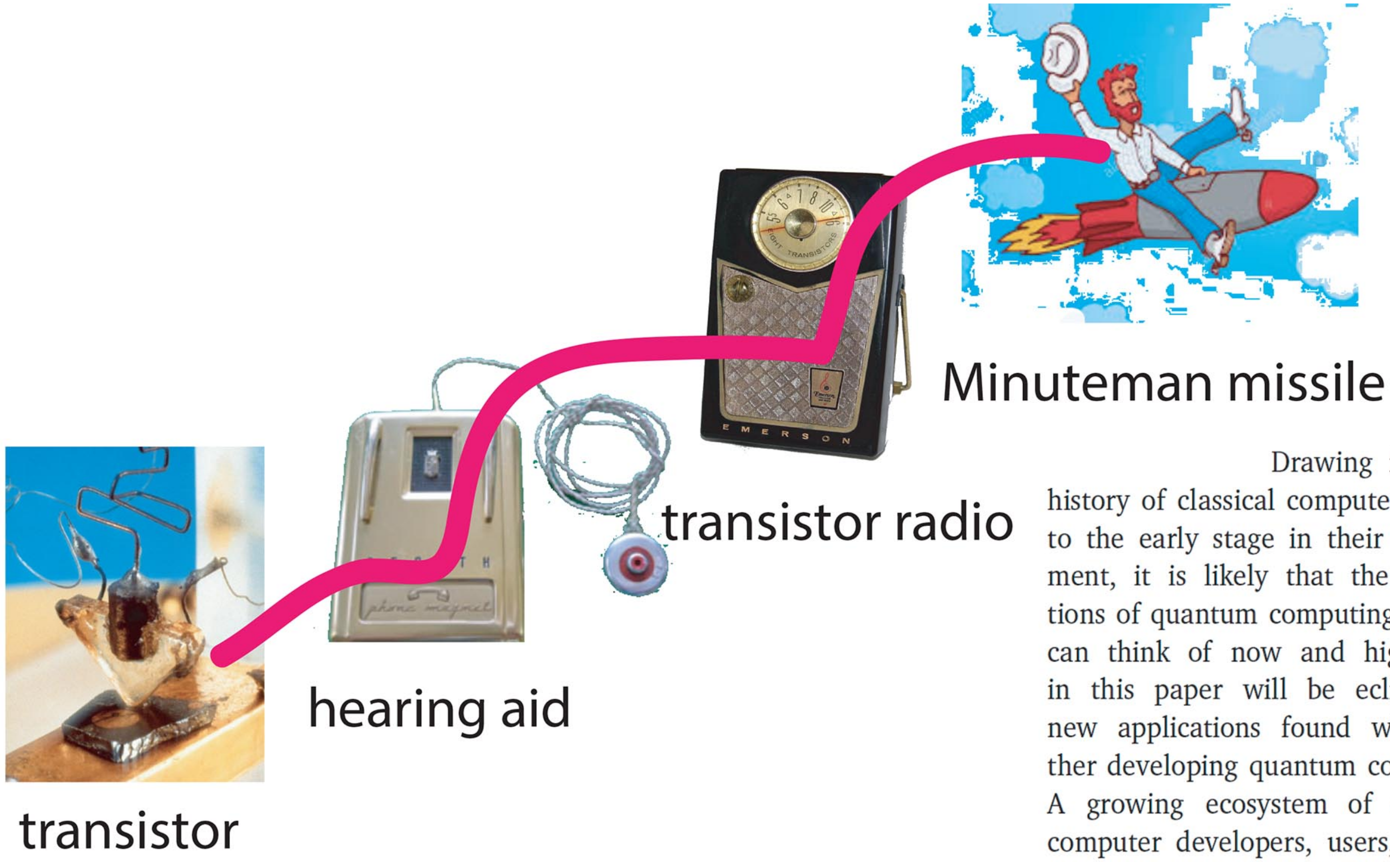
A computationally universal phase of quantum matter

Robert Raussendorf, UBC

joint work with D.-S. Wang, D.T. Stephen, C. Okay, and H.P. Nautrup

The hearing aid story



Drawing from the history of classical computers dating to the early stage in their development, it is likely that the applications of quantum computing that we can think of now and highlighted in this paper will be eclipsed by new applications found while further developing quantum computers. A growing ecosystem of quantum computer developers, users, and an educational system training necessary workforce will be critical in enabling a vibrant future quantum computing industry.

D. Maslov, Y. Nam and J. Kim, *An Outlook for Quantum Computing*,
Proc. IEEE **107**, 5 - 10 (2019).

A different story: GPS

- GPS requires GR, GR requires non-Euclidean geometry
- Was more than 2000 years in the making

Why have so few quantum algorithms been found?

The first possible reason is that quantum computers operate in a manner so different from classical computers that our techniques for designing algorithms and our intuitions for understanding the process of computation no longer work. The second reason is that there really might be relatively few problems for which quantum computers can offer a substantial speed-up over classical computers, and we may have already discovered many or all of the important techniques for constructing quantum algorithms.

P.W. Shor, *Why Havent More Quantum Algorithms Been Found?*, JACM 50, 87-90 (2003).

