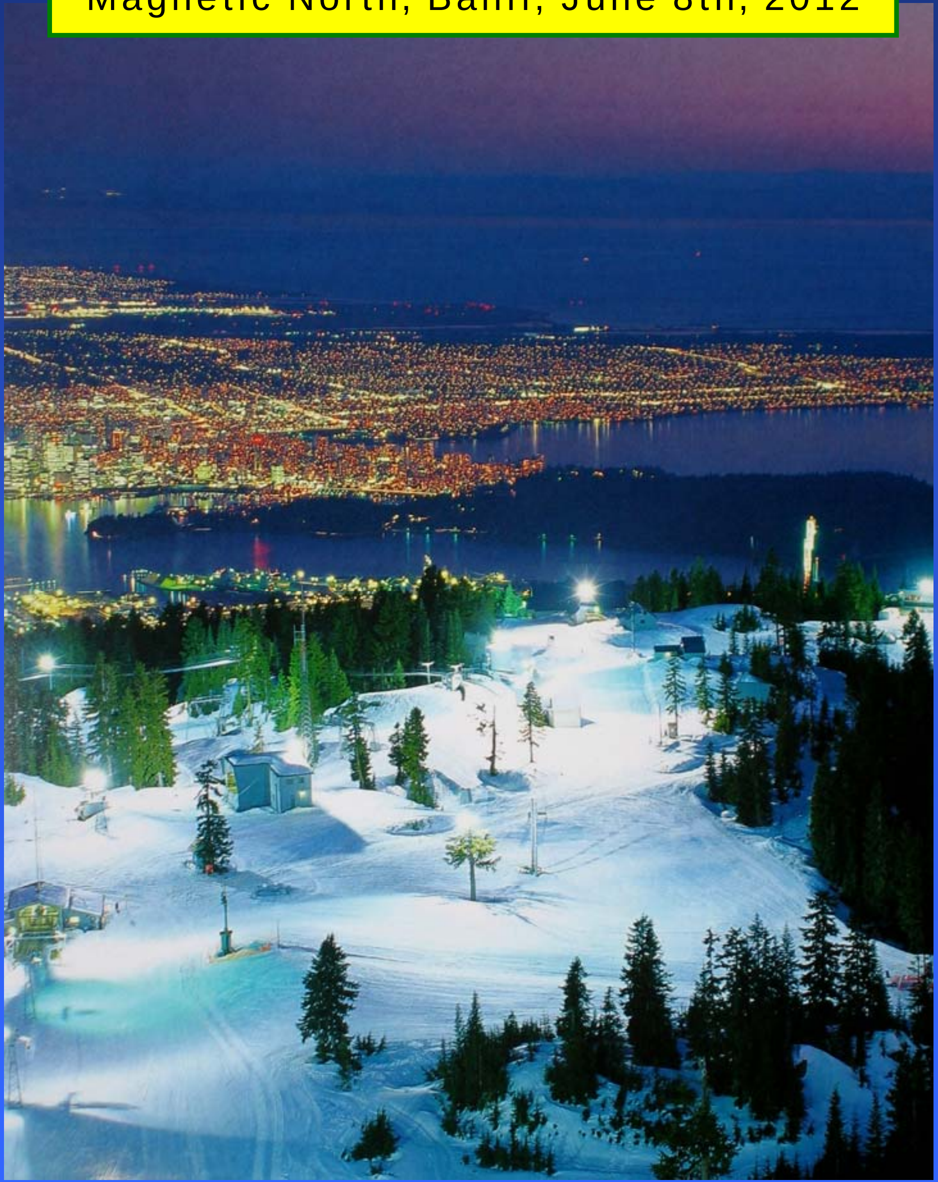
PCE STAMP

LARGE-SCALE COHERENCE & DECOHERENCE

Magnetic North, Banff, June 8th, 2012



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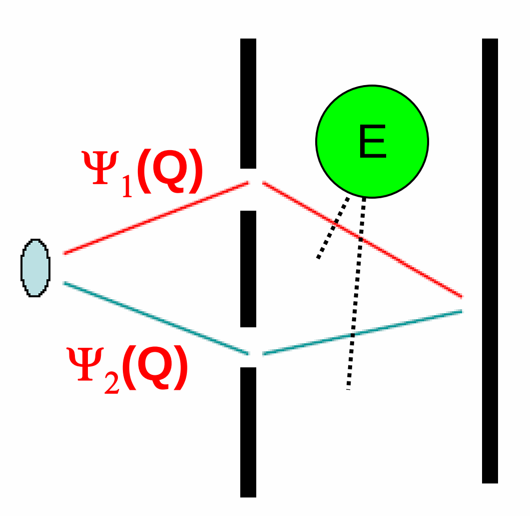
PART 1:

MECHANISMS
of
ENVIRONMENTAL DECOHERENCE

ENVIRONMENTAL DECOHERENCE 100



Some quantum system with coordinate Q interacts with any other system (with coordinate x) ; typically they then form an entangled state

Example: **In a 2-slit expt., the particle coordinate Q couples to photon coordinates, so that:**

$$\Psi_o(Q) \;\; \Pi_q \, \phi_q^{in} \;\; \rightarrow \;\; [a_1 \Psi_1(Q) \, \Pi_q \, \phi_q^{(1)} \;\; + \;\; a_2 \Psi_2(Q) \, \Pi_q \, \phi_q^{(2)} \,]$$

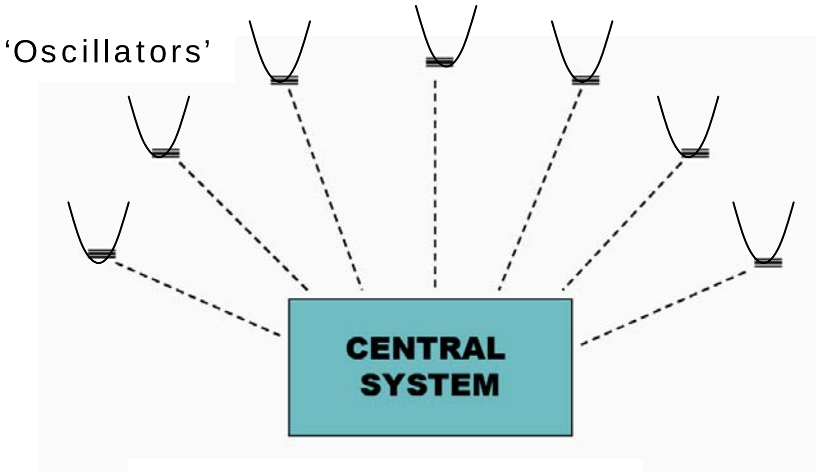
Now suppose we have no knowledge of / control over, the photon states – we then average over these states, consistent with the experimental constraints. In the extreme case this means we lose all information about the PHASES of the coefficients a_1 & a_2 (and in particular the relative phase between them).

This process is called DECOHERENCE

NB 1: **No requirement for energy to be exchanged between the system and the environment – only a communication of phase information.**

NB 2: **Nor does phase interference between the 2 paths have to be associated with a noise coming from the environment- what matters is entanglement - that the state of the environment be CHANGED according to the what is the state of the system.**

CURRENT MODELS of ENVIRONMENTAL DECOHERENCE



$$H_{\rm eff}^{\rm osc} = H_0 + H_{\rm int} + H_{\rm env}^{\rm osc}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left(\frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2 \right)$$

Int: $$H_{\rm int}^{\rm osc} = \sum_{q=1}^{N} [F_q(Q) x_q + G_q(P) p_q]$$

Very SMALL (~ $O(1/N^{1/2})$

Phonons, photons, magnons, spinons,
Holons, Electron-hole pairs, gravitons,..

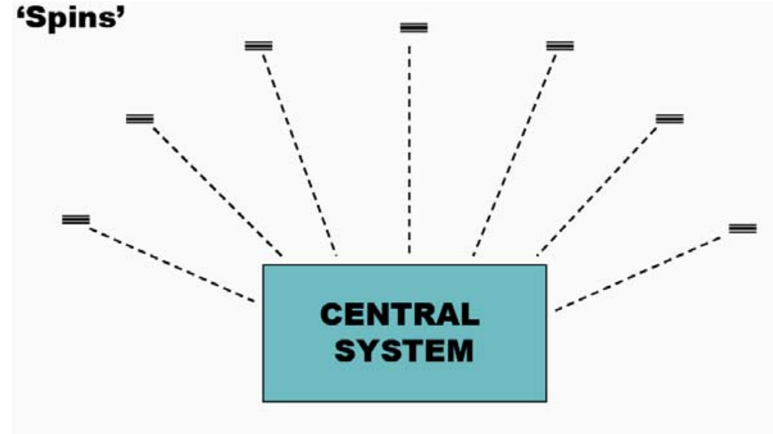
Feynman & Vernon, Ann. Phys. 24, 118 (1963)

Caldeira & Leggett, Ann. Phys. 149, 374 (1983)
AJ Leggett et al, Rev Mod Phys 59, 1 (1987

DELOCALIZED
BATH MODES

↓

OSCILLATOR
BATH



$$H_{\rm eff}^{\rm sp}(\Omega_0) = H_0 + H_{\rm int}^{\rm sp} + H_{\rm env}^{\rm sp}$$

Bath: $$H_{\rm env}^{\rm sp} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$

Interaction: $$H_{\rm int}^{\rm sp} = \sum_{k}^{N_s} \boldsymbol{F}_k(P,Q) \cdot \boldsymbol{\sigma}_k$$

NOT SMALL !

Defects, dislocation modes, vibrons,
Localized electrons, spin impurities,
nuclear spins, ...

LOCALIZED
BATH MODES

↓

SPIN BATH

(1) P.C.E. Stamp, PRL 61, 2905 (1988)
(2) NV Prokof'ev, PCE Stamp, J Phys CM5, L663 (1993)
(3) NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

MECHANISMS of ENVIRONMENTAL DECOHERENCE: a SIMPLE PICTURE

Easiest to visualize this in path integral theory:

(1) OSCILLATOR BATH Oscillator Lagrangian: $L_q(x_q, \dot{x}_q; t) = \dfrac{m_q \dot{x}_q^2}{2} - \Upsilon_q(t) x_q$

Each oscillator is subject to a force $\Upsilon_q(t) = m_q \omega_q^2 x_q - F_q(Q(t))$

Problem is exactly solvable (Feynman). Each oscillator very weakly coupled to system, and slowly entangles with it…weak excitation of oscillator

(2) SPIN BATH Each bath spin has the Lagrangian

$$L(\boldsymbol{\sigma}_k, \dot{\boldsymbol{\sigma}}_k; t) = \mathscr{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{d\tau} - \boldsymbol{\Upsilon}_k(t) \cdot \boldsymbol{\sigma}_k$$

with the force: $\boldsymbol{\Upsilon}_k(t) = \mathbf{h}_k + \mathbf{F}_k(t) + \boldsymbol{\xi}_k(t)$

Entanglement with system via $\boldsymbol{F}_k(P, Q) \cdot$ (not weak)

This problem is highly non-trivial (in general UNSOLVABLE even for spin-1/2 !).

