Fluctuations in Glassy Systems

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Ludovic Jaubert - ENS Lyon
Gabriel Fabricius - UNLP

Review in JSTAT (2007)

PRL 89, 217201 (2002)
PRL 88, 237201 (2002)
PRB 68, 134442 (2003)
J. Chem. Phys. 121, 10120 (2004)
JSTAT P01006 (2006)
JSTAT P05001 (2007)

NSF
DMR 0305482
DMR 0128922

BOSTON UNIVERSITY

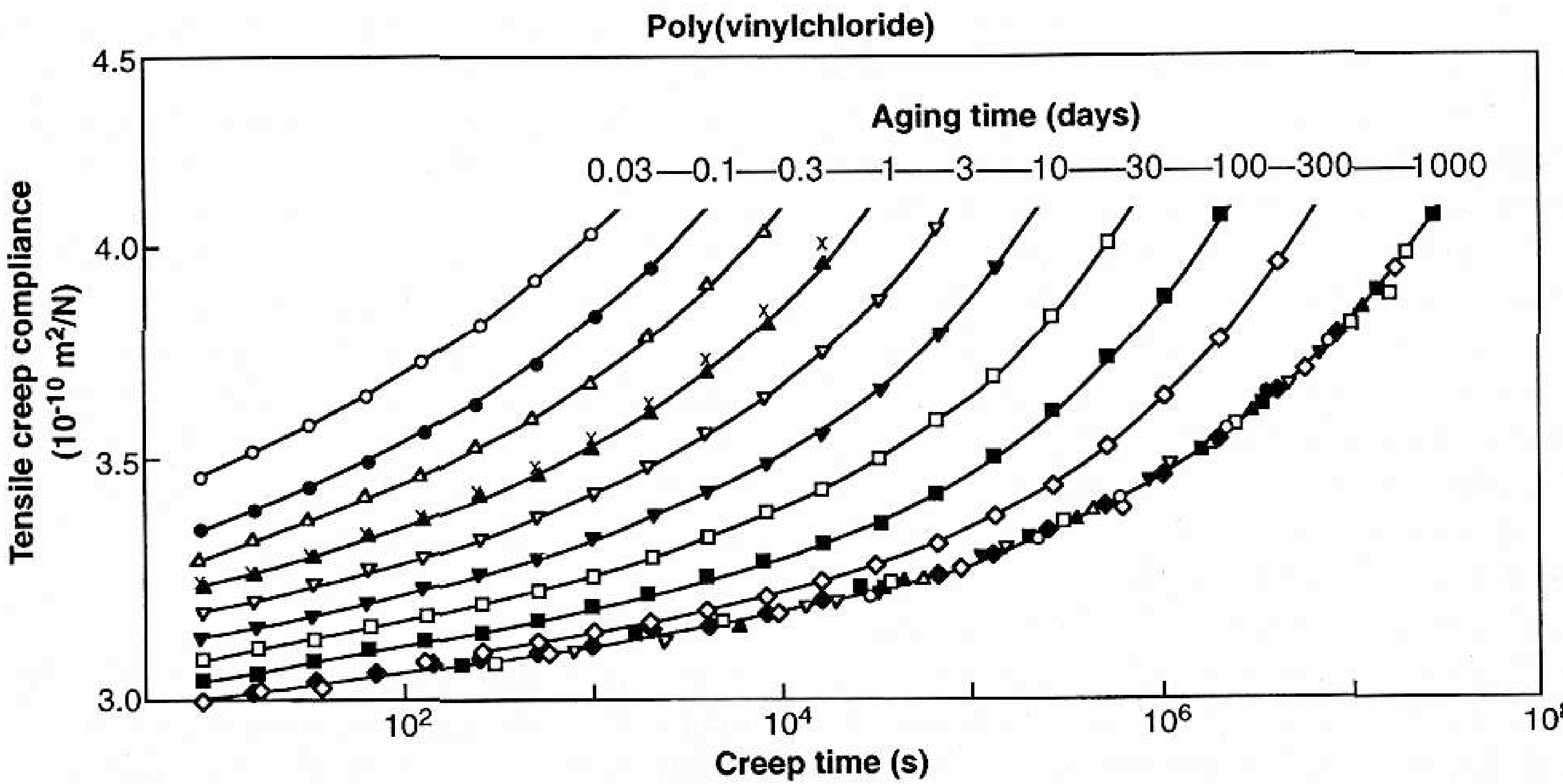
So ... Why Glasses?

Do we really know?

Do we need to fully answer *why glasses?* before we really further our understanding of glassy dynamics?

Here we take the point of view that, by starting from the fact that glassy systems exist (as nature presents us with concrete examples), we can then attempt to characterize whatever possible universal properties there are in glassy dynamics.

Physical aging



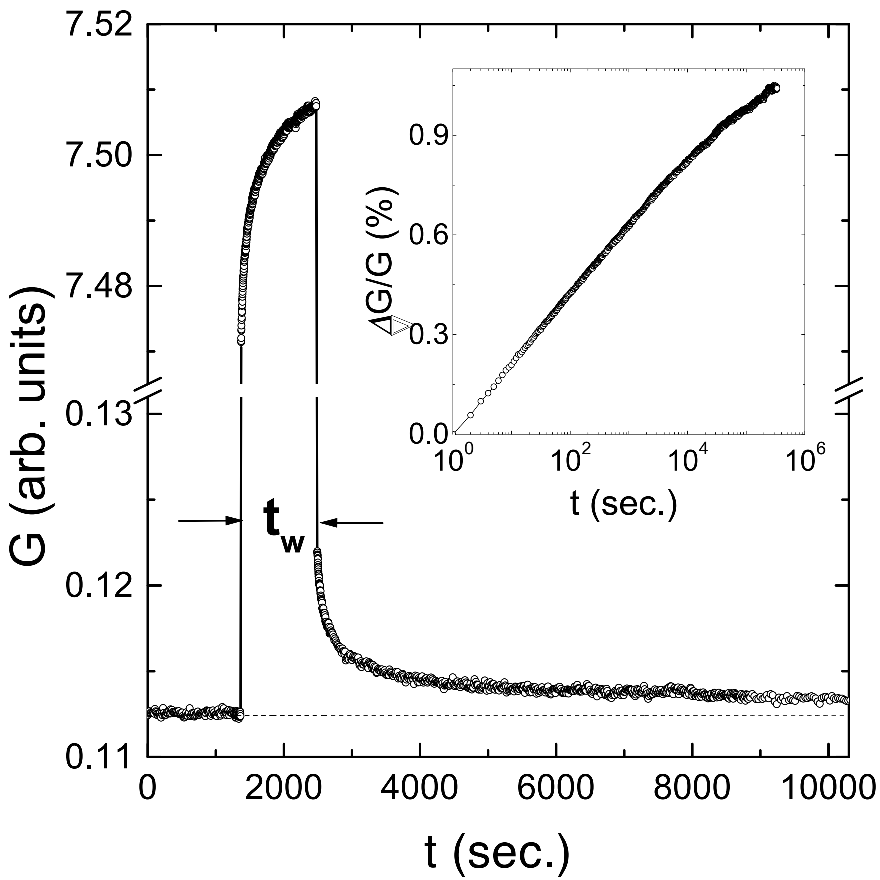
Source: L.C.E. Struik, *Physical aging in amorphous polymers and other materials*, Elsevier, Amsterdam (1978)