A computationally universal phase of quantum matter

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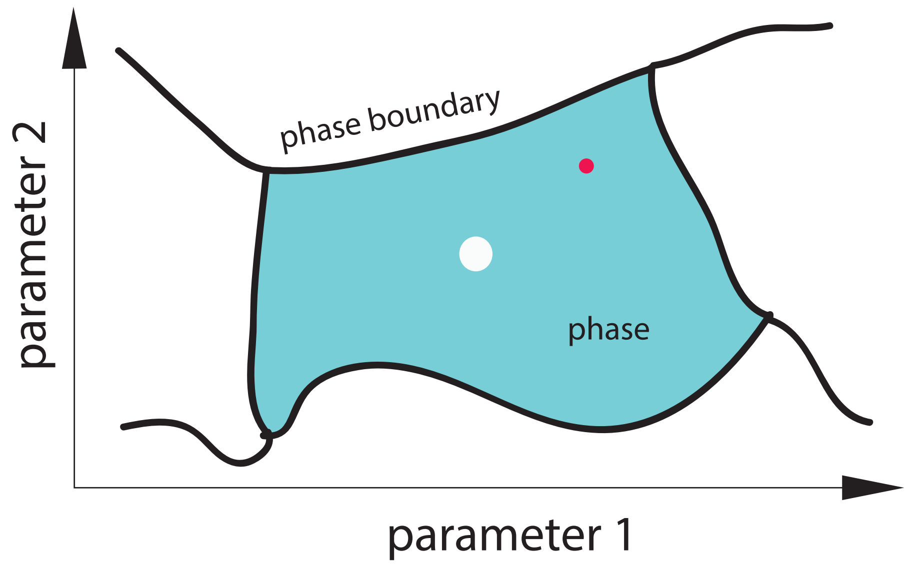
The 2D cluster state is a computationally universal “material”

The computational power of cluster states is utilized by measurement-based quantum computation.

An entire physical phase surrounding the cluster state is computationally universal

A quantum phase of spins in 2D

... which supports universal quantum computation

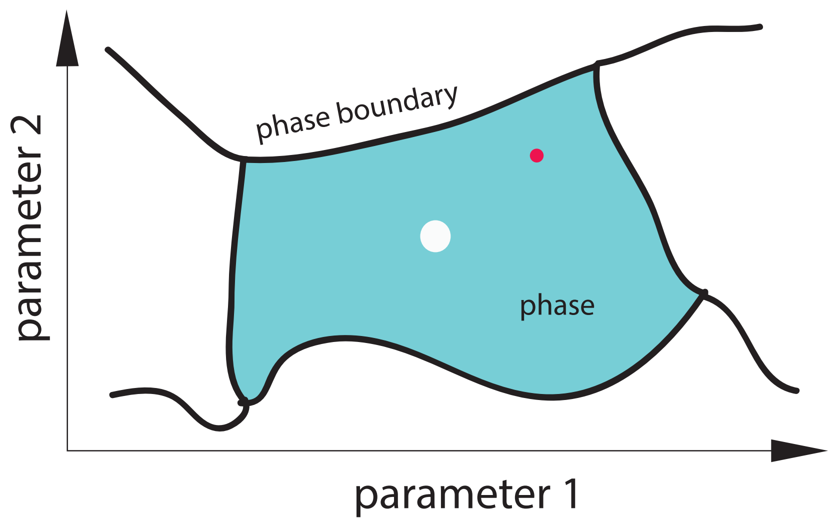


We consider:

- Phases of unique ground states of spin Hamiltonians, at $T = 0$,
- In the presence of symmetry,
- In spatial dimension 2 (a lattice of spin 1/2 particles)

A quantum phase of spins in 2D

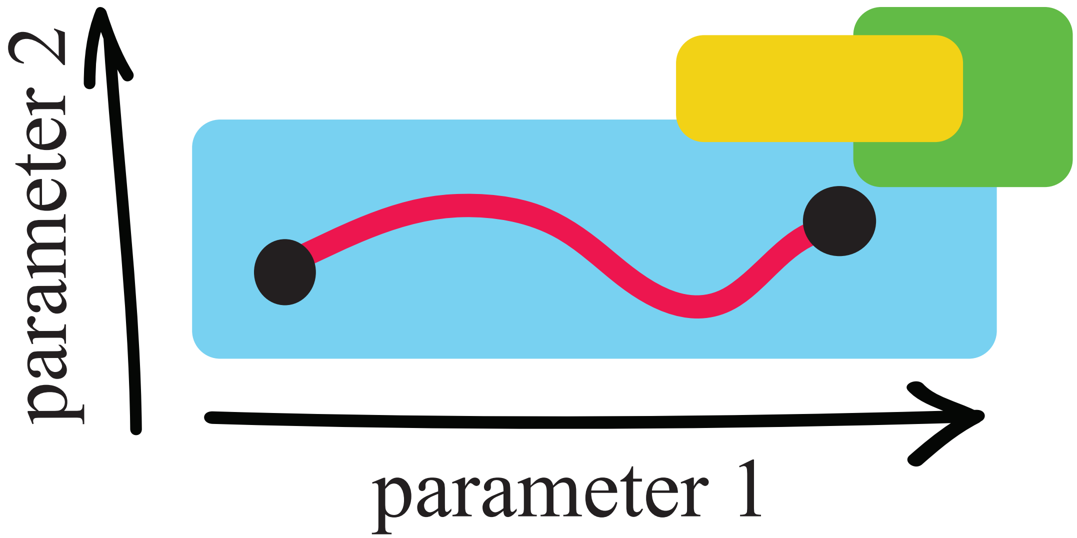
... which supports universal quantum computation



We show: for measurement-based quantum computation,

- There exists a quantum phase of matter which is universal for quantum computation
- The computational power is *uniform* across the phase.