We also have a small 'internal noise' term, of form

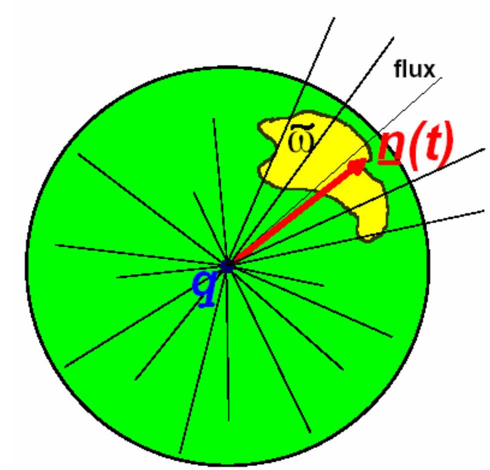
$$\xi_k^\alpha(t) = \sum_{k'} V_{kk'}^{\alpha\beta} \sigma_{k'}^\beta(t) \sim \sum_{k'} V_{kk'}^{\alpha\beta} \langle \sigma_{k'}^\beta(t) \rangle$$

The 'topological term'

$$\phi_B = q/\hbar \int \mathcal{A} \cdot d\mathbf{n}$$

gives an added phase

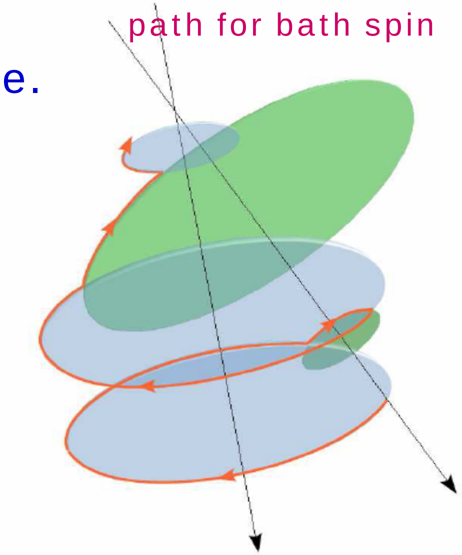
$$\phi_B = \varpi S$$



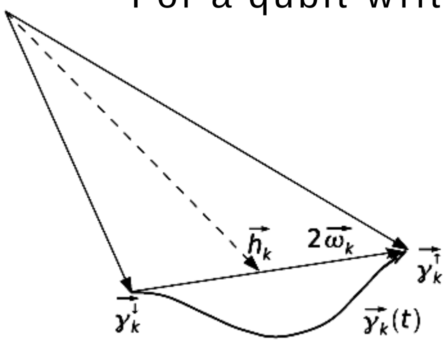
EXAMPLE: CENTRAL SPIN DYNAMICS

A beautiful test case for decoherence theory – which predicts Qubit dynamics is dominated by 'precessional decoherence' from the spin bath. This precessional decoherence has NO DISSIPATION – it is invisible in energy relaxation, but causes very strong decoherence.

Precessional path for bath spin



For a qubit write: $$\hat{H}_{QB} = H^0_{QB}(\vec{\tau}) + \sum_k (\vec{\gamma}_k + \xi_k)\cdot\vec{\sigma}_k$$



where $$\gamma_k^\alpha = h_k^\alpha + \sum_\beta \omega_k^{\beta\alpha}\tau_\beta$$

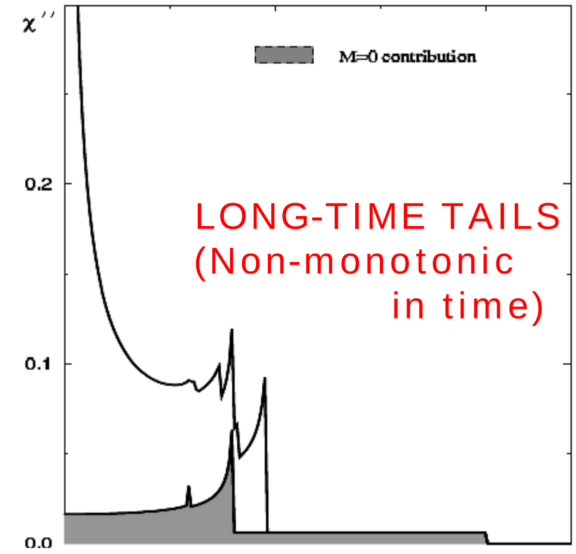
$$\xi_k^\alpha = \sum_{k'}\sum_\beta V_{kk'}^{\alpha\beta}\sigma_{k'}^\beta$$

Path of field on bath spin from Qubit – it jumps quickly between 2 orientations

Precessional decoherence rates are

$$\Gamma_\phi^P \sim 1/2\sum_k (\omega_k/h_k)^2; \omega_k \ll h_k$$ High field

$$\Gamma_\phi^P \sim 1/2\sum_k (h_k/\omega_k)^2; h_k \ll \omega_k$$ Low field



LONG-TIME TAILS (Non-monotonic in time)

The only dissipation in the problem comes from the departures from the sudden approximation, & from the weak bath noise term – both very small.

The lineshape is not conventional at all; see RIGHT:

For Central Spin model, see:

NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

PART 2:

So much for the Theory..
Now let's go back to the

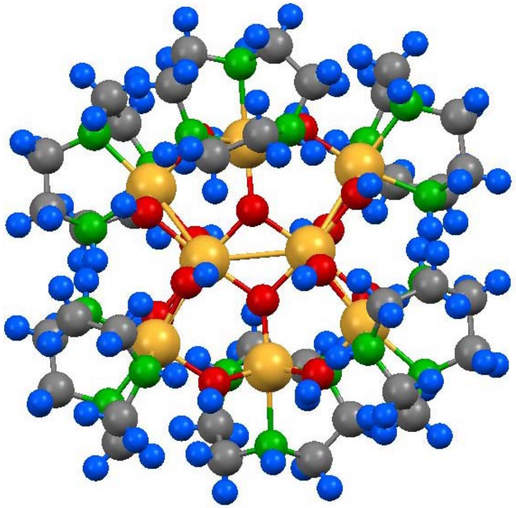
REAL WORLD

APPLICATION
to
EXPERIMENT

Only wimps specialize in the general case. Real scientists pursue examples.

MV Berry: Ann NY Acad Sci 755, 303 (1995)

EXAMPLE 1: The Fe-8 MOLECULE – A SPIN QUBIT



Fe_8 S = 10

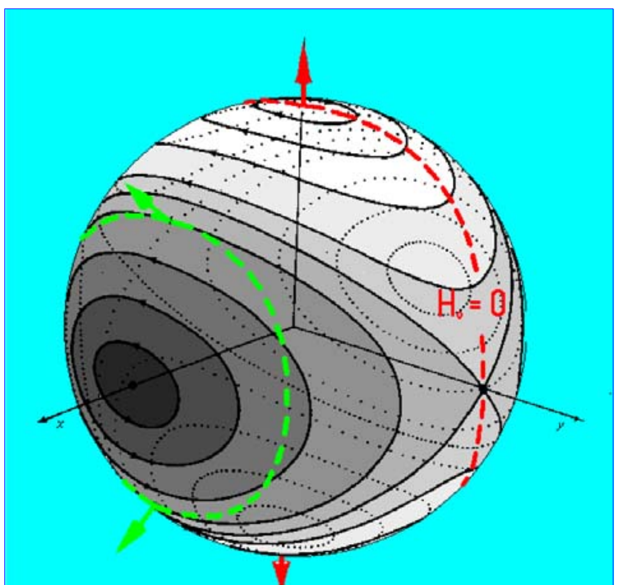
Low-T Quantum regime- effective Hamiltonian (T < 0.36 K):

$$\mathcal{H}_o(\hat{\tau}) = (\Delta_o \hat{\tau}_x + \epsilon_o \hat{\tau}_z)$$

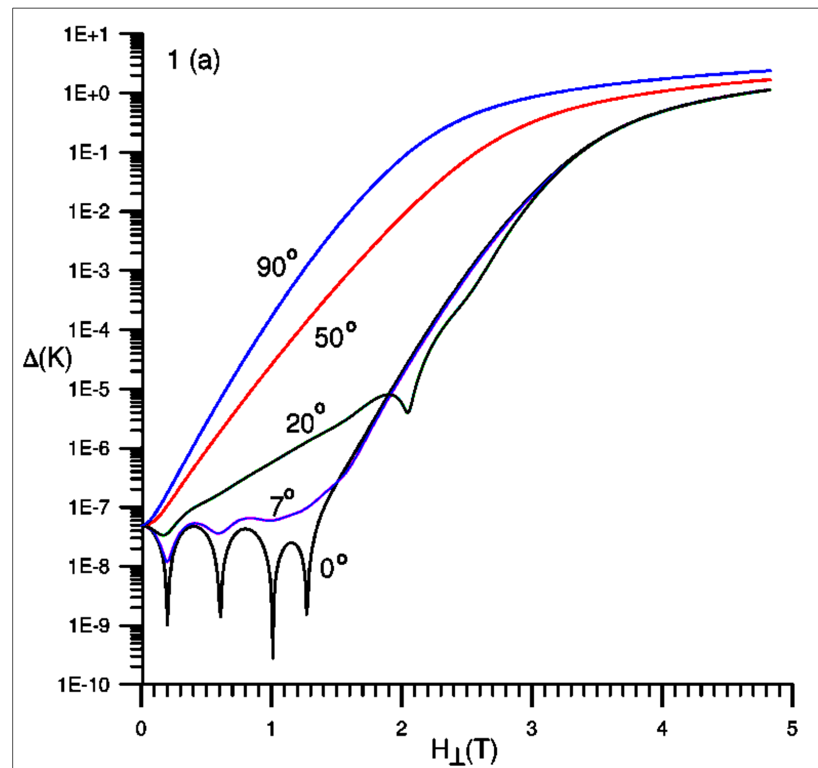
Longitudinal bias: $\epsilon_o = g\mu_B S_z H_o^z$

Eigenstates: $|\pm\rangle = [|\uparrow\rangle \pm |\downarrow\rangle]/\sqrt{2}$

defines orthonormal states: $|\uparrow\rangle, |\downarrow\rangle$



Feynman Paths on the spin sphere for a biaxial potential. Application of a field pulls the paths towards the field

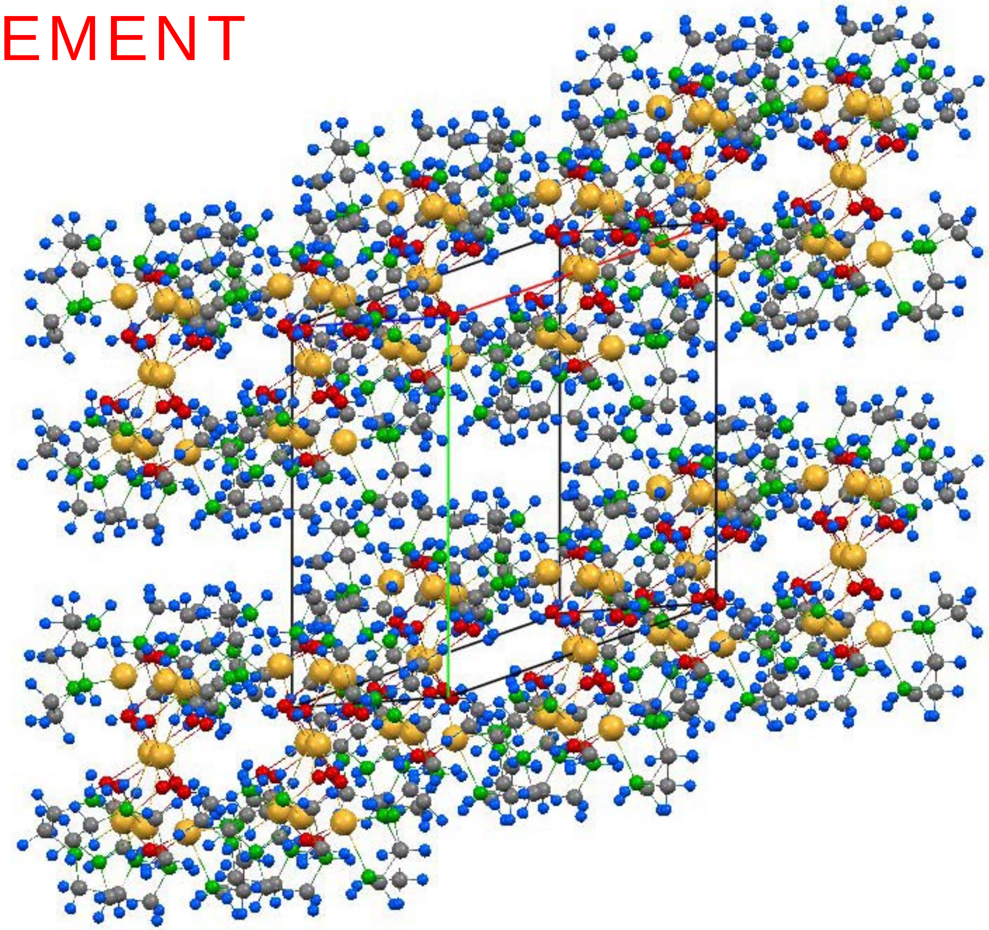


GEOMETRICAL ARRANGEMENT

We have:

