Electrodynamic response of coexisting amorphous magnetic and superconducting phases

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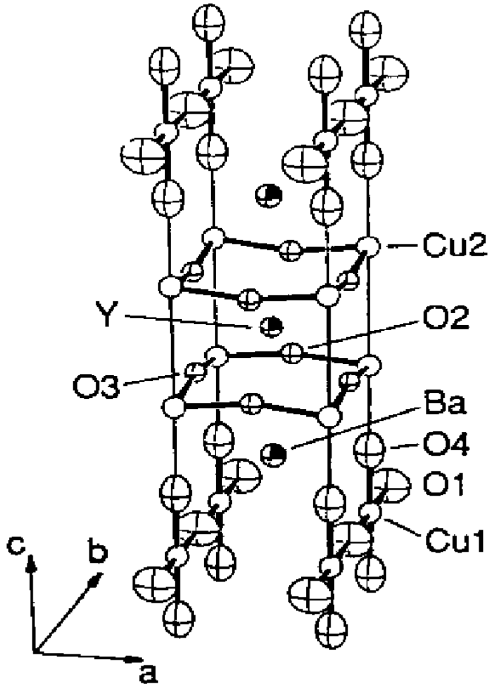
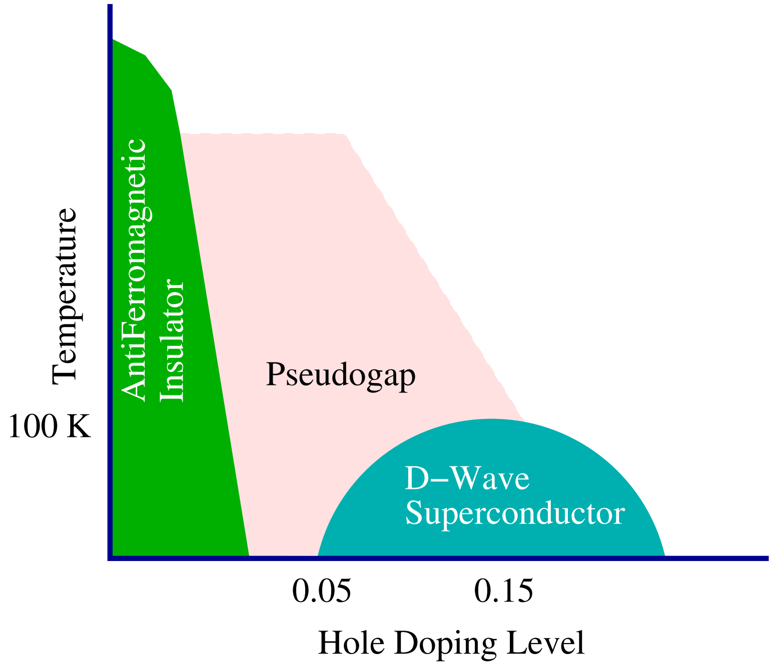
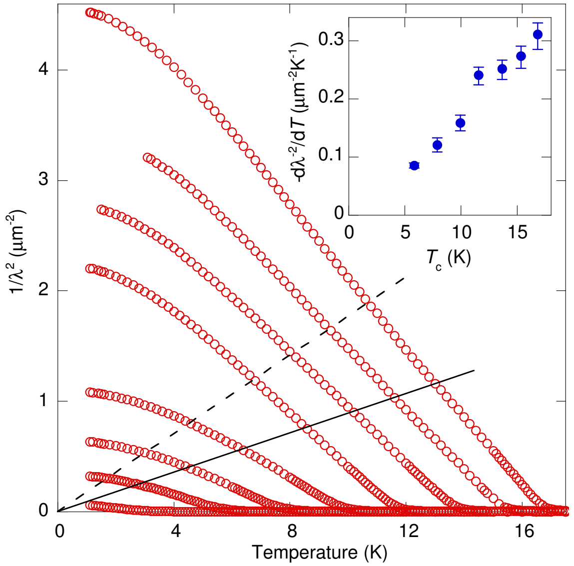
Phase Diagram

Main focus: Influence of coexisting phases on density of states and on the penetration depth (or superfluid density).