Symmetry-protected topological order



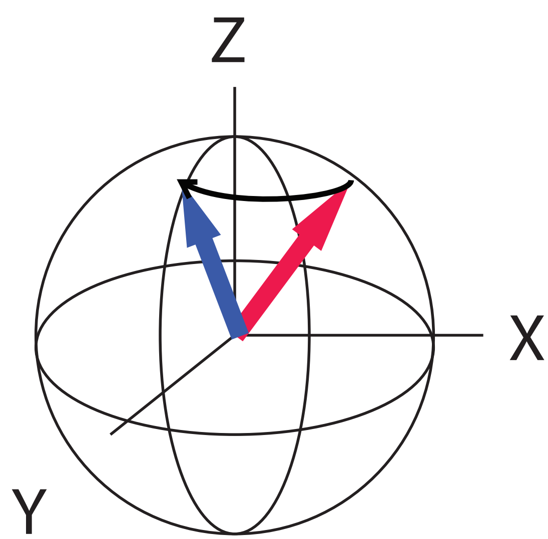
Two points in parameter space lie in the same SPT phase iff they can be connected by a path of Hamiltonians such that

1. At every point on the path, the corresponding Hamiltonian is invariant under G.

2. Along the path the energy gap never closes.

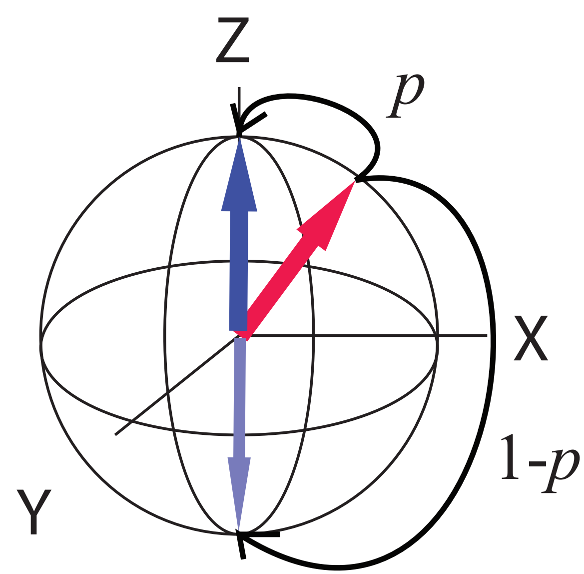
Measurement-based quantum computation

Unitary transformation



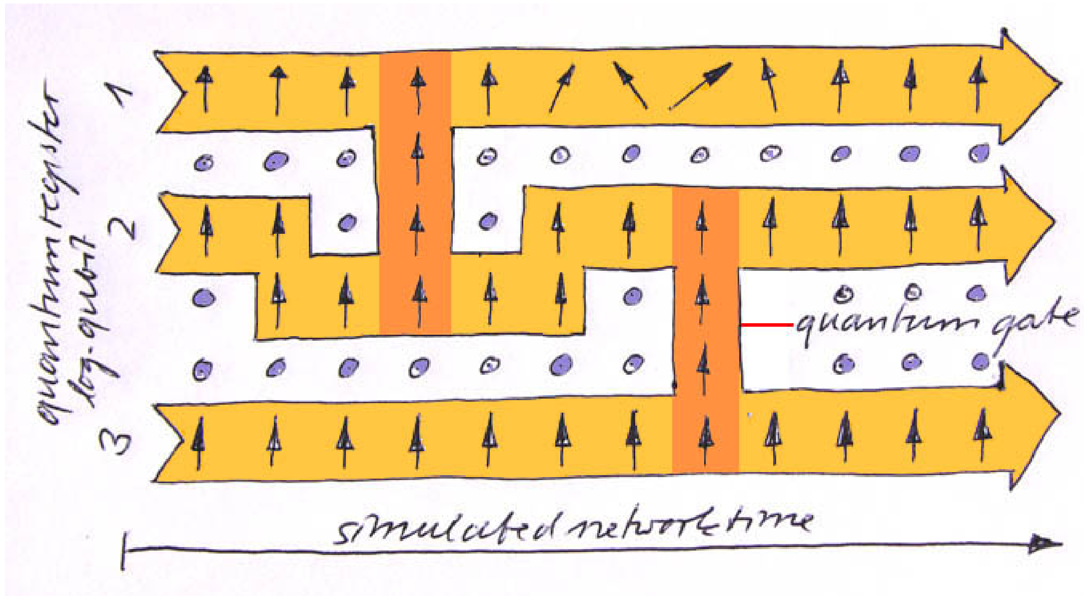
deterministic,
reversible

Projective measurement



probabilistic,
irreversible

Measurement-based quantum computation



measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha\, X + \sin\alpha\, Y$ ($\nearrow$)

- Information written onto the resource state, processed and read out by one-qubit measurements only.
- Universal computational resources exist: cluster state, AKLT state.

R. Raussendorf, H.-J. Briegel, Physical Review Letters 86, 5188 (2001).

Outline

1. "Computational phases of quantum matter":
   - Our motivation
   - A short history of the question

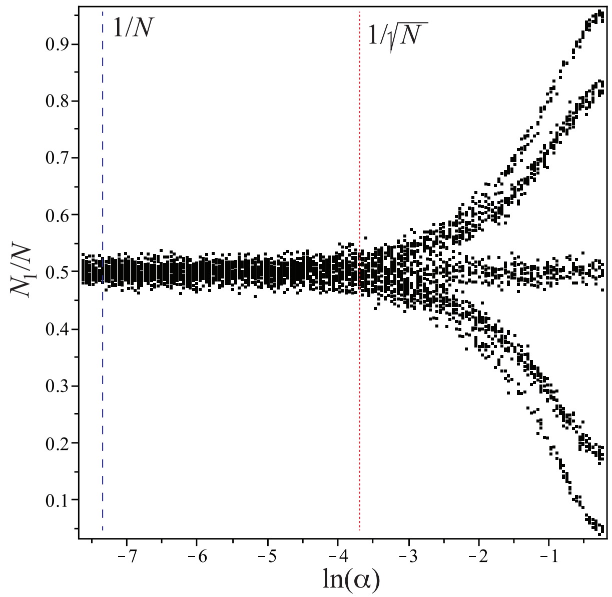
2. A computationally universal phase of matter in 2D