(i) Intermolecular dipolar coupling
(ii) Hyperfine coupling
(iii) Spin-phonon coupling

Each causes decoherence



NB: this is not the only kind of process that can occur in a qubit network. In, eg., a ‘topological quantum computer’, One has non-local decoherence processes

IS Tupitsyn, A Kitaev, NV Prokof’ev, PCE Stamp
Phys Rev B82, 085114 (2010)

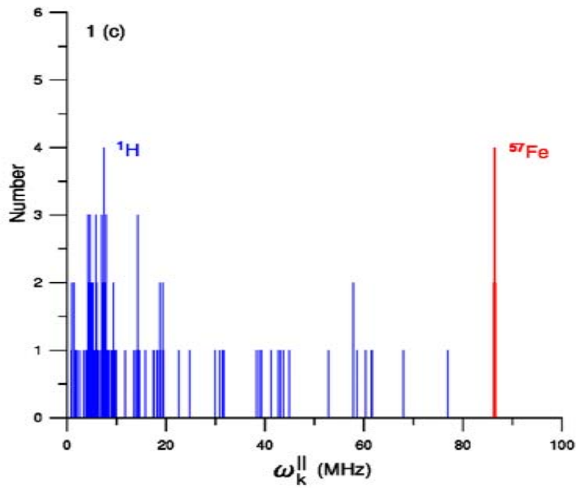
QUANTUM COHERENCE REGIME: quantitative predictions made long before experiments:

A Morello et al., PRL 97, 207206 (2006)

DECOHERENCE IN Fe-8 SYSTEM

(A) Nuclear Spin Bath

Hyperfine couplings of all 213 nuclear spins are well known



$$H_{eff}^{CS} = [\Delta_o \hat{\tau}_+ e^{-i\sum_k \boldsymbol{\alpha}_k \cdot \boldsymbol{\sigma}_k} + H.c.]$$

$$+ \hat{\tau}^z(\epsilon_o + \sum_k \boldsymbol{\omega}_k \cdot \boldsymbol{\sigma}_k) + H_{env}^{sp}([\boldsymbol{\sigma}_k])$$

Nuclear spin decoherence rate

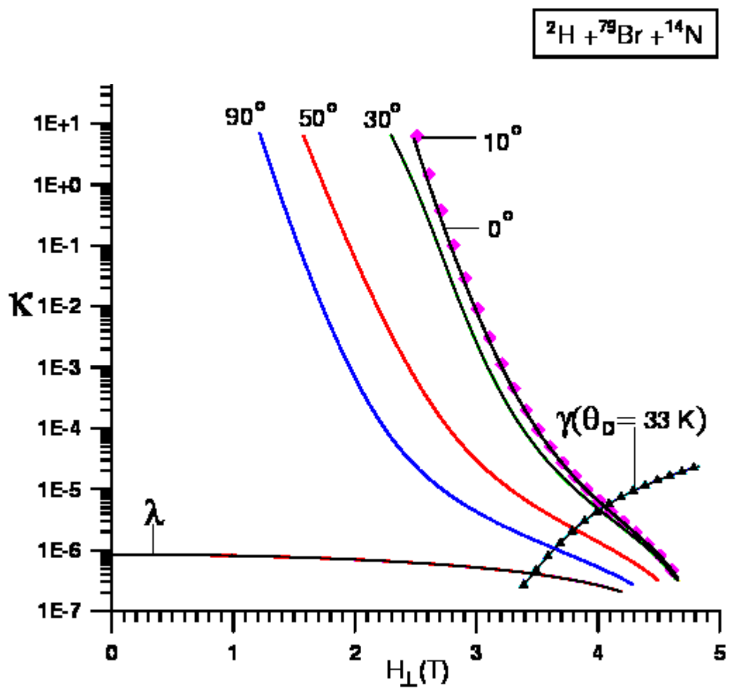
$$\gamma_\phi^{NS} = E_0^2/2\Delta_0^2 \quad \text{where} \quad E_o^2 = \sum_k \frac{I_k+1}{3I_k}(\omega_k^{\parallel} I_k)^2$$

(b) Phonon Bath

Phonon spectrum and spin-phonon couplings are known. Phonon decoherence rate is:

$$\gamma_\phi^{ph} = \frac{\mathcal{M}_{\mathcal{AS}}^2 \Delta_0^2}{\pi \rho c_s^5 \hbar^3} \coth\left(\frac{\Delta_0}{k_B T}\right)$$

$$\mathcal{M}_{\mathcal{AS}}^2(H_y) \approx \tfrac{4}{3} D^2 |\langle \mathcal{A}| S_y S_z + S_z S_y |\mathcal{S}\rangle|^2$$



Total SINGLE QUBIT decoherence rate shown in Figure at right:

(c) Dipolar Decoherence

This is an example of "correlated errors" caused by inter-qubit interactions. It turns out to be very serious.

The high-T (van Vleck) limiting form is

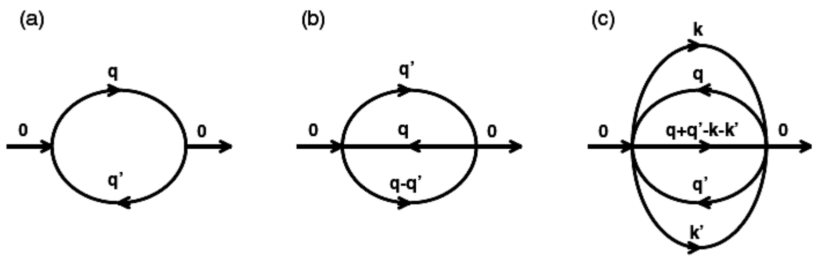
$$(\gamma_\phi^{\mathrm{vV}})^2 \approx \left[1 - \tanh^2\!\left(\frac{\Delta_0}{k_B T}\right)\right] \sum_{i \neq j} \left(\frac{\mathcal{A}_{yy}^{ij}}{\Delta_0}\right)^2,$$

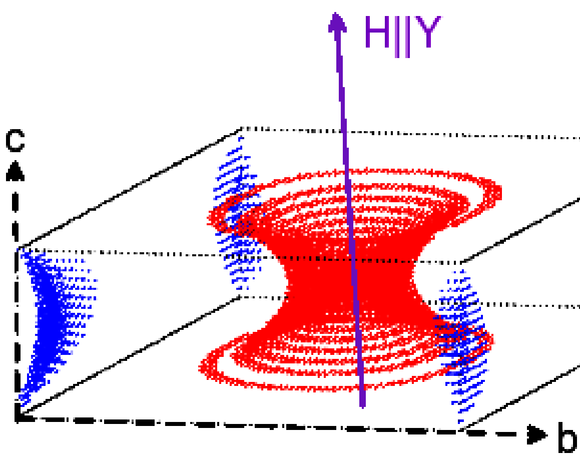
$$\mathcal{A}_{yy}^{ij} = \frac{U_d}{(2 g_e S)^2}\left[(2\tilde{g}_y^2 + \tilde{g}_z^2)\mathcal{R}_{yy}^{ij} - (\tilde{g}_x^2 - \tilde{g}_z^2)\mathcal{R}_{xx}^{ij}\right],$$

$$\mathcal{R}_{\mu\nu}^{ij} = \mathcal{V}_c(|\mathbf{r}^{ij}|^2 \delta_{\mu\nu} - 3 r_\mu^{ij} r_\nu^{ij})/|\mathbf{r}^{ij}|^5$$

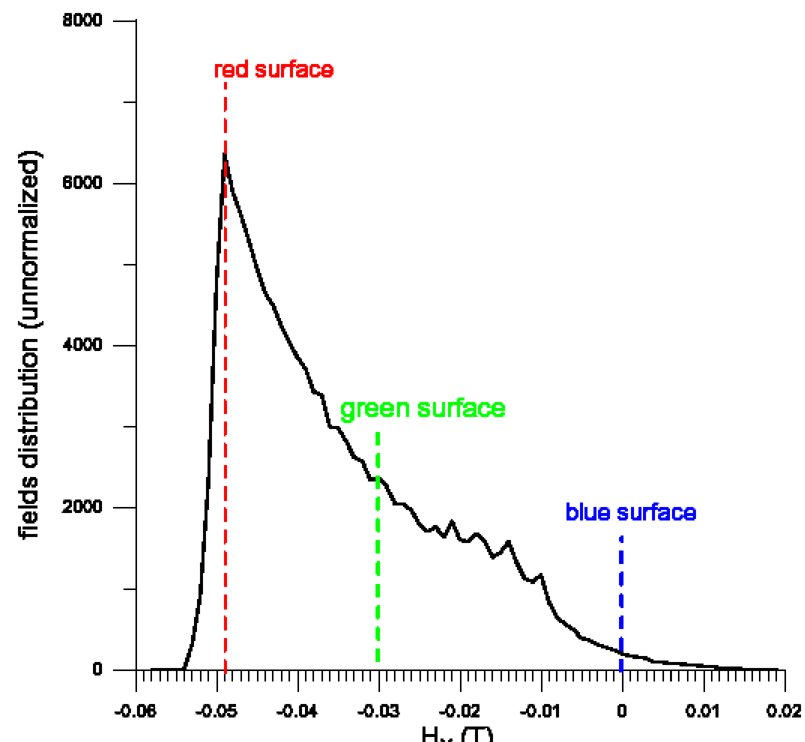
At low T one gets a quite different form

$$\gamma_\phi^m = \frac{2\pi}{\hbar\Lambda} \sum |\Gamma_{\mathbf{q}\mathbf{q}'}^{(4)}|^2 \mathcal{F}[\bar{n}_{\mathbf{q}}]\delta(\omega_0 + \omega_{\mathbf{q}} - \omega_{\mathbf{q}'} - \omega_{\mathbf{q}-\mathbf{q}'}).$$