Penetration Depth of $\mathrm{YBa_2Cu_3O_{7-\delta}}$



`Broun et al, PRL` **`99`**`, 237003 (2007).`
`c.f. Zuev et al, PRL` **`95`**`, 137002 (2005)`

The penetration depth $\lambda(T)$ is the length scale over which a static applied magnetic field decays in a superconductor.

The superfluid density or superfluid stiffness is $\rho_s(T) \propto \lambda^{-2}(T)$

In a BCS superconductor:

$$\rho_s(T) = \frac{4\pi n e^2}{m c^2} - \delta\rho_s(T)$$

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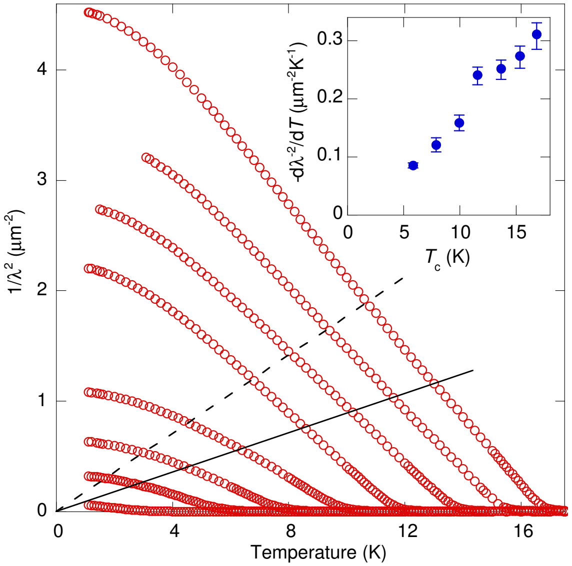
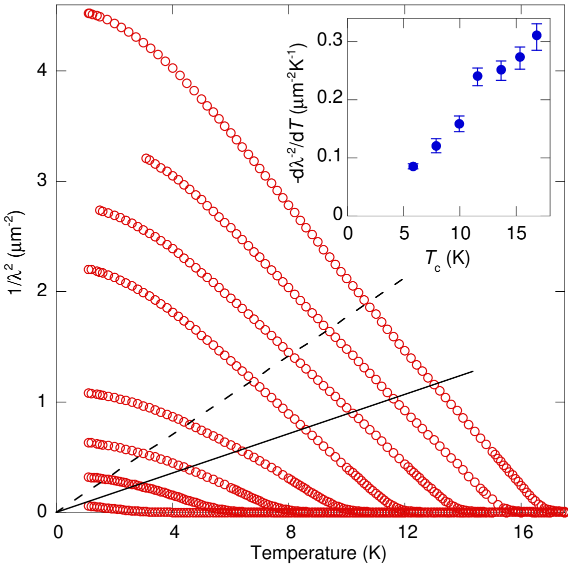


- Strong Correlation Effects. Divergent effective mass, renormalization of QP spectral weight. Kinetic energy operator in Gutzwiller approximation:

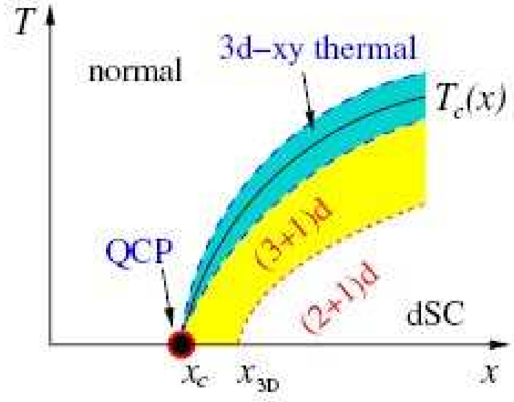
$$\hat{T} = \hat{T}_0 \frac{2p}{1+p}$$

Anderson, Science **235** 1196 (1987); Lee & Wen, PRL **78**, 4111 (1997);Zhang et al, Supercond. Sci. Technol. **1**, 36 (1988); Paramekanti et al, Phys. Rev. Lett. **87**, 217002 (2001).

