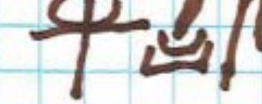
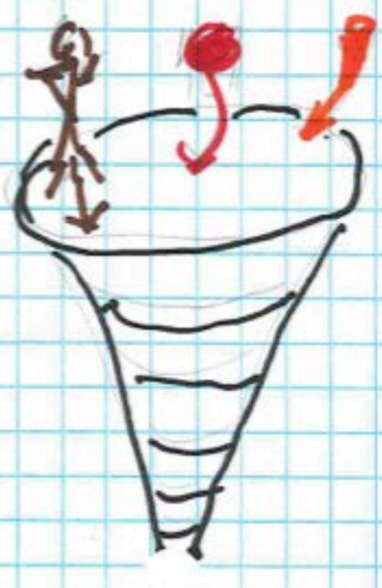
# Old/Past/Ancient/Historic Frontiers in Black Hole [©1963] Astrophysics [©1869]


Virginia Trimble

UCalif Irvine



Queen Jadwiga Obs.
Rzepiennik POLAND

IRS

May 2017
6 1/2Pines

IAUS 324
~~August~~ Sept 2016
Ljubjana
Republika Slovenija

BASIC QUESTIONS

1. What is a Black Hole? (the name?)

   One wrong answer: "It folds a blanket of space-time around itself and quietly goes to sleep." Thornton Leigh Page (and close to Eddington, who said "nonsense")

2. Do black holes exist?
   This depends a good deal on answer to (1)

3. Where/when/why/how formed, what are they good for?
   This is (mostly) not history

If Black Hole = escape velocity ≥ c, two essential concepts:

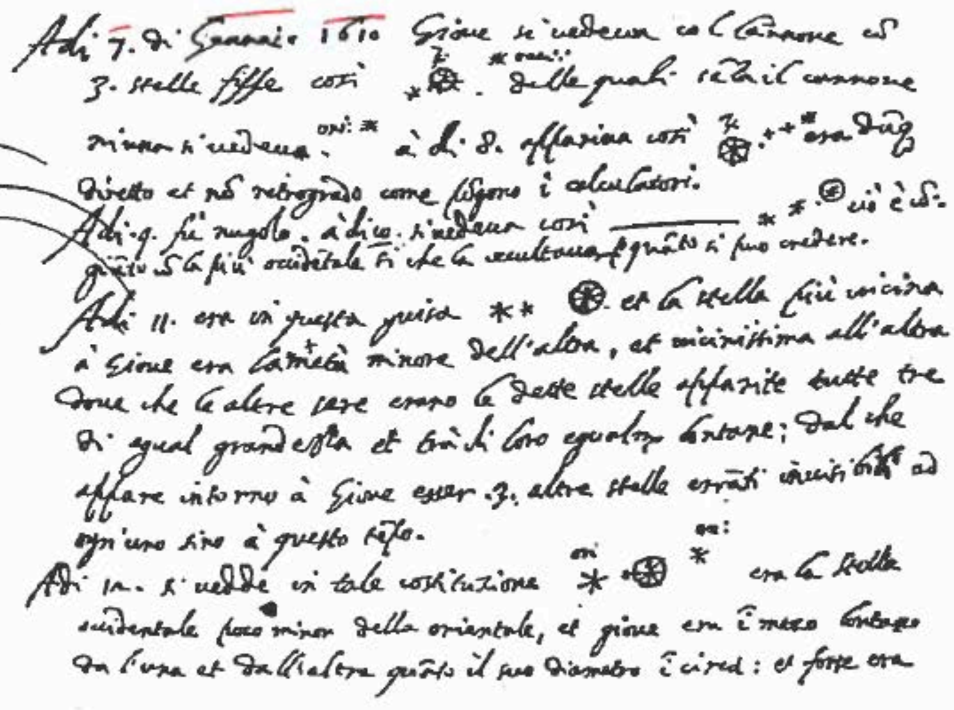
1. Finite speed of light: Galileo, 1638, Discorsi e dimostrazioni matematiche, intorno a due nuove scienze attenenti all mecanica & movimenti local; Salviati (GG's alter ego) on Day 1, the lanterns in the mountains experiment ("large if not infinite")

200,000 km/s (NOT in those units!) 1672, Io occultations, Ole Rømer)

301,000 km/s, James Bradley, 1728, aberration of starlight

2. Escape velocity: Newton 1687, Principia
1704 Opticks (light is a particle, which is good enough for small gravitational redshifts).