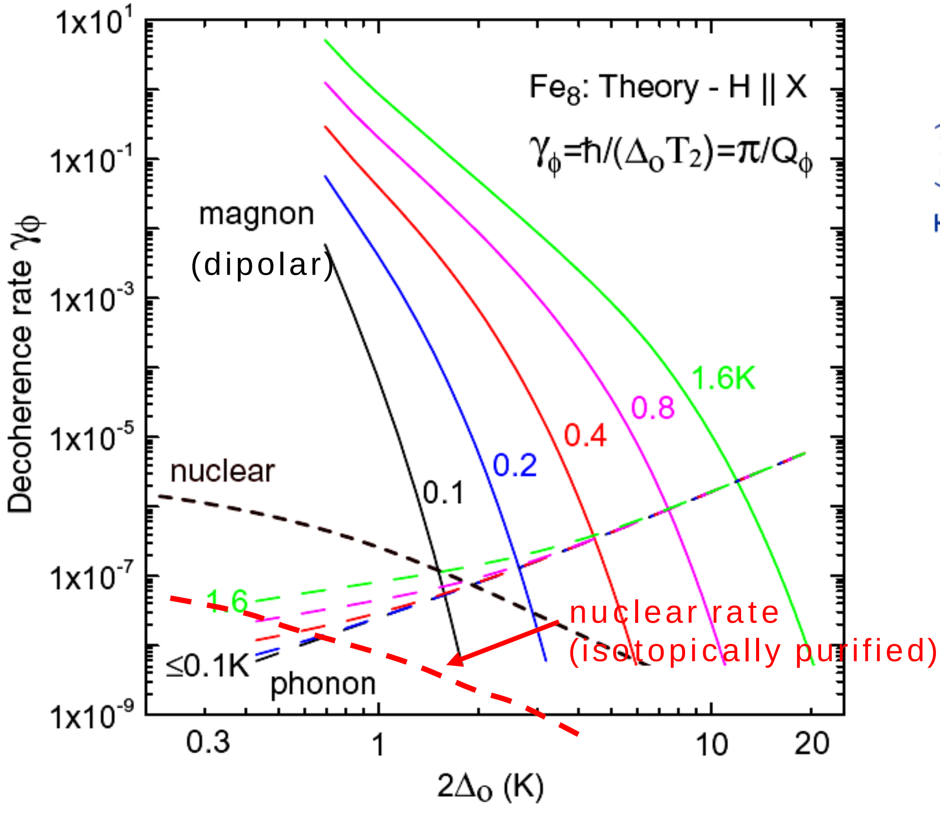


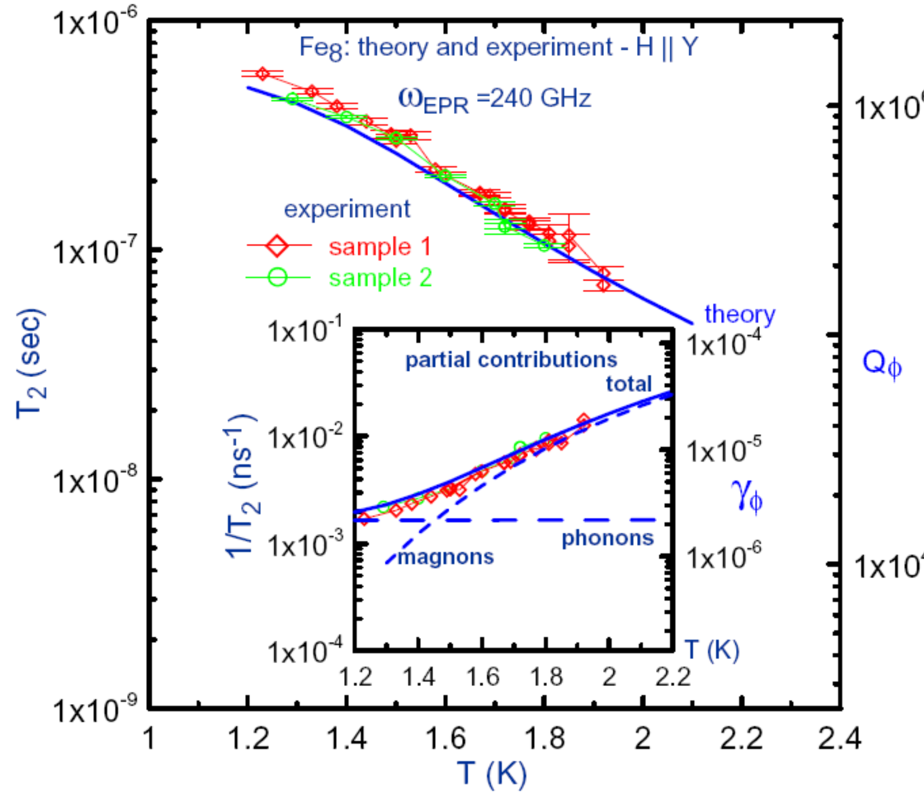
RESONANT SURFACES



THEORY vs. EXPERIMENT

Let's summarize the theoretical predictions, for an experiment finally performed in 2011:

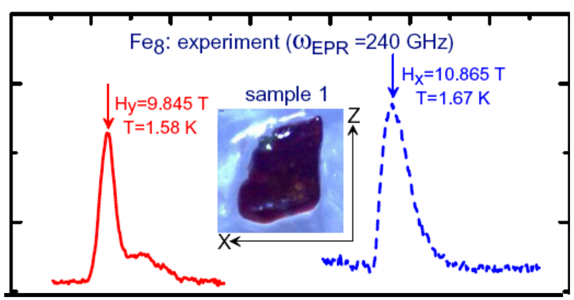




1. First detection of macroscopic spin precession of qubits
2. Lowest decoherence rate ever seen in molecular spins.
3. First measurement of dipole decoherence in qubit array
4. First controlled mmt of decoherence rates from spin bath, oscillator bath, & dipolar interactions (agreed with theory)

EXPERIMENT

Used high-field Hahn echo ESR at 240 GHz, on 2 different samples in various field orientations.



S Takahashi et al., Nature 476, 76 (2011)

EXAMPLE 2: DYNAMICS of a QUANTUM VORTEX

STANDARD "HVI" PHENOMENOLOGY: $M_v \ddot{\boldsymbol{r}}_v - \boldsymbol{f}_M - \boldsymbol{f}_{qp} - \boldsymbol{F}_{ac}(t) = 0$

with the Magnus force: $\boldsymbol{f}_M = \rho_s \boldsymbol{\kappa} \times (\dot{\mathbf{R}}_v - \boldsymbol{v}_s)$

+ quasiparticle force: $\boldsymbol{f}_{qp} = D_0(\boldsymbol{v}_n - \dot{\mathbf{R}}_v) + D_0' \hat{\mathbf{z}} \times (\boldsymbol{v}_n - \dot{\mathbf{R}}_v)$

$D_0'(T) = -\kappa \rho_n(T)$ (Iordanskii)

This gives MUTUAL FRICTION: $\boldsymbol{F}_{\mathrm{sn}} = -n_{\mathrm{L}} \kappa \rho_{\mathrm{s}} \alpha (\boldsymbol{v}_{\mathrm{s}} - \boldsymbol{v}_{\mathrm{n}}) + n_{\mathrm{L}} \kappa \rho_{\mathrm{s}} \alpha' [\hat{z} \times (\boldsymbol{v}_{\mathrm{s}} - \boldsymbol{v}_{\mathrm{n}})]$

where $\alpha = \dfrac{d_\parallel}{d_\parallel^2 + (1 - d_\perp)^2}$ $\quad 1 - \alpha' = \dfrac{1 - d_\perp}{d_\parallel^2 + (1 - d_\perp)^2}$ $\quad d_\perp = \dfrac{D'}{\kappa \rho_{\mathrm{s}}}$ $\quad d_\parallel = \dfrac{D}{\kappa \rho_{\mathrm{s}}}$

These phenomenological eqtns have been very controversial. They are applied to both superfluids and superconductors (one simply has to find the coefficients in each case).

H.E. Hall and W.F. Vinen, Proc. R. Soc. A **238**, 204 (1956).

S.V. Iordanskii, Ann. Phys. (N.Y.) **29**, 335 (1964); Sov. Phys. JETP **22**, 160 (1966).

D.J. Thouless, P. Ao, and Q. Niu, Phys. Rev. Lett. **76**, 3758 (1996).

E.B. Sonin, Phys. Rev. B **55**, 485 (1997).

D.J. Thouless and J.R. Anglin, Phys. Rev. Lett. **99**, 105301 (2007).

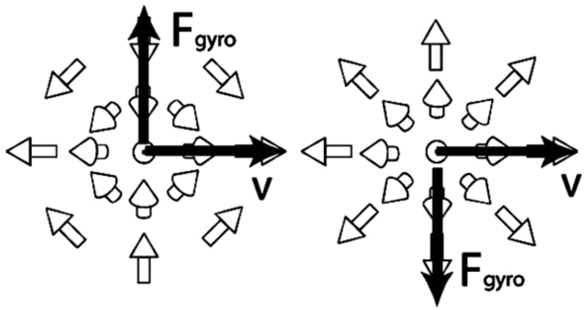
SPIN VORTEX DYNAMICS

One arrives at a similar classical equation of motion:

$$M_V \ddot{\mathbf{r}}_V(t) + \eta_m \dot{\mathbf{r}}_V(t) = \mathbf{F}_g - \nabla V_{bc}$$

The gyrotropic (Magnus) force is $\mathbf{F}_g = \dfrac{\pi S p q}{a^2} \hat{\mathbf{z}} \times \partial_t \mathrm{R}(t)$

and p, q, are the core polarization and winding number.



THESE EQUATIONS CANNOT BE RIGHT FOR A QUANTUM VORTEX !

VORTEX DYNAMICS: the PROBLEM

1) FULLY QUANTUM TREATMENT

Given an N-particle wave-fn $\Psi(\{\mathbf{r}_j\}) = \langle\{\mathbf{r}_j\}|\Psi\rangle$ (with $j = 1, 2, ...N$)

and density operator: $\hat{\boldsymbol{\rho}}_N = |\Psi\rangle\langle\Psi|$

Define the reduced density matrix
$$\bar{\boldsymbol{\rho}}(\boldsymbol{R}, \boldsymbol{R}', t) = \mathrm{Tr}_{\mathbf{q_k}}\, \boldsymbol{\rho}_N(\{\boldsymbol{R}, \mathbf{q}_k\}; \{\boldsymbol{R}', \mathbf{q}_k\})$$
$$= \prod_k \int d\mathbf{q}_k\, \boldsymbol{\rho}_N(\{\boldsymbol{R}, \mathbf{q}_k\}; \{\boldsymbol{R}', \mathbf{q}_k\})$$

$\{\mathbf{q}_k\}$: the N-1 collective coordinates adapted to the position R of the vortex node

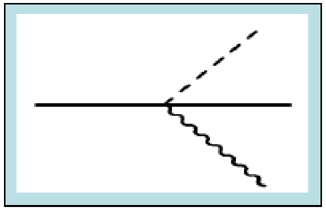
Propagator of density matrix:
$$K(2,1) = \int_{\boldsymbol{R}_1}^{\boldsymbol{R}_2} \mathcal{D}\boldsymbol{R}(t) \int_{\boldsymbol{R'}_1}^{\boldsymbol{R'}_2} \mathcal{D}\boldsymbol{R}'(t) e^{\frac{i}{\hbar}(\tilde{S}_V^0[\boldsymbol{R}] - \tilde{S}_V^0[\boldsymbol{R}'])} \mathcal{F}[\boldsymbol{R}(t), \boldsymbol{R}'(t)]$$
(Non-Perturbative)

where
$$\mathcal{S}_V^0 = \frac{1}{2}\int dt \left[\rho_s(\boldsymbol{\kappa} \times (\dot{\boldsymbol{R}} - \boldsymbol{v}_s)) \cdot \boldsymbol{R} + M_v^0 \dot{R}^2\right]$$
is a bare vortex action

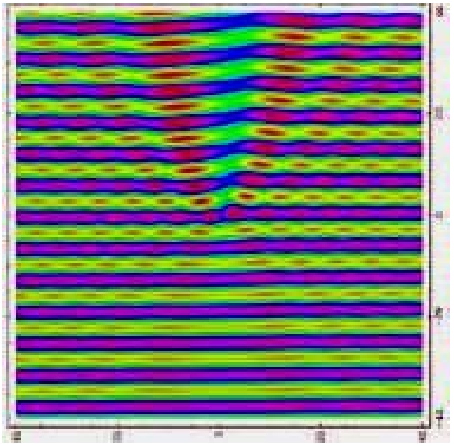
2) SOLITONIC FIELD THEORY

The 2nd subtlety – the vortex soliton interacts not with bare quasiparticles, but with 'renormalized' modes orthogonal to the soliton. These modes are quite different from the original quasiparticles. The scattering matrix element is IR divergent:

$$\Lambda_{kq}^{\sigma 0} = \frac{k+q}{4\sqrt{kq}}\delta(k-q) + \frac{a_0}{4}\begin{cases} \frac{\sigma}{2}\left(\frac{k}{q}\right)^{\frac{3}{2}} + \sigma\sqrt{kq}\frac{k}{q(k-q)} & \text{if } k < q \\ \frac{k+q}{16\sqrt{kq}}a_0 q + \frac{\sigma}{2}\left(\frac{q}{k}\right)^{\frac{1}{2}} + \sigma\sqrt{kq}\frac{q}{k(k-q)} & \text{if } q \le k \end{cases}$$