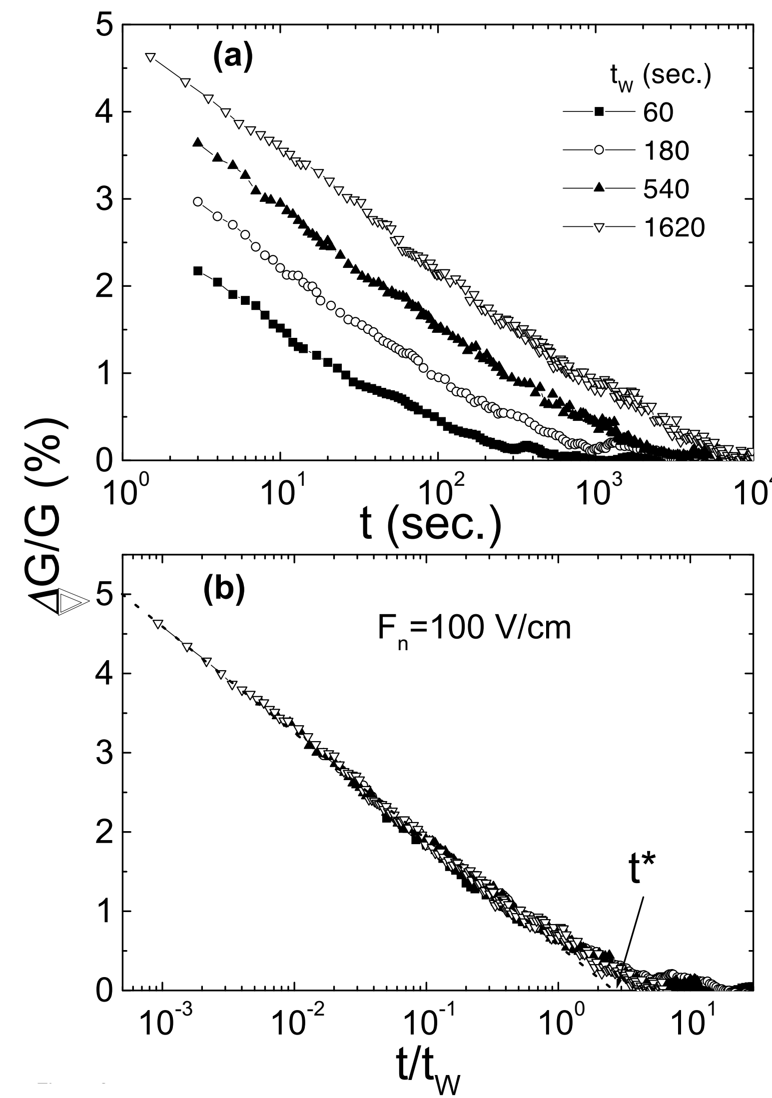
Physical aging II

2D electron glass

Orlyanchik & Ovadyahu, PRL (2004)

2D thin films of crystalline In_2O_{3-x}





Source: Orlyanchik & Ovadyahu, PRL (2004)

Conceptual approach:

Analogy:

Phil Anderson in
Concepts in Solids

Why crystals?

Even without the answer,

If crystals exist, then one has

Bloch states,
quasimomentum,
phonon spectrum,...

Why glasses?

Even without the answer,

If glass exist, then one has

Aging dynamics,
spatial heterogeneities,
universal scalings,...

Guiding principle: symmetry

Invariance of an effective dynamical action under uniform reparametrizations of the time scales

Invariance at the level of eqs. of motion was long noted

S. L. Ginzburg, Zh. Eksp. Teor. Fiz. 90, 754 (1986) [Sov. Phys. JETP 63, 439 (1986)].
L. B. Ioffe, Phys. Rev. B 38, 5181 (1988).
L. F. Cugliandolo and J. Kurchan, J. Phys. A 27, 5749-5772 (1994).

Time reparametrization: $t \to h(t)$

$$C_{AG}(t_1,t_2) \to \tilde{C}_{AG}(t_1,t_2) = C_{AG}(h(t_1),h(t_2))$$

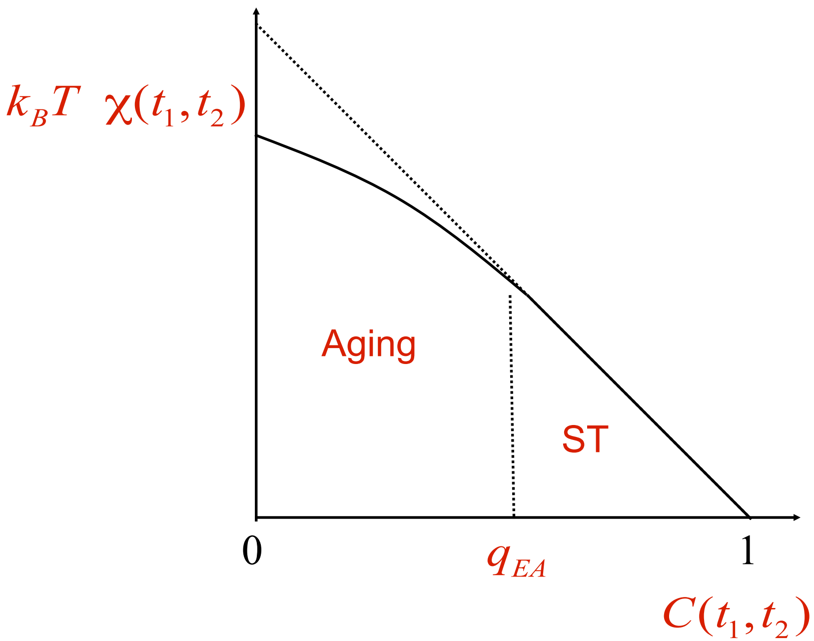
$$R_{AG}(t_1,t_2) \to \tilde{R}_{AG}(t_1,t_2) = \frac{\partial h}{\partial t_2} R_{AG}(h(t_1),h(t_2))$$

“Many solutions” of eq. of motion!

This “annoyance”, we claim, has physical meaning and consequences

FDT plots:

Susceptibility vs. Correlation $\chi(t_1,t_2) = \int_{t_1}^{t_2} dt' R(t,t')$ vs. $C(t_1,t_2)$



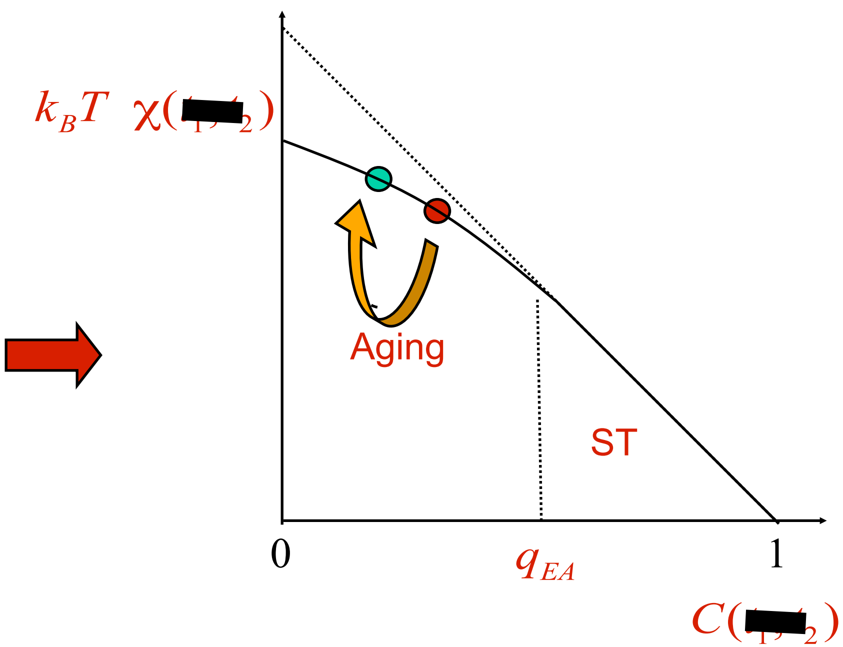
$$R_{AG}(t_1,t_2) = \beta_{\text{eff}}(C_{AG})\,\frac{\partial}{\partial t_2} C_{AG}(t_1,t_2)$$

Off-Equilibrium Fluctuation Dissipation Relation - OEFDR

Time reparametrization: $t \to h(t)$

$$C_{AG}(t_1,t_2) \to \tilde{C}_{AG}(t_1,t_2) = C_{AG}(h(t_1),h(t_2))$$

$$R_{AG}(t_1,t_2) \to \tilde{R}_{AG}(t_1,t_2) = \frac{\partial h}{\partial t_2} R_{AG}(h(t_1),h(t_2))$$



Reparametrization invariance holds at the level of the *action*, not only *eqs. of motion*
True for ***both*** *short* range and *infinite* range models.

Chamon, Kennett, Castillo, and Cugliandolo - PRL 2002a

under $t \to h(t)$

$$Q_i(t_1,t_2) \to \tilde{Q}_i(t_1,t_2) = \left(\frac{\partial h}{\partial t_1}\right)^{\Delta_A} \left(\frac{\partial h}{\partial t_2}\right)^{\Delta_R} Q_i(h(t_1),h(t_2))$$

Δ_A advanced dimension
Δ_R retarded dimension

$$\Downarrow$$

Reparametrization Group (RpG)

Kennett and Chamon - PRL 2001

$$S_{AG}[Q_{AG}] \to \tilde{S}_{AG}[\tilde{Q}_{AG}] = S_{AG}[\tilde{Q}_{AG}]$$

Two assumptions: 1) unitarity and causality and 2) separation of time scales

Detailed proof for soft spin version of Edwards-Anderson model

Castillo, in preparation

Consequences of Reparametrization Invariance ...