Penetration Depth of $\mathrm{YBa_2Cu_3O_{7-\delta}}$



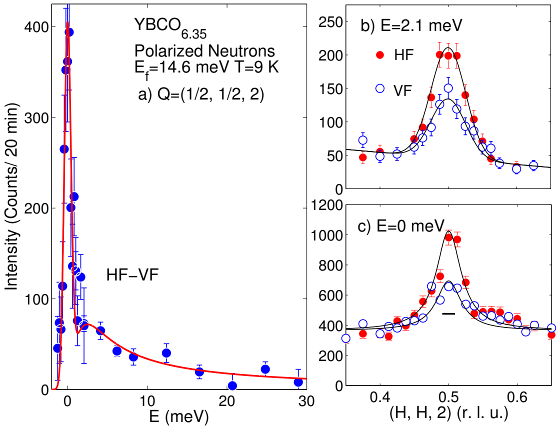
- Phase Fluctuations. Emery & Kivelson, Roddick & Stround, Herbut & Case, Franz & Iyengar.



Franz & Iyengar, PRL 96, 047007

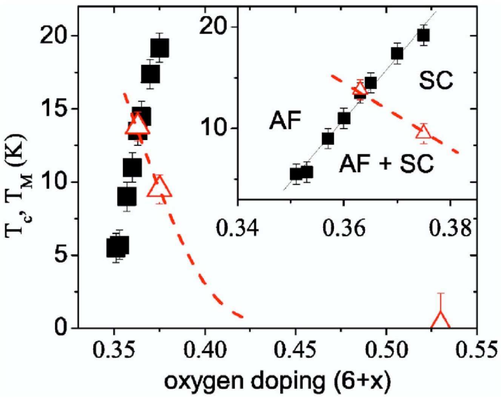
- Phase transition is 3+z dimensions ⇒ mean-field.
- Linear T-dependence from QP excitations
- Overall renormalization of $\rho_s(0)$ by Coulomb interaction.

Neutron Scattering



C. Stock *et al.*, Phys. Rev. B 73, 100504(R) (2006).

Muon Spin Rotation



R. I. Miller *et al.* PRB **73**, 144509 (2006).

Magnetic Effects

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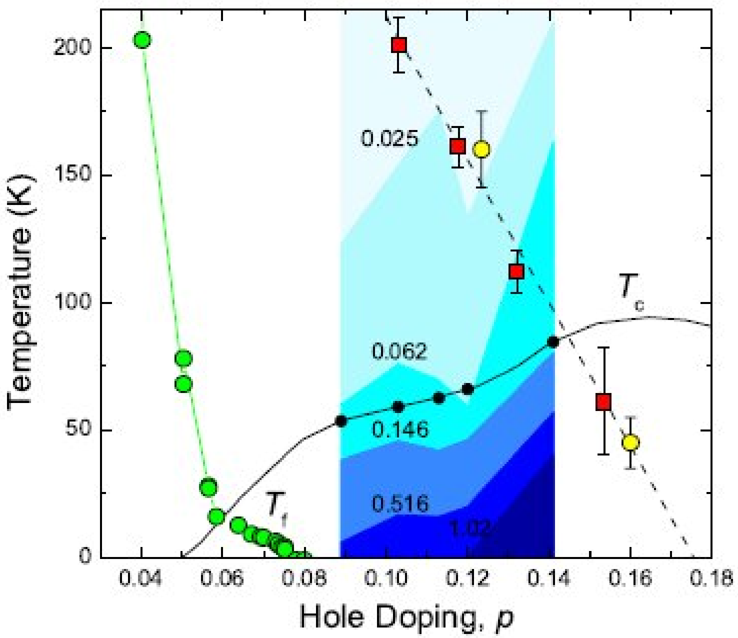
- C. Panagopoulos, J. L. Tallon, B. D. Rainford, J. R. Cooper, C. A. Scott, and T. Xiang, Solid State Commun. **126**, 47 (2003); C. Panagopoulos and A. P. Petrovic and A. D. Hillier and J. L. Tallon and C. A. Scott and B. D. Rainford, Phys. Rev. B **69**, 144510 (2004).
- J. A. Hodges and Y. Sidis and P. Bourges and I. Mirebeau and M. Hennion and X. Chaud, Phys. Rev. B **66**, 020501 (2002).
- C. Stock *et al.*, Phys. Rev. B 73, 100504(R) (2006).
- R. I. Miller *et al.* PRB **73**, 144509 (2006).

