



*Pacific Institute of Theoretical Physics, July, 2007*



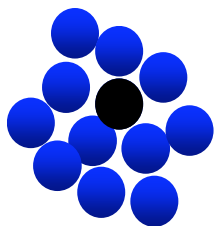
# Glassy Dynamics & Mechanical Properties of Quiescent and Stressed Particle Suspensions

**Kenneth S. Schweizer**

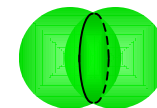
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University of Illinois @ Urbana-Champaign*

*Collaborators : Erica Saltzman, Vladimir Kobelev, Galina Yatsenko*

**GOALS** : “Simple”, Microscopic, Predictive Dynamic Theory  
Beyond MCT : Activated Barrier Hopping + Nongaussian Dynamic Fluctuations  
Nonlinear Viscoelasticity, NONspherical Object  
*HARD SPHERES : Quantitative confrontation with Experiment & Simulation*



*Quiescent* → **Driven (deformed)**

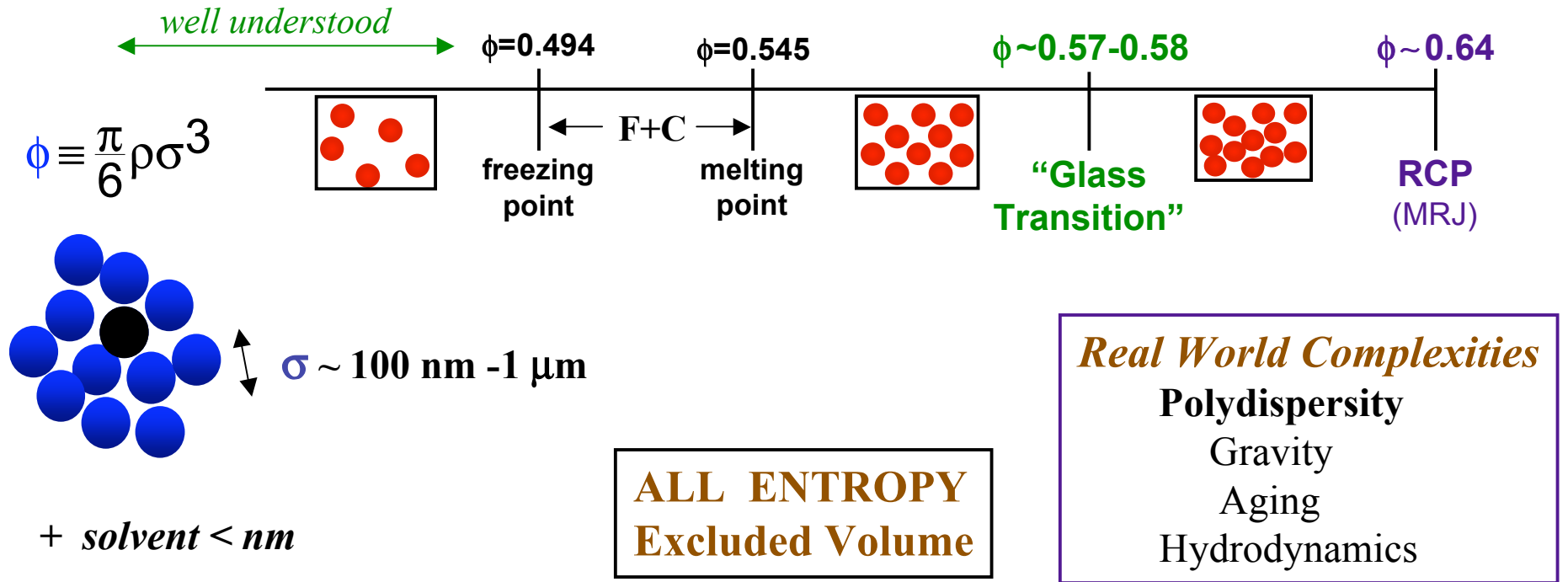


**Hard Spheres** → “Colloidal Molecules”



*models for other systems : Atomic Liquids ; microgels, pastes, emulsions, charged, ...*

# “Athermal” Glassy HARD SPHERE Colloidal Suspensions



**“GLASS TRANSITION”** : kinetic crossover...ala thermal glass  $T_g$   
 vs. *True Singularity?*

*Relaxation Time > Expt time scale ~ 1000 - 10,000 secs....No Singularities below RCP?*

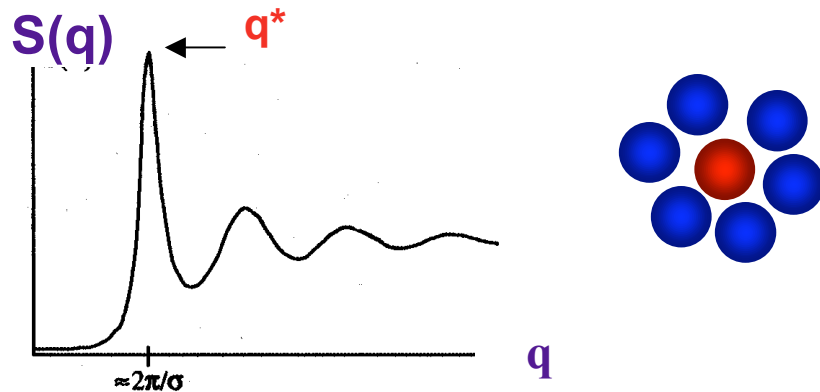
**Experiments (and simulations) probe “precursor” regime**

Colloids:  $10^{3-4}$  change in  $\eta, D$  vs.

Thermal liquids:  $10^{14}$  *Brownian time*  $\tau_0 = \sigma^2 / D \sim 0.01-10 \text{ sec... very large}$

# “Average” Dynamical Behavior: Ideal Mode Coupling Theory

Gotze, Sjogren, Leutheusser, .....1986-



SLOW Cage Scale  $(\delta\rho)^2$

Relate Local Structure,  $S(q)$ , & Dynamics

Gaussian Mean Field Theory for  $S(q,t)$

**PREDICTS** “ideal” dynamic glass : a strict LOCALIZATION transition  $\ll$  RCP

**v/v EXPTS** : GOOD for many AVERAGE properties **if FIT** location of singularity

$$\tau \sim \eta \sim D^{-1} \sim (\phi_C - \phi)^{-\nu}$$

$$\nu \sim 2.5 \quad , \quad \phi_C \sim 0.57-0.58$$

“critical power law” over few orders magnitude

**Shear Effects** : 2002- ; Fuchs/Cates, Reichman.....Useful But BUILT on Singularity

...Divergence (“ideal glass”) NOT real....rather signals **Dynamic Crossover**  
*new mode of transport : Activated Barrier Hopping.....when important ?*

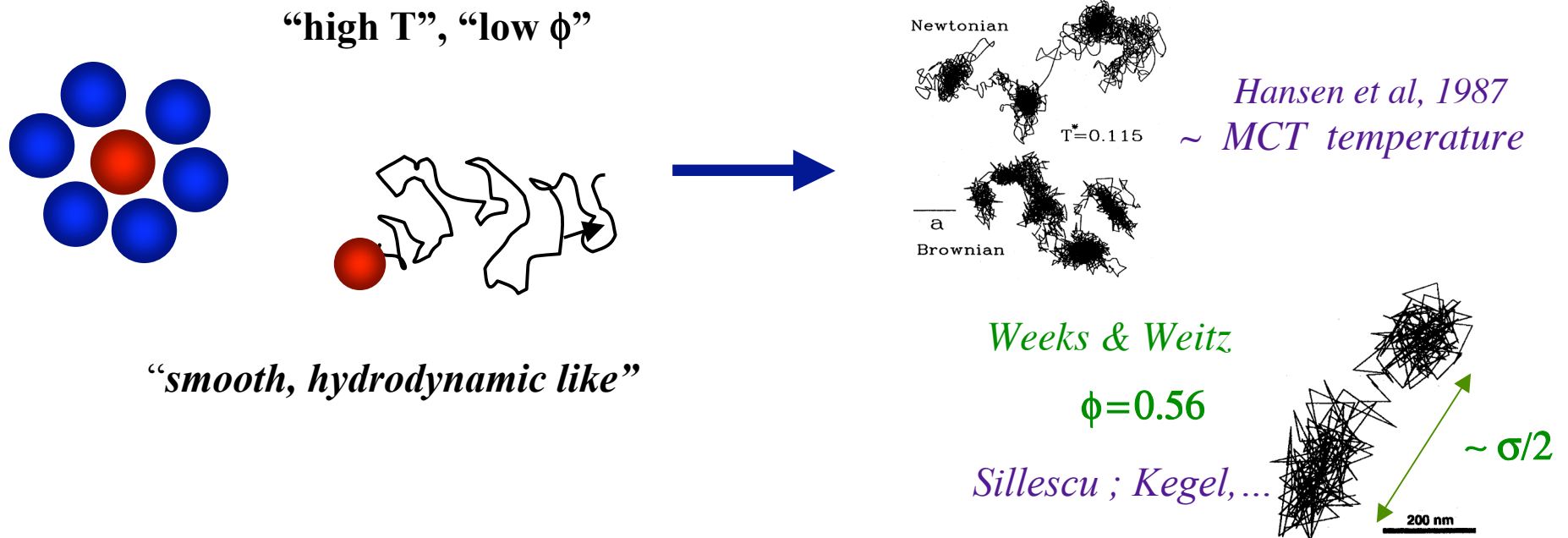
# Nongaussian Effects and Rare Events

*Multiple Simulations* (Reichman ; Heuer; Berthier...) + *Confocal Colloid Expts see :*

\* **HOPPING** processes important *BEFORE* empirically deduced (fit) MCT transition

*Trajectories change character*

*“Solid - Like”...intermittent hopping*



\* Multiple Strong “Dynamic Heterogeneity” Effects @ Single Particle level

.....*NOT predicted by MCT*

**OUR GOAL :** Build on MCT but :

*treat Entropic Barriers & Hopping, AVOID Fitting, NO Singularities below RCP*

# Activated Barrier Hopping Theory

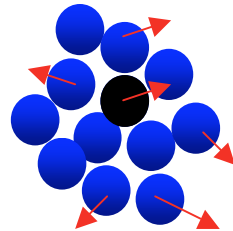
DERIVATION :  
KSS, JCP, 2005

Simplest **SINGLE Particle** Dynamics Level

Seek Stochastic Equation of Motion **NOT** closed equation for time correlation functions

Dynamic DFT

$$\hat{\rho}_s(\vec{r}, t) = \delta(\vec{r} - \vec{r}_i(t))$$



$\mathbf{r}(t)$  = scalar displacement from initial position

Solid State View

Local Equilibrium Approx + “on average” Caging constraints via  $S(q)$

$$\frac{\partial \hat{\rho}_s(\vec{r}, t)}{\partial t} = D_s \nabla^2 \hat{\rho}_s(\vec{r}, t) + D_s \nabla \hat{\rho}_s(\vec{r}, t) \int d\vec{r}' \hat{\rho}_s(\vec{r}', t) \nabla V(\vec{r} - \vec{r}') + \eta_i \nabla \hat{\rho}_s(\vec{r}, t)$$

CONTRACT to lowest level + Average over local packings + Local Equilibrium Approx

$$\frac{\partial}{\partial t} r_i^2(t) = 6D_s + 2r_i(t)\eta_i(t) - 2r_i(t) \int d\vec{r} \int d\vec{r}' \overline{\hat{\rho}_s(\vec{r}, t) \hat{\rho}_s(\vec{r}', t)} \nabla V(\vec{r} - \vec{r}')$$

Local Equilibrium

$$\frac{\rho^{(2)}(\vec{r}, \vec{r}'; t)}{\rho^{(1)}(\vec{r}; t)} \approx \rho g(|\vec{r} - \vec{r}'|)$$

Sum Rule



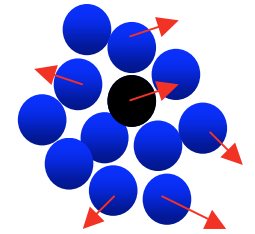
$$-2r_i(t) \int d\vec{r} \bar{\rho}_s(\vec{r}, t) \nabla \frac{\delta \bar{f}_{exc}}{\delta \bar{\rho}_s(\vec{r})}$$

Instantaneous Caging Force

ala Ramakrishnan-Yusoff DFT **BUT** “coarse-grained” NOT ensemble-averaged

$$\bar{f}_{exc} \stackrel{RY}{\equiv} -\frac{1}{2} \int \frac{d\vec{q}}{(2\pi)^3} C(q) N^{-1} \sum_{i \neq j} \frac{\approx \rho h(q)}{e^{i\vec{q} \cdot R_{ij}^0} e^{-q^2 r^2 / 3}} \quad \text{Intermolecular liquid pair correlation}$$

ala Einstein solid  
**Dynamic Mean Field**  
"closure"