What are the consequences of reparametrization invariance?

basis for a
Dynamical theory of local aging in glasses

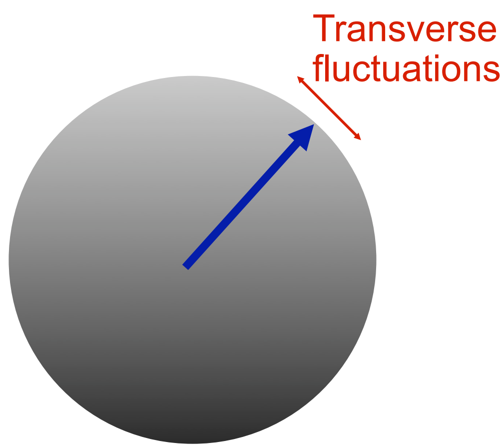
Global symmetry

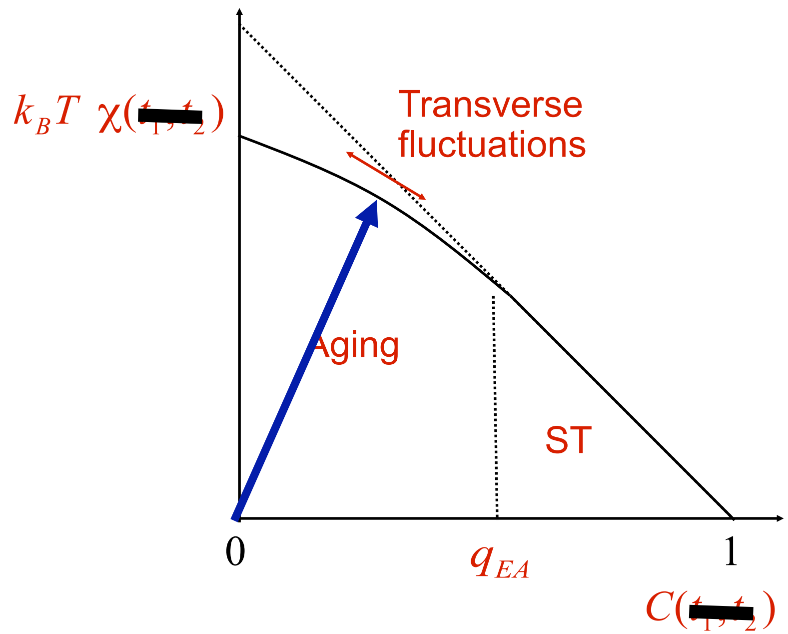
$t \to h(t)$

Slow spatial fluctuations
local reparametrizations

$h(t) \to h(t, \vec{x})$

$O(N)$





$$C_{\vec{x}}(t_1, t_2) = C\big(h(t_1, \vec{x}), h(t_2, \vec{x})\big)$$

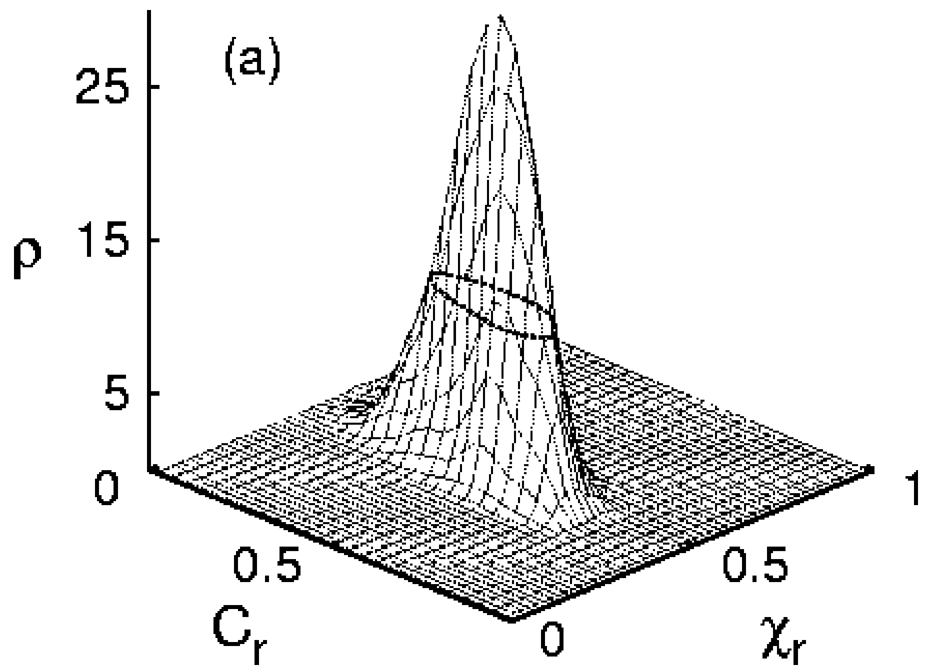
$$\chi_{\vec{x}}(t_1, t_2) = \chi\big(h(t_1, \vec{x}), h(t_2, \vec{x})\big)$$

What are the consequences of reparametrization invariance?

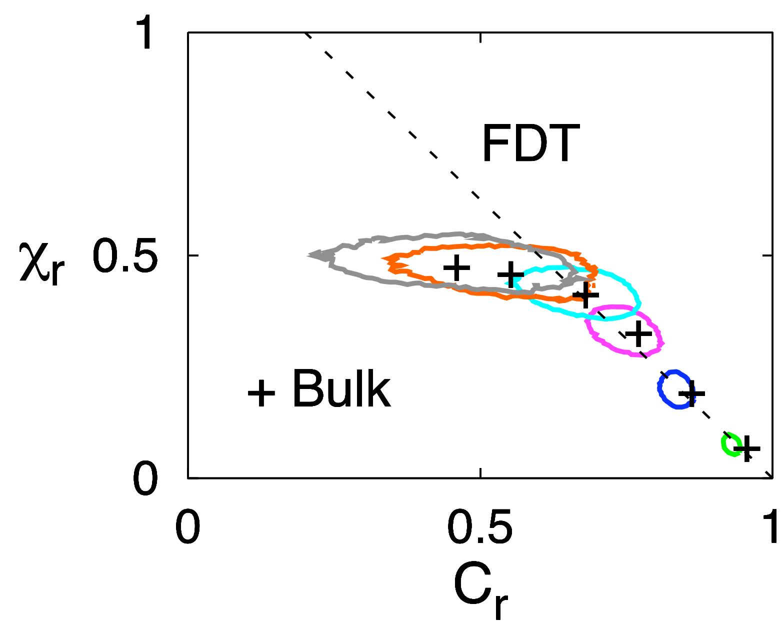
1. A growing dynamical correlation length.
2. Scaling of the pdf of local two-time functions.
3. Functional form of the pdf of local two-time functions.
4. Triangular relations between two-time functions.
5. Scaling relations for general multi-time functions.
6. Local fluctuation-dissipation relations.
7. Infinite susceptibilities.

Joint probability distribution function $\rho(C_{\vec{r}}, \chi_{\vec{r}})$.

$L = 64,\ T = 0.72\,T_c,\ V = 13^3,\ t_w = 4 \times 10^4$ MC steps.



Surface plot of the joint PDF for $t/t_w = 16$. Points within the shown contour account for 2/3 of the total probability.

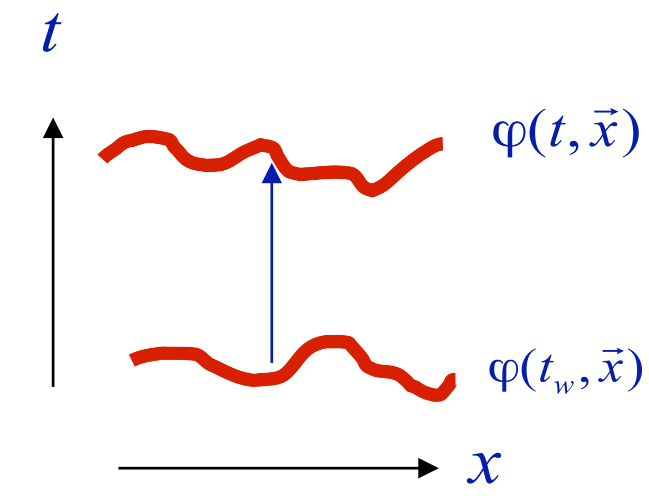


Time evolution of $\rho(C_r(t,t_w), \chi_r(t,t_w))$, for $t/t_w = 1.00005,\ 1.001,\ 1.06,\ 2,\ 8,\ 32$ (from right to left, 2/3-probability contours). The crosses are the bulk $\chi(C)$ for the same pairs of times.