This is a very peculiar field theory, which has to be solved non-perturbatively



Renormalized quasiparticles

SOLUTION TO THE PROBLEM

L Thompson, PCE Stamp, PRL 108, 184501 (2012)

Let's first Fourier transform: $R_v^i(\Omega) = A^{ij}(\Omega, n_{\boldsymbol{q}}) F^j(\Omega)$

HVI eqtns become:

driving force: $\mathbf{F} = \mathbf{F}_{ac}(\Omega) - q_v \boldsymbol{\kappa} \times \mathbf{J}(\Omega)$ where $\mathbf{J} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$

"admittance" matrix: $\boldsymbol{A}_o(\Omega) = \dfrac{1}{\mathbb{D}_o(\Omega)} \begin{pmatrix} -\Omega^2 M_v + i\Omega D_o & -i\kappa\rho\Omega \\ i\kappa\rho\Omega & -\Omega^2 M_v + i\Omega D_o \end{pmatrix}$

CORRECT FULL EQUATION of MOTION

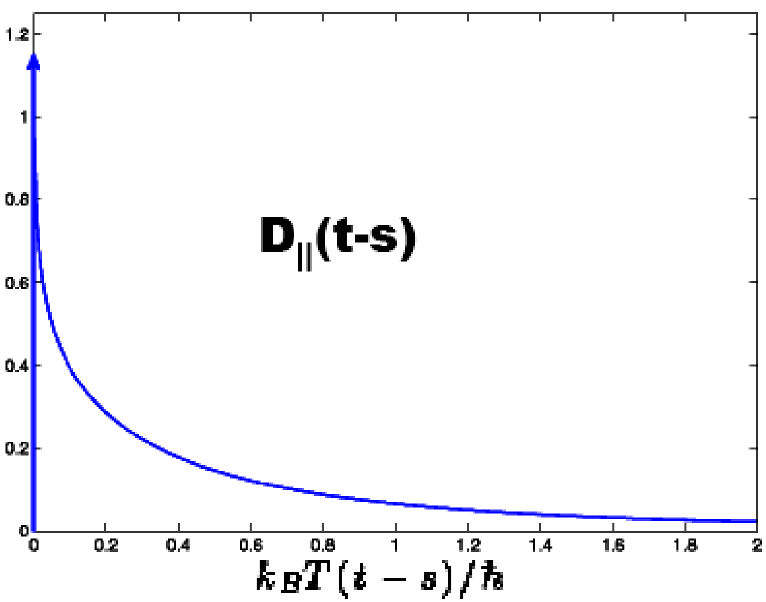
$$\boldsymbol{A}^R(\Omega, [n_{\boldsymbol{q}}]) = \frac{1}{\mathbb{D}_R(\Omega)} \begin{pmatrix} -\Omega^2 M_v(\Omega) + i\Omega D_{\parallel}(\Omega) & -i\kappa\rho\Omega - |\Omega| d_{\perp}(\Omega) \\ i\kappa\rho\Omega - |\Omega| d_{\perp}(\Omega) & -\Omega^2 M_v(\Omega) + i\Omega D_{\parallel}(\Omega) \end{pmatrix}$$

new driving force: $\mathbf{F} = [\mathbf{F}_{ac}(\Omega) + \mathbf{f}_{\perp} + \boldsymbol{F}_{fl}(\Omega, n_{\boldsymbol{q}})]$

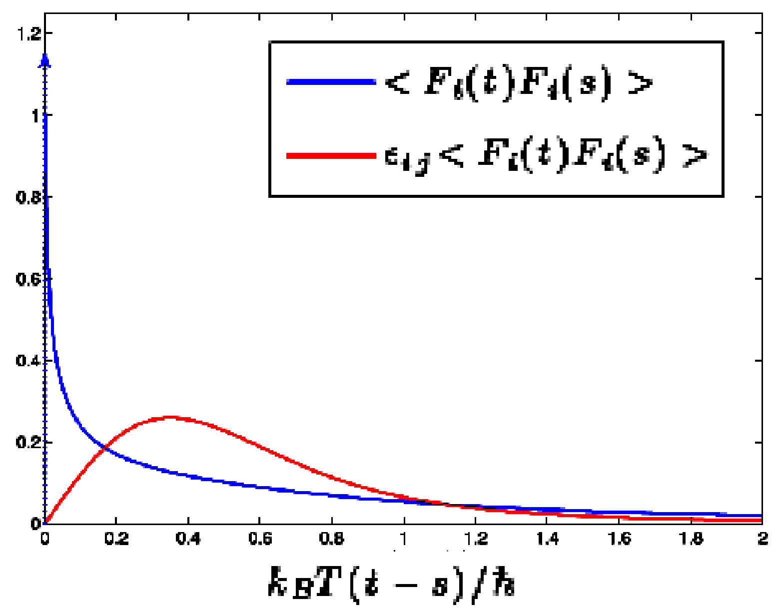
Quantum Regime:

$\boxed{\tilde{\Omega} \gg 1}$

$\tilde{\Omega} = \hbar\Omega / k_B T$



Longitudinal Drag (non-local in time)



Noise correlator (very non-Markovian)

Quantum Vortex in 2D Easy-plane Ferromagnet

Hamiltonia $\mathcal{H} = -\frac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \sum_i \sum_{a,b=x,y,z} K_{ab}S_{ia}S_{jb} - \frac{\mu_0\gamma^2}{4\pi}\sum_{i,j}\frac{3(\mathbf{S}_i\cdot\hat{\mathbf{e}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{e}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j}{r_{ij}^3}$

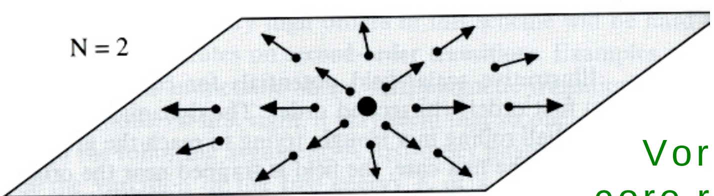
$\longrightarrow$ **Continuum** $\frac{L_z}{M_s^2}\int d^2r A(\nabla\mathbf{M})^2 + \frac{\mu_0}{4\pi}\int d^3r\, \mathbf{M}(\mathbf{r})\cdot\nabla\int d^3r' \frac{\mathbf{M}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$

The action is: $\mathcal{S} = \omega_B - \int dt\mathcal{H}$ where $\omega_B = -S\int\frac{d^2r}{a^2}\cos\theta\dot{\phi}$ **(Berry**

The vortex is a 'skyrmion', with profile:

$$\phi_v(\mathbf{r}) = q\xi + \delta$$

$$\cos\theta_v(\mathbf{r}) = p\begin{cases} 1 - c_1\left(\frac{r}{r_v}\right)^2, & r\to 0;\\ c_2\sqrt{\frac{r_v}{r}}\exp(-\frac{r}{r_v}), & r\to\infty.\end{cases}$$



Vortex core radius $r_v^2 = J/2K$

BARE MAGNON SPECTRUM

$\omega = ckQ$ with Spin Wave velocity: $c = SJa^2$

$Q^2 = k^2 + k_o^2$ with $k_o^2 = 2K/J$

VORTEX-MAGNON INTERACTION

Basic arguments go through as before.

Interaction action: $\tilde{S}_2^{int} = \frac{L_z\mu_0 M_s}{\gamma}\int dt\int d^2r\,(\eta\nabla\phi\cdot\dot{\mathbf{r}}_V)$

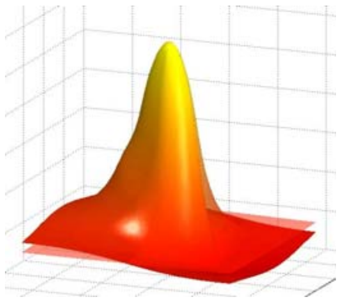
Magnon eqtns of motion:

$$\frac{\sin\Theta_V}{c_m a_m}\frac{\partial\phi}{\partial t} = \nabla^2\eta + \cos 2\Theta_V\left(\frac{1}{a_m^2} - (\nabla\Phi_V)^2\right)\eta - \sin 2\Theta_V\nabla\phi\cdot\nabla\Phi_V$$

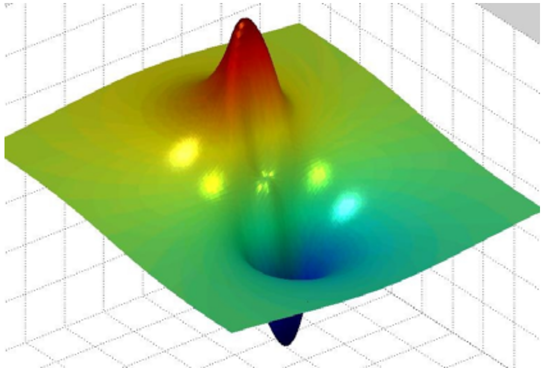
$$\frac{1}{c_m a_m}\frac{\partial\eta}{\partial t} = -\sin\Theta_V\nabla^2\phi - 2\cos\Theta_V\left(\nabla\Theta_V\cdot\nabla\phi + \nabla\eta\cdot\nabla\Phi_V\right)$$

MAGNETIC VORTICES
INTERACTION WITH MAGNONS

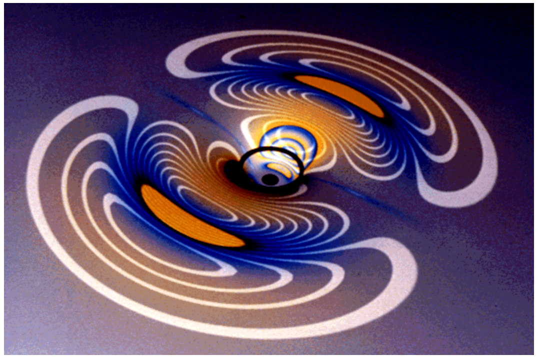
The results can be physically interpreted in terms of interaction between travelling spinning gyroscope (the magnon) and a massive spinning string (the vortex).



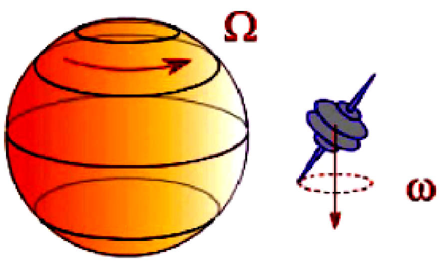
Mz (moving Vortex)



Metric Curvature (moving vortex)



Metric curvature (static vortex)



The deSitter (geodesic) and Lense-Thirring effects cancel – there is no Iordanski force!