## Nonlinear Stochastic Langevin Equation...force balance

$$r(t=0) = 0 \quad \zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial \vec{r}} F_{eff}(r(t)) + \eta(t) \quad \text{white noise}$$

$$\beta F_{eff}(r) = -3 \ln(r) - \frac{1}{3} \int \frac{d\vec{q}}{(2\pi)^3} C^2(q) \rho S(q) e^{-q^2 r^2 / 3} \equiv F_{ideal} + F_{cage}$$

**Time Local Displacement-Dependent Trapping "Field"**

**Liquid Structure Caging Forces**

**Favors: Delocalized Liquid Localized Solid**

$\phi$ -dependent competition

Source of material specific predictive power...ala MCT

$$S^{-1}(q) = 1 - \rho C(q)$$

# Analogies & Reduction to *Naïve Ideal MCT*

$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t) \quad \langle \eta(t) \eta(0) \rangle = 2\zeta_s k_B T \delta(t)$$

Stochastic Trajectories...*destroys Ideal Glass State, **Dynamical Crossover..”onset”***

Resembles **Kramers** theory of chemical reactions, **Model A** of dynamic critical phenomena  
 ...BUT  $F_{eff}$  *not an Equilibrium Free Energy nor Potential-of-Mean Force*

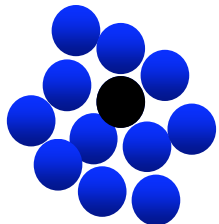
\* **RECOVER *Naïve Ideal MCT* Transition IF :**

*Kirkpatrick & Wolynes  
PRA, 1987*

*Drop Noise OR Gaussian approximation for  $\langle r^2(t) \rangle$*

➔ *Dynamic Order Parameter : Mean Localization Length*

$$r_{LOC}^2 \equiv \langle r^2(t \rightarrow \infty) \rangle$$



*Debye-Waller*

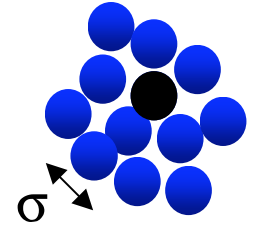
$$\frac{1}{r_{LOC}^2} = \frac{1}{18\pi^2} \int_0^\infty dq q^4 C^2(q) \rho S(q) e^{-\frac{q^2 r_{LOC}^2}{6} (1+S^{-1}(q))}$$

**IDEAL  
GLASS  
at  $\phi_C$**

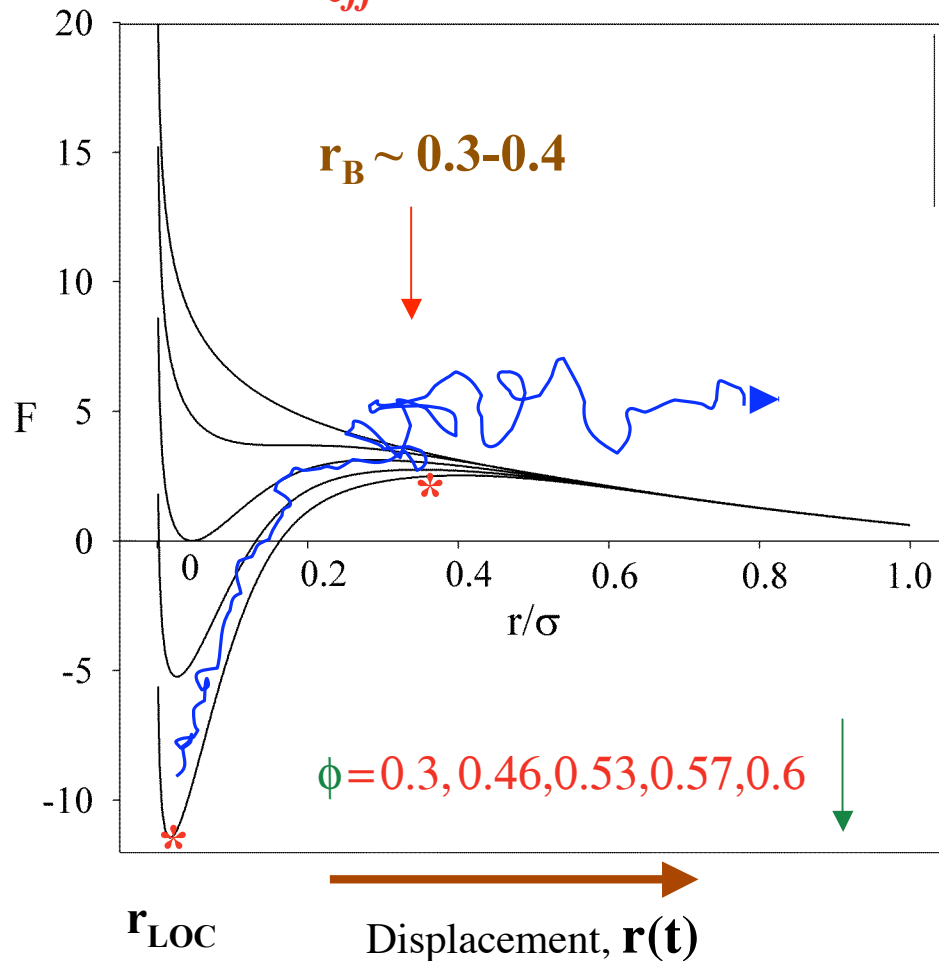
# Nonequilibrium “Free Energy” and Dynamical Crossover

$\phi < \phi_C \sim 0.432$  (PY input) → Diffusive, smooth motion

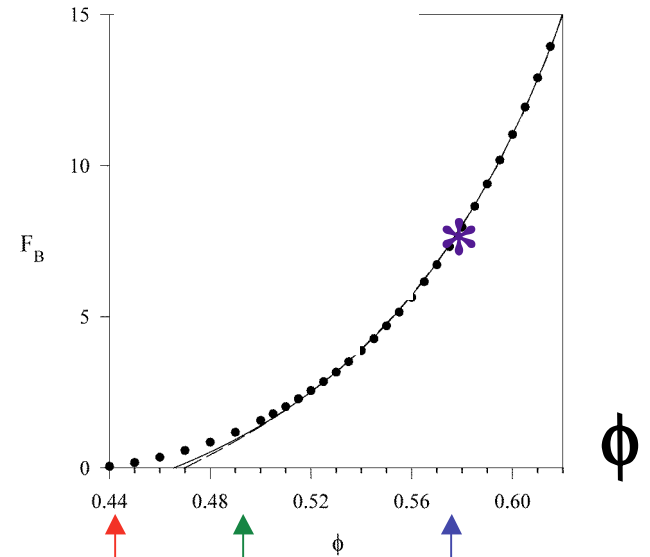
$\phi > \phi_C$  → Activated Hopping, abrupt / intermittent displacements



$F_{eff}(r)$  ...”scalar landscape”



Entropic  
barrier  
 $F_B(\phi)$



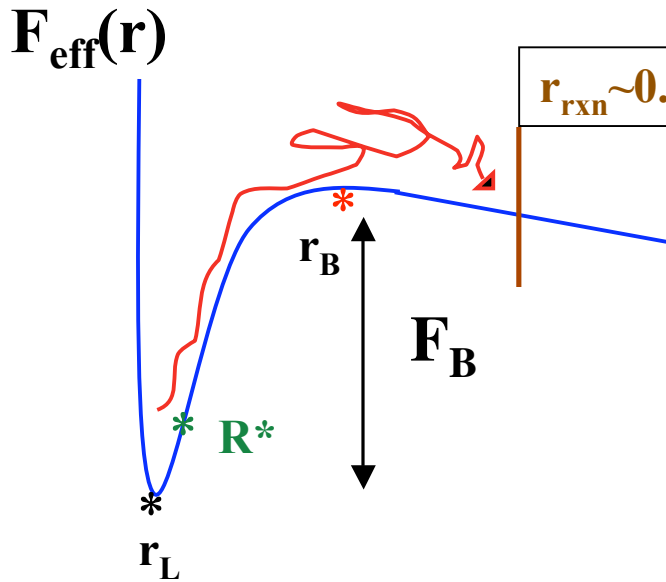
nMCT Freezing Kinetic Vitrify

~ Negligible  $\phi < 0.49$  ....normal fluid  
 ~7 @ EXPT “kinetic glass transition”  
*Glassy Dynamics : hopping over “low” barriers*



# Rich “Self - Dynamics”

$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t)$$



## 4 Local Length Scales

**Reaction Point** : Cage Escape, negligible localizing force  
**Crossover to 3-d Fickian Diffusion**

$$\zeta_{tot} = \zeta_s + \zeta_{HOP} \quad , \quad D_{HOP} = r^{+2} \langle \tau_{rxn}^{-1} \rangle / 2 \equiv k_B T \zeta_{HOP}^{-1}$$

Maximum Caging Force at R\*

....related to “yielding”

**Connections between :** Transient Localization  
 Early Cage Escape (late  $\beta$ )  
 final Alpha Process

Average Scalars (analytic)

Brownian Trajectories

Time Correlations

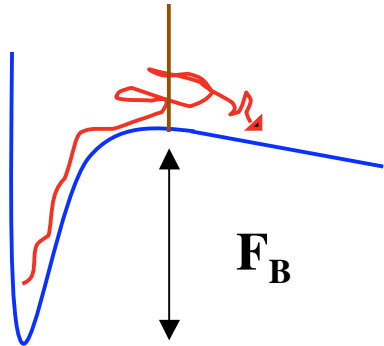
Nongaussian effects

Nonlinear Viscoelasticity, Colloidal Molecules,....

Purely Dynamical  
 NOT structure fluctuations

# Quasi-Analytic Theory for Average Scalars

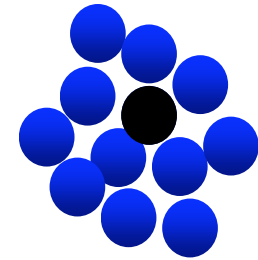
JCP, 2003



Diffusion-Controlled *Kramers theory*

$$\frac{\bar{\tau}_{hop}}{\tau_0} = \frac{2\pi (\zeta_s / \zeta_0) e^{F_B}}{\sqrt{K_0 K_B}}$$

~ *alpha or flow time*



$\tau_0 = \sigma^2 / D_0$  *Brownian time*

## Transport Coefficients (*Green-Kubo*) : Bridge multiple regimes

$$D = \frac{k_B T}{\zeta}$$

**Cohen et al theory** (PRE, 1997)

$$\zeta = \zeta_s + \frac{1}{3} \beta^{-1} \int_0^\infty dt \int \frac{d\vec{q}}{(2\pi)^3} q^2 C^2(q) \rho S(q) \Gamma_s(q,t) \Gamma_c(q,t)$$

“binary collision in a mean field cage approximation”

$$\eta = \eta_\infty + \frac{k_B T}{60\pi^2} \int_0^\infty dt \int d\vec{q} q^4 \left( \frac{\partial}{\partial q} \ln S(q) \right)^2 \Gamma_c^2(q,t)$$

works well in  
“normal fluid”  
regime :  $\phi < 0.5$

Include “hopping friction” :

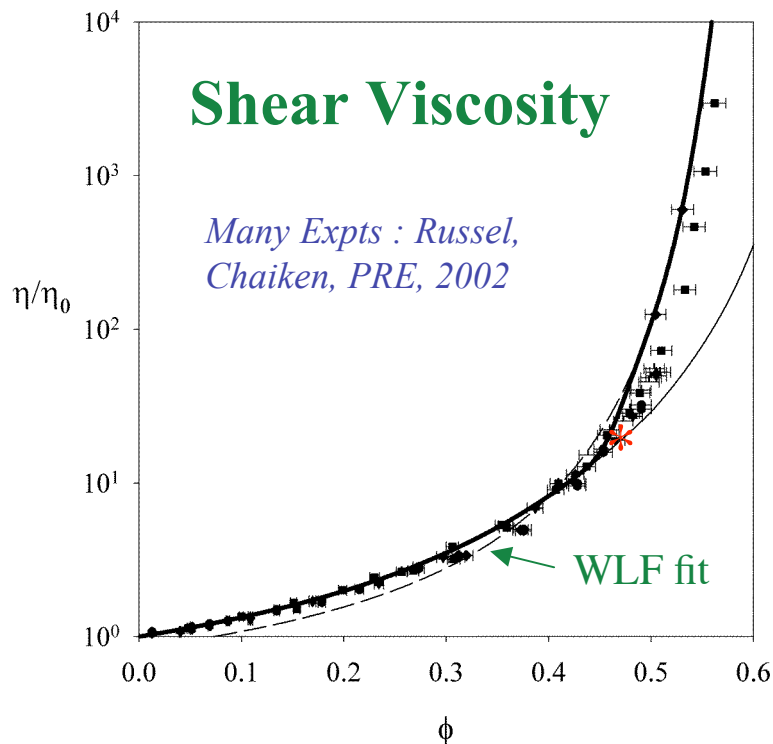
$$\zeta_s \rightarrow \zeta_s + \zeta_{HOP}$$

