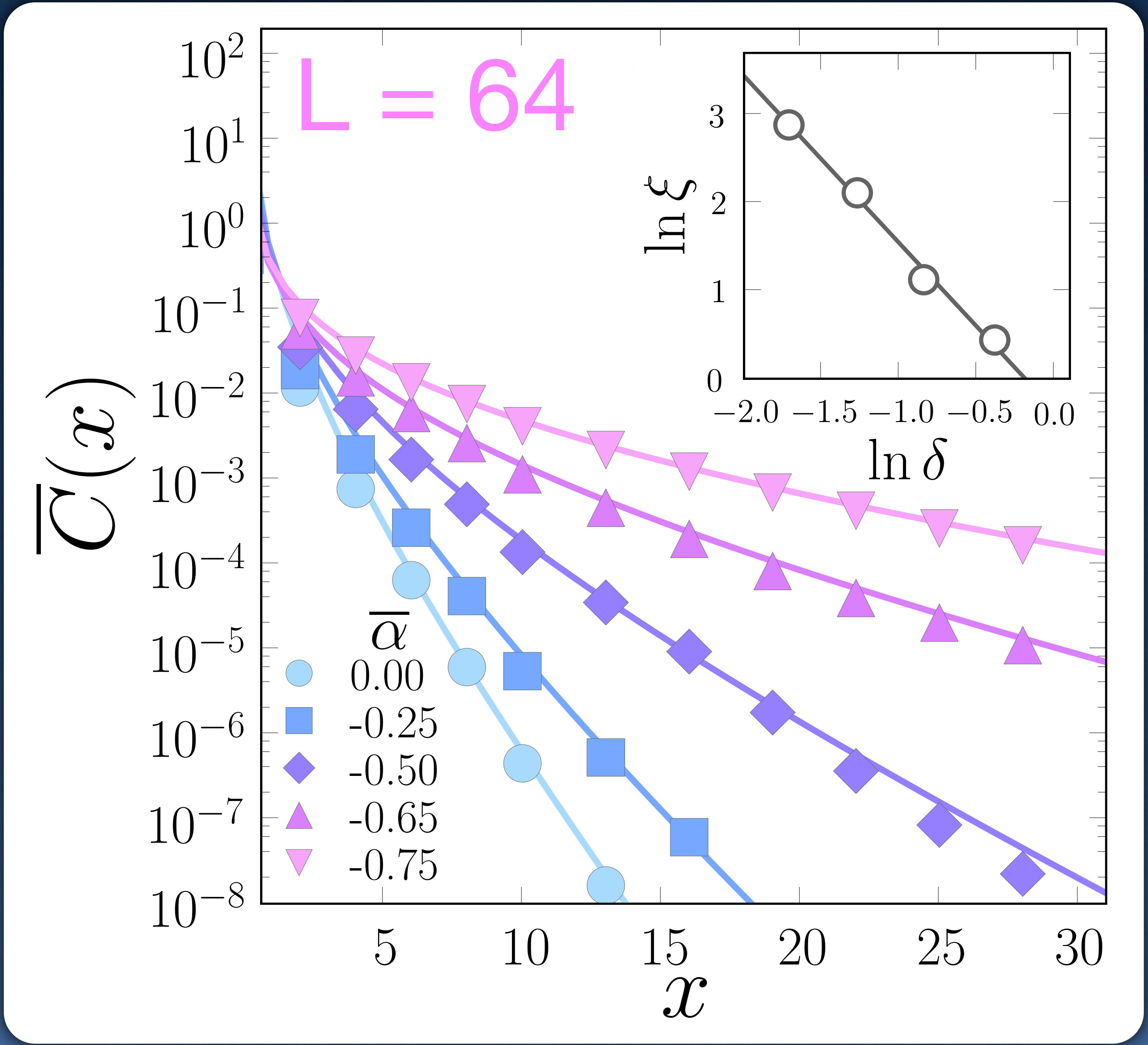
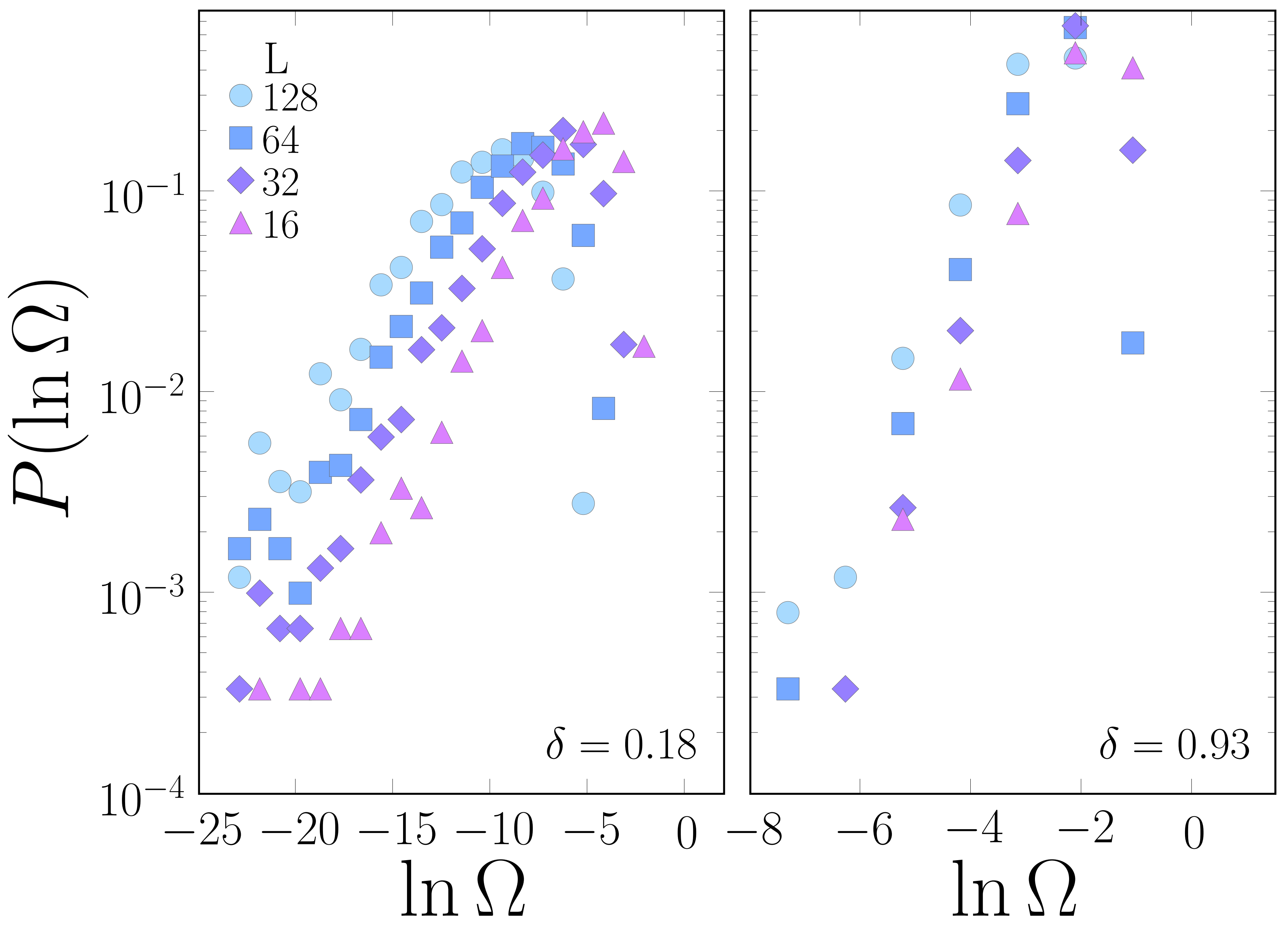