Random surfaces and local correlations

local correlations:

$$C_r^{\mathrm{S}}(t,t_w) \sim f\left(\frac{h_r(t_w)}{h_r(t)}\right) = f\left(e^{-[\phi_r(t)-\phi_r(t_w)]}\right) = f\left(e^{-\Delta\phi_r|_{t_w}^{t}}\right)$$



i) The action must be invariant under a global time reparametrization $t \to s(t)$.

ii) If our interest is in short-ranged problems, the action must be written using local terms. The action can thus contain products evaluated at a single time and point in space of terms such as $\phi_r(t)$, $\dot{\phi}_r(t)$, $\nabla\phi_r(t)$, $\nabla\dot{\phi}_r(t)$, and similar derivatives.

iii) The scaling form of C_r^S is invariant under $\phi_r(t) \to \phi_r(t) + \Phi_r$, with Φ_r independent of time. Thus, the action must also contain this symmetry.

iv) The action must be positive definite.

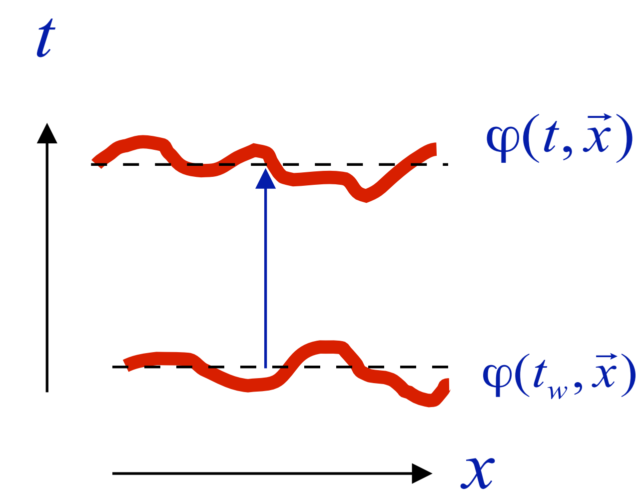
$$\Rightarrow S_{\mathrm{EFF}} = K \int d^d r \int dt \, \frac{\left(\nabla\dot{\phi}_r(t)\right)^2}{\dot{\phi}_r(t)}$$

Random surfaces and local correlations

$$S_{\rm EFF} = K \int d^d r \int ds \, \frac{(\nabla \dot{\varphi}_r(s))^2}{\dot{\varphi}_r(s)} = K \int d^d r \int ds \, (\nabla \psi_r(s))^2$$

where $\psi_r = \sqrt{\dot{\phi}_r}$

$s \equiv \ln h(t)$



$$\Delta\phi_r|_{t_w}^{t} = \int_{s(t_w)}^{s(t)} ds \, \dot{\phi}_r(s) = \int_{s(t_w)}^{s(t)} ds \, \psi_r^2(s)$$

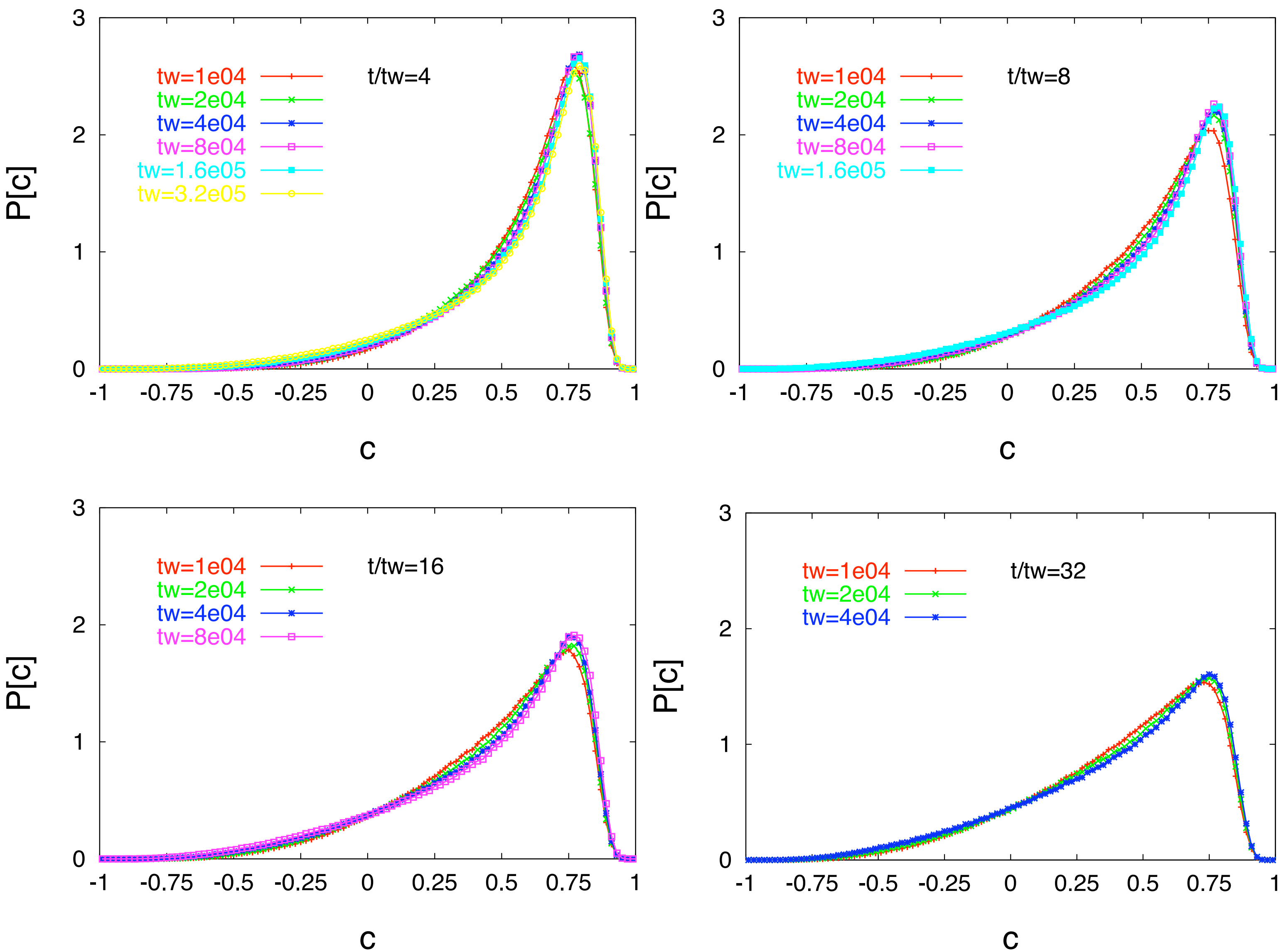
from which: $\langle \Delta\phi_{r_1}|_{t_w}^{t} \, \Delta\phi_{r_2}|_{t_w}^{t} \cdots \Delta\phi_{r_N}|_{t_w}^{t} \rangle_c = [s(t) - s(t_w)] \, \mathcal{F}(r_1, r_2, \ldots, r_N)$