*NOT rigorous, but introduces NO adjustable parameters*

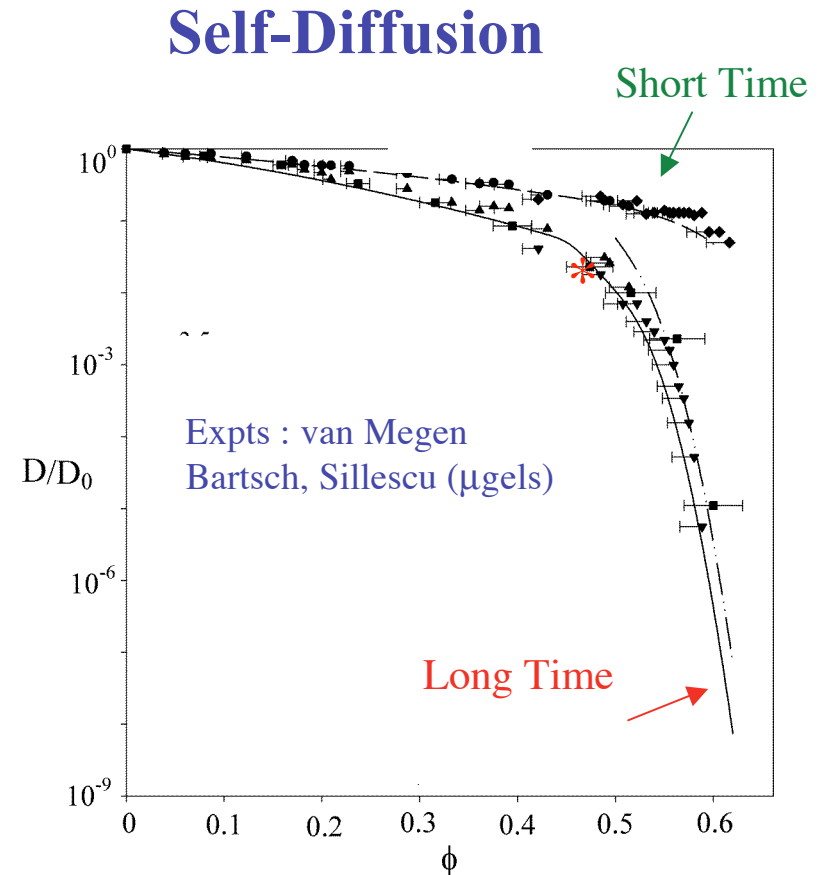
# Theory vs. Experiment

**NO fitting  
Parameters**

Single Theory for **ALL** Volume Fractions



“low”  
Barriers  
Control  
 $\phi > 0.5$



FITS to (*essential*) **Singular** forms  
Adams-Gibbs Entropy

Free Volume  
Ideal MCT

$$\exp\left(\frac{C}{\Delta S_{config}}\right), \exp\left(\frac{B}{\phi - \phi_C}\right), (\phi_C - \phi)^{-2.5}$$

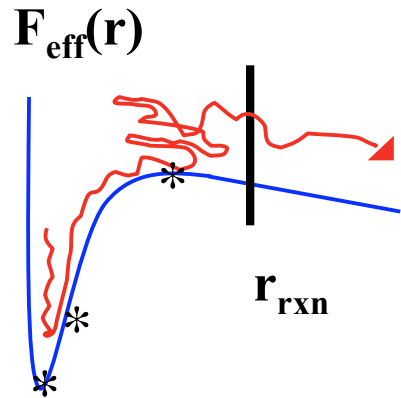
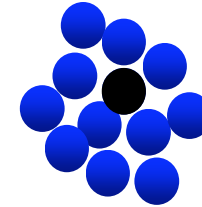
**FIT  $\phi_c$**

# Full Numerical Solution

Saltzman & KSS  
*JCP & PRE, 2006*

*Brownian Trajectories*

$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t)$$

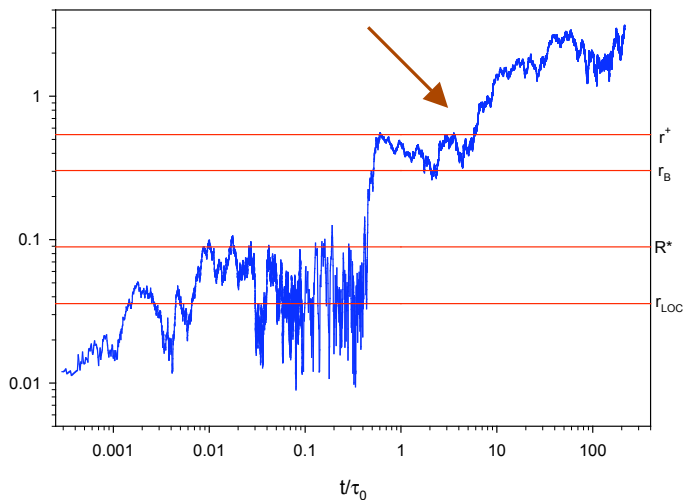


Relaxation Time Distribution  
*"Heterogeneity"*  
 Intermittent dynamics



**ALL Single Particle Time Correlations**

$$r(t)/\sigma$$

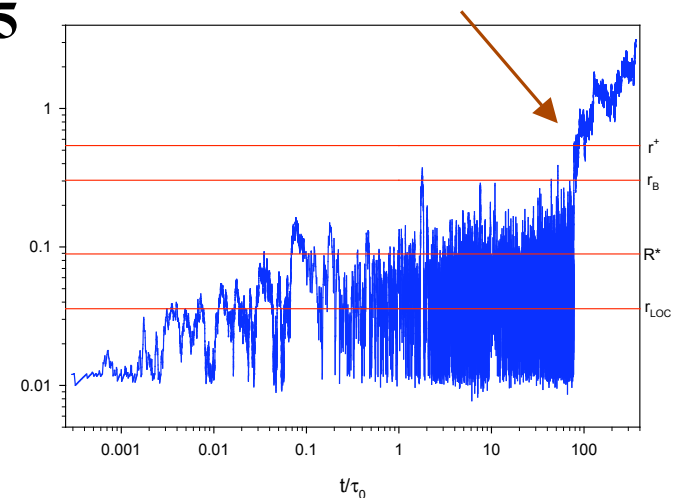


$\phi=0.55$  ; Barrier  $\sim 5$

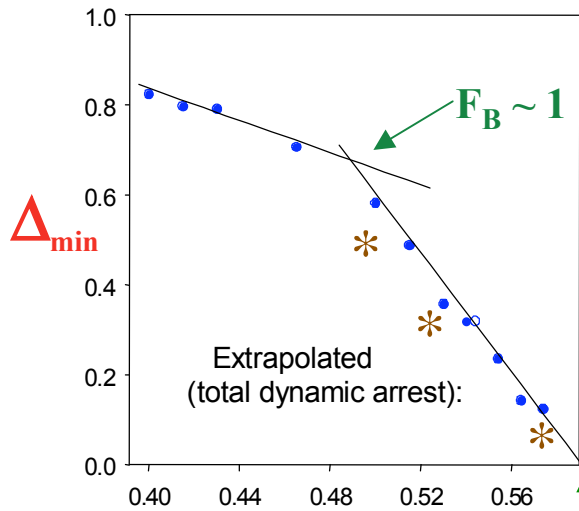
*Reaction point*  
 Barrier  
 Maximum force  
 Localization length

*Re-crossings*  
*Large Fluctuations*

FAST "reaction"



# Mean Square Displacement & Anomalous Diffusion

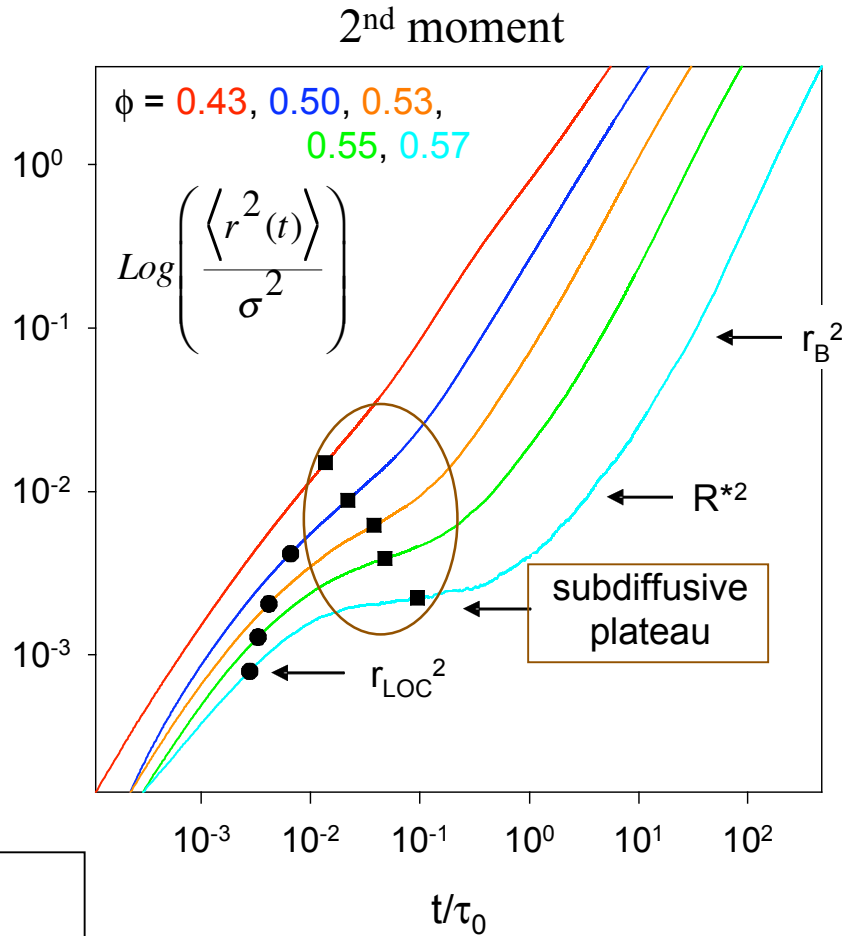


*Anomalous Diffusion*

$\phi$

$$\langle r^2(t) \rangle \propto t^{\Delta_{\min}}$$

$\phi_c \sim 0.58$   
 $\sim EXPT + MCT$  empirical fits

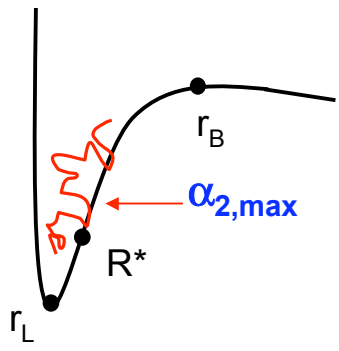
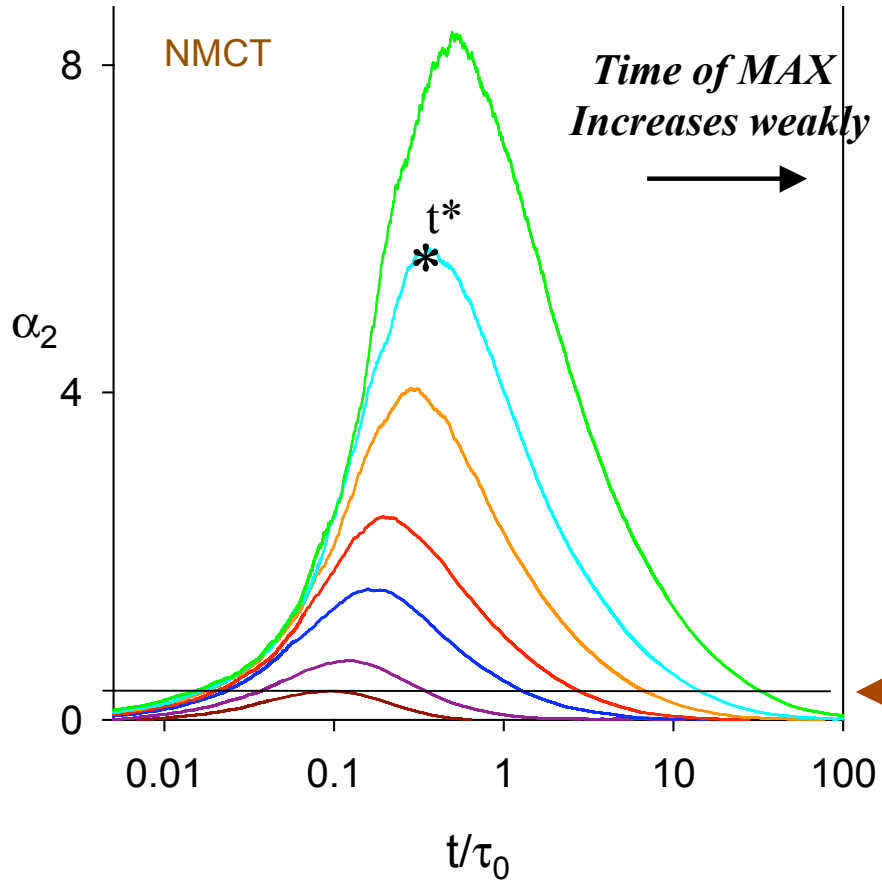


van Megen Dynamic Scattering Expts \*

PRE, 2005  $\sim$  linear reduction of minimum  $\Delta(\phi)$

# NONgaussian Parameter

$\phi = 0.43, 0.465, 0.5, 0.515, 0.53, 0.54, 0.55$



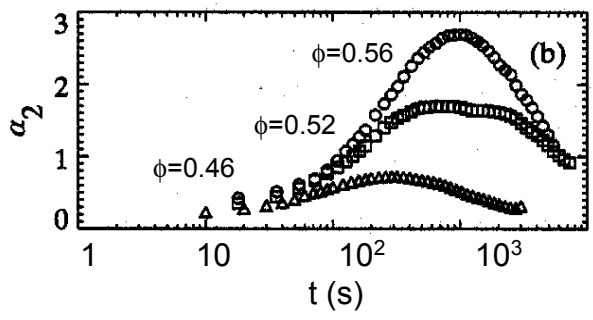
$$\alpha_2(t) \equiv \frac{3}{5} \frac{\langle r^4(t) \rangle}{\langle r^2(t) \rangle^2} - 1$$