$$\xi \sim \delta^{-\nu}$$

$$\alpha_c \simeq -0.93$$
$$\nu = 1.9(2)$$

Energy gap statistics

$$\delta \sim \overline{\alpha} - \overline{\alpha}_c$$



P. Young and H. Rieger, PRB (1996)

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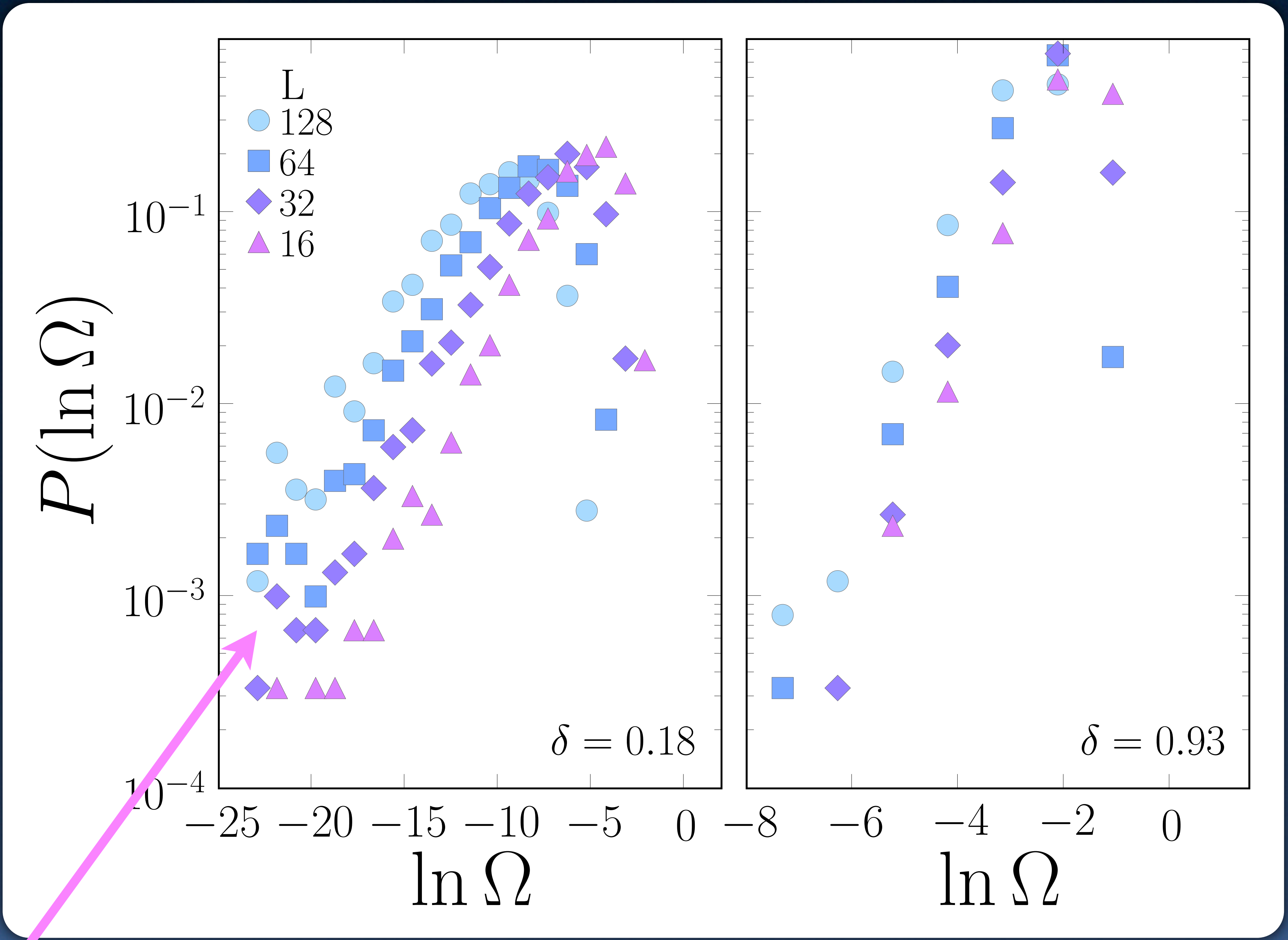