Part I:

A short history of

"computational phases of quantum matter"

Motivation #1: MBQC and symmetry

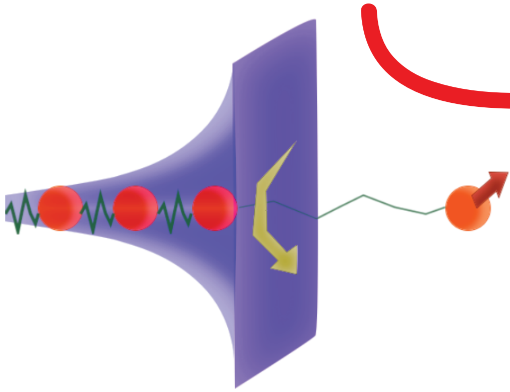


Can MBQC schemes be classified by symmetry, in a similar way as, say, elementary particles can?

If so, does this have a bearing on quantum algorithms?

1. Symmetry protects computation

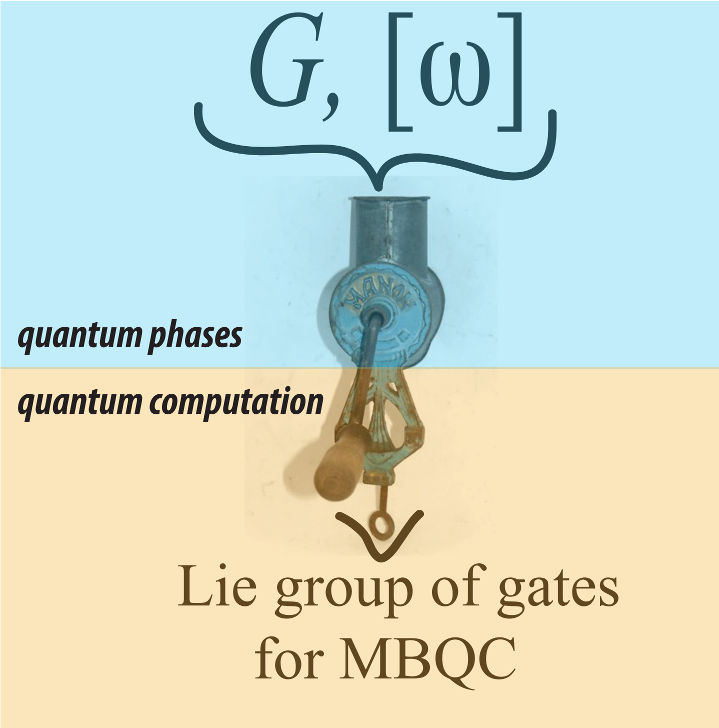
we observe low-maintenance features of the ground-code MQC in that this computation is doable without an exact (classical) description of the resource ground state as well as without an initialization to a pure state. It



It turns out these features are deeply intertwined with the physics of the 1D Haldane phase (cf. Fig. 1), that is well characterized as the symmmetry-protected topological order in a modern perspective [6, 7]. We believe our approach must bring the study of MQC, conventionally based on the analysis of the model entangled states (e.g., [1, 8, 9]), much closer to the condensed matter physics, which is aimed to describe characteristic physics based on the Hamiltonian.

A. Miyake, Phys. Rev. Lett. 105, 040501 (2010).

2. The SPT⇒MBQC meat grinder



Hints at the classification of MBQC schemes by symmetry.

J. Miller and A. Miyake, Phys. Rev. Lett. 114, 120506 (2015) [first 1D comp. phase].

A. Prakash and T.-C. Wei, Phys. Rev. A (2016).

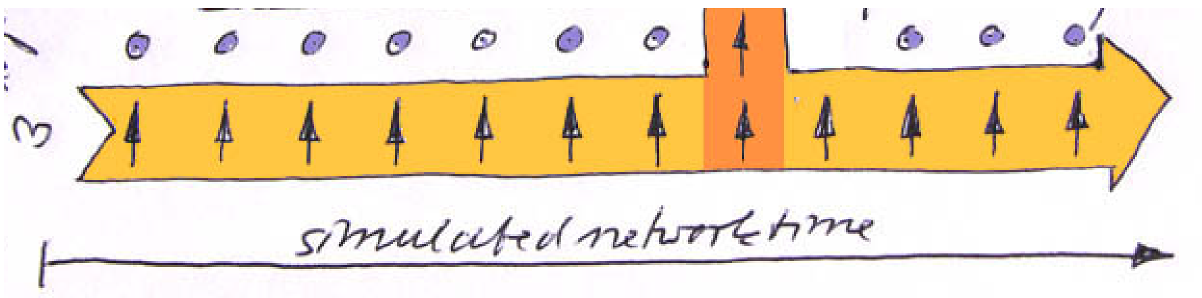
RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017).

Inspection

The above waypoints are about 1D systems.

1D is not sufficient for universal MBQC

here is why:



- MBQC in spatial dimension D maps to the circuit model in dimension $D-1$

$\Rightarrow$ Require $D \geq 2$ for universality.

Are there computationally universal quantum phases in two dimensions?