**Max NGP correlated with**

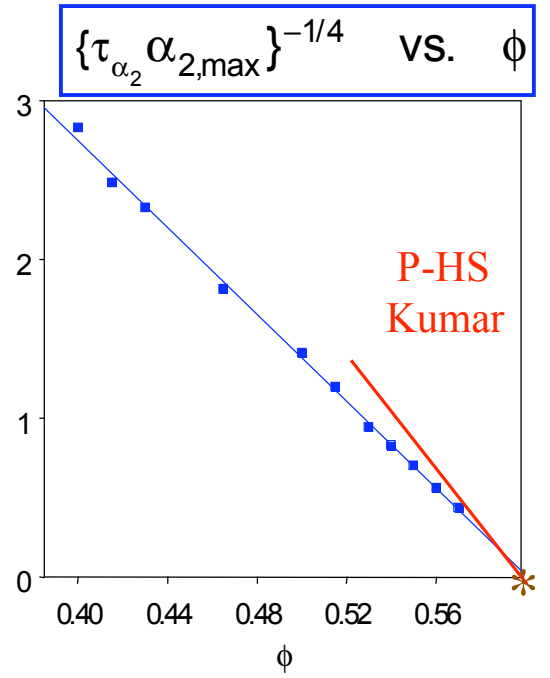
- \* Late  $\beta$  / early  $\alpha$  (Weitz expt)  
 $r(t^*) \sim 0.3 \rightarrow 0.1$
- \* Cage-Restoring Force :  
 $\alpha_{2,max} \sim (f^*)^{7/4} \sim e^{31\phi}$

**Full MCT :**  
 $\alpha_{2,max} \sim 0.33$   
*~ OUR result at NMCT*

EXPTS : Weeks, Weitz, PRL, 2002



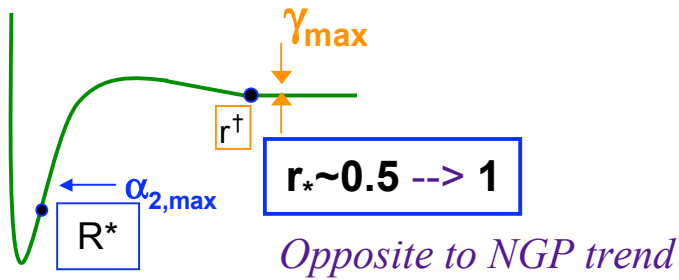
**P-HS Simulations**  
*Reichman et. al.*  
 $\alpha_{2,max} \sim 5, \phi \sim 0.55$



# NEW Nongaussian Parameter

$$\gamma(t) = \frac{1}{3} \langle r(t)^2 \rangle \left\langle \frac{1}{r(t)^2} \right\rangle - 1$$

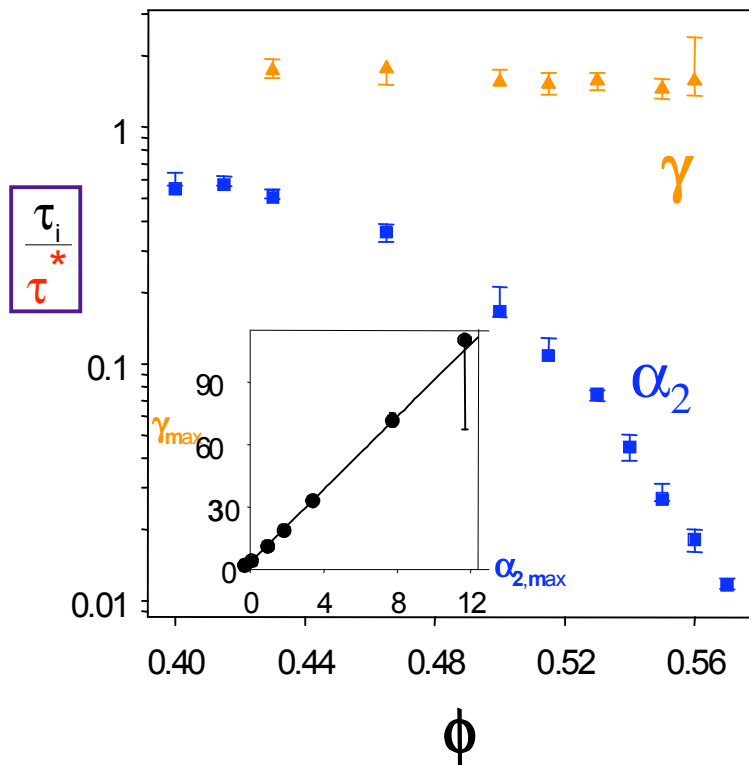
Flenner & Szamel PRE, 2005



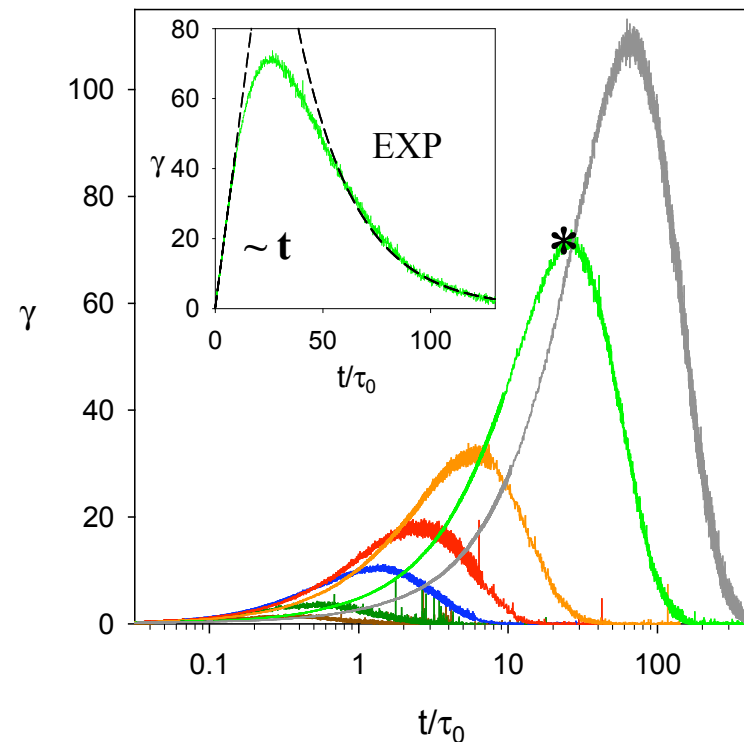
*Probes LONG time alpha process*

*Full Simulations find :*

- Timescale of MAXIMUM tracks  $\sim 2\tau^*$  ...ala  $\chi_4(t)$
- Amplitude LARGER than classic NGP
- **Different Shape** : sharp long time cutoff



$\phi = 0.43, 0.465, 0.5, 0.515, 0.53, 0.55, 0.57$



**“Heterogeneity” of EARLY Cage Escape & FINAL Alpha Relaxation strongly COUPLED**

# Incoherent Dynamic Structure Factor :

$$F_S(\vec{q}, t) = \langle \exp[i\vec{q} \cdot \vec{r}(t)] \rangle = \text{F.T.} \langle \delta(\vec{r} - \vec{r}_j(t)) \rangle$$

## Mean Alpha Time

$$F_S(q^*, \tau^*(\phi)) \equiv e^{-1}$$

**Power Law fit**  $\tau \sim (\phi_c - \phi)^{-\gamma}$

assume :  $\phi_c = 0.57, 0.58$

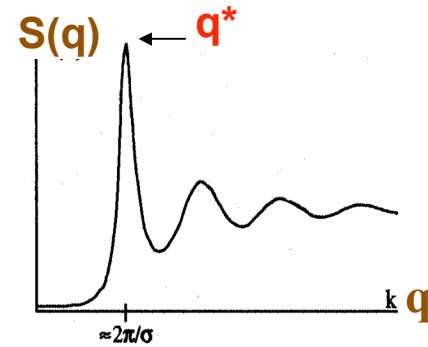
$\gamma = 2.4 - 3.0$

vs. *full MCT* :  $\gamma = 2.6$

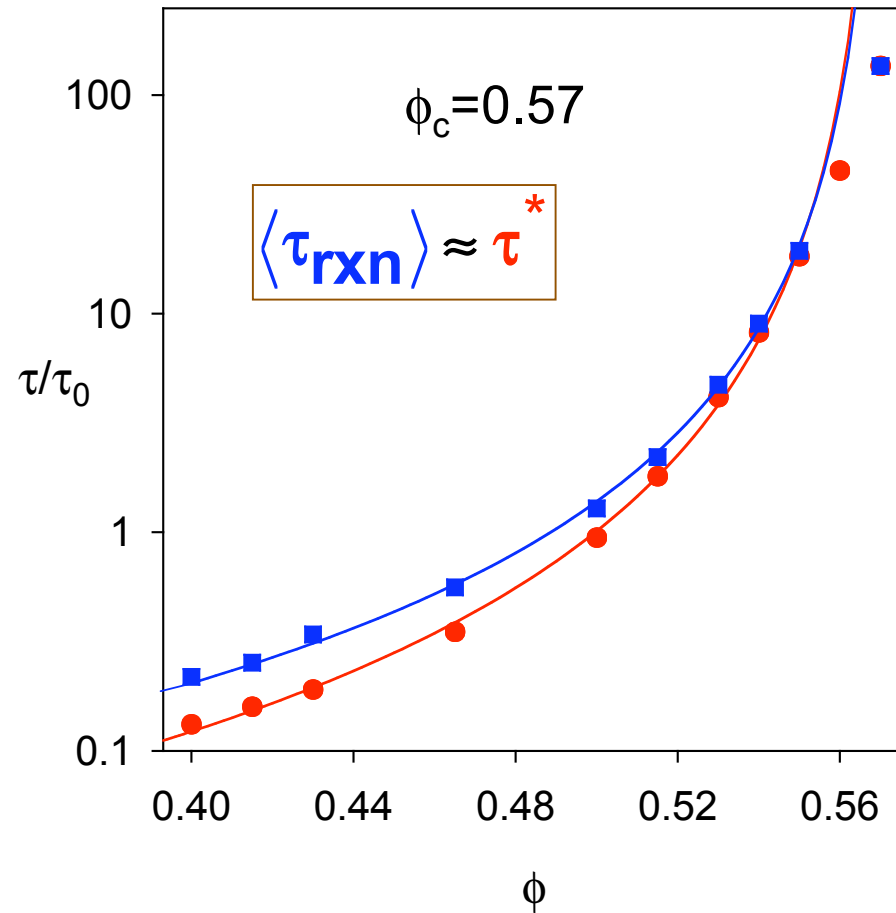
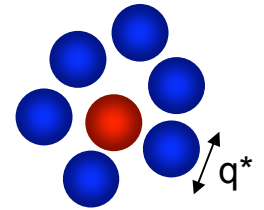
## Free Volume : "Perfect" FIT

$$\tau \sim e^{B/(\phi_c - \phi)}, \phi_c \approx 0.62 \pm 0.02$$

**Theory: No Divergence below RCP**  $\tau \sim e^{F_B(\phi)}$



## CAGE Scale



*Fluctuation effects ?*



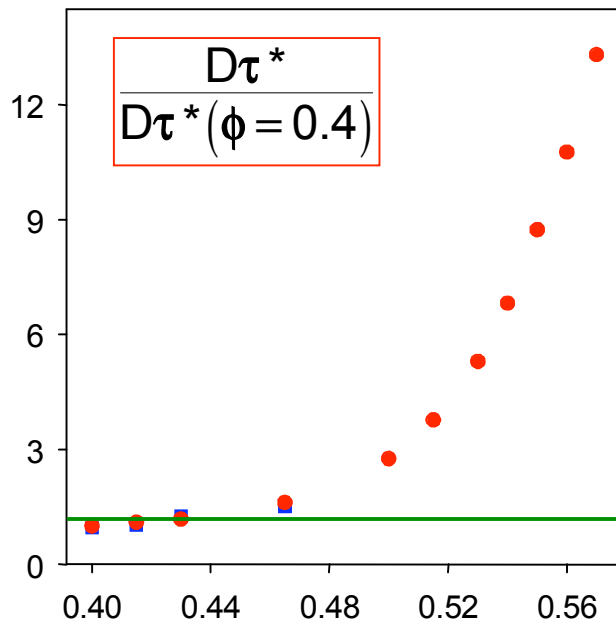
# Diffusion vs. Relaxation Decoupling

**Stochastic Hopping**  
Distribution of  
Relaxation times

$$\langle \tau_j^{-1} \rangle > \frac{1}{\langle \tau_j \rangle}$$

$$\langle \tau_{\text{rxn}}^{-1} \rangle \propto \langle \tau_{\text{rxn}} \rangle^{-0.77}$$