Revised Phase Diagram



Sonier et al, PRL **101**, 117001 (2008).

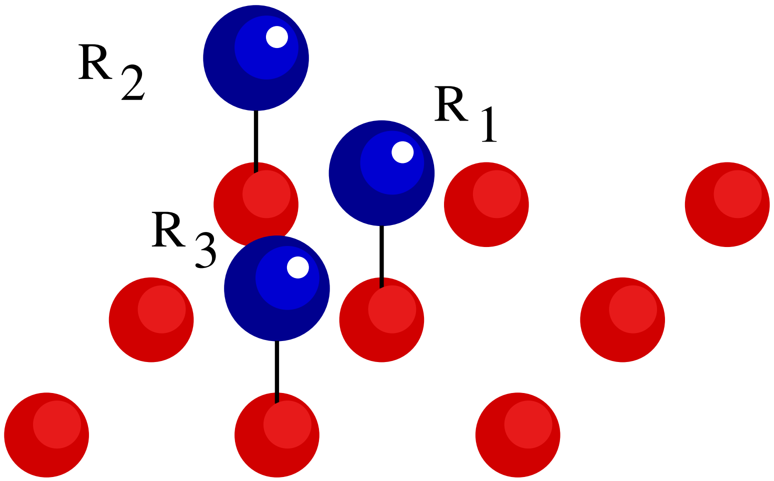
Self-Consistent Model

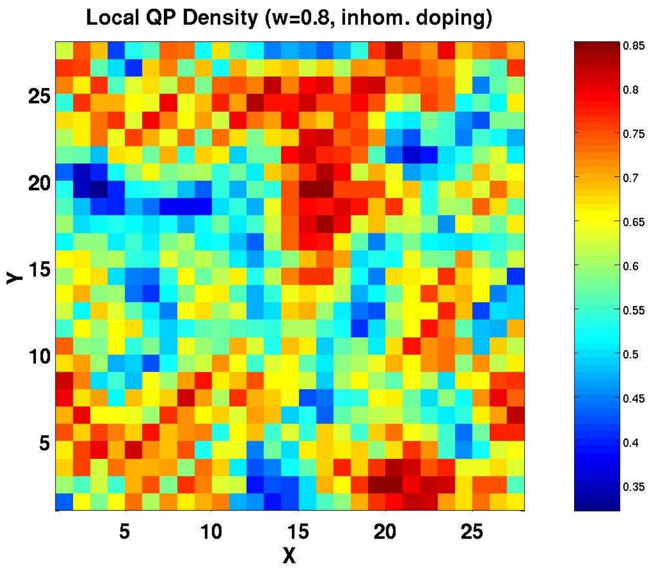
$$\begin{aligned} \hat{H}_H &= w \sum_{ij\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_{i\sigma} \hat{n}_{i\sigma} n_{i\bar{\sigma}} + \sum_{\langle i,j \rangle} (\Delta_{ij} \hat{f}_{ij} + \Delta^*_{ij} \hat{f}^\dagger_{ij}), \\ \hat{H}_c &= \sum_{i \neq j} V(\mathbf{r}_i - \mathbf{r}_j) \hat{n}_i n_j - Z \sum_{i,\mathbf{R}} V(\mathbf{r}_i - \mathbf{R}) \hat{n}_i, \end{aligned}$$

$$n_{i\sigma} = \langle \hat{n}_{i\sigma} \rangle \qquad \Delta_{ij} = -J \langle \hat{f}_{ij} \rangle, \qquad \hat{f}_{ij} = (c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow})/2$$