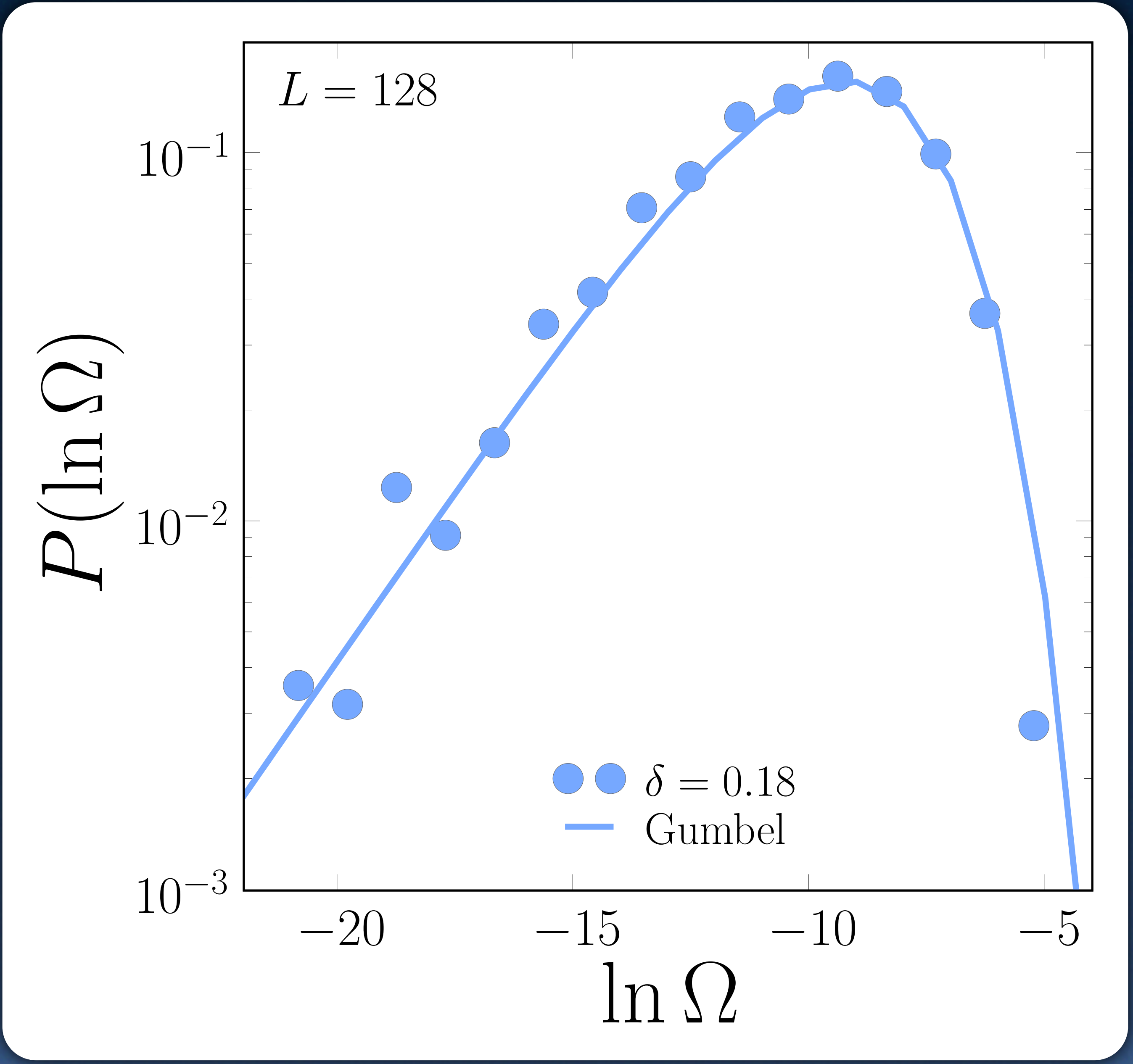
distribution broadens with increasing system size near criticality