EXAMPLE: PERMALLOY

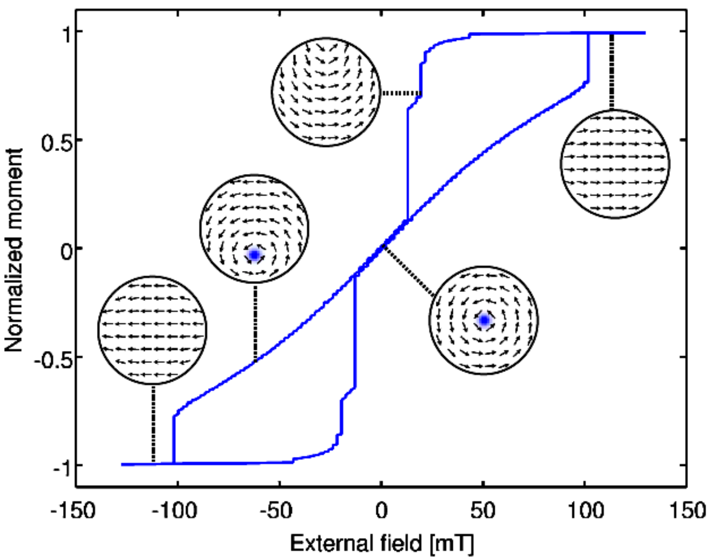
We have

$$\begin{aligned} M_s &= 8.6 \times 10^5 \text{ A/m} \\ A &= 1.3 \times 10^{-11} \text{ J/m} \\ \gamma &= 2.2 \times 10^5 \text{ m/As} \end{aligned}$$

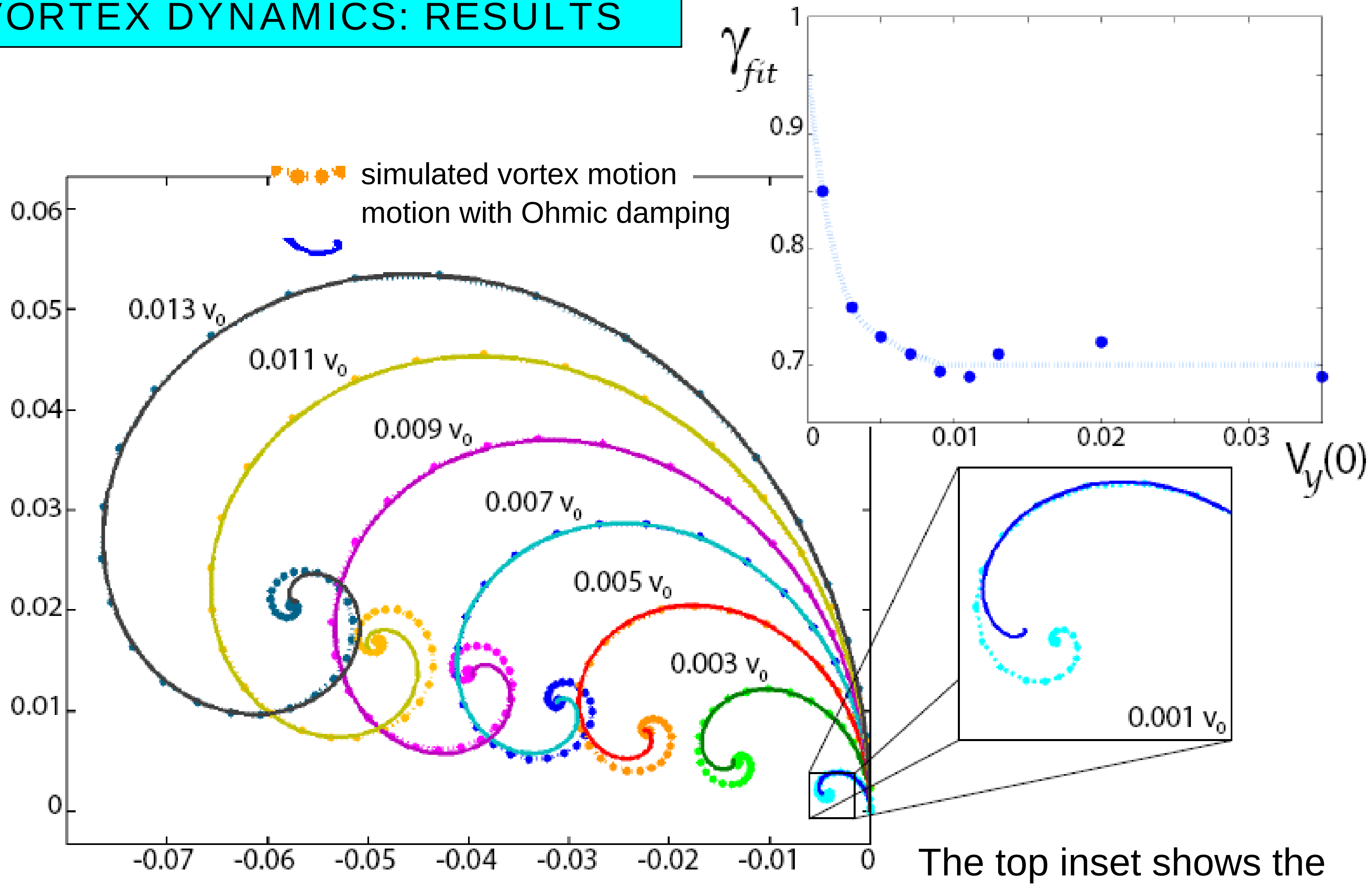
RIGHT: high-T experiments. Low-T regime is below 8K

Key parameters: $c_m = \dfrac{2A}{a_m \mu_0 M_s / \gamma}$ get $c_m = 500 \text{ m/s}$

$a_m = \sqrt{\dfrac{2A}{\mu_0 M_s^2}}$ get $a_m = 5.3 \text{ nm}$



VORTEX DYNAMICS: RESULTS



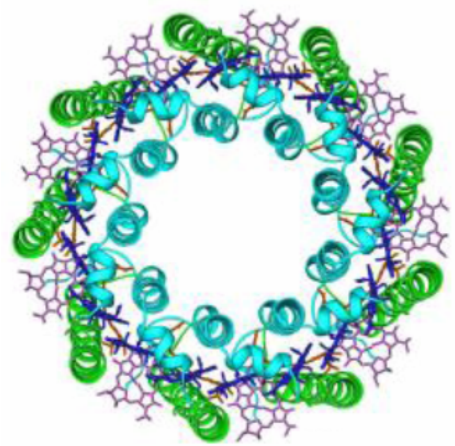
If we set the vortex into motion with a δ-kick, we find decaying spiral motion dependent on the initial vortex speed (shown in fractions of $v_0 = c/r_v$)

The top inset shows the necessary γ of Ohmic damping to fit full simulated motion. *Note the strong upturn at low speeds!*

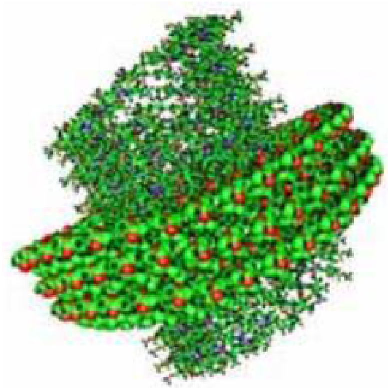
EXAMPLE 2: COHERENCE/DECOHERENCE in BIOMOLECULES

LIGHT-HARVESTING MOLECULES

LH2 (purple bacteria)

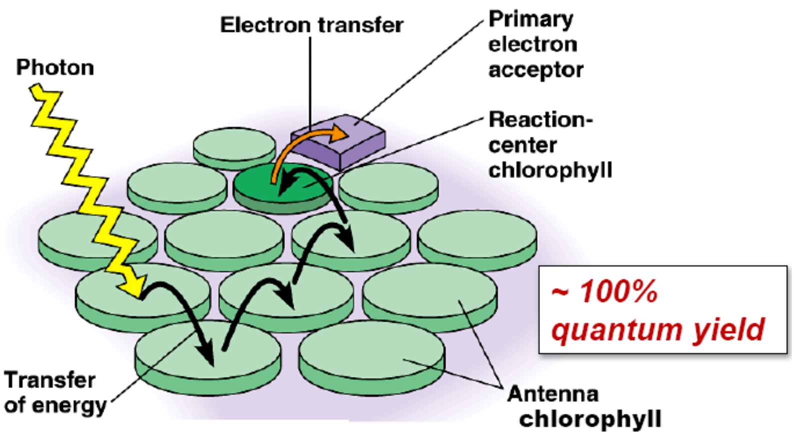


chlorosome (green sulfur bacteria)



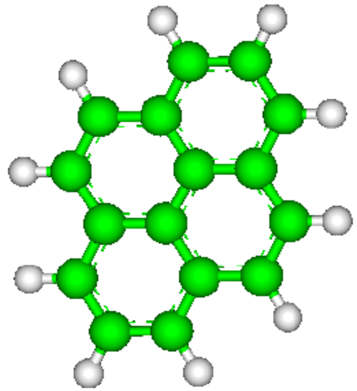
Energy is transported around RINGS by excitons, created by low-intensity (but high energy) sunlight. Quantum yield is ~ 98%. Old model of incoherent dynamics is wrong.

Need new models & results.

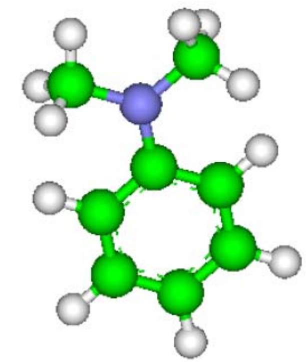


CHEMICAL COMPASSES

pyrene (Py-h_{10})



N,N-dimethylaniline (DMA-h_{11})



Light absorption creates spin pair of entangled radicals – one or both move around rings. Their recombination controls chemical reaction in eye → navigation. (Spin dynamics controlled by nuclear spins + Earth's field).

Need proper understanding of dynamics – which is controlled by nuclear spin bath

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E Collini et al, Nature 463, 644 (2010)
GS Engel et al, Nature 446, 782 (2007)
H Lee et al, Science 316, 1462 (2007)

K Maeda et al, Nature 453, 387 (2008)
CT Rodgers, PJ Hore, PNAS 106, 353 (2009)

DECOHERENCE of CHARGE CARRIERS in BIOMOLECULES

Charge carriers in biomolecules: Polarons & Excitons. Dynamics is poorly understood. Even simple polaron has surprises. Consider non-diagonal coupling to phonons:

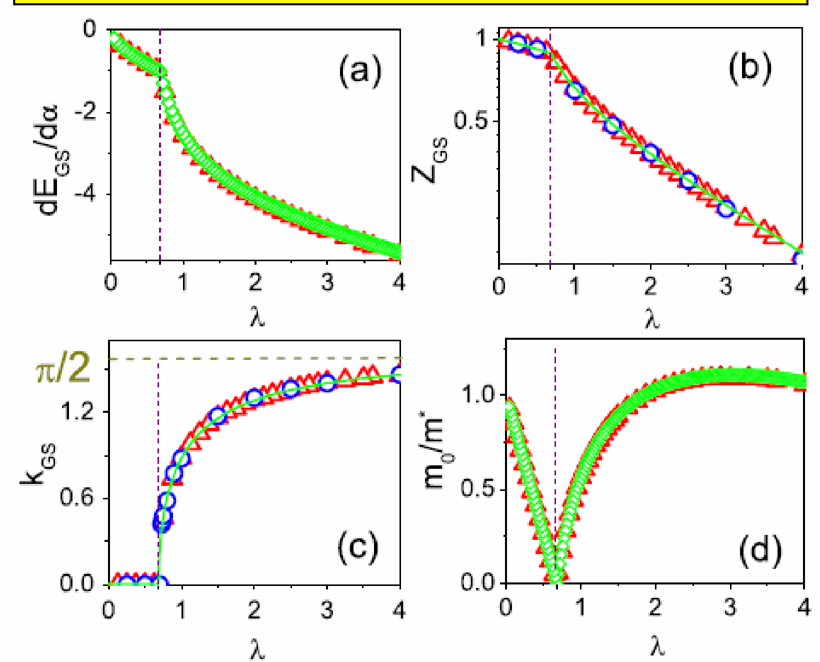
$$\left.\begin{aligned} V &= -\tilde{\alpha} t_0 \sum_i (\hat{X}_i - \hat{X}_{i+1})(c_i^\dagger c_{i+1} + \text{H.c.}) \\ &= N^{-1/2} \sum_{k,q} M(k,q) c_{k+q}^\dagger c_k (b_{-q}^\dagger + b_q), \end{aligned}\right\}$$

Dimensionless coupling $\lambda = 2\alpha^2/(t_0 \omega_{\text{ph}})$

The effect of this is dramatic – led to our discovery of sharp 'critical transition' in dynamics (no analogue for diagonal couplings).