$$= \ln\left(\frac{h(t)}{h(t_w)}\right) \mathcal{F}(r_1, r_2, \ldots, r_N)$$

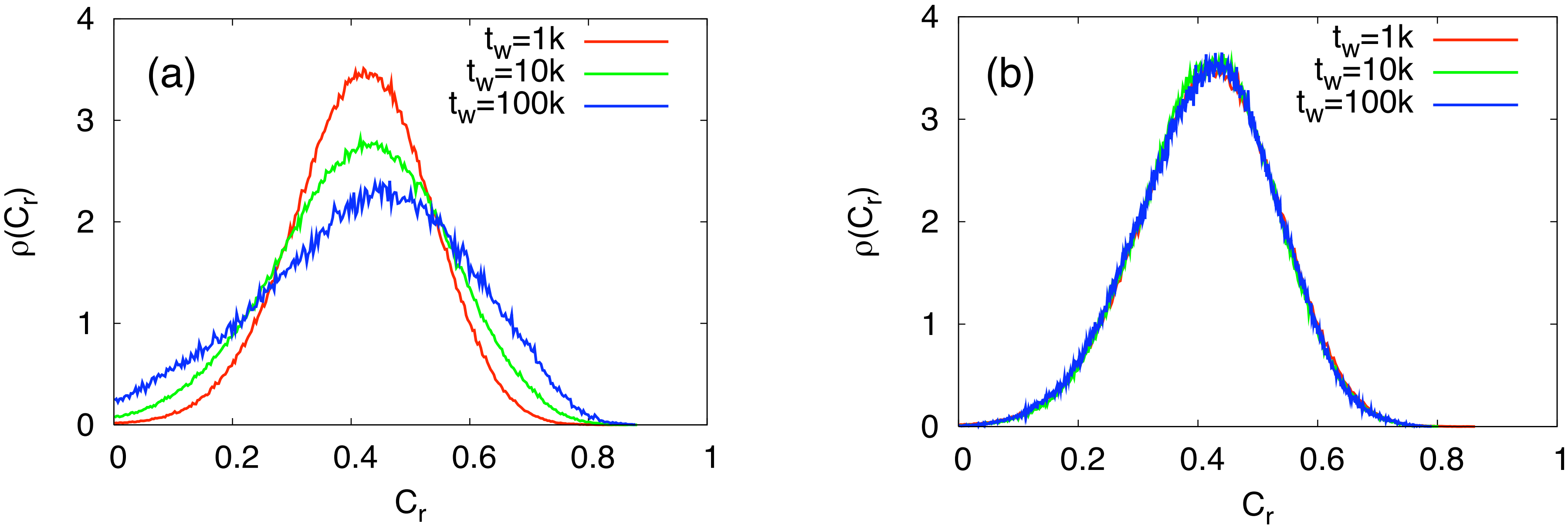
collapse of PDFs at same global correlation

Scaling and collapse of the distribution of local correlations

Bulk correlations: $C(t,t_w) \approx f(t/t_w)$
Local correlation distribution: $P_{t,t_w}[C_{\text{local}}] \approx P_{t/t_w}[C_{\text{local}}]$



Correction from the growing correlation length



PDF of local coarse-grained correlations Cr at different times t and tw in the 3d Edwards-Anderson model with L = 100 at T = 0.6 < Tc. The waiting-times are given in the key and the global correlation is fixed to C = 0.4 < q_{EA}.

(a) The coarse-graining boxes have linear size ℓ = 9 in all cases. The curves do not collapse, a slow drift with increasing tw is clear in the figure.

(b) Variable coarse-graining length ℓ chosen so as to held ℓ/ξ approximately constant. The collapse improves considerably with respect to panel (a).

Triangular relations

$$C_{\vec{x}}(t_1, t_2) \approx q_{EA} \times f\left(\frac{h(\vec{x}, t_1)}{h(\vec{x}, t_2)}\right) \qquad \text{or} \qquad \frac{h(\vec{x}, t_1)}{h(\vec{x}, t_2)} \approx f^{-1}\left(C_{\vec{x}}(t_1, t_2)/q_{EA}\right)$$

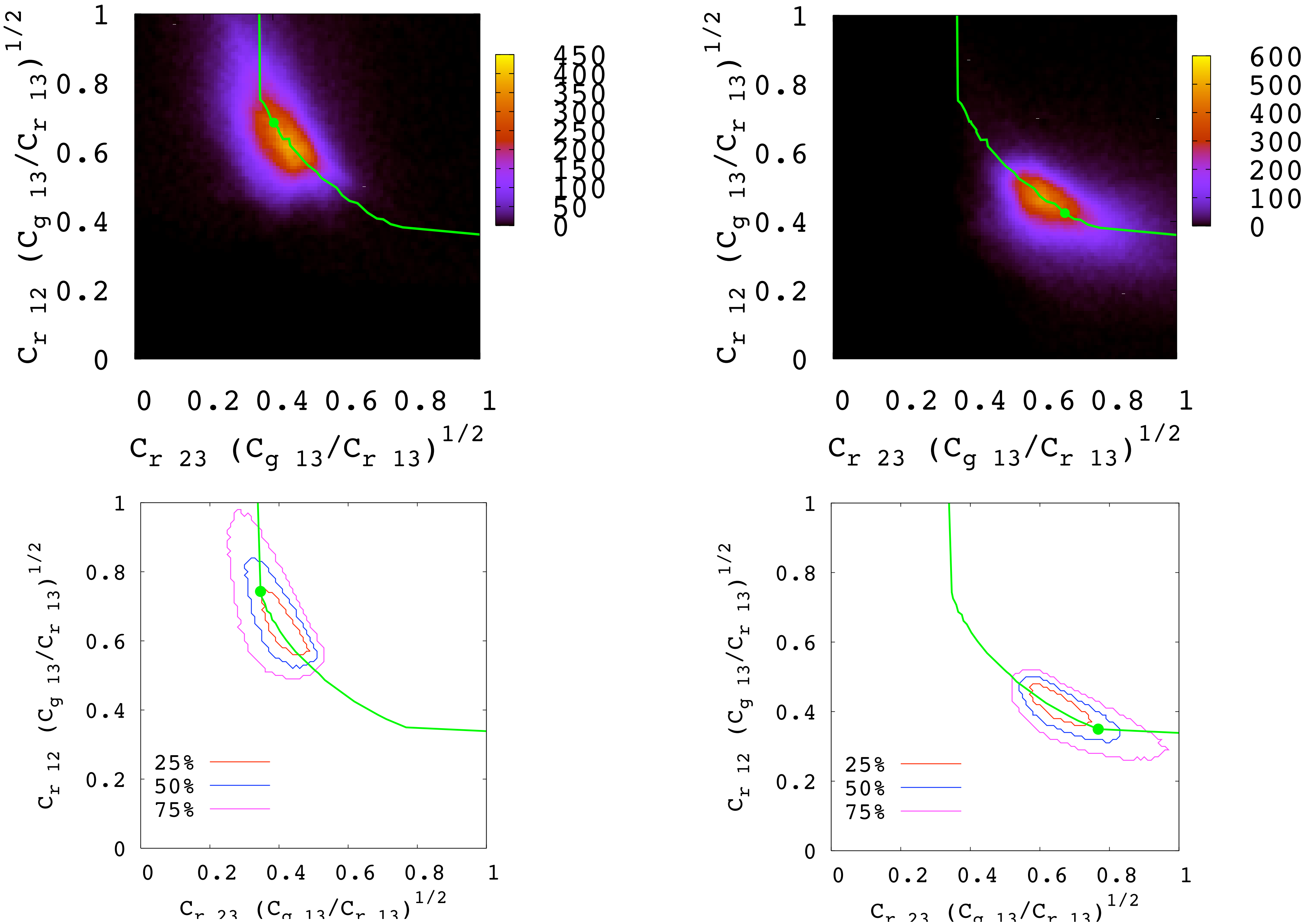
Take $t_1 < t_2 < t_3$

$$1 = \frac{h(\vec{x}, t_1)}{h(\vec{x}, t_2)} \times \frac{h(\vec{x}, t_2)}{h(\vec{x}, t_3)} \times \frac{h(\vec{x}, t_3)}{h(\vec{x}, t_1)} \approx \frac{f^{-1}\left(C_{\vec{x}}(t_1, t_2)/q_{EA}\right) \times f^{-1}\left(C_{\vec{x}}(t_2, t_3)/q_{EA}\right)}{f^{-1}\left(C_{\vec{x}}(t_1, t_3)/q_{EA}\right)}$$

If $f^{-1}(z) \sim z^{\lambda}$

$$\frac{C_{\vec{x}}(t_1, t_2) \times C_{\vec{x}}(t_2, t_3)}{C_{\vec{x}}(t_1, t_3)} \approx q_{EA}$$