P. Young and H. Rieger, PRB (1996)

Extreme value distribution

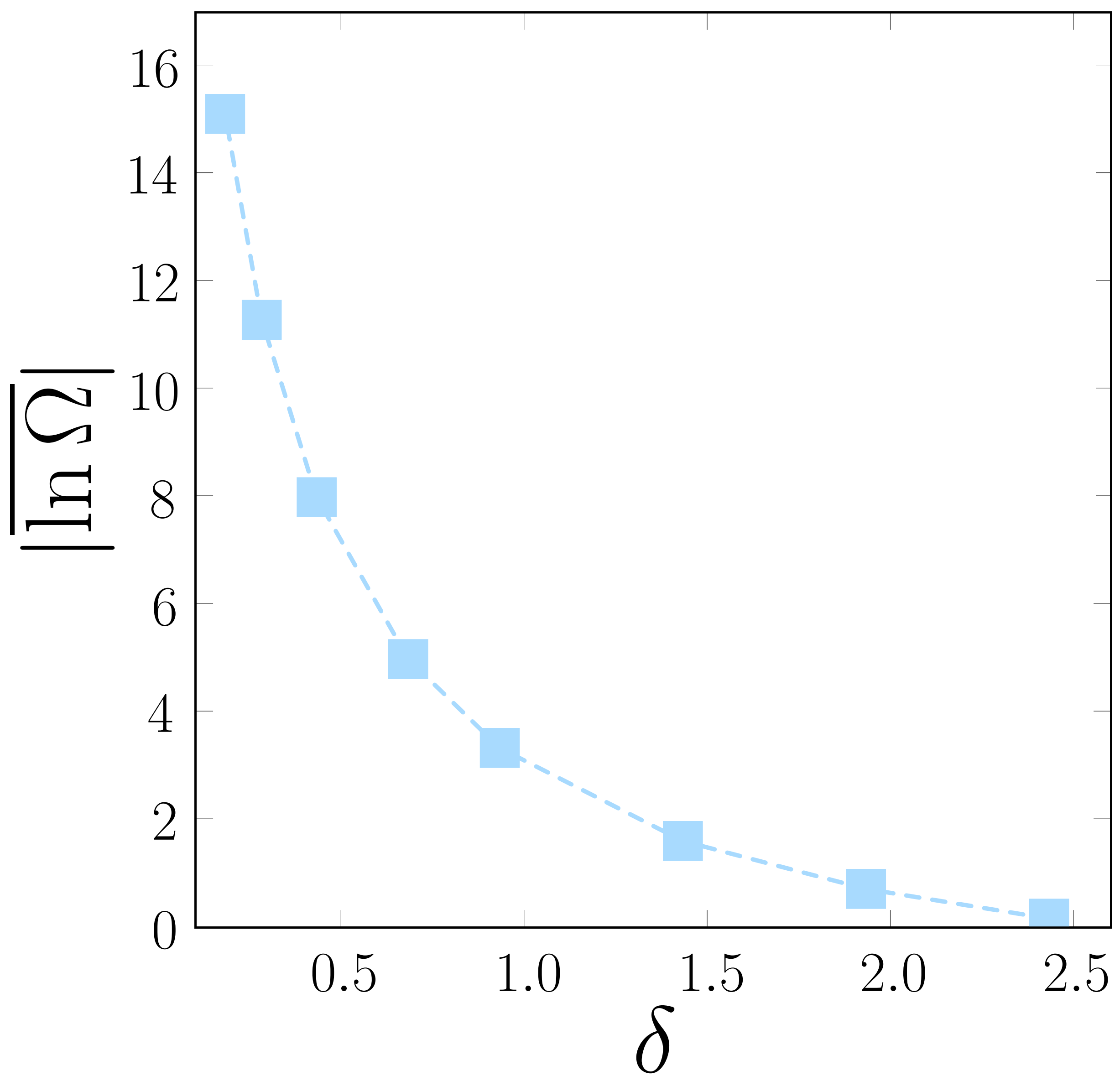
- The minimum excitation energy is due to a rare event, an extremal value

$$P(x) = \frac{1}{\beta} e^{\left[\left(\frac{x-\mu}{\beta} \right) - e^{\left(\frac{x-\mu}{\beta} \right)} \right]}$$



Juhász, Lin and Iglói (2006)

Activated scaling



$$|\ln \Omega| \sim L^{\psi} \sim \delta^{-\nu\psi}$$

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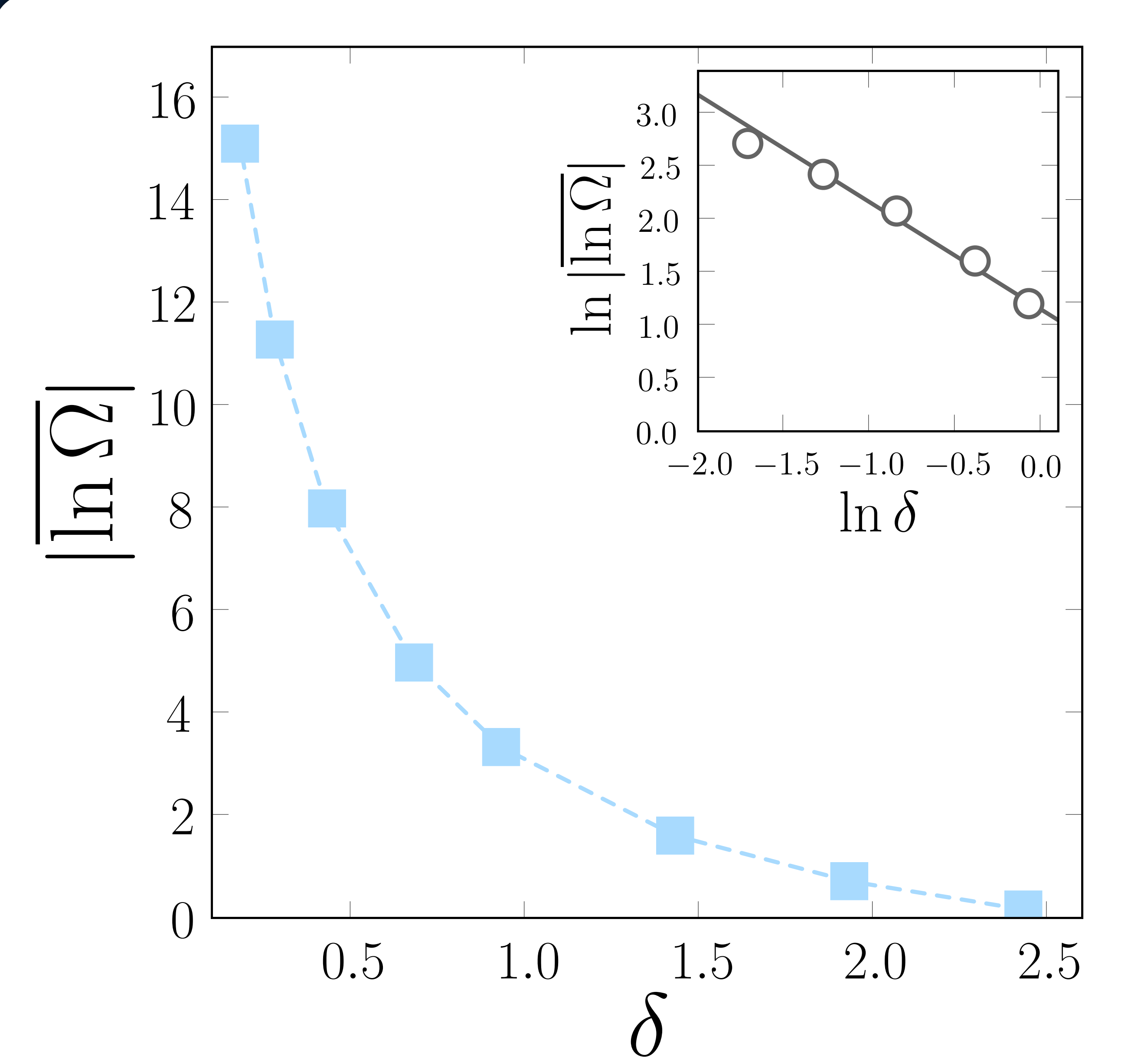