- Total doping is $Z N_I$, N_I is number of donor impurities.
- Strong correlations are assumed to renormalize KE via w. $\boxed{x \longrightarrow w}$.
- Screened Coulomb $V(r) = e^2 \exp(-r/\Lambda)/\epsilon r$, $\Lambda = 20 a_0$; $e^2/\epsilon a_0 = 1.5$.
- Choose 8-16 random initial moment configurations and iterate to self-consistency.
- Self-consistency is non-trivial to obtain. Thomas-Fermi-Pulay iteration scheme usually works. Even then, convergence is slow. [D. Raczkowski et al, Phys. Rev. B **64**, 121101 (2001).]

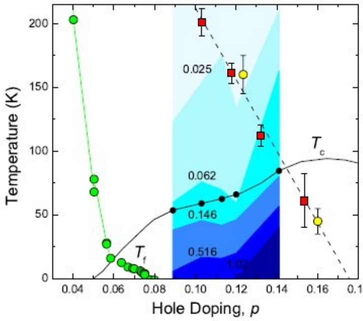
Inhomogeneous Doping

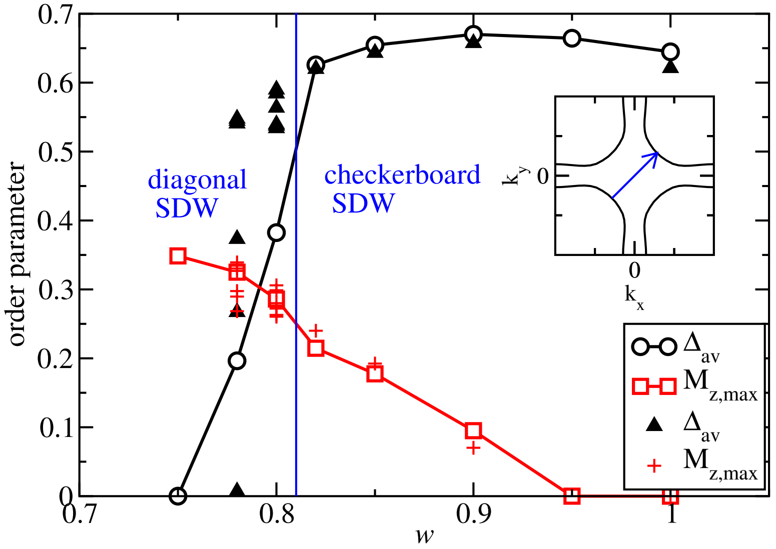


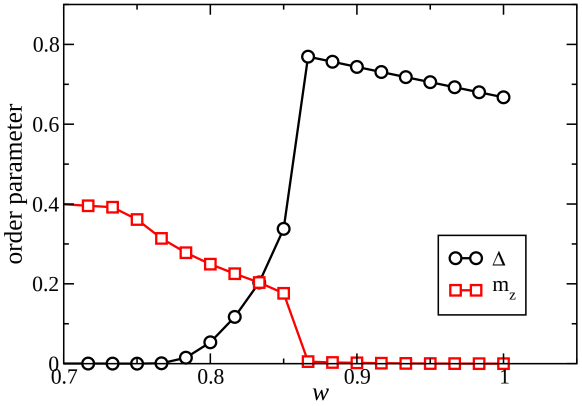


Total QP doping is $n = ZN_I/N$ so we can adjust inhomogeneity keeping n fixed.
Homogeneous doping means $N_I = N$.
Inhomogeneous doping means $N_I/N = 0.4 \Leftrightarrow$ YBCO$_{6.35}$.