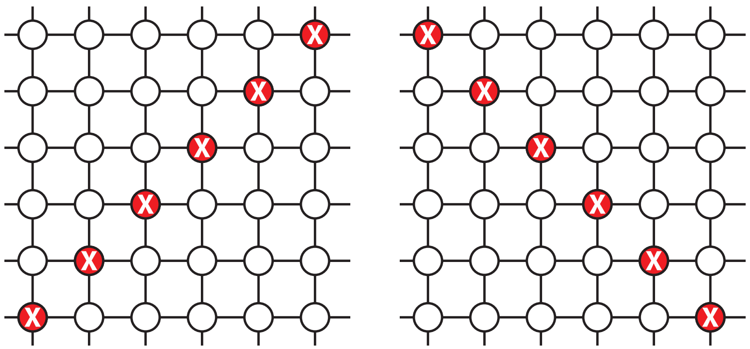
This talk describes one.

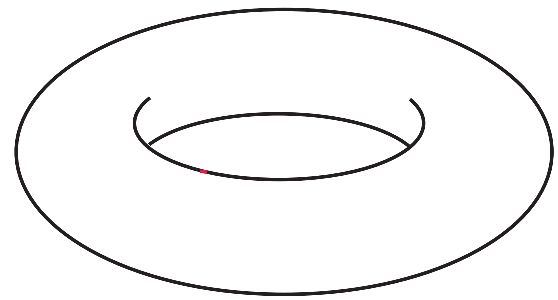
Part II:

A computationally universal SPT phase in 2D

Description of the 2D phase & result

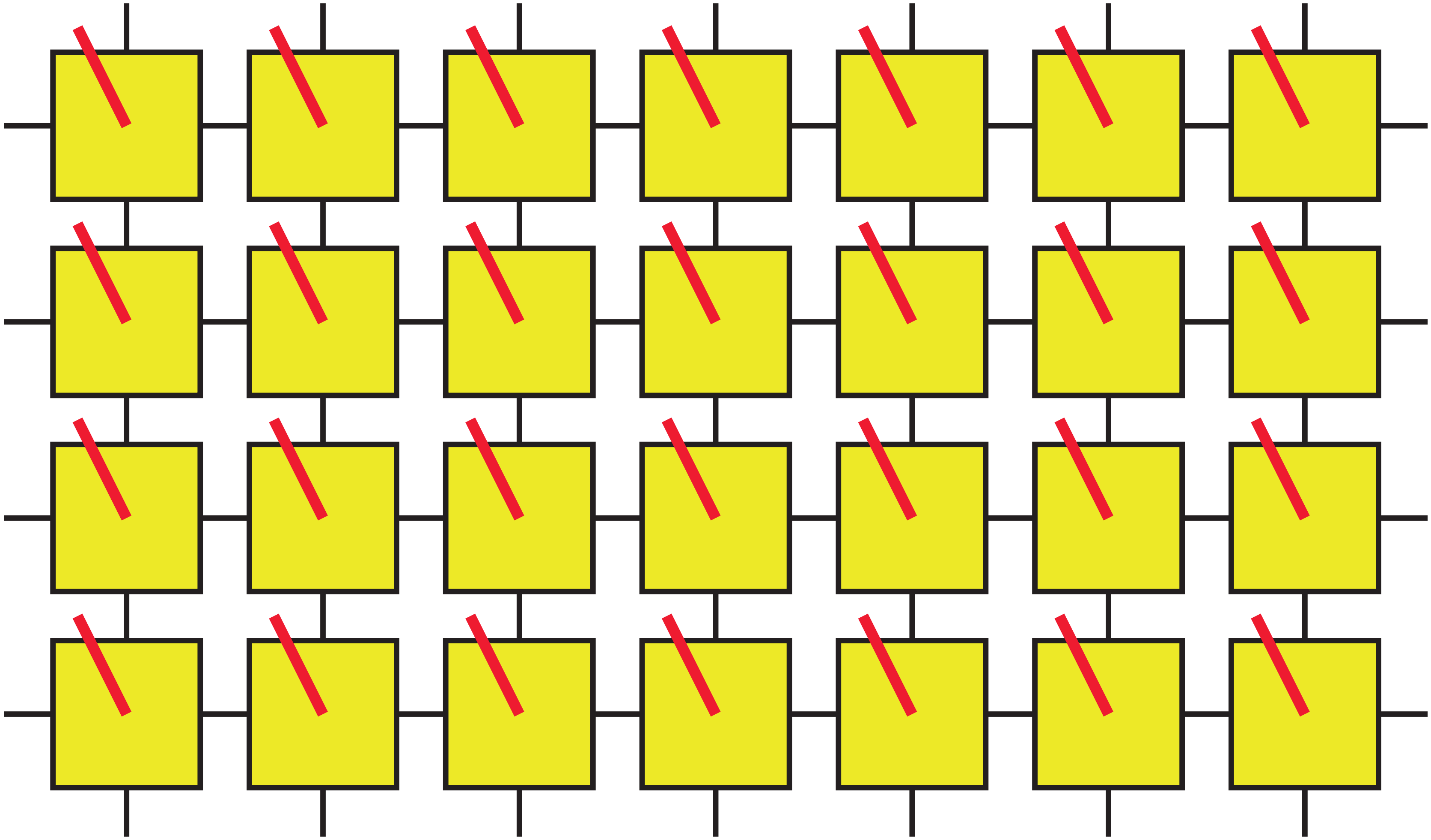
- The symmetries of the phase are





- The 2D cluster state is inside the phase

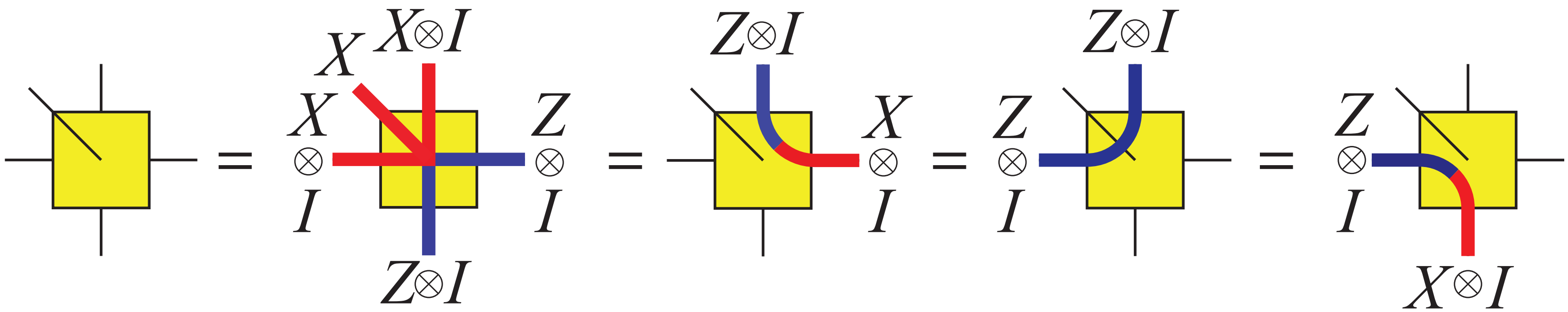
Result. For a spin-1/2 lattice on a torus with circumferences n and Nn, with n even, all ground states in the 2D cluster phase, except a possible set of measure zero, are universal resources for measurement-based quantum computation on $n/2$ logical qubits.



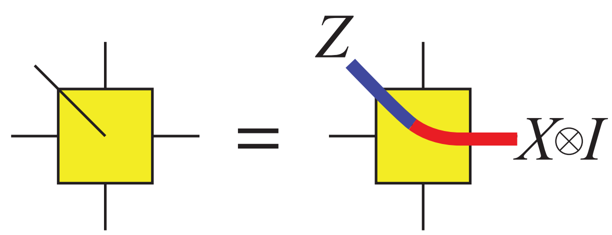
Consider MBQC resource states as tensor networks

Cluster-like states

... have PEPS tensors with the following symmetries



The cluster states have the additional symmetry



(We do not require the latter symmetry for cluster-like states)

Splitting the problem into halves

Part A:

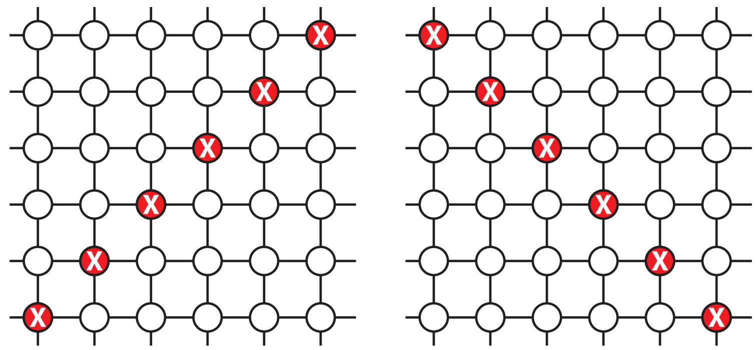
Lemma 1. All states in the 2D cluster phase are cluster-like.

Part B:

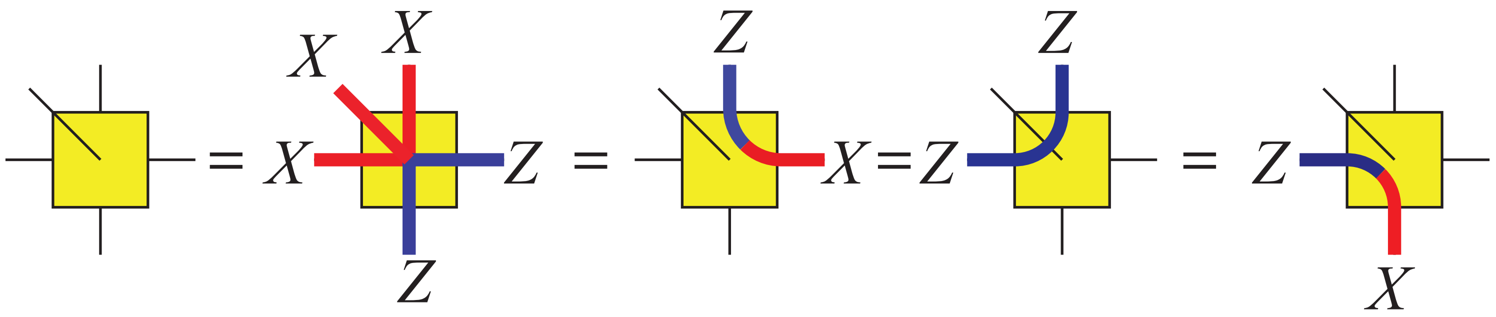
Lemma 2. All cluster-like states, except a set of measure zero, are universal for MBQC.