$$|\ln \Omega| \sim L^{\psi} \sim \delta^{-\nu\psi}$$

$$\psi = 0.53(6)$$

Dynamic susceptibilites

- We have direct access to real dynamical quantities

$$\mathrm{Im}\,\overline{\chi}(\omega) = \frac{1}{L}\sum_x \mathrm{Im}\,\left.\overline{\langle \Psi_x^*(i\omega)\Psi_0(i\omega)\rangle}_{\mathcal{S}_N}\right|_{i\omega\to\omega+i\epsilon}$$

$$\mathrm{Im}\,\overline{\chi_{\mathrm{loc}}}(\omega) = \mathrm{Im}\,\left.\overline{\langle \Psi_0^*(i\omega)\Psi_0(i\omega)\rangle}_{\mathcal{S}_N}\right|_{i\omega\to\omega+i\epsilon}$$

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real frequency

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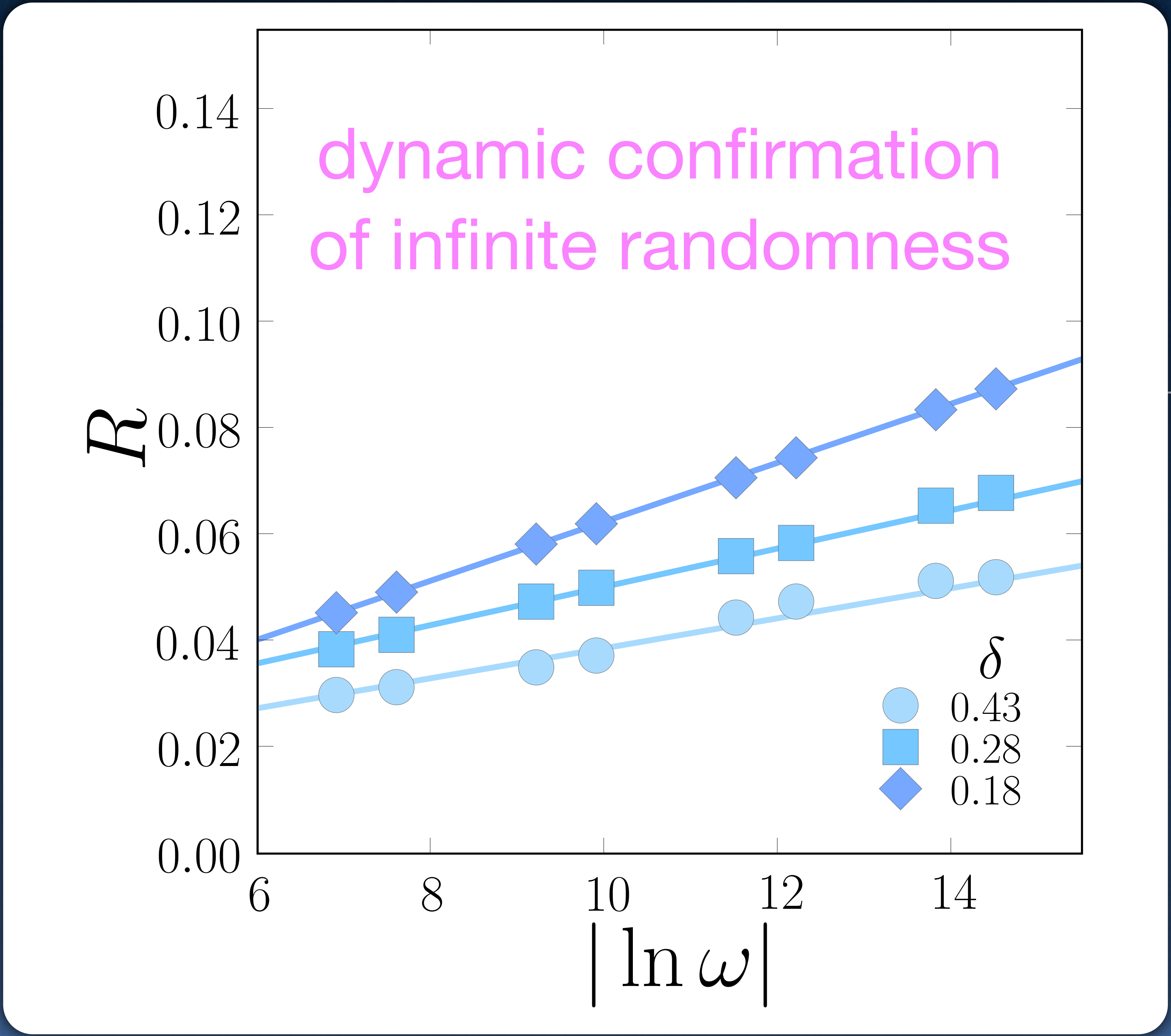
real frequency