Phase Diagram





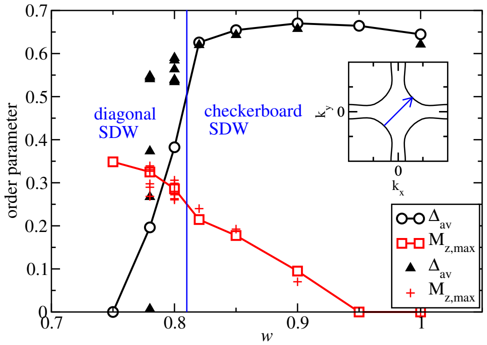


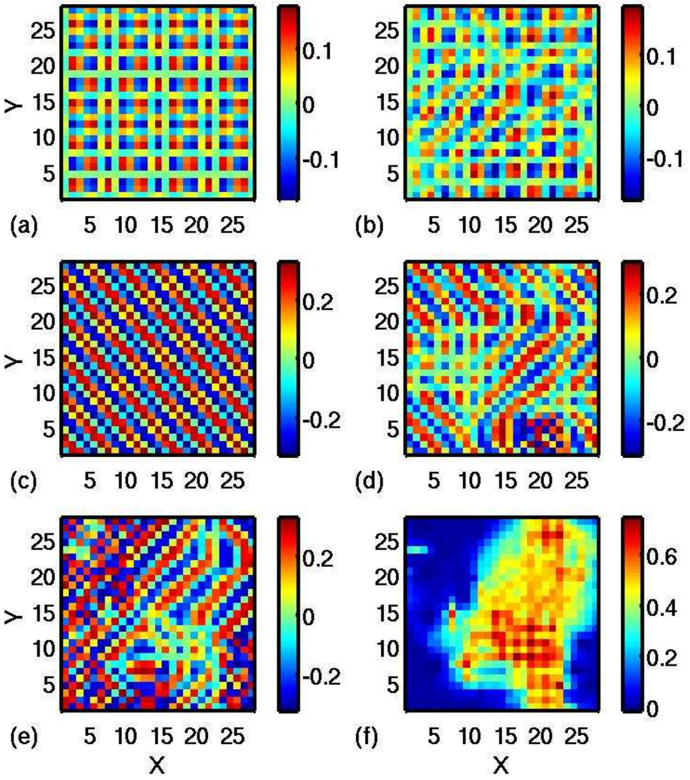
Commensurate AF

w : KE renormalization

$U = 3.2 \quad J = 1.5 \quad t_0 = 1.7$

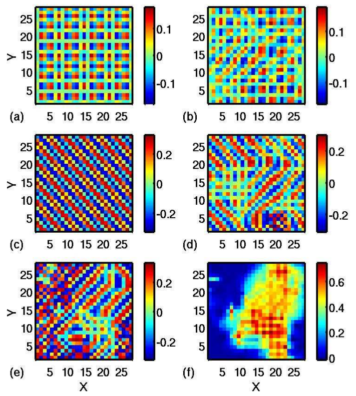
Self-consistent Phases

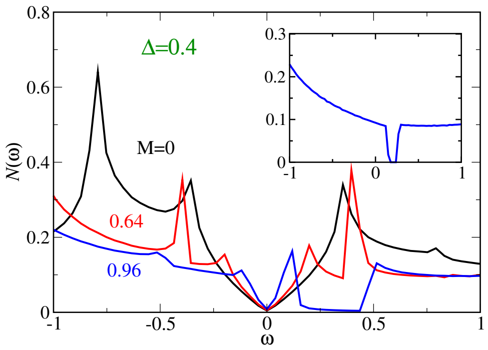


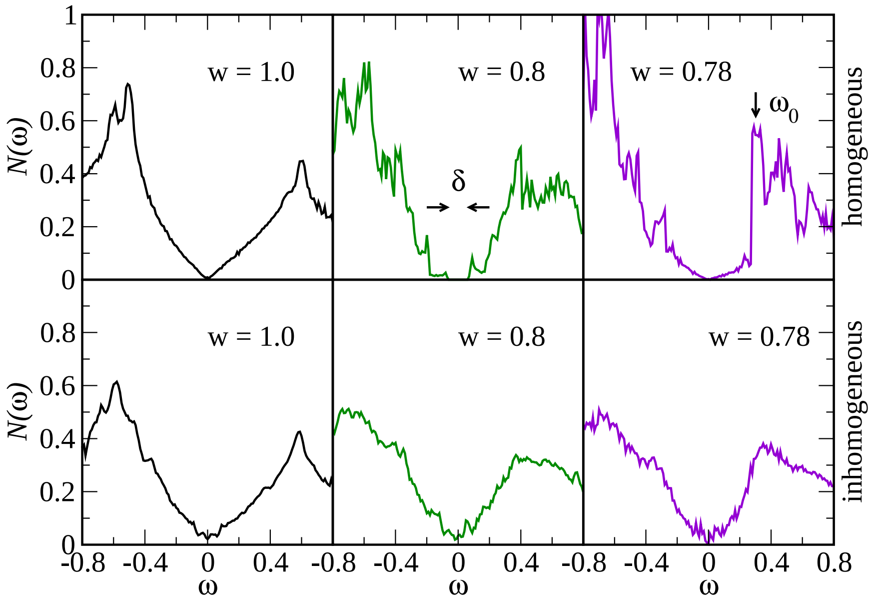


Top: m_z^Q for $w = 0.85$, hom. & inhom.
Middle: m_z^Q for $w = 0.8$ hom. & inhom.
Bottom: m_z^Q & Δ for $w = 0.78$, inhom.