*Faster Rate*



*Order of Magnitude*

*Kumar et al ; Truskett et al  
polydisperse-HS SIMULATION*

$$\frac{D\tau^*}{(D\tau^*)_0} \approx 10 - 20 ; \phi = 0.58 - 0.59$$

*~ MCT ....tiny ..... < 10-15% effect*

$\phi$

**Mean Square Displacement @  $\tau^*$**

$$\sqrt{\langle r^2(\tau^*) \rangle} \cong 0.4 \quad , \quad \phi \leq 0.5$$

$$\rightarrow 0.9, \quad \phi = 0.57$$

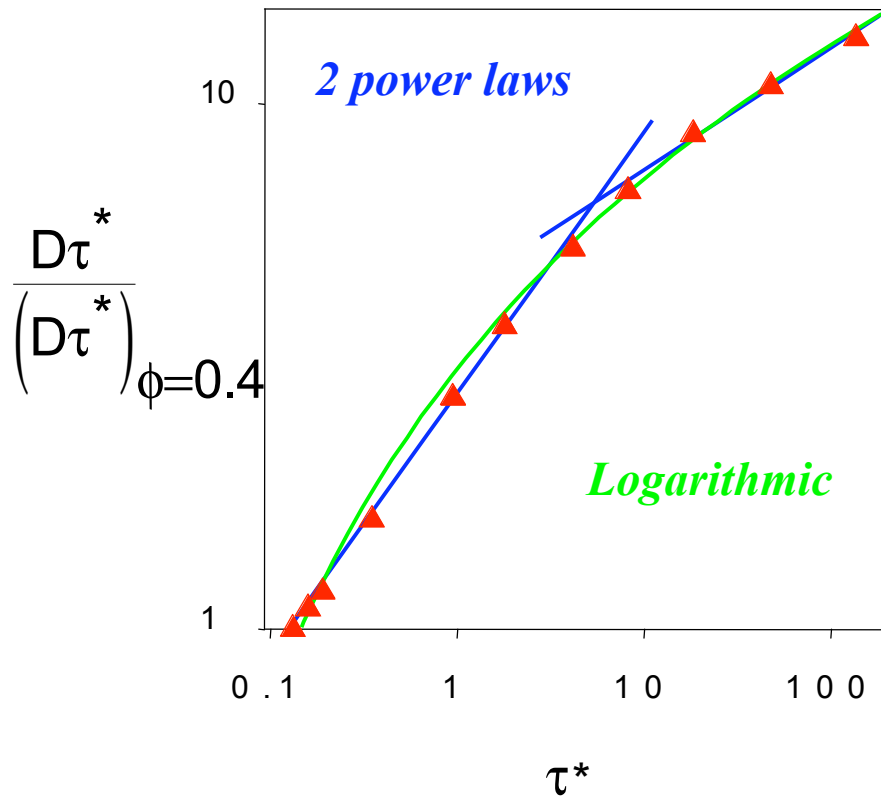


*NOT activated  
~ Stokes-Einstein*

*Mass transport enhanced at fixed "relaxation" time*

# Diffusion vs. Relaxation Decoupling : Length Scale

## Scaling with Relaxation time



$$\xi \equiv \sqrt{D\tau^*} \propto \ln(\tau^*) \propto F_B(\phi)$$

or

$$\propto (\tau^*)^{0.25} \quad \text{lower } \phi$$

$$\propto (\tau^*)^{0.115} \quad \text{high } \phi$$

**WEAK  
Growth**

ala BLJM Sims  
Szamel, Yamamoto,....

implies  $D \sim (\tau^*)^{-0.77}$

*Fractional Stokes-Einstein behavior*

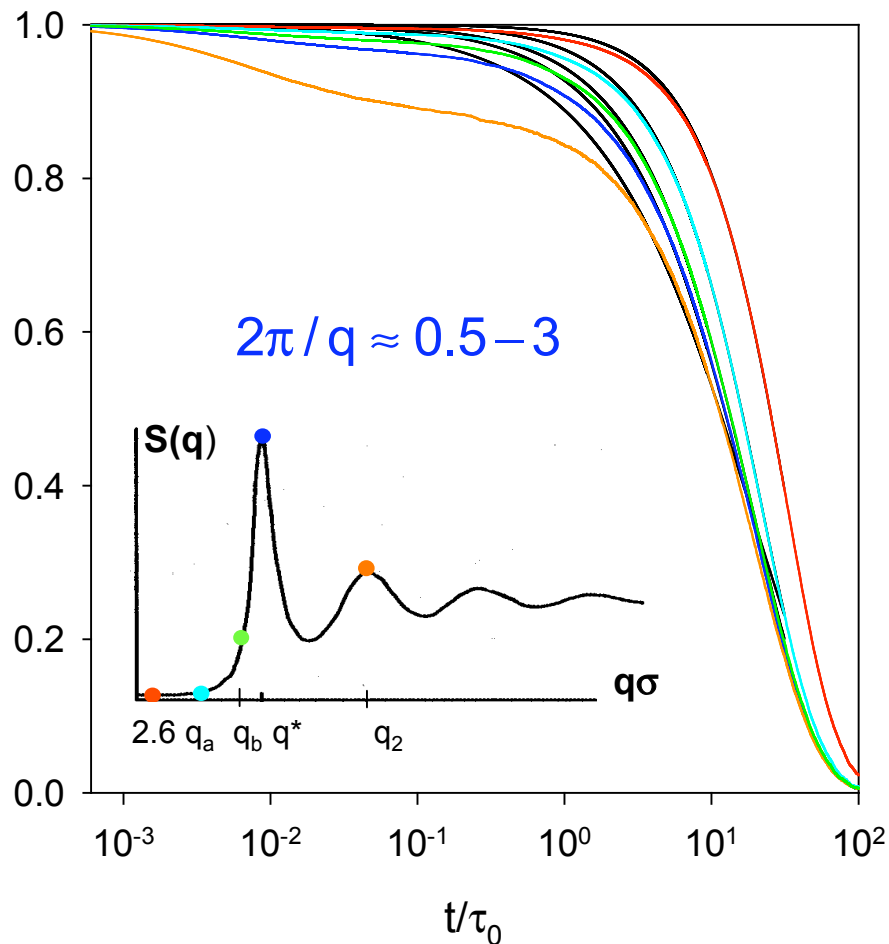
# Spatial Dependence of Relaxation

$$F_S(q,t) = \langle \exp[i\vec{q} \cdot \vec{r}(t)] \rangle$$

$\phi = 0.55$

**q-dependence**

$$F_S(q, \tau(q)) \equiv e^{-1}$$



## Relaxation Times

*Extremely NONdiffusive, NonFickian*

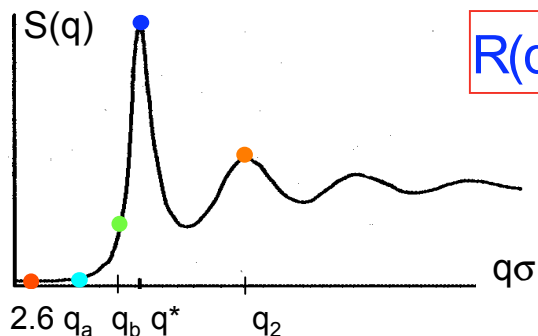
*not*  $\exp(-q^2Dt)$

*Slow Relaxation ~ q-INdependent*

*Why?....intermittent "hopping"*

*Length Scale for Crossover to Fickian ?*

# Quantify Space-Time NONgaussian Aspects



$$R(q) \equiv q^2 D \tau(q) \rightarrow 1, \text{ Gaussian} \approx \text{MCT}$$

*Growing length scale for recovery of Fickian diffusion*

Derived *ANALYTIC*  
 “Jump Diffusion” model

$$\frac{1}{\tau(q)} = \frac{q^2 D}{1 + (q \xi_D)^2}$$

*Viscoelastic  
 Length scale*

$$\xi_D \propto \sqrt{D \tau^*}$$

*...use to fit numerical theory*

$$R(q) \sim 1 + \left( \frac{L_D q}{2\pi} \right)^2$$



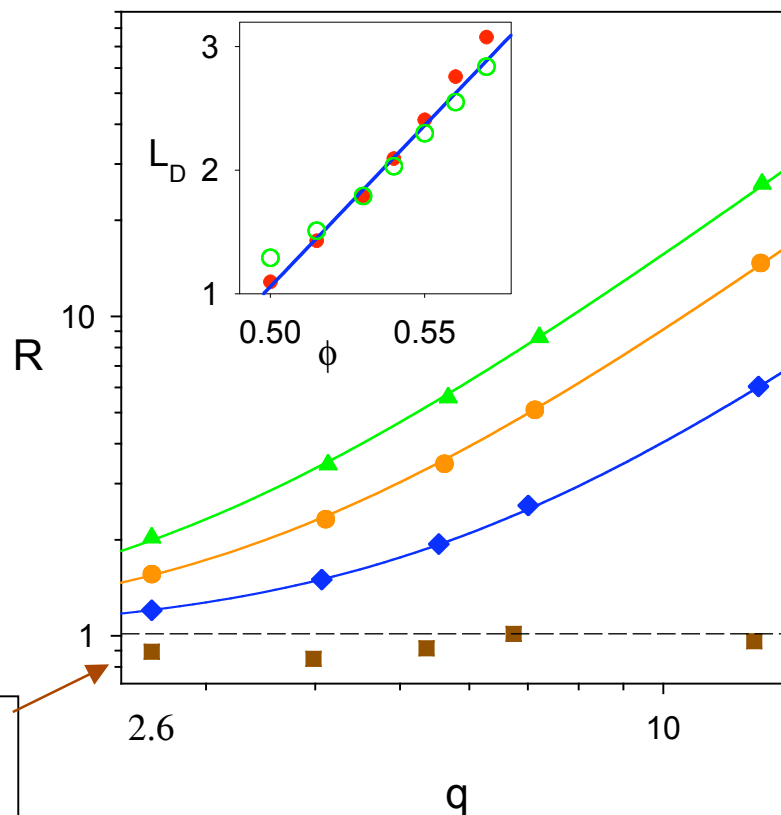
$$L_D \propto \sqrt{D \tau^*}$$

$$\propto \phi \approx 1-3$$

*agrees  
 with  
 analytics*

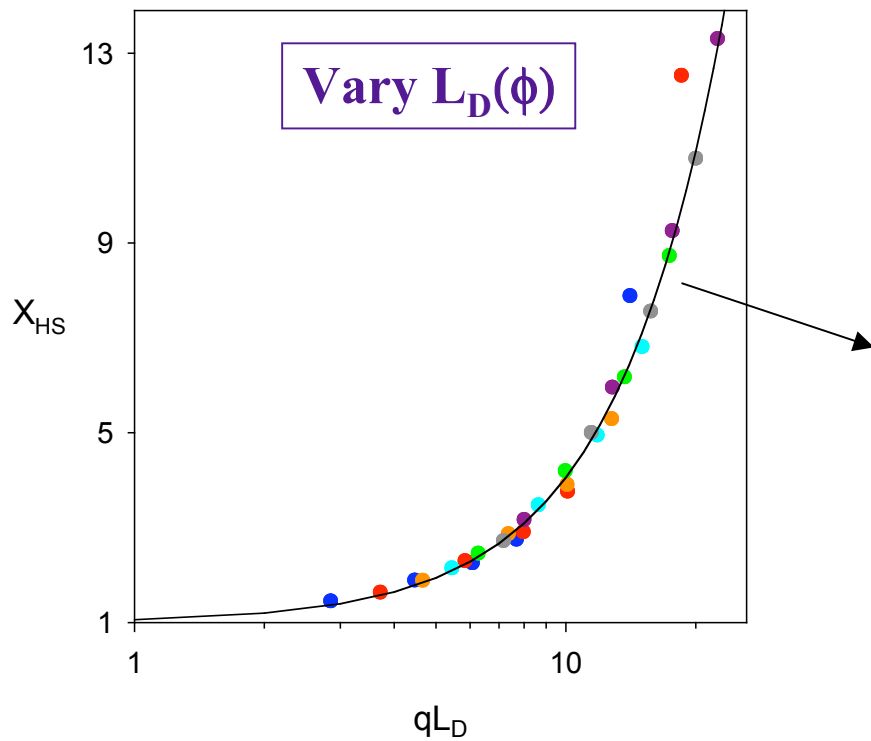
*Fickian  
 ala MCT  
 ~10% effect*

$\phi = 0.43, 0.5, 0.53, 0.55$



*Consistent with BLJM Simulations,  
 Flenner & Szamel, PRE, 2005*

# Dynamical Scaling : Collapse of ALL $\phi$ and $q$ dependences



$$X_{HS} = \frac{\tau(q, \phi) D(\phi)}{\tau(q, \phi = 0.4) D(\phi = 0.4)} \quad \leftarrow \text{normal regime}$$