- The ratio of the average to local susceptibility is related to the *average cluster moment*

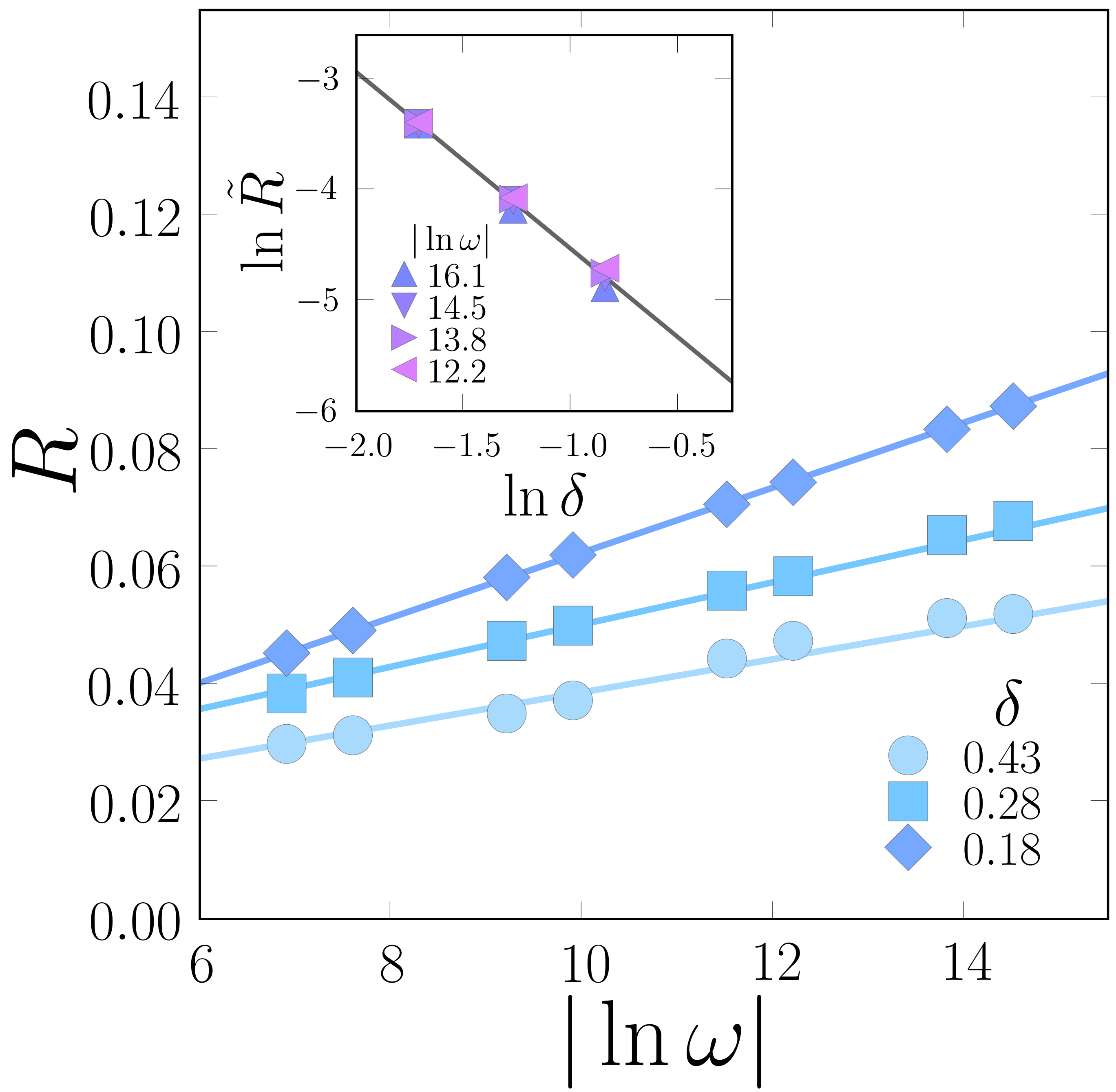
Activated Dynamical Scaling



- Can derive a scaling form for the cluster moment

$$R(\omega) = \frac{\mathrm{Im}\overline{\chi}(\omega)}{\mathrm{Im}\overline{\chi_{\mathrm{loc}}}(\omega)} \sim \delta^{\nu\psi(1-\phi)}|\ln\omega|$$