Part A: PEPS tensor symmetries

The physical symmetries

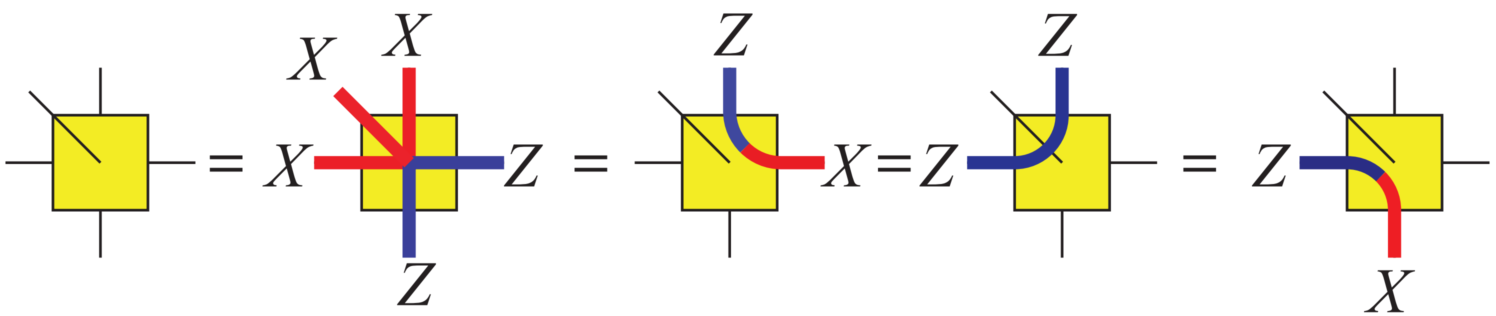


in the 2D cluster phase imply the local PEPS tensor symmetries,



Part B: Symmetry Lego

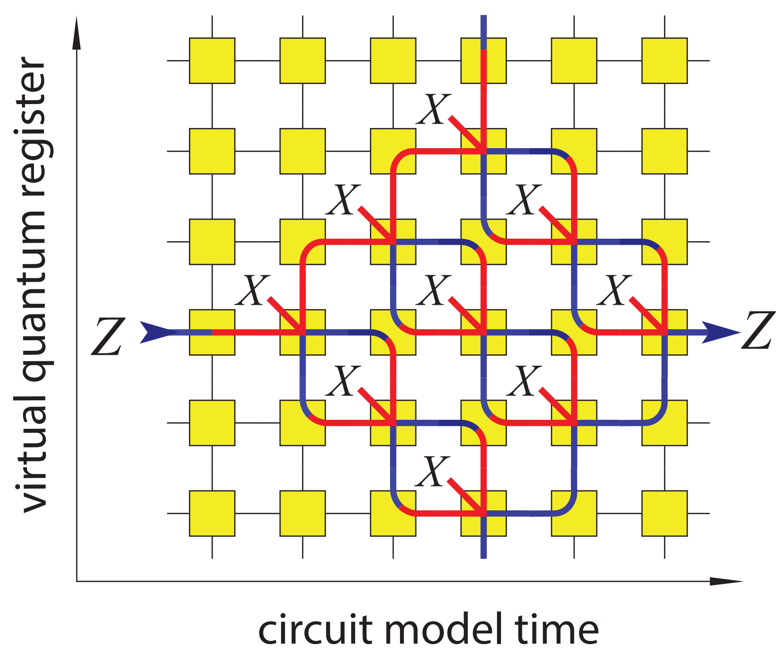
Now weave the PEPS tensor symmetries

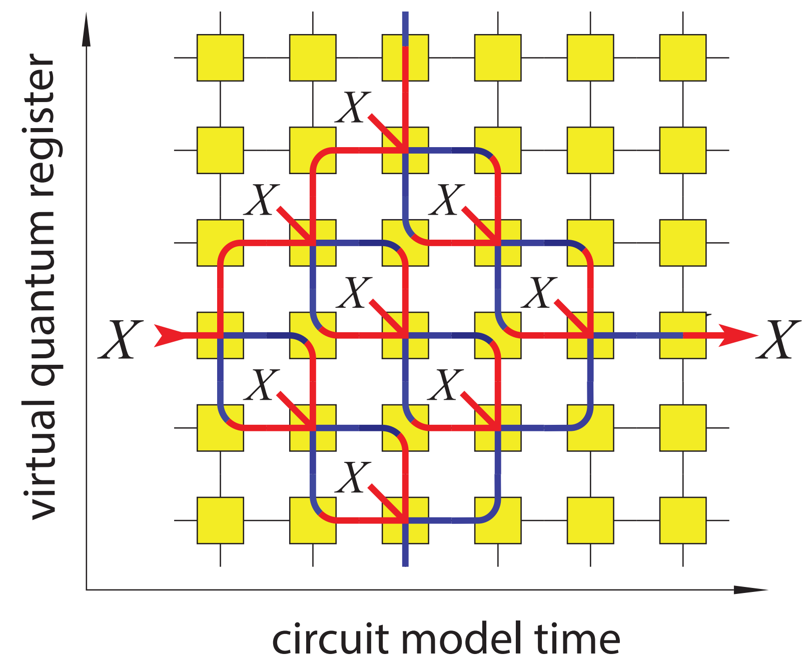


into larger patterns.

B: Cluster-like $\Rightarrow$ universal

The clock cycle:

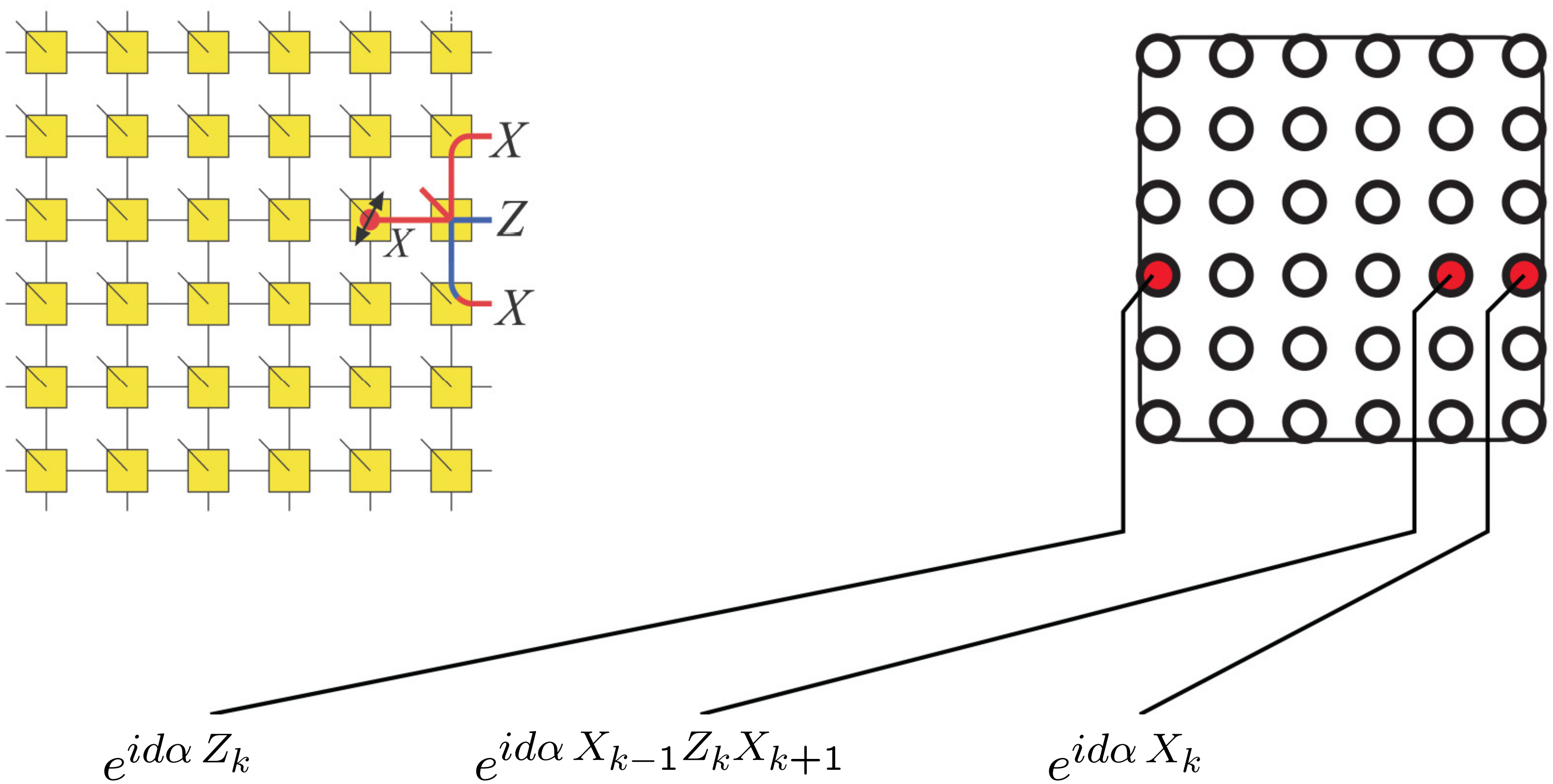




- Every byproduct operator is mapped back to itself after n columns (n = circumference).

$\Rightarrow$ If a gate can be done once, it can be done many times.

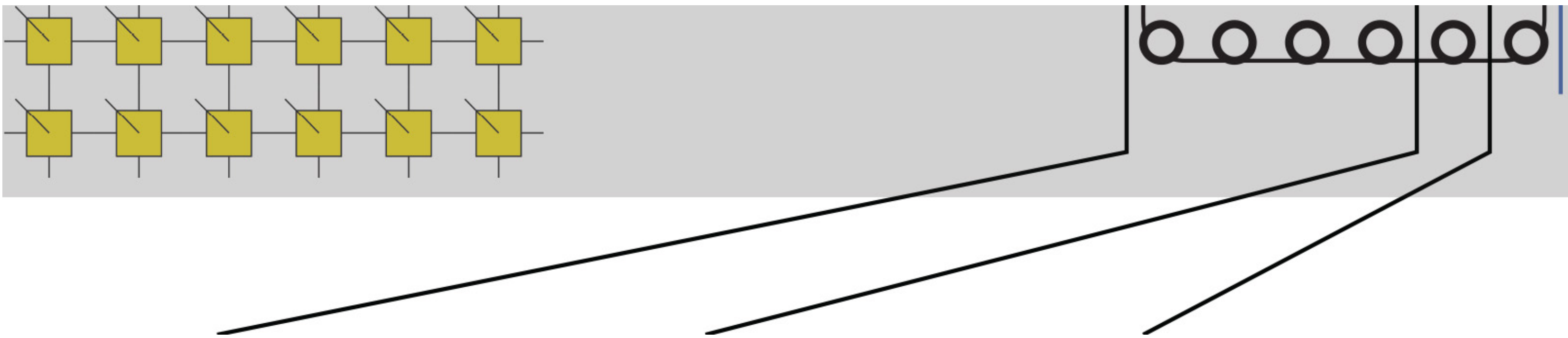
B: Cluster-like ⇒ universal



Universal gate set on $n/2$ qubits

B: Cluster-like $\Rightarrow$ universal

2D cluster state:



$e^{id\alpha\, Z_k}$ $\qquad$ $e^{id\alpha\, X_{k-1}Z_kX_{k+1}}$ $\qquad$ $e^{id\alpha\, X_k}$

Throughout the phase:

$e^{i|\nu|d\alpha\, Z_k}$ $\qquad$ $e^{i|\nu|d\alpha\, X_{k-1}Z_kX_{k+1}}$ $\qquad$ $e^{i|\nu|d\alpha\, X_k}$

$$|\nu| \leq 1$$

(ν depends on the location in the phase)

About ν: RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017).

Summary and outlook

- There exists a symmetry-protected phase in 2D with uniform universal computational power for MBQC.
- *Can we have a classification of MBQC schemes in 2D, based on symmetry?*
- Symmetry Lego is fun—Try it!

PRL 122, 090501 (2019)

Related: **Quantum 3, 162 (2019)**