Numerical check of triangular relations



Colloidal Glasses

Data from Weeks *et al.*

Particle position to spin mapping

Lagrange *vs.* Euler

$$\vec{R}_\alpha(t) \to S_i(t)$$

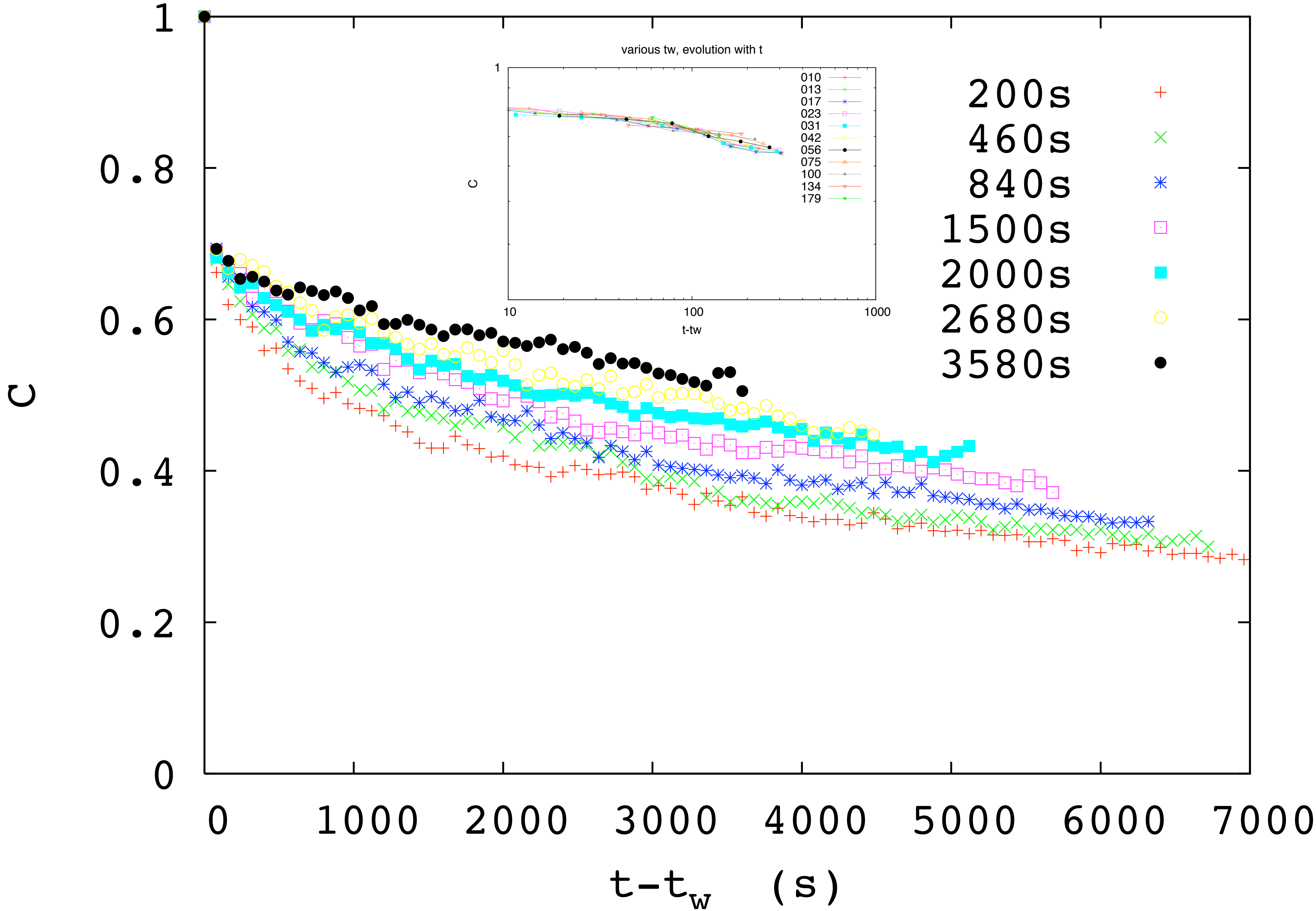
Lack of positional order ➡ $\langle S_i(t) S_j(t) \rangle \to 0$

exponentially fast in $|i - j|$

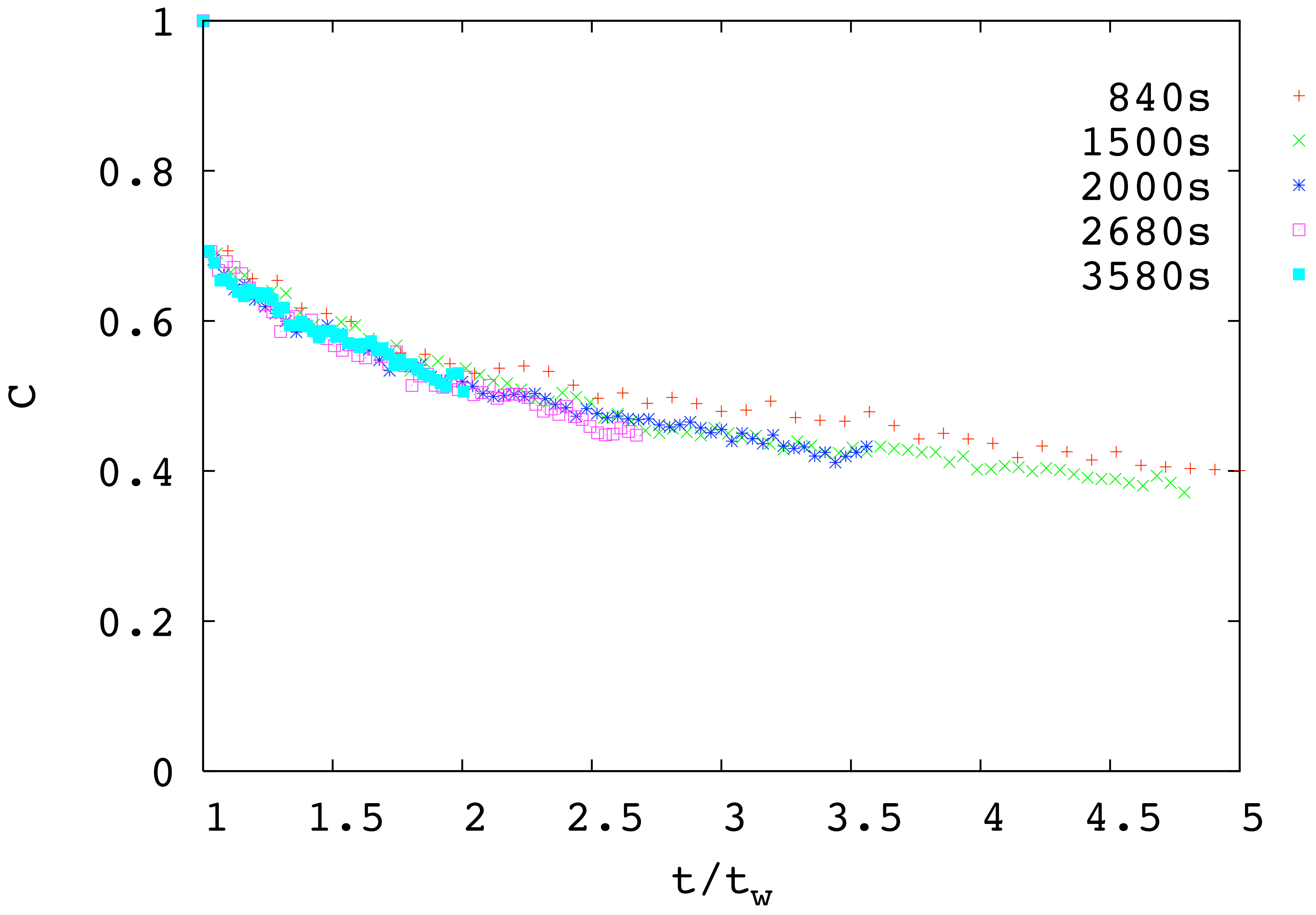
Glass type order ➡ $\langle S_i(t) S_i(t_w) \rangle$