Activated Dynamical Scaling



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$$\phi = 1.6(2)$$

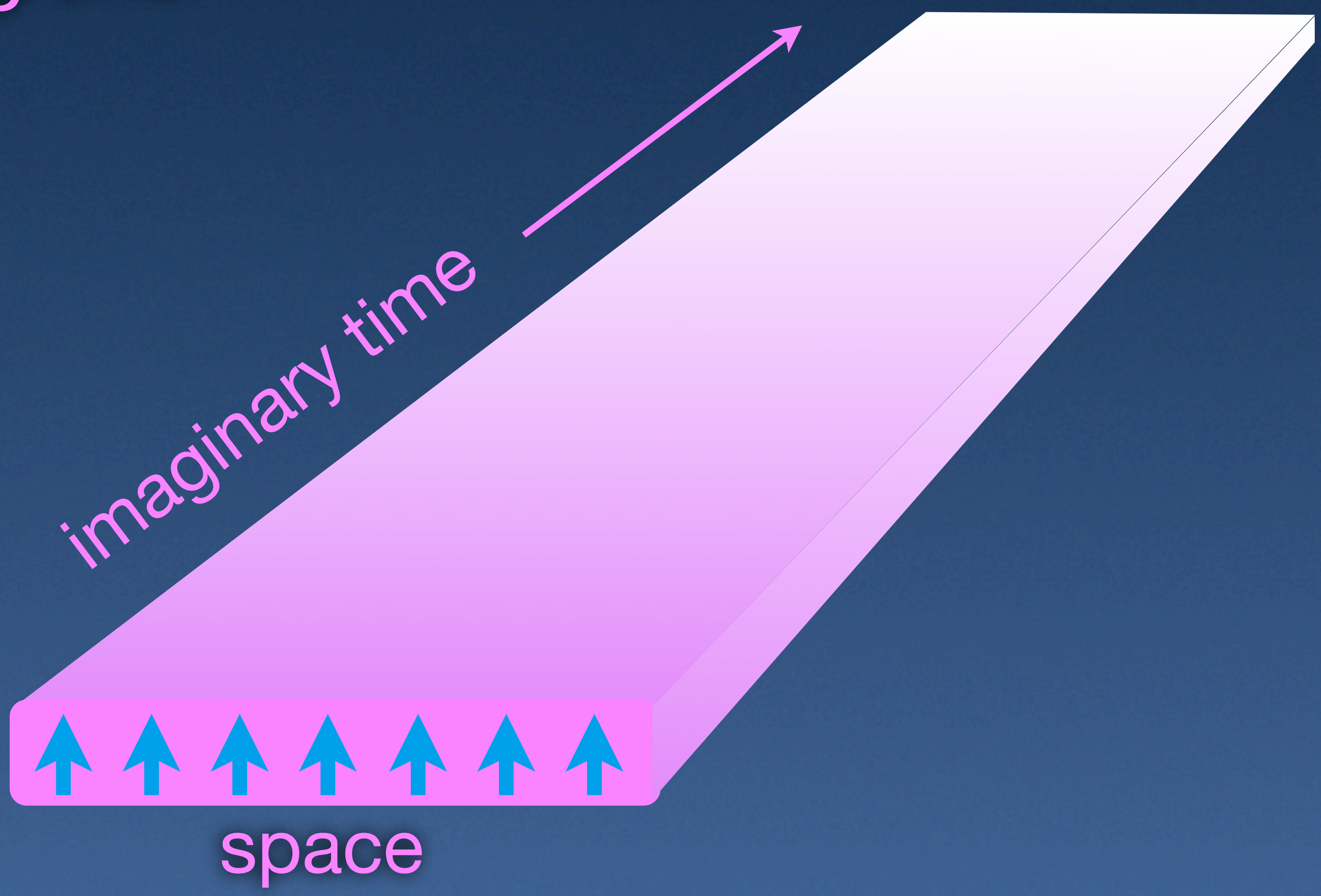
Putting it all together

	ν	ψ	ϕ
RTFIM	2	1/2	$(1+\sqrt{5})/2$
SMT	1.9(2)	0.53(6)	1.6(2)

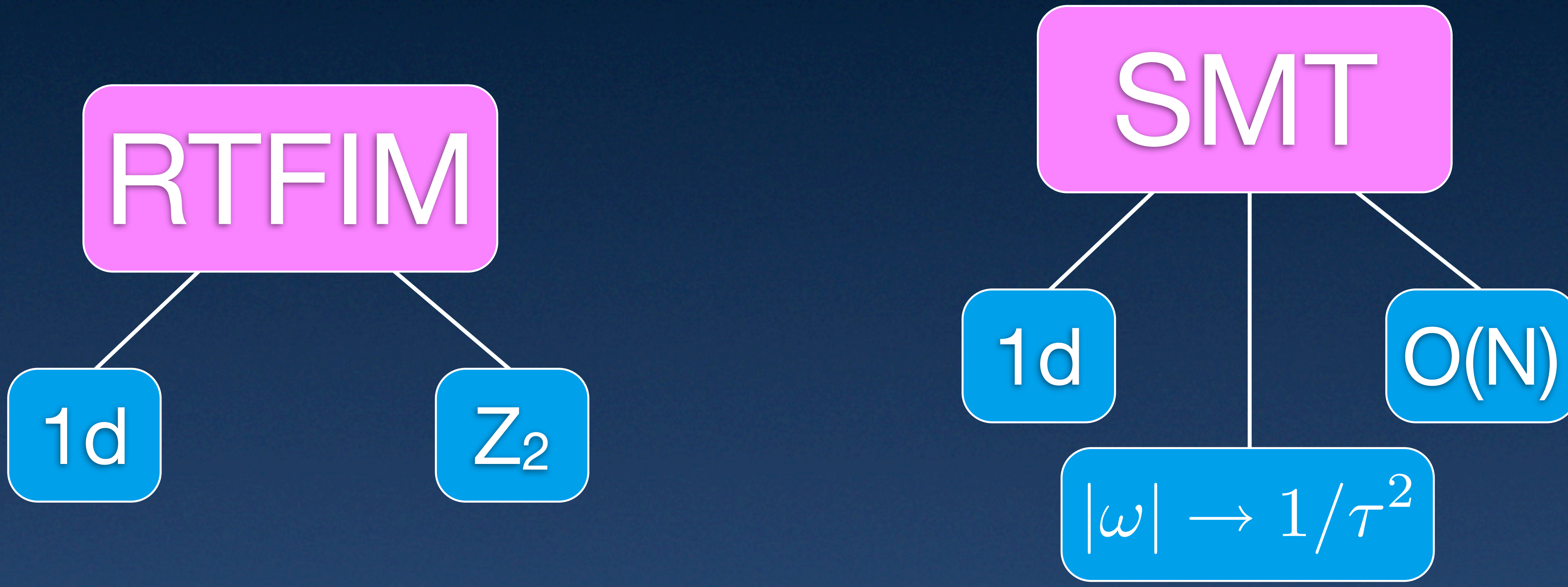
Numerical confirmation of the strong disorder RG calculations of Hoyos, Kotabage and Vojta!