A1



Adi 7. di Gennaio 1610 Giove si vedeva col Cannone con 3. stelle fisse così delle quali senza il cannone niuna si vedeva. a dì 8. appariva così era dunque diretto et non retrogrado come pongono i calculatori.
Adi 9. si nugolò. a dì 10. si vedeva così cioè congiunto con la più occidentale sì che la occultava per quanto si può credere.
Adi 11. era in questa guisa et la stella più vicina à Giove era la metà minore dell'altra, et vicinissima all'altra dove che le altre sere erano le dette stelle apparite tutte tre di egual grandezza et tra di loro egualmente lontane; dal che appare intorno à Giove esser 3. altre stelle erranti invisibili ad ogni uno sino à questo tempo.
Adi 12. si vedde in tale costituzione era la stella occidentale poco minor della orientale, et giove era in mezzo lontano da l'una et dall'altra quanto il suo diametro in circa: et forse era

THE

# CHRONOLOGY

OF

# ANCIENT KINGDOMS

# AMENDED.

To which is Prefix'd,

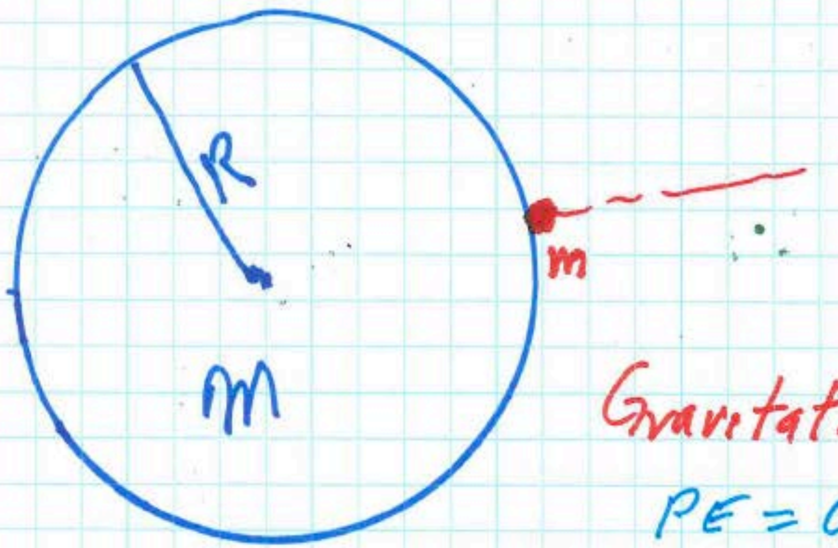
*A* SHORT CHRONICLE *from the First Memory of Things in* Europe, *to the Conquest of* Persia *by* Alexander *the Great.*

By Sir *ISAAC NEWTON.*

*LONDON:*

Printed for J. TONSON in the *Strand*, and J. OSBORN and T. LONGMAN in *Pater-noster Row.*

MDCCXXVIII. 1728




$V_{esc} = \frac{2GM}{R} \to c, \quad R \leq \frac{2GM}{c^2}$

Gravitational z: $KE = \frac{1}{2}mv^2$, $\frac{1}{2}mc^2$ TO START

$PE = \frac{GMm}{R} \to 0$

SO: $\frac{1}{2}mv^2 = \frac{1}{2}mc^2 - \frac{GMm}{R}$ $\quad v^2 = c^2 - \frac{2GM}{r}$