Edwards-Anderson type ordering

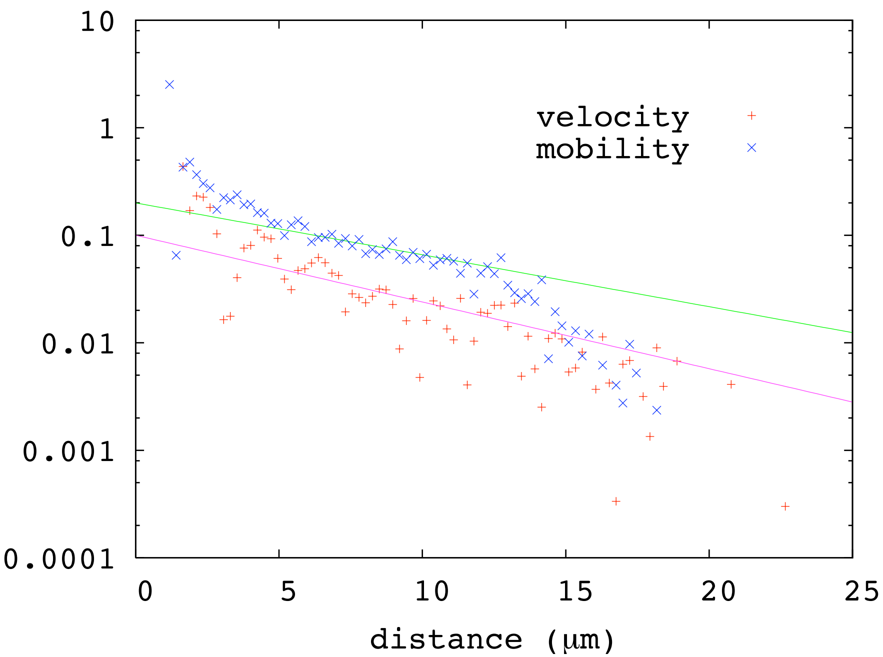
Global correlation and aging

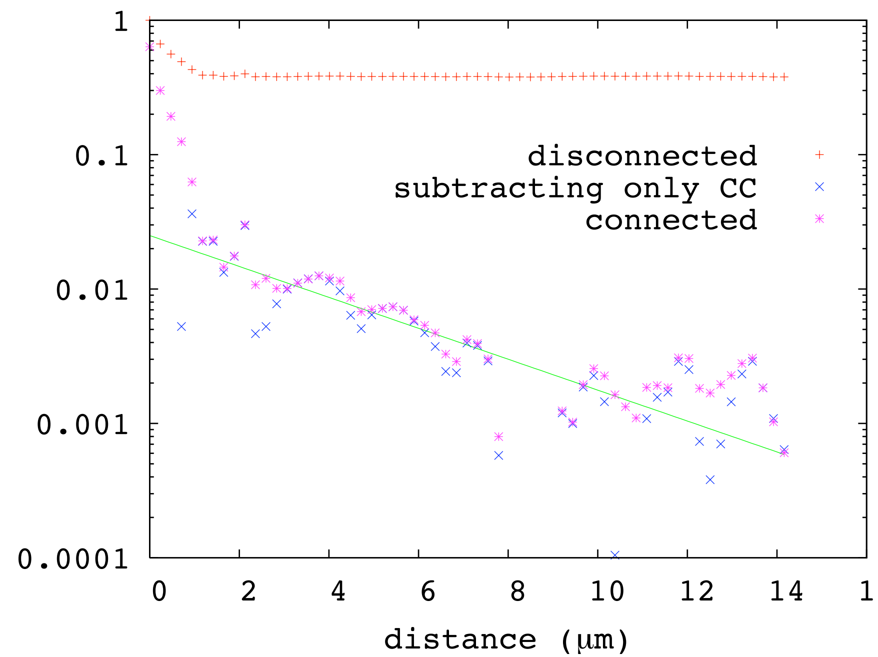


Global correlation and aging



Correlation length





Weeks, Crocker & Weitz, JPCM 07 -- Lagrange approach

$$S_{\vec{u}}(t_w, t; \Delta r) = \frac{1}{\mathcal{N}_{\vec{u}}} \sum_{|\vec{r}_\alpha - \vec{r}_\beta| = \Delta r} \vec{u}_\alpha \cdot \vec{u}_\beta$$

$$S_{\delta u}(t_w, t; \Delta r) = \frac{1}{\mathcal{N}_{\delta u}} \sum_{|\vec{r}_\alpha - \vec{r}_\beta| = \Delta r} \delta u_\alpha \ \delta u_\beta$$

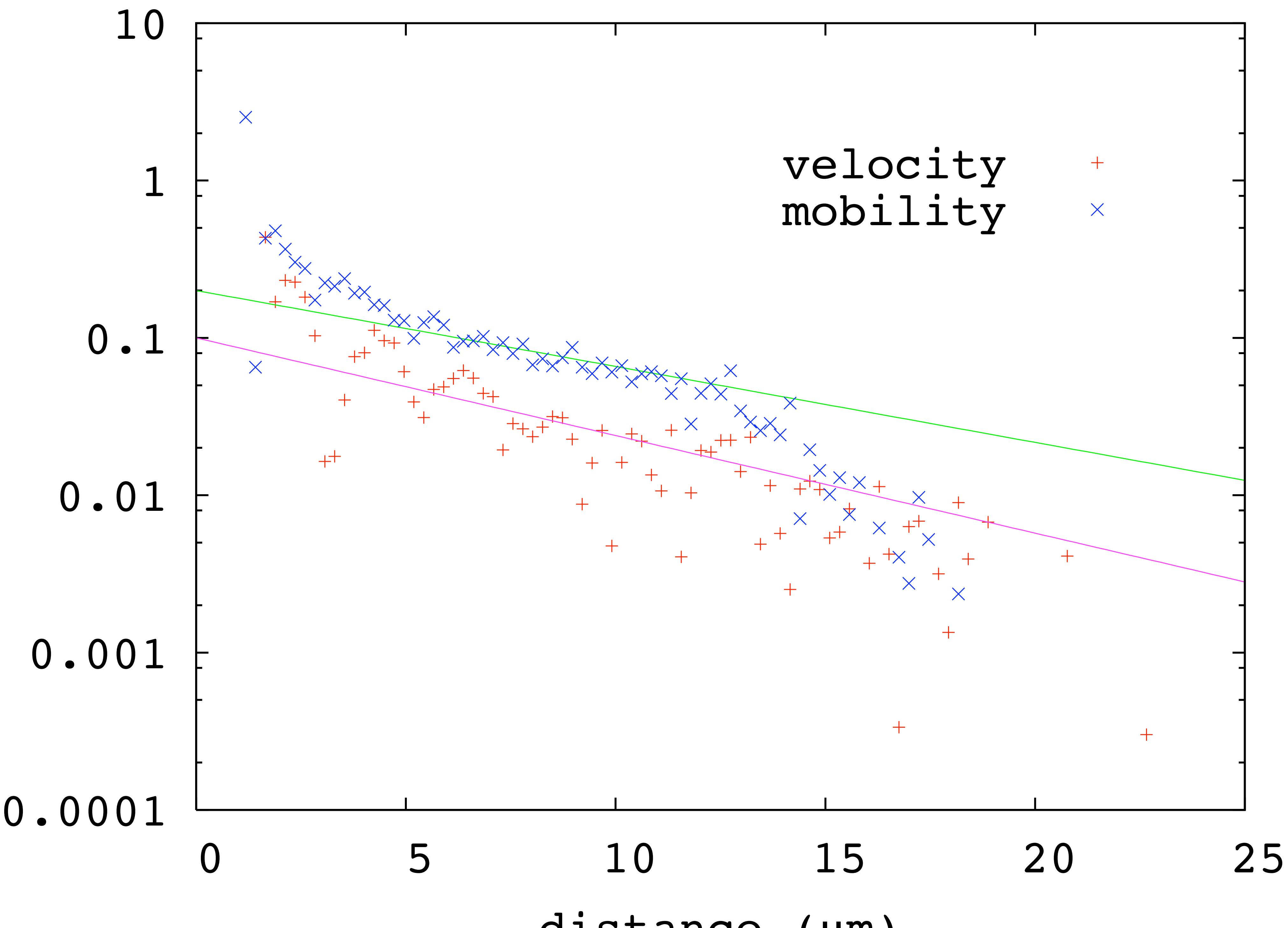
$$\delta u_\alpha = |\vec{u}_\alpha| - \frac{1}{N_{\rm part}} \sum_\alpha |\vec{u}_\alpha|$$