Origin of correspondence

- The physics of infinite randomness comes from the slow dynamics and large contribution of rare regions



Effective classical theories



T. Vojta and J. Schmalian (2005); T. Vojta (2006)

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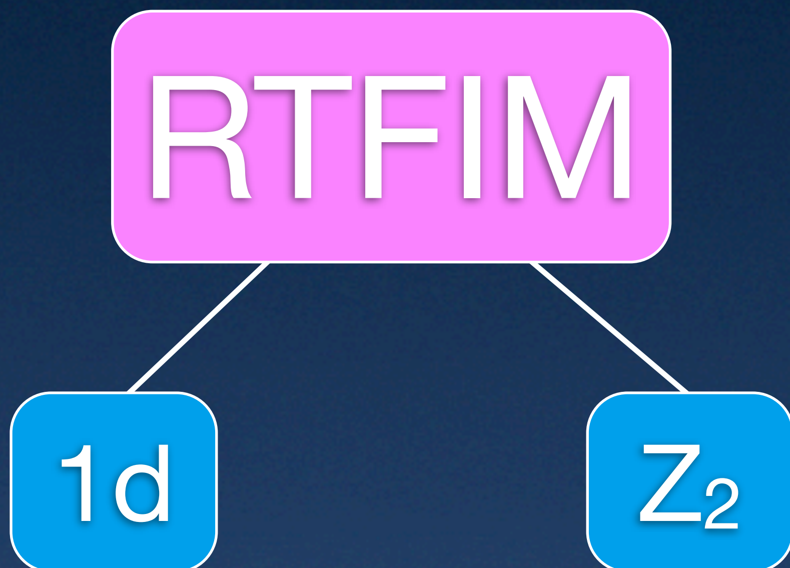

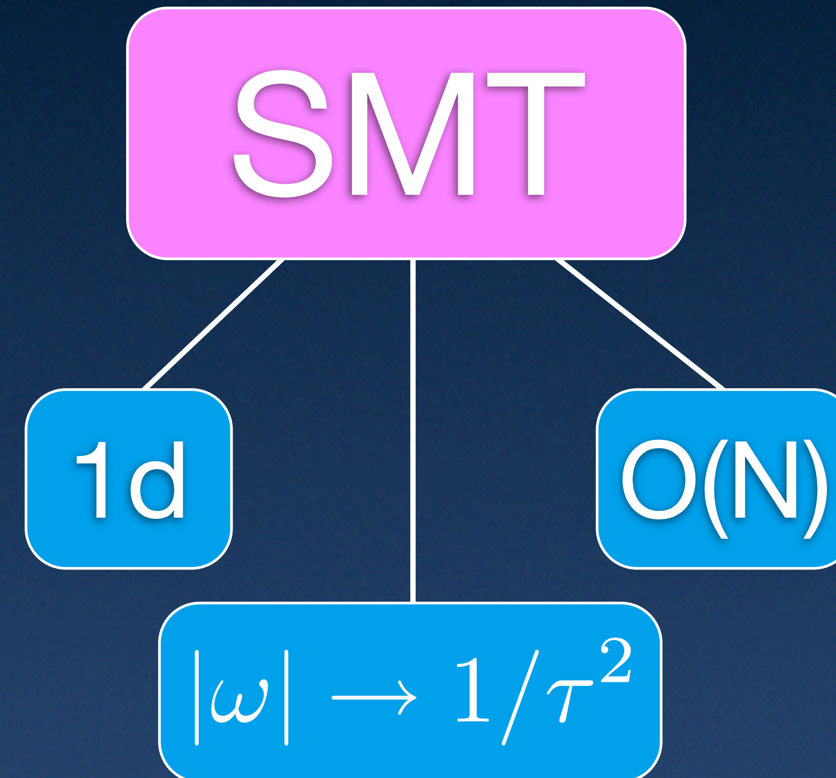

rare regions are marginal!

T. Vojta and J. Schmalian (2005); T. Vojta (2006)

Conclusions

- First dynamical confirmation of activated scaling from numerical simulations

- SMT has an infinite randomness fixed point in the RTFIM universality class

- Scratching the surface of transport calculations near a strong disorder fixed point