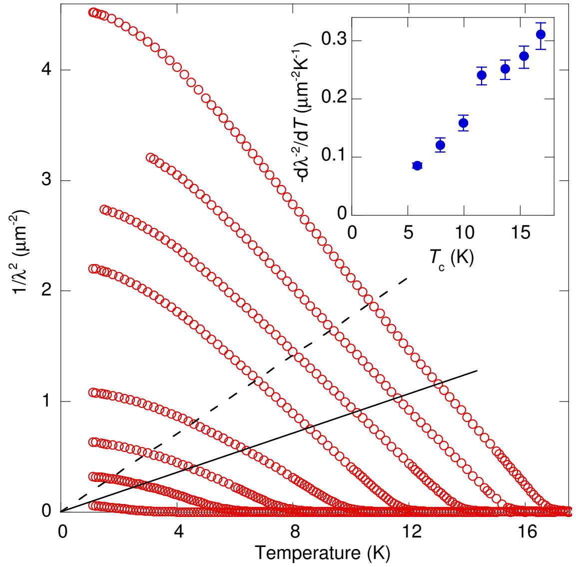
Self-consistent Phases

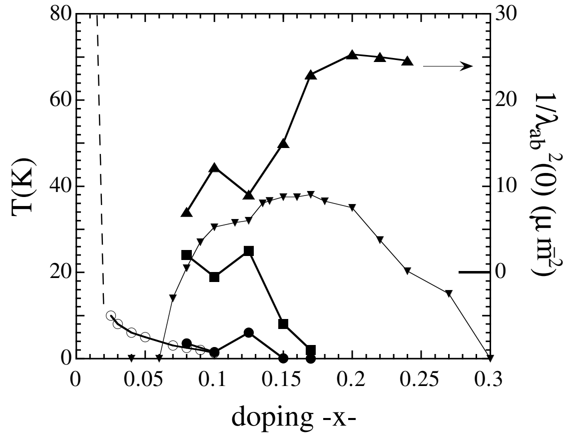






The Penetration depth

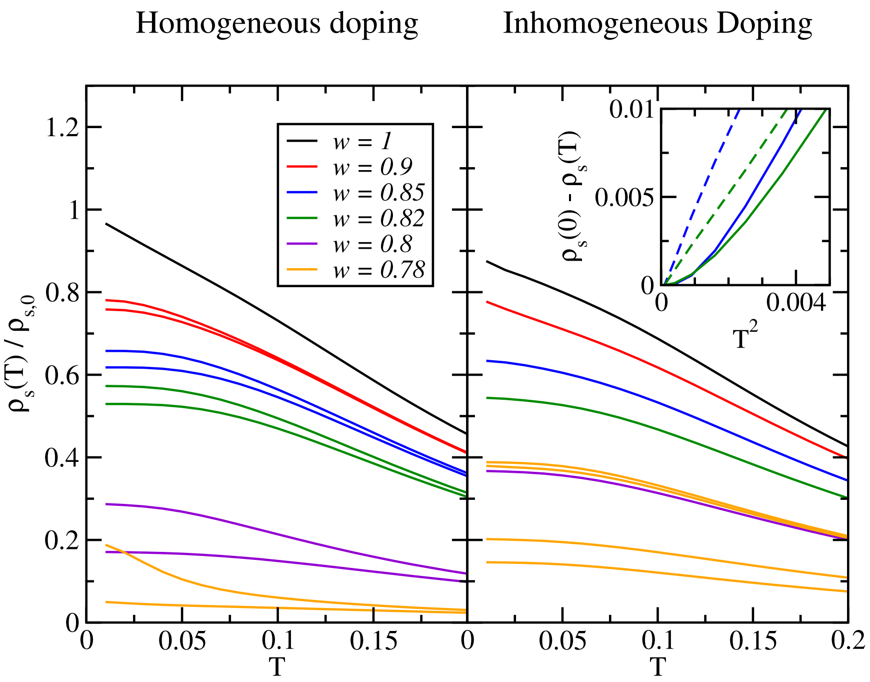




μSR measurements in polycrystaline La$_{2-x}$Sr$_x$CuO$_4$, Panagopoulos et al, PHYSICA C **341** 843-846 (2000).

- $\lambda^{-2}(T=0)$ (triangle up)
- T_c (triangle down)
- $T_{\rm sf}$ (square)
- $T_{\rm sg}$ (circle)

Low T Superfluid Density

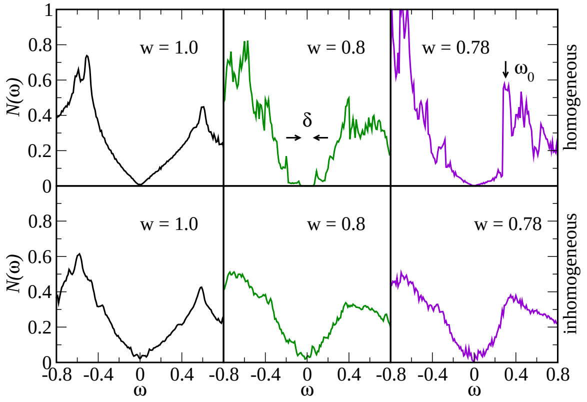


- Homogeneous case:

$$\rho_S(w,T) = \rho_S(w,0) - \alpha\exp(C/T).$$

- Inhomogeneous case:

$$\rho_S(w,T) = \rho_S(w,0) - \alpha T^2.$$



Inhomogeneity is important!

Why are magnetic correlations so effective at suppressing superfluidity?

In BCS theory charge is not conserved but current is.
When SDW is mixed in, current is no longer a conserved quantity.
e.g. Commensurate antiferrromagnetism mixes $\mathbf{k}$ and $\mathbf{k}+\mathbf{Q}$. The current vertex is

$$\gamma_x(\mathbf{k}) = \left[\begin{array}{cc} v_x(\mathbf{k})\tau_0 & 0 \\ 0 & v_x(\mathbf{k}+\mathbf{Q})\tau_0 \end{array} \right]$$

and $[\gamma_x, H] \neq 0$.
Quasiparticles do not have a well-defined current.

Conclusions

- Incommensurate, disordered spin density waves can coexist with d-wave superconductivity.
- DOS that of dirty d-wave superconductor.
- Superfluid density dramatically reduced with little effect on gap.

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