For  $\phi > 0.5$  ( $F_B > 1$ )

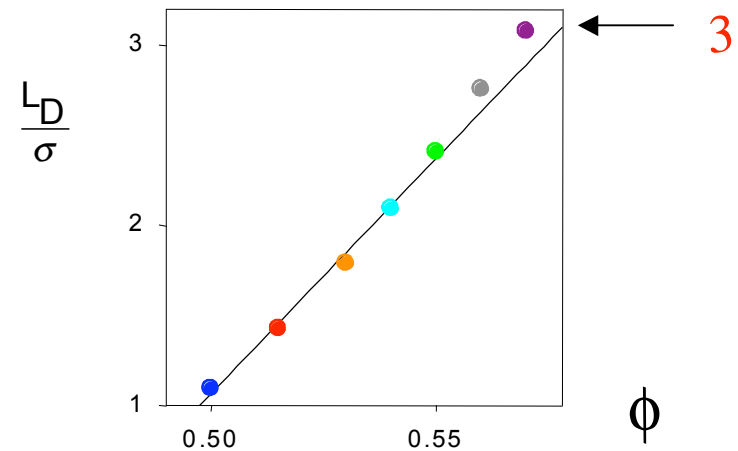
$$X(qL_D) \cong 1 + (qL_D / 2\pi)^\beta \quad \beta \sim 1.8 \pm 0.2$$

(best fit)

ala BLJM thermal liquid Simulation  
L. Berthier, PRE (2004)

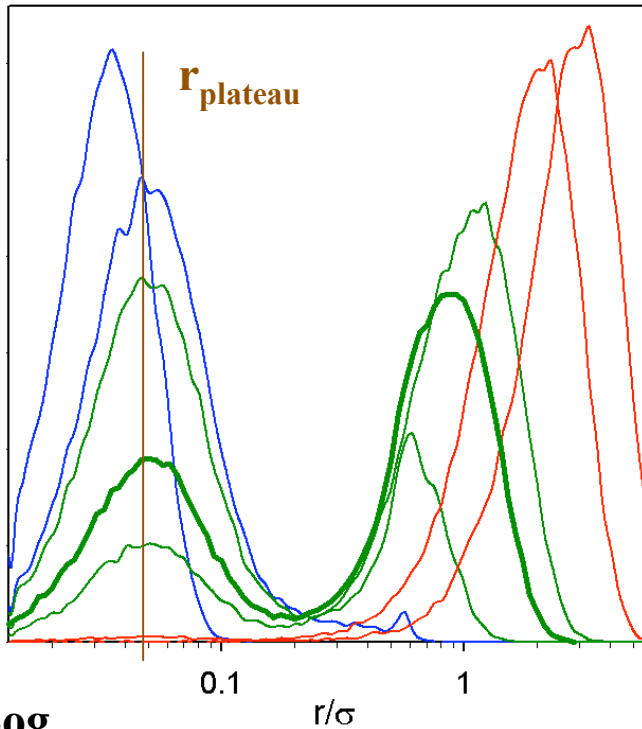
$$X(q, T) = \frac{\tau(q, T) D(T)}{\tau(q, T_0) D(T_0)}$$

*dynamic length scale*



# Displacement Distribution $P(\log|r(t)|)$ & Mobility Bifurcation

$\phi = 0.55$   $F_B \sim 5$



Log

increasing time

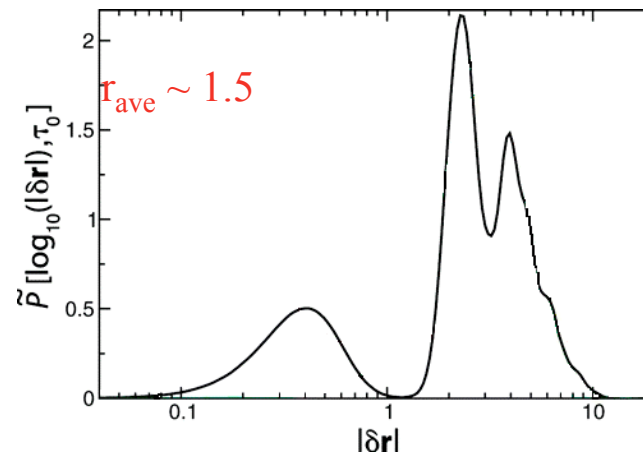
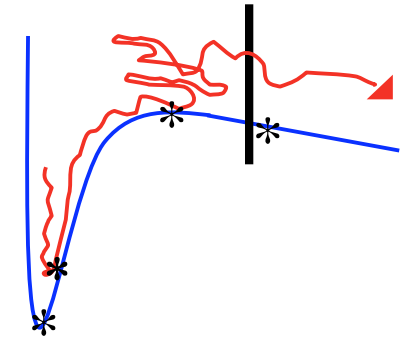
$r_{loc}, R^*, r_B, \tau^*, \sigma, 2\sigma, 3\sigma$

- Gaussian (Fickian) at **short & long** times
- **Intermediate times**: BIMODAL..more so as  $\phi$  increases
- **Strongest** near  $t = \tau^* \sim \tau_{rxn}$

*Fast & Slow Populations*

\* *Slow population ~ pinned at MSD plateau location (max anomalous diffusion)*

\* *Fast population diffuses*



$\phi = 0.55$

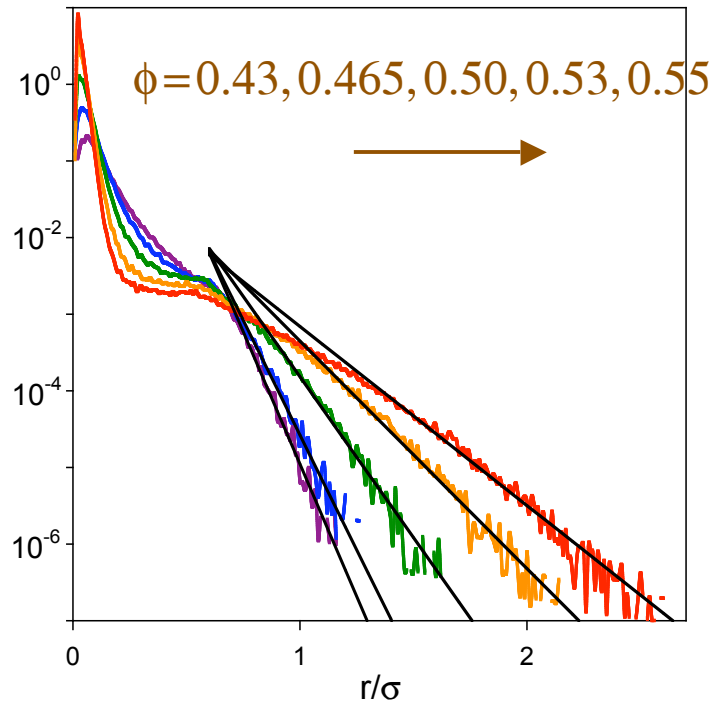
*P-HS simulation*

Reichman et al, JPCB, 2005

*Exponential Tails of  $G_s(r,t)$  .....van Hove predicted by "Jump Diffusion" model Underlies Decoupling,.....*

# Van Hove Representation and Exponential Tails

$\text{Log}\{G_s(r, t=\tau^*)\}$  @ mean  $\alpha$ -time



*Analytic “Jump Diffusion”*

$$F_s(q, t) \cong \exp\left(-q^2 D t / (1 + q^2 \xi^2)\right)$$

$$\xi \propto \sqrt{D \tau^*}$$

$$\tau_\alpha \equiv \xi^2 D^{-1} \propto \tau^*$$

$t < \tau^*$

$r$  NOT too small

$$G_s(r, t) \cong \delta(\vec{r}) e^{-t/\tau_\alpha} + \frac{1}{4\pi\xi^2} \frac{t}{\tau_\alpha} e^{-t/\tau_\alpha} \frac{e^{-r/\xi}}{r}$$

Exponential Tail

$\xi = \text{Length scale} \sim 0.06-0.20$

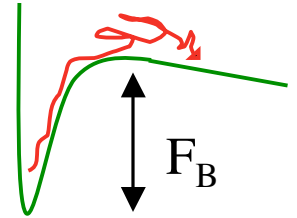
*slow  
localized  
particles*

*fast particles*

*~ Exponential Tail*

## Comments

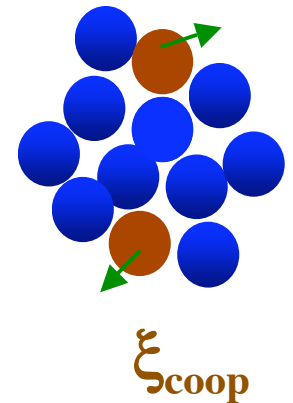
- “MCT-like” aspects... ***BUT hopping over low barriers & NO singularity***
- **MANY** NonGaussian, Non-MCT effects even in “*precursor regime*”  
***all emerge from stochastic activated dynamics on LOCAL scales***
- Connections between dynamics on different time & (local) length scales
- Consistent with Simulations & Expts, **BUT** NEED more for Hard Spheres



## Theory Simple / Multiple Limitations

.....addresses only *self-dynamics*

- \* Generalization to *Collective* Dynamic Fluctuations,  $S(\mathbf{q},t)$  ?
- \* *Many Correlated Hops* , Space-Time Mobilities, 4-point Susceptibilities ?



## *Virtues of Simplicity*

**SAME** *predictive & experimentally testable approach applies to :*

**Colloid Gels**

**Polymer Melts & Glass**

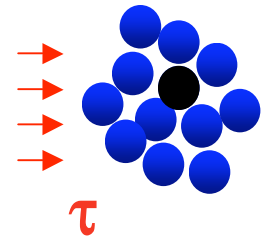
***External Stress / Mechanics***

***“Molecules”***



# Nonlinear Viscoelasticity : *Stress Rheology Perspective*

**Classic Idea : External Deformation Reduces Barriers to Flow**



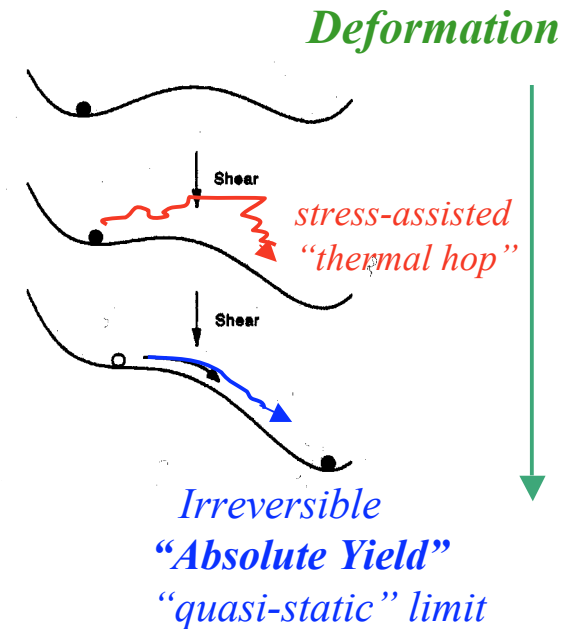
\* **Eyring (1936)** *Arrhenius viscoplastic flow*  
*Frenkel (crystals)*

**Mechanical Work**

$$E_B(\tau) \approx E_B(0) - \tau V_A$$

• Phenomenological “**Soft Glassy Rheology**” or “**Trap**” models  
 irreversible “barrier hops”, ... ..Cates, Sollich, Bouchaud, ...