Euler approach

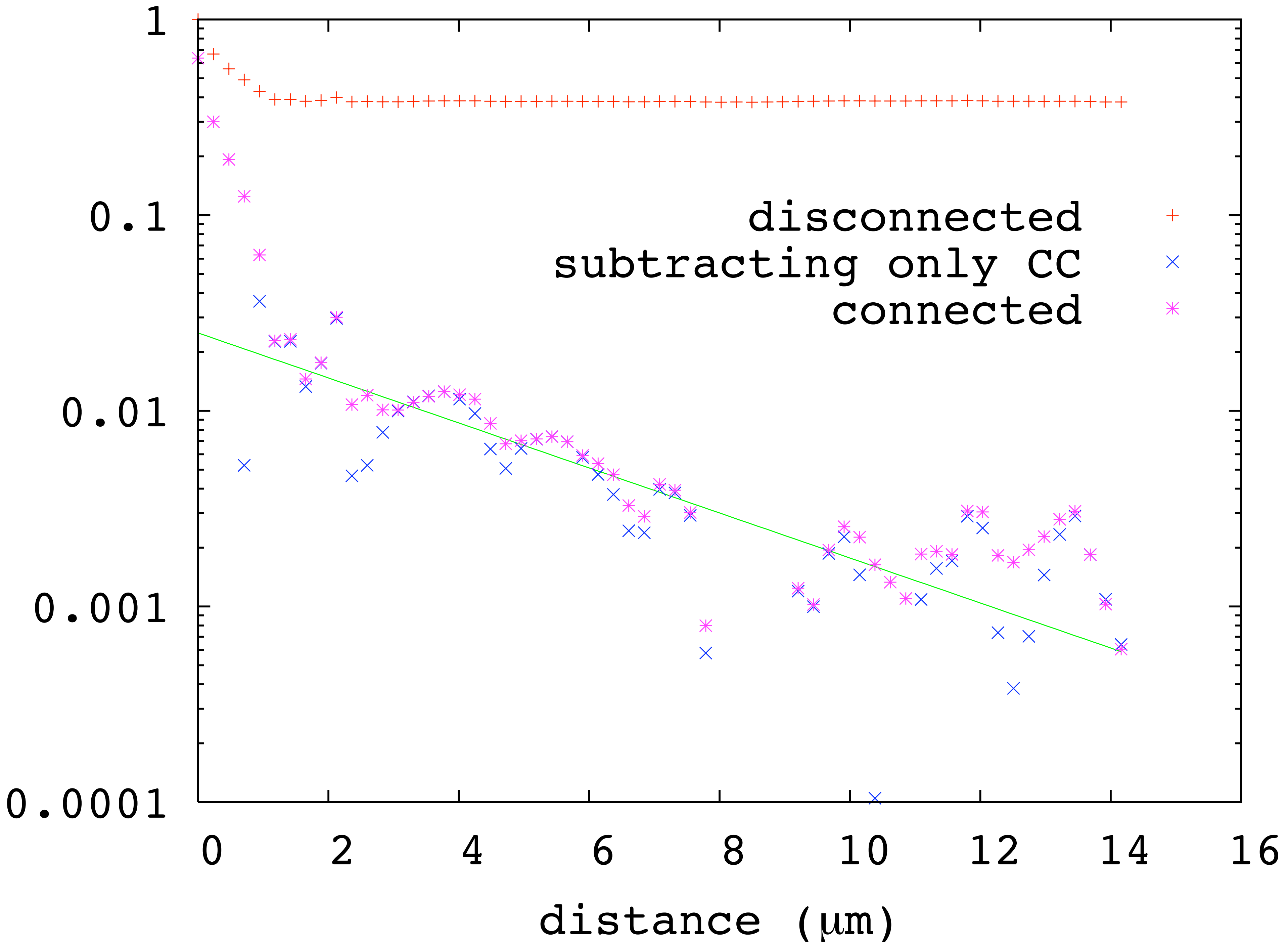
$$S_4(t_w, t; \vec{\Delta r}) = \frac{1}{\mathcal{N}_{\rm sites}} \sum_{\vec{r}_i - \vec{r}_j = \vec{\Delta r}} S_i(t)\, S_i(t_w)\, S_j(t)\, S_j(t_w)$$

$$C_2(t_w, t; \vec{\Delta r}) = \frac{1}{\mathcal{N}_{\rm sites}} \sum_{\vec{r}_i - \vec{r}_j = \vec{\Delta r}} S_i(t)\, S_j(t_w)$$

$$\begin{aligned} S_4^c(t_w, t; \vec{\Delta r}) = S_4(t_w, t; \vec{\Delta r}) \quad & -[C_2(t_w, t; 0)]^2 \\ & - C_2(t_w, t; \vec{\Delta r})\ C_2(t_w, t; -\vec{\Delta r}) \\ & - C_2(t_w, t_w; \vec{\Delta r})\ C_2(t, t; \vec{\Delta r}) \end{aligned}$$



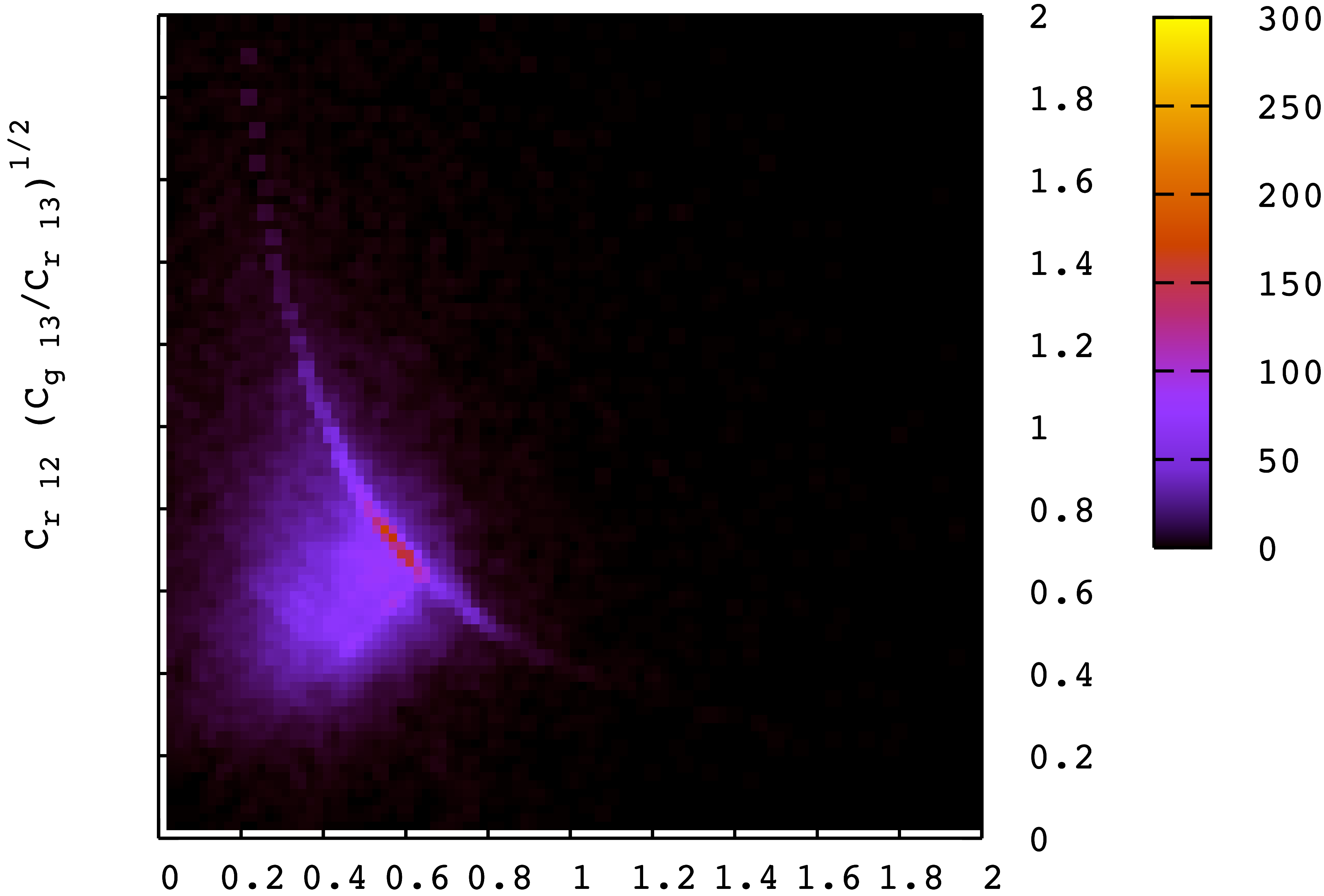
10
1
0.1
0.01
0.001
0.0001
0
5
10
15
20
25
velocity
mobility
distance (μm)



1
0.1
0.01
0.001
0.0001
0
2
4
6
8
10
12
14
16
distance (μm)
disconnected
subtracting only CC
connected

Triangular relations

$$t_1 = 1500s \quad t_3 = 6400s$$
$$t_2 = 1600s, 3600s, 6300s$$



Conclusions:

- Approach: understand dynamics *once* glasses are presented by nature
- Reparametrization invariance as guiding symmetry
- Soft reparametrization modes: aging is spatially heterogeneous in glasses
- Distribution of ages shows universality - collapse of PDF of local correlations
- Connection between the distribution of local correlations and the theory of random dynamical manifolds
- Triangular relations
- Colloidal glasses vs. Spin glasses

The End