$\frac{v}{c} = \left(1 - \frac{2GM}{Rc^2}\right)^{1/2} \simeq 1 - GM/Rc^2$

White dwarf

$M \sim 2 \times 10^{33}\ g$

$R \sim 10^9\ cm$

$G = 6.67 \times 10^{-8}$

$c = 3 \times 10^{10}$

$\frac{GM}{Rc^2} \sim 1.48 \times 10^{-4}$

$\Delta v \simeq (1.48 \times 10^{-4})(3 \times 10^5)$

$\simeq 45\ km/sec$

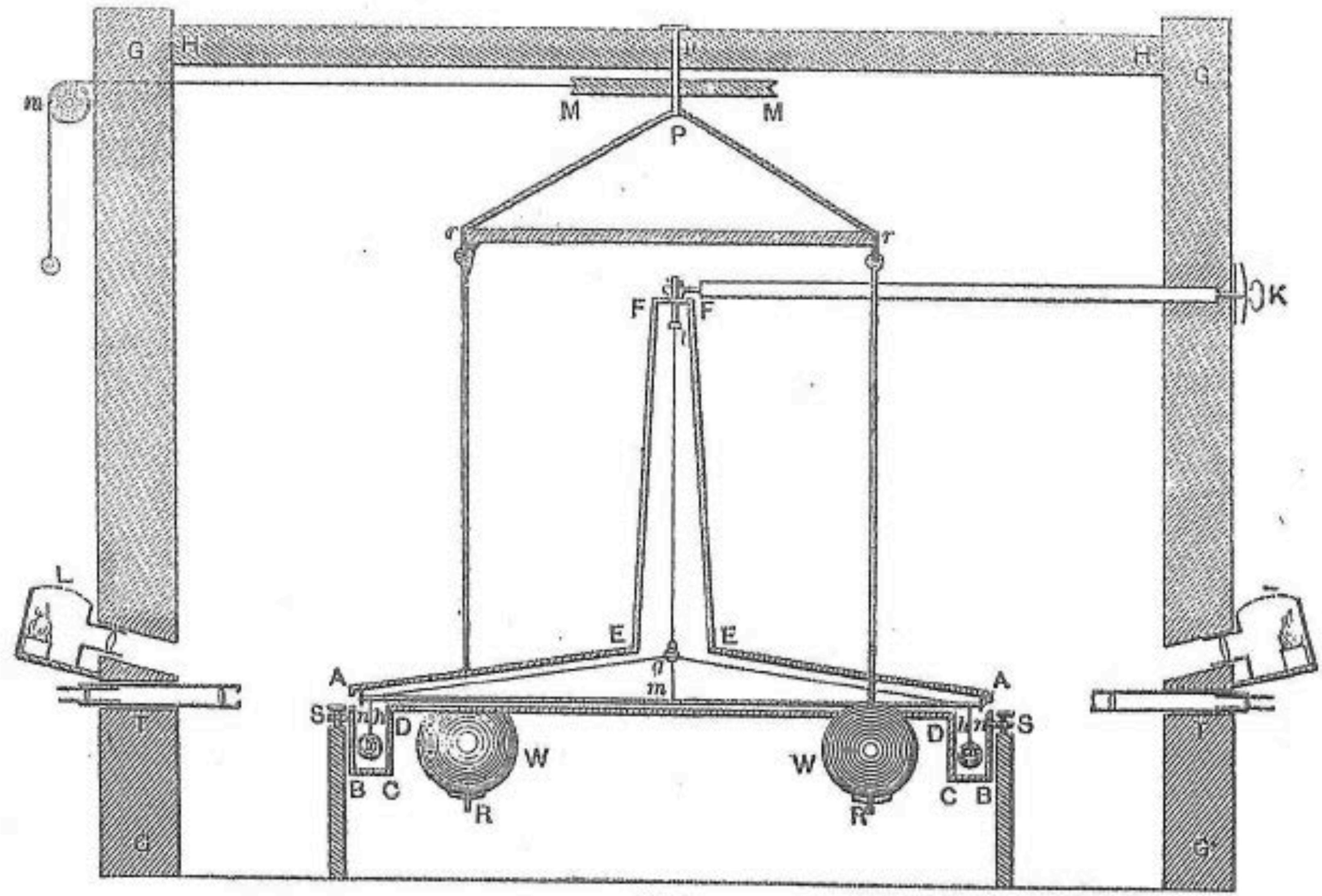



*Fig.* 1

The torsion balance, based on a design by John Michell, that was used by Henry Cavendish in 1797–98 to weigh the Earth. (*Philosophical Transactions of the Royal Society of London*)

∫ ∓ f

later page
$V_{esc} \to c$
for ★ w/
$R \sim 1AU$

VII. *On the Means of diſcovering the Diſtance, Magnitude,* &c. *of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

Read November 27, 1783.

DEAR SIR, Thornhill, May 26, 1783.

THE method, which I mentioned to you when I was laſt in London, by which it might perhaps be poſſible to find the diſtance, magnitude, and weight of ſome of the fixed ſtars, by means of the diminution of the velocity of their light, occurred to me ſoon after I wrote what is mentioned by Dr. PRIESTLEY in his Hiſtory of Optics, concerning the diminution of the velocity of light in conſequence of the attraction of the ſun; but the extreme difficulty, and perhaps impoſſibility, of procuring the other data neceſſary for this purpoſe appeared to me to be ſuch objections againſt the ſcheme, when I firſt thought of it, that I gave it then no farther conſideration. As ſome late obſervations, however, begin to give us a little more chance of procuring ſome at leaſt of theſe data, I thought it would not be amiſs, that aſtronomers ſhould be apprized of the method, I propoſe (which, as far as I know,

 has

# EXPOSITION

## DU SYSTEME

# DU MONDE,

PAR PIERRE-SIMON LAPLACE,
de l'Institut National de France, et
du Bureau des Longitudes.

TOME SECOND.

1795

deleted when light → wave
(Th Young etc)

A PARIS,

De l'Imprimerie du CERCLE-SOCIAL, rue du
Théâtre Français, N°. 4.