• **Inherent Structure / Landscape simulations...** Dan Lacks  
*stress, strain reduce and destroy barriers...ala Eyring*



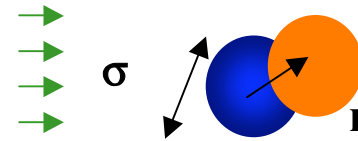
**Macroscopic Rheology**  $\longleftrightarrow$  **local, cage scale physics**  
 usual “mean field” assumption ala “**MicroRheology**”

• **Simulations : Dynamics** ~ *Isotropic on CAGE scale*

# SIMPLE Incorporation of External Stress

Instantaneous Mechanical work

$\tau = \text{Applied Stress}$



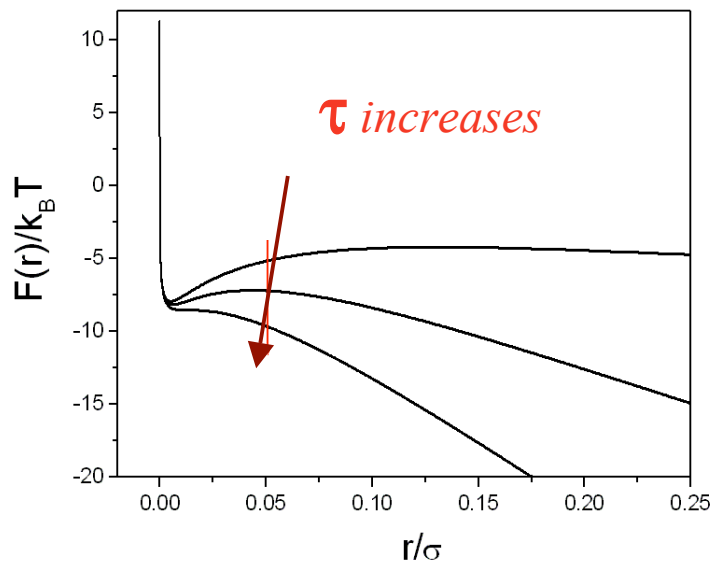
$$-\tau \Delta V(r) \cong -\lambda \sigma^2 \tau r \propto -f \cdot r \quad \boxed{\text{force on particle}}$$

$S(q)$  remains quiescent

*O(1) "stress transmission" factor*

*...factor  $\sim 2$  uncertainty in local  $\tau$*

$$F(r; \tau) = F(r; \tau=0) - \lambda \sigma^2 \tau r$$



Stress softens localization, Reduces Barrier

*NONlinear  $F_B(\tau)$  vs. linear Eyring*

*Reduces Modulus & Accelerates Relaxation*

➔ "Absolute YIELD" ↔ Barrier destroyed

$$\frac{\bar{\tau}_{hop}}{\tau_0} = \frac{2\pi (\zeta_s / \zeta_0)}{\sqrt{K_0(\tau) K_B(\tau)}} e^{F_B(\tau)}$$

Glassy  
Shear  
Modulus

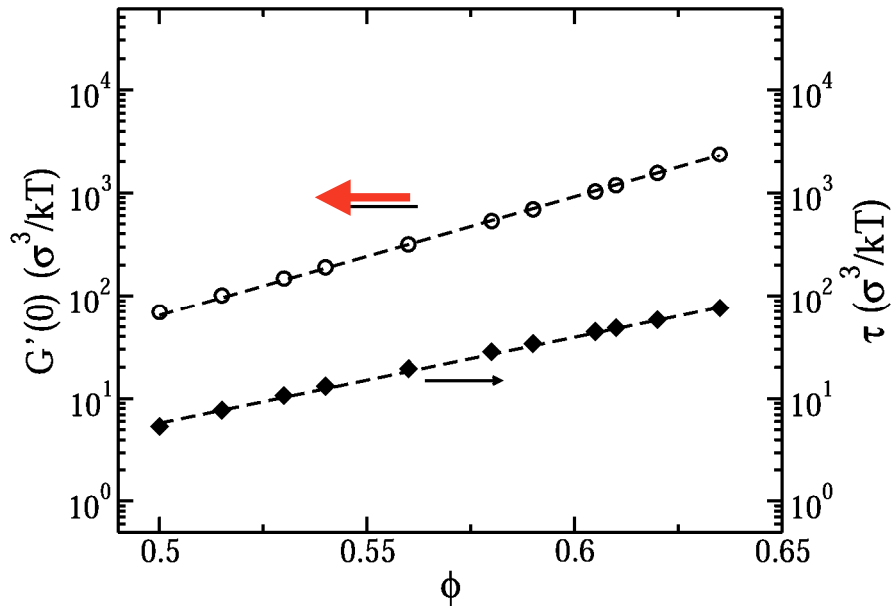
$$G'(\tau) = \frac{1}{60\pi^2} \int_0^\infty dq q^4 \left( \frac{\partial \ln S(q)}{\partial q} \right)^2 e^{-q^2 r_{LOC}^2(\tau) / 3S(q)}$$

# Shear Modulus, Absolute Yield Stress, Alpha Time

Units :  $kT/\sigma^3 = 4 \text{ Pa}$  for 100 nm

## Linear Shear Modulus & Absolute Yield Stress

*hopping NOT an issue*



## Exponential or High Power Laws

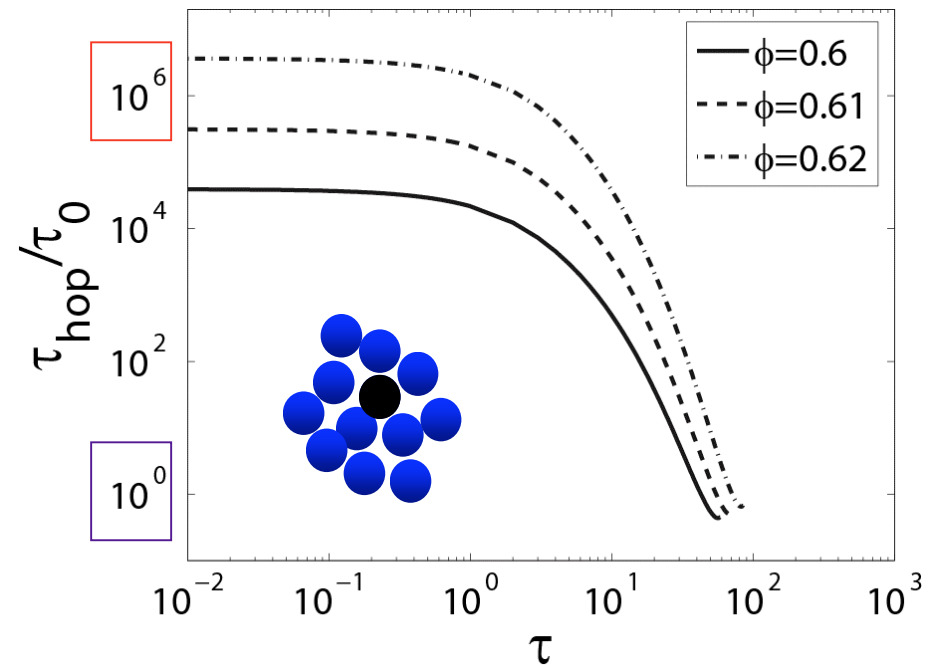
$$G' \propto e^{27\phi} \propto \phi^{14}, \quad \tau_{y,abs} \propto e^{19\phi} \propto \phi^{11}$$

Yield STRAINS ~ 20%

*Broadly consistent with variety of Expts*

## Stress-Accelerated Relaxation

*barrier hopping event ~  $\alpha$ -time*



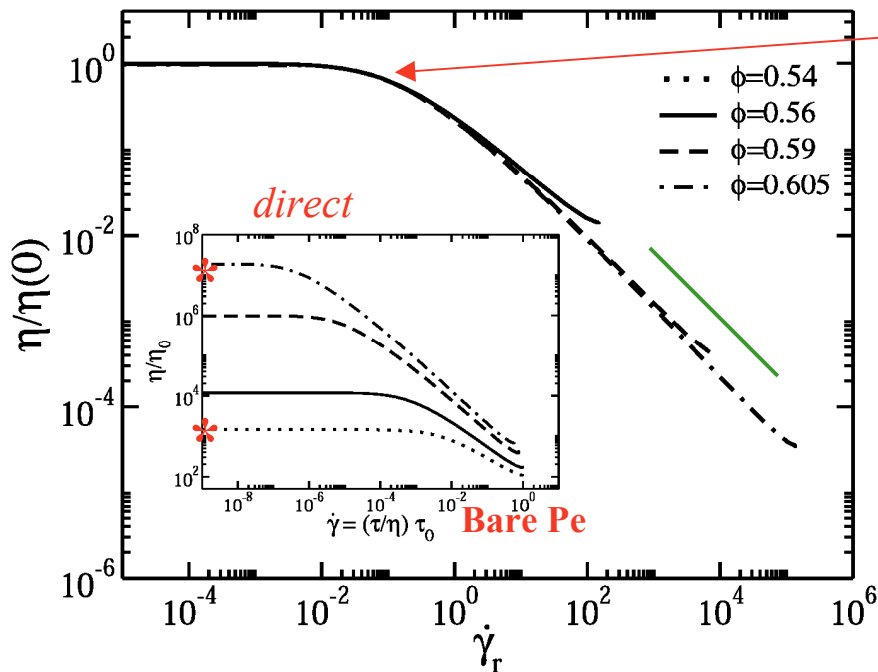
*Entropic Barriers  
Massively Reduced*

# Viscosity Thinning & Flow Curves : *beyond MCT...hopping*

$$\tau = \eta(\tau) \dot{\gamma}$$

$\phi$
0.54
0.56
0.59
0.605

$\eta/\eta_s$   
 $10^3-10^7$



$$\dot{\gamma}_{r,crit} \approx 0.3-0.4 \sim \text{Expts}$$

$$\dot{\gamma}_r \equiv \dot{\gamma} \left( \beta R^3 \eta(0) \right) \text{ Dressed Peclet}$$

Near collapse

“stress relaxation time”

$$\dot{\gamma} \tau_0 = \text{Bare Peclet}$$

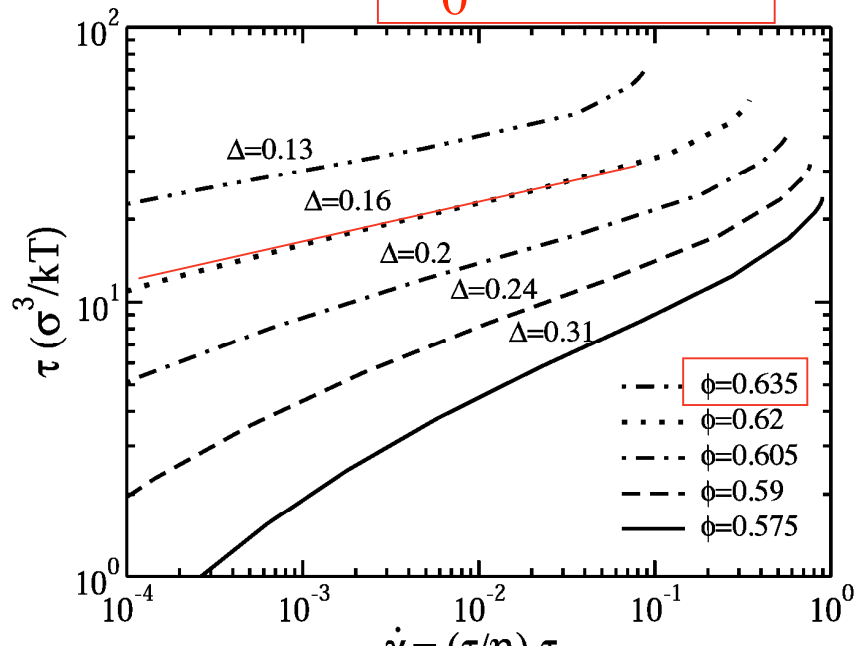
## FLOW CURVES

*No true plateau (hopping)*

APPARENT power law regime

$$\tau \equiv \dot{\gamma}^\Delta$$

$\Delta \sim 0.1 - 0.3$



# SELF-Motion Under Shear

Besseling, Weeks, Poon, PRL, July, 2007

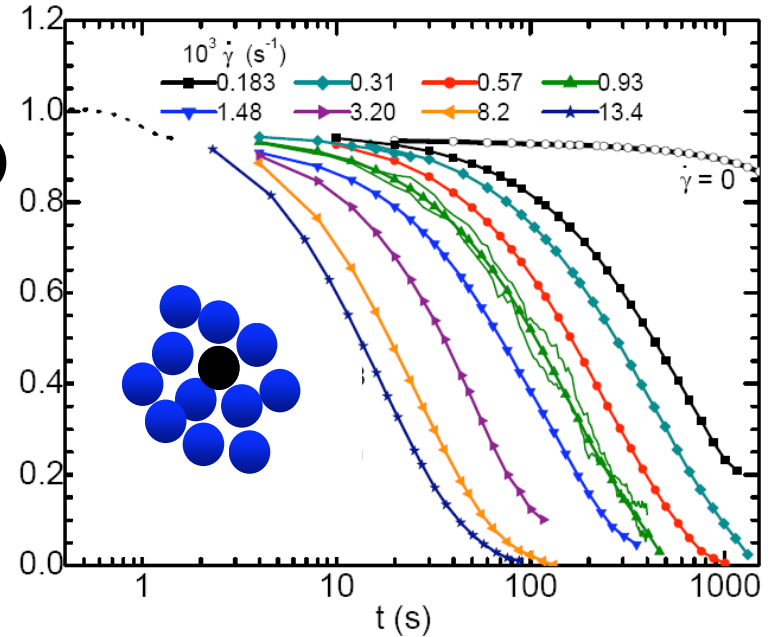
*Confocal* : direct microscopic probe of theory