- Important for biomolecules

DJ Marchand et al., PRL 105, 266605 (2010)



MODEL of CHARGE DYNAMICS

Polarons/excitons interact with delocalized & localized phonons. Key model:

$$H = H_{band} + H_{SB}$$

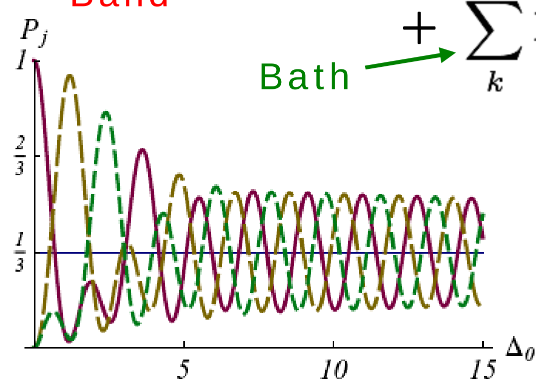
$$= \sum_{ij} [t_{ij} c_i^\dagger c_j e^{iA_{ij}^o + i\sum_k (\boldsymbol{\phi}_k^{ij} + \boldsymbol{\alpha}_k^{ij} \cdot \boldsymbol{\sigma}_k)} + H.c.]$$

$$+ \sum_j (\varepsilon_j + \sum_k \boldsymbol{\gamma}_k^j \cdot \boldsymbol{\sigma}_k) c_j^\dagger c_j$$

Band

$$+ \sum_k \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$

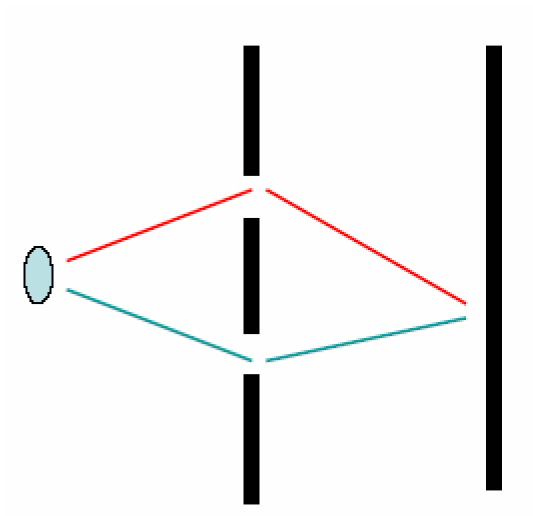
Bath



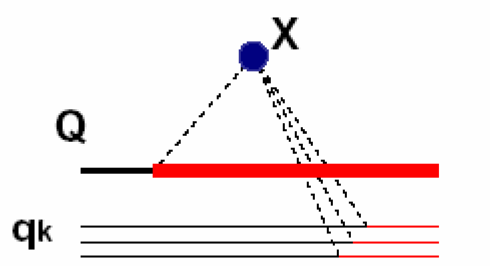
Very interesting dynamics!

Z Zhu, A Aharony, O Entin, PCE Stamp, PR A81, 062127 (2010)

3rd PARTY DECOHERENCE:



This is decoherence in the dynamics of a system A (with coordinate **Q**) caused by *indirect* entanglement with an environment E- the entanglement is achieved via a 3rd party B (coordinate **X**).



Ex: Buckyball decoherence

Consider the 2-slit expt with buckyballs. The COM coordinate Q of the buckyball does not couple directly to the vibrational modes {qk } of the buckyball - by definition. However BOTH couple to the slits in the system, in a distinguishable way.

Note: the state of the 2 slits, described by a coordinate X, is irrelevant- it does not need to change at all. We can think of it as a scattering potential, caused by a system with infinite mass. It is a PASSIVE 3rd party. We can also have ACTIVE 3rd parties

PCE Stamp, Stud. Hist Phil Mod Phys **37**, 467 (2006)

See also PCE Stamp, WG Unruh, in preparation

PART 3:

INTRINSIC DECOHERENCE

INTRINSIC DECOHERENCE

This has nothing to do with environments at all – IT AMOUNTS TO A BREAKDOWN OF QUANTUM MECHANICS.

Such possibilities have been advocated for reasons, notably because they purport to

(i) Solve the Quantum Mmt problem

(ii) Reconcile Quantum Mechanics and Gravity

NON-LINEAR SCHRODINGER EQTN

Lots of different suggestions here (Pearle, Ghirardi et al, Milburn, Diosi, etc..
One of the more precise suggestions results in

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\tau\hbar^{-2}[H,[H,\rho]]$$

So far only definitive tests of Weinberg’s 1989 suggestion – QM won out

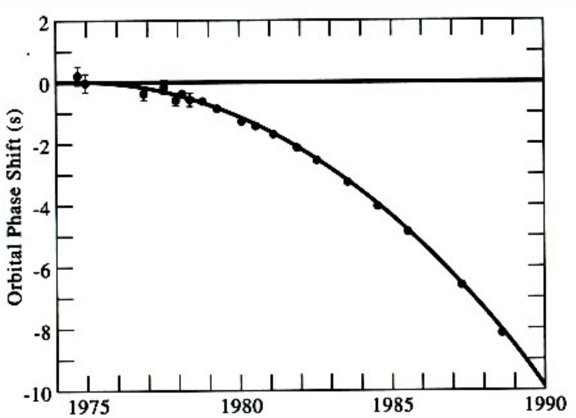
GRAVITATIONAL DECOHERENCE

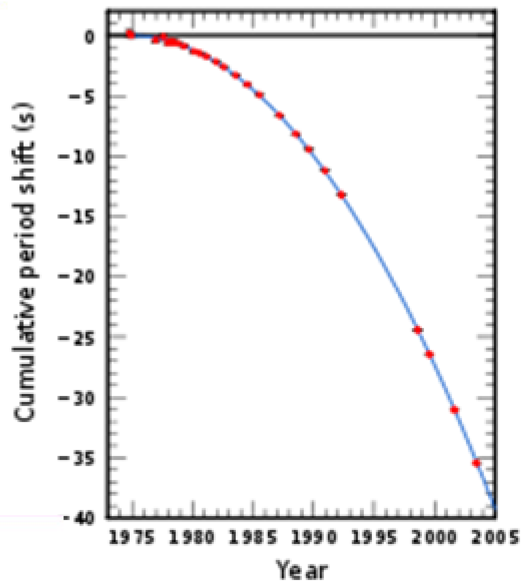
(i) Decoherence from quantum fluctuations in Spacetime (Hawking, Ellis, ..)
(ii) Decoherence from implementation of the ‘Holographic principle (‘t Hooft)
(iii) Decoherence from uncertainty in quantum time (Diosi, Penrose,...)

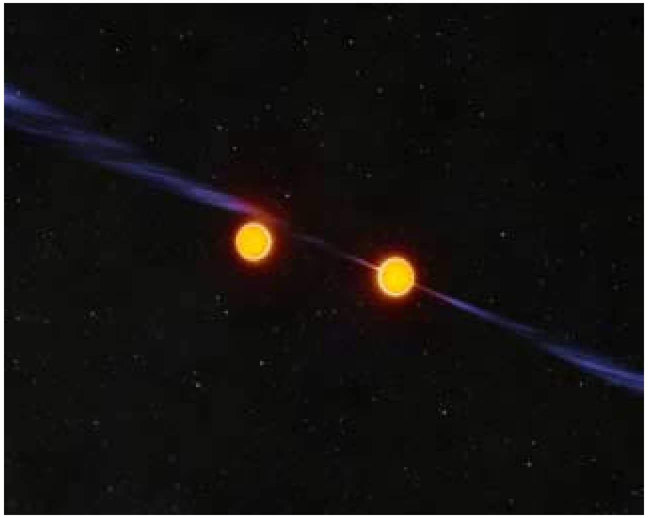
We will look at option (iii).

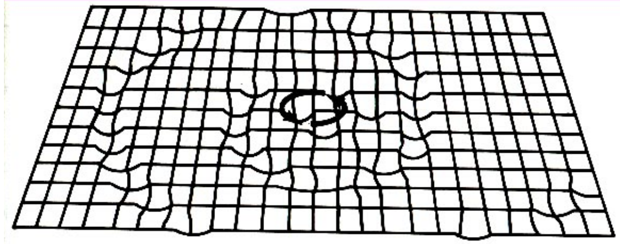
SOME TESTS of Gen Relativity

BINARY PULSAR

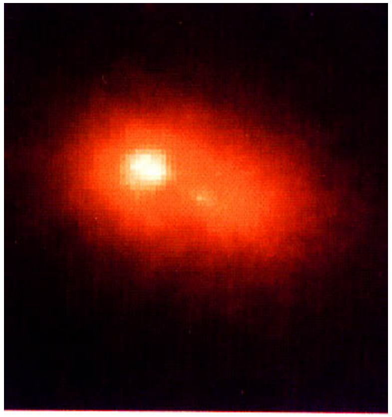








MASSIVE BLACK HOLES

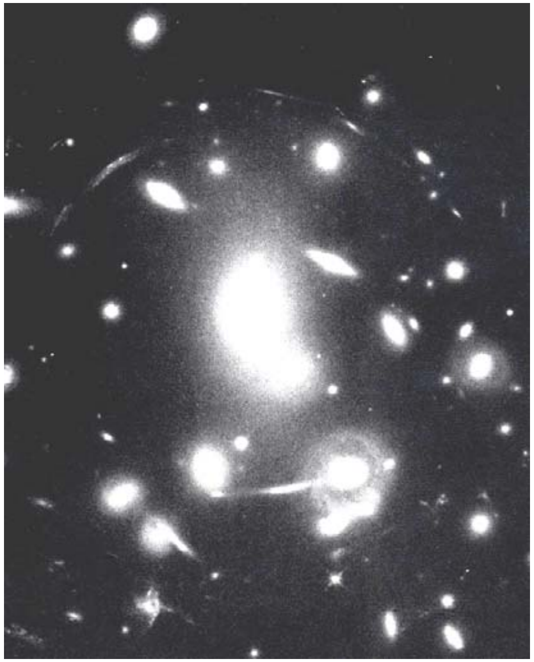


40 light years

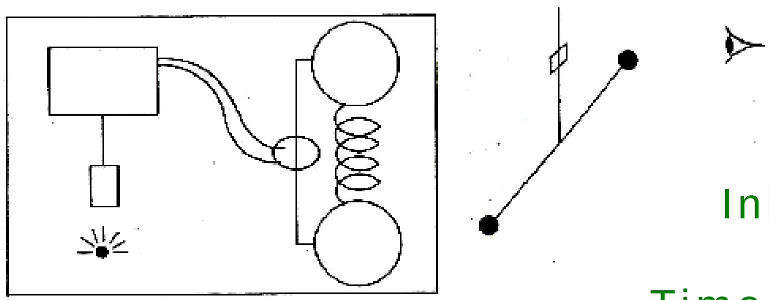
Now in excess of 10^7 solar masses

GRAVITATIONAL LENSING

Once a curiousity, these lenses have become an essential tool in observational cosmology



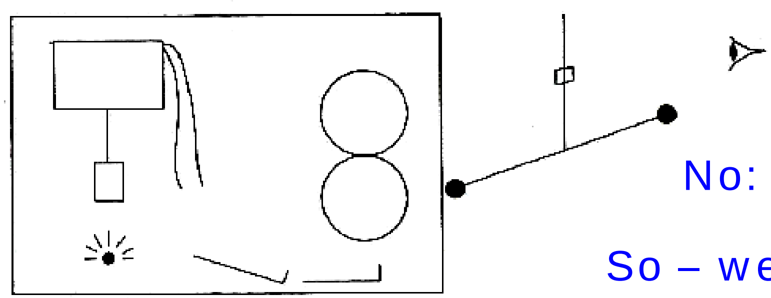
GRAVITATIONAL DECOHERENCE



There are v good arguments to show we need to quantize gravity. Consider superposition at left

Initial state: $\langle T_{\mu\nu} \rangle_0 = \langle \updownarrow | T_{\mu\nu} | \updownarrow \rangle$

Time evolution: $|\phi\rangle = \alpha(t)|\updownarrow\rangle + \beta(t)|\not\updownarrow\rangle$



Suppose now we perform a measurement – does the energy-momentum tensor ‘collapse’?