$$\phi = 0.62$$

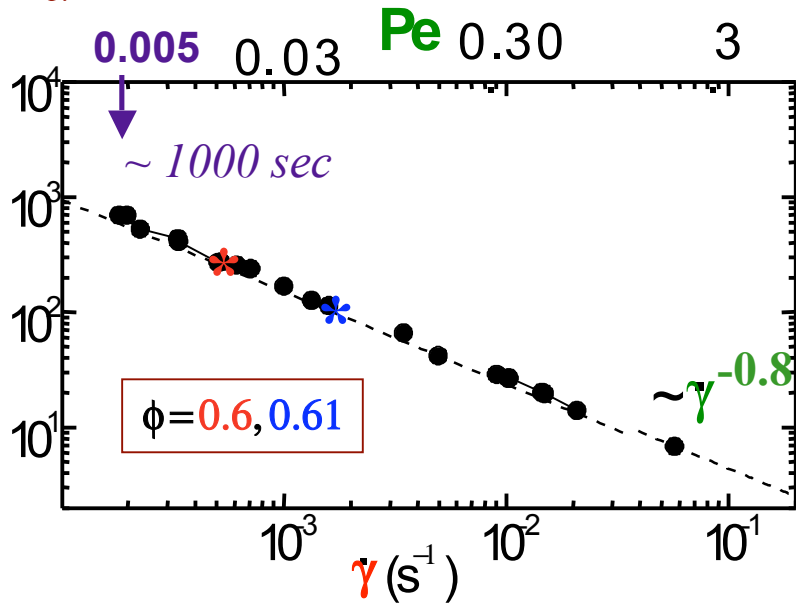
Alpha Time  $\sim 1/(\text{shear rate})^{0.8}$

for bare  $Pe \equiv \dot{\gamma} \tau_0 \approx 0.005 - 1$

$F_s(\mathbf{q}^*, t)$



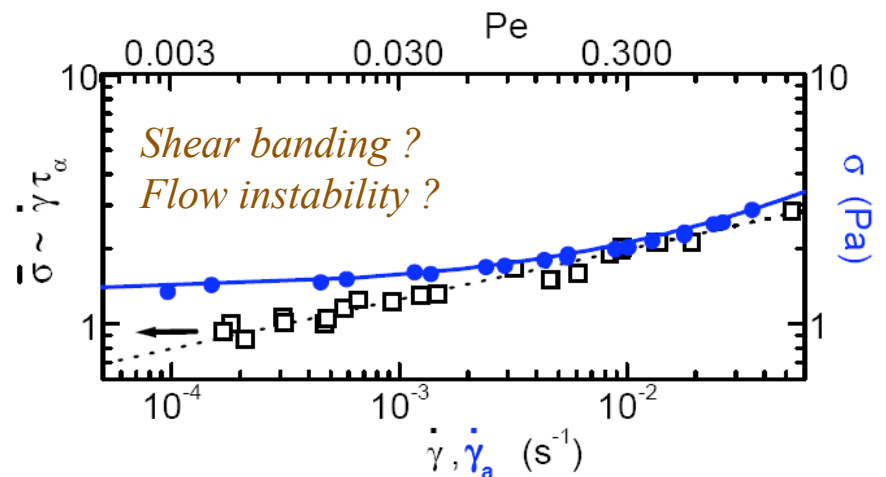
$\tau_\alpha$  (sec)



“microscopic flow curve”...NO plateau

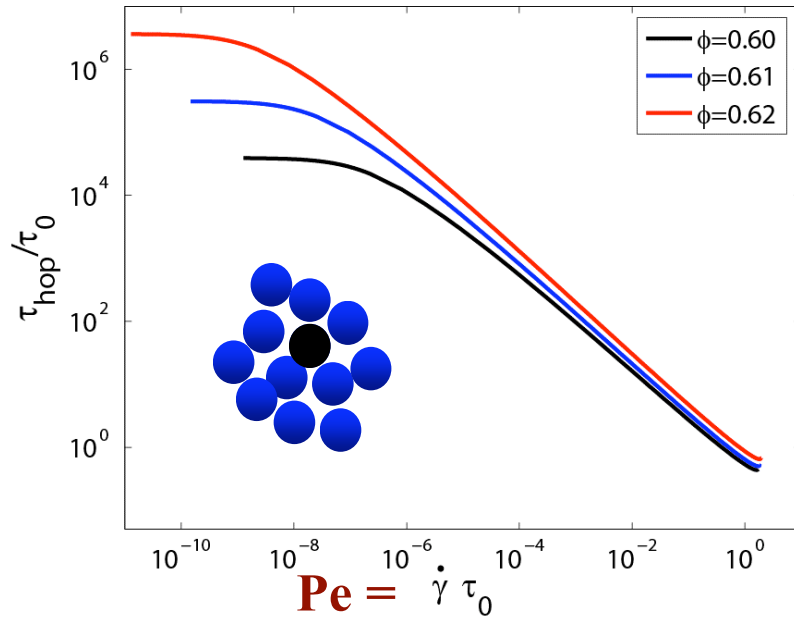
*Disagrees with Bulk Rheology*

*Hershel-Buckley...yield stress ?*



# Theoretical Predictions

Kobelev & KSS, PRE, 2005



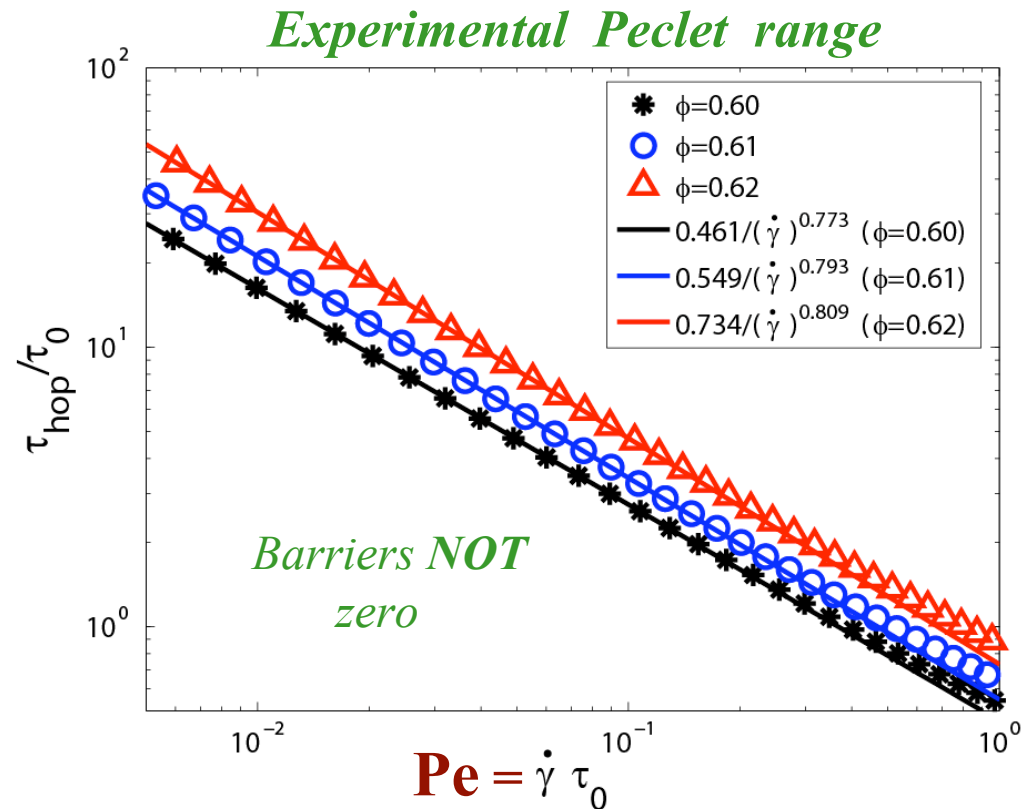
*Hopping*

$\tau_0 \sim 30 \text{ secs}$   $\xrightarrow{\phi=0.62}$   $\tau_\alpha \sim 60 \text{ million secs}$   
 $\sim 2 \text{ Years}$

**AT lowest  $Pe = 0.005$  : 900 secs  $\sim$  EXPT**

*“shear thins” by  $\sim 5$  orders of magnitude !*

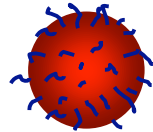
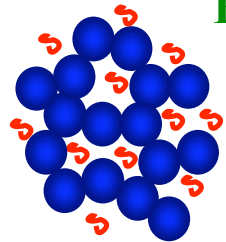
Quiescent differ by  $\sim 100$   
 ...**BUT** High Shear < **factor 2**  
 $\sim 1/(\text{shear rate})^{0.8}$   
 $\sim$  **EXPERIMENT**



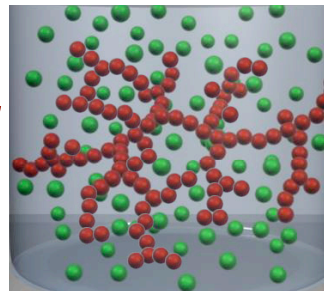
# Ongoing & Future Directions : Fluids, Glasses, Gels

\* **Glassy Colloid Nonlinear Mechanics** : Step-Strain, Creep + Recovery,....

\* **Polymer-Colloid Depletion Gels**  
**Brushed Coated Thermal Gel Formers**

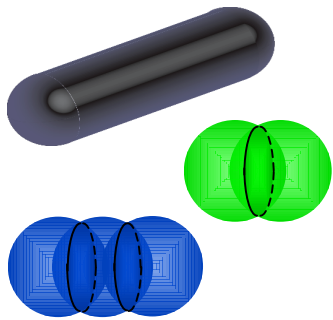


*Hopping, Heterogeneity*  
*Elasticity, Yielding*

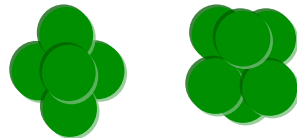


\* **Biphasic Repulsive-Sticky Mixtures**

• **Molecular Colloids**



“Pineoids”

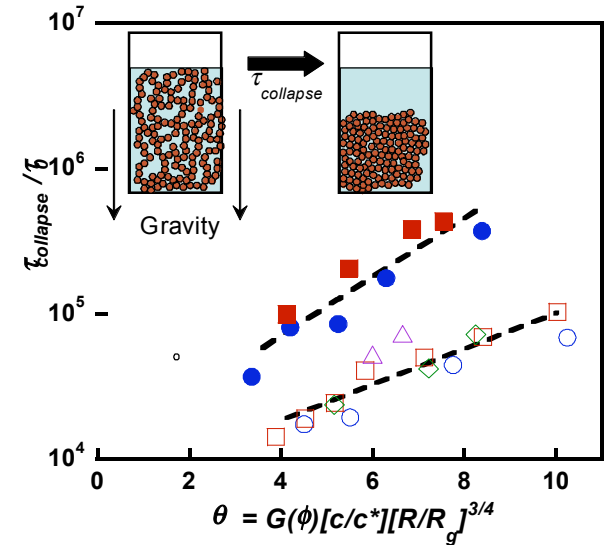


“Patchy”



**Transient Gel Collapse**

*Zukoski, KSS, JPCM, 2006*



**Polymer Melts and Glasses**

*Aging, Stress, Yield*  
*Strain Harden,*  
*Rejuvenate, ...*