No: causes discontinuity, violating $\langle T_\mu{}^\nu \rangle_{;\nu} = 0$

So – we cannot write $G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$

Rough argument for the existence of an ‘intrinsic gravitational decoherence’:

$$E_{i,j} = -G \int\int \mathrm{d}\vec{r}_1 \mathrm{d}\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

Gravitational interaction energy:

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}$$

Energy Uncertainty

L Diosi, Phys Rev A40, 1165 (1989)
R Penrose, “Shadows of the Mind” (OUP, 1994)

W. Marshall, C. Simon, R. Penrose, D. Bouwmeester: PRL 91, 130401 (2003).

Then there will be a timescale of loss of phase coherence, given by Time-energy uncertainty relation.

INTRINSIC DECOHERENCE: a THEORETICAL FORMULATION

PCE Stamp, Phil Trans Roy Soc A (in press); arXiv 1205.5307

“A theory is not a theory until it produces a number”

R.P. Feynman (Lectures on Physics, 1965)

Path integral QM: $G_0(2,1) = G_0(\mathbf{r}_2\mathbf{r}_1; t_2, t_1) = \int_1^2 \mathcal{D}\mathbf{r}\; e^{\frac{i}{\hbar}\int_1^2 dt\, L(\mathbf{r},\dot{\mathbf{r}};t)}$

Let’s now introduce a modification of QM, as follows:

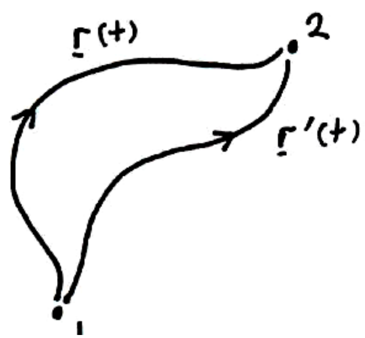
Let: $\mathcal{G}(2,1) = G_0(2,1) + \Delta\mathcal{G}_0(2,1)$

where $\Delta\mathcal{G}_0(2,1) = \frac{1}{2}\int_1^2 \mathcal{D}\mathbf{r}\int_1^2 \mathcal{D}\mathbf{r}'\; K_0(\mathbf{r},\mathbf{r}';t)\; e^{\frac{i}{\hbar}\left(S_0[\mathbf{r}] + S_0[\mathbf{r}']\right)}$

For gravitational decoherence, we introduce

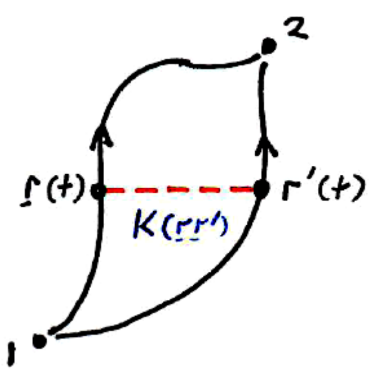
$$\kappa[\mathbf{r},\mathbf{r}'] = \exp\int^t d\tau\, \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} - 1$$

The picture is then:



Standard QM

Modification to QM



The effect of this term on the propagator is just a renormalization; eg., for a free particle we have:

$$\Delta\mathcal{G}(X,X') \propto \int \mathcal{D}\mathbf{x}_1(\tau) \int \mathcal{D}\mathbf{x}_2(\tau)\, \kappa[\mathbf{x}_1,\mathbf{x}_2] \exp \frac{i}{2\hbar} \int d\tau \frac{m}{2}(\dot{\mathbf{x}}_1^2 + \dot{\mathbf{x}}_2^2)$$

$$\propto \mathcal{A}(0,0;t,t')G_o(X,X')$$

containing a 'Coulomb' renormalization.

However the effect on the density matrix evolution is more profound. Writing $\rho(2) = \int d1 \mathcal{K}(2,1)\rho(1)$

for the density matrix propagation, we now have

$$\mathcal{K}(X,Y;X'Y') = \bar{K}(X,Y;X'Y') + \Delta\mathcal{K}(X,Y;X'Y')$$

with(in the case of gravitational decoherence) a correction:

$$\Delta\mathcal{K}(X,Y;X'Y') \sim \int_{\mathbf{X}'}^{\mathbf{X}} \mathcal{D}\mathbf{x}(\tau) \int_{\mathbf{Y}'}^{\mathbf{Y}} \mathcal{D}\mathbf{y}(\tau) \left[\exp \int d\tau \frac{8i\pi G m^2}{|\mathbf{x}(\tau)-\mathbf{y}(\tau)|} - 1\right] \times \exp \frac{i}{2\hbar} \int d\tau \frac{m}{2}(\dot{\mathbf{x}}^2 - \dot{\mathbf{y}}^2)$$

and this causes intrinsic decoherence

All of this is for a single particle. We also, if we want to compare with experiments, do this for a larger body. How do we do this?

SLOW & FAST VARIABLES in this THEORY

The easiest way to treat real experiments in this formalism is to make a Born-Oppenheimer separation between slow and fast variables. Recall that in ordinary QM we have

$$G_o(2,1) = \int_1^2 \mathcal{D}\mathbf{R}\, e^{\frac{i}{\hbar}\int dt L_o(\mathbf{R},t)} G_o^f(2,1)$$

where the fast propagator is

$$G_o^f(2,1) \;=\; G_o^f(\{x_k^{(2)}, x_k^{(1)}\}; t_2, t_1 | [\mathbf{R}(t)])$$

If we now integrate out the fast variables, we get an effective propagator for the slow variables which contains extra terms. Instead of just the slow Lagrangian, conventional QM gives

$$L_o(\mathbf{R}) - \epsilon_n(\mathbf{R}) - i\hbar\dot{\mathbf{R}} \cdot \langle n | \nabla_{\mathbf{R}} m \rangle$$

and then the extra gravitational term gives a contribution:

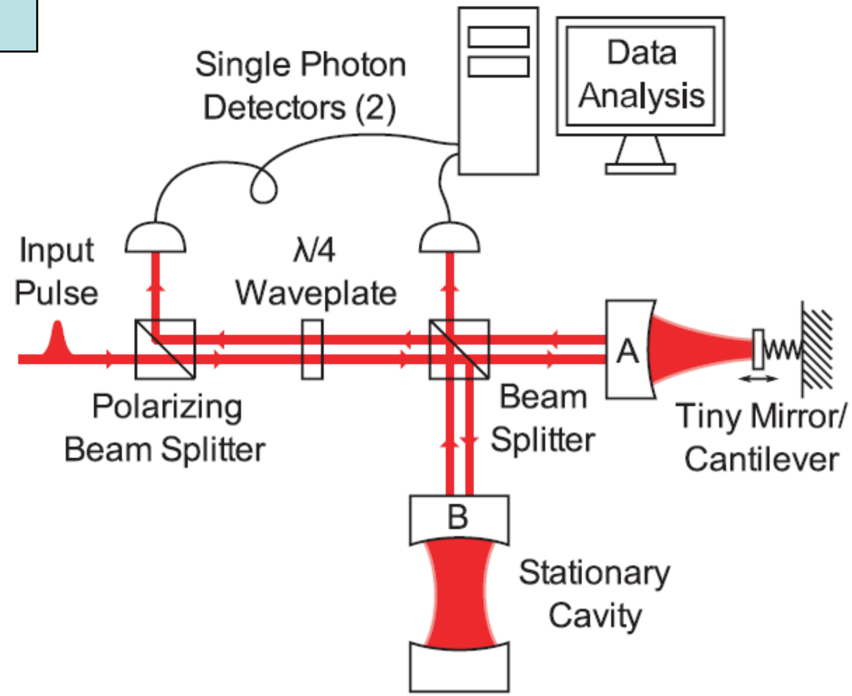
$$\Phi_{nm}[\mathbf{R}, \mathbf{R}'] \;=\; \int dt\, \dot{\mathbf{R}} \cdot \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | m(\mathbf{R}) \rangle\, \chi_{mn}[\mathbf{R}, \mathbf{R}']\, \langle n(\mathbf{R}') | \nabla_{\mathbf{R}'} | m(\mathbf{R}') \rangle \cdot \dot{\mathbf{R}}'$$

where $\chi_{mn}[\mathbf{R}, \mathbf{R}'] \;=\; |m(\mathbf{R})\rangle\, \exp \dfrac{4\pi i G m^2}{|\mathbf{R}(\tau) - \mathbf{R}'(\tau)|}\, \langle n(\mathbf{R}')|$

So – now we need an experiment to look at!

BOUWMEESTER EXPERIMENT: DETAILS

If we ignore environmental decoherence in this experiment, we have a system in which we have a photon in a superposition of cavity A and cavity B states, with an entanglement to a cantilever vibrational mode C, via the small mirror M on C. The Hamiltonian is taken to be



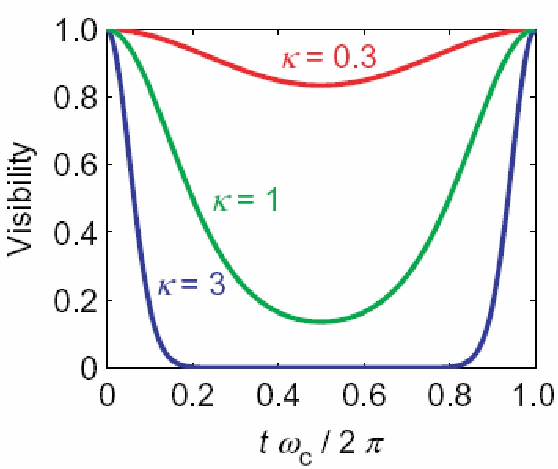
$$H = \hbar\omega_a \left[a^\dagger a + b^\dagger b\right] + \hbar\omega_c \left[c^\dagger c - \kappa a^\dagger a \left(c + c^\dagger\right)\right]$$

where $$\kappa = \frac{\omega_a}{L\omega_c}\sqrt{\frac{\hbar}{2m\omega_c}} = \frac{\sqrt{2}Nx_0}{\lambda}$$

The if at *t = 0* we are in the state $|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$

the system evolves to time *t* to the state:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_C + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B\,|\kappa(1 - e^{-i\omega_m t})\rangle_C],$$



with off-diagonal matrix element $v(t) = \mathrm{e}^{-\kappa^2(1-\cos(\omega_c t))}$

Environmental decoherence...almost solved

D Kleckner et al., N J Phys 10, 095020 (2008)

Intrinsic Decoherence ... stay tuned.....

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A Morello (UNSW)
S Takahashi (UCSB/USC)