L'AN IV DE LA RÉPUBLIQUE FRANÇAISE.



aussi sensibles à la distance qui nous en sépare ; et combien ils doivent surpasser ceux que nous observons à la surface du soleil ? Tous ces corps devenus invisibles, sont à la même place où ils ont été observés, puisqu'ils n'en ont point changé, durant leur apparition ; il existe donc dans les espaces célestes, des corps obscurs aussi considérables, et peut être en aussi grand nombre, que les étoiles. Un astre lumineux de même densité que la terre, et dont le diamètre serait deux cents cinquante fois plus grand que celui du soleil, ne laisserait en vertu de son attraction, parvenir aucun de ses rayons jusqu'à nous ; il est donc possible que les plus grands corps lumineux de l'univers, soient par cela même, invisibles. Une étoile qui, sans être de cette grandeur, surpasserait considérablement le soleil ; affaiblirait sensiblement la vîtesse de la lumière, et augmenterait ainsi l'étendue de son aberration. Cette différence dans l'aberration des étoiles ; un catalogue de celles qui ne font que paraître, et leur position observée au moment de leur éclat passager ; la détermination de toutes les étoiles changeantes,

# WHAT IS A BLACK HOLE?

1. Size close to the gravitational, or Schwarzschild radius (Michell, LaPlace)

2. Horizon (K. Schwarzschild; Kerr), $R = 2GM/c^2$

3. Stuff disappears at horizon, including information/entropy (accretion-dominated advection; advection dominated accretion)

4. Information/stuff hangs up at/around horizon

5. Worldlines end (singularity) at $r = 0$, or prevented by quantum effects. This was what most seemed to have bothered Einstein, leading to his 1939, wrong, paper.

*A. Einstein 1879–1955*

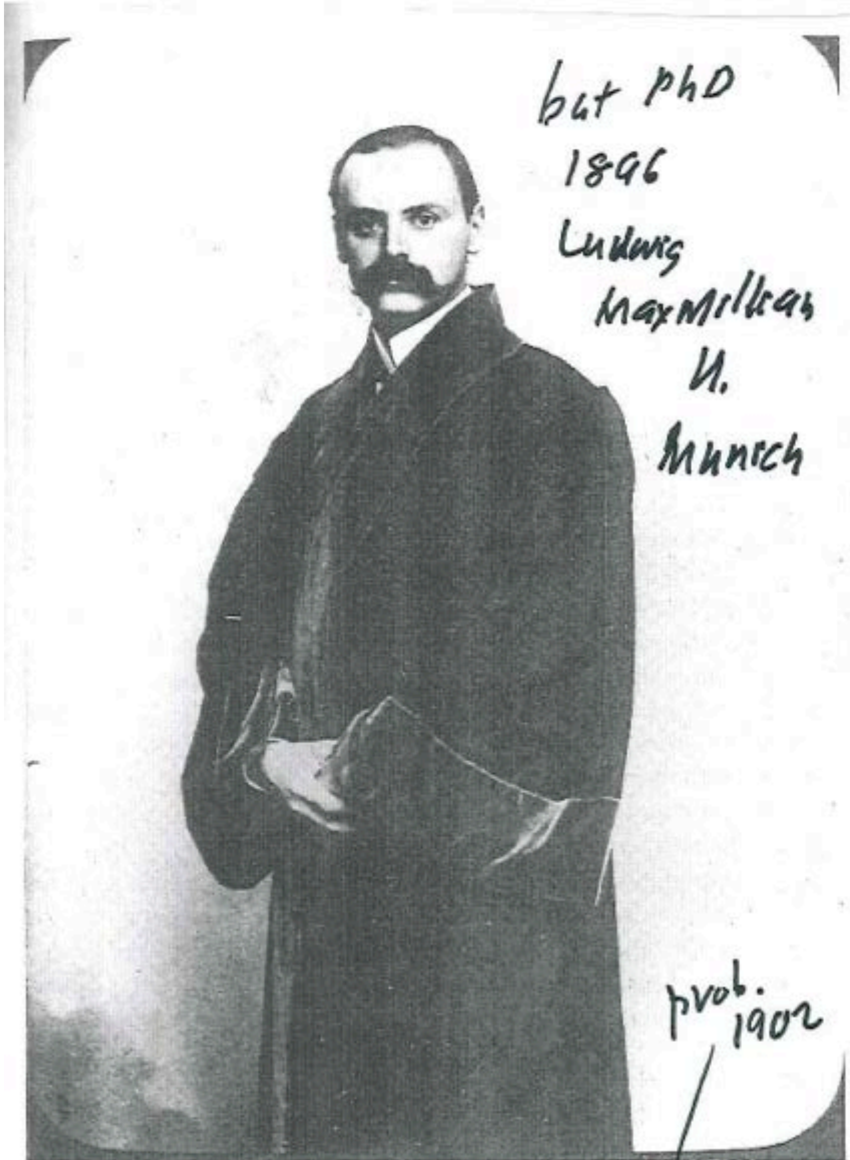
# GENERAL [cut off] AND GRAV[cut off] WAVES

J. WEBER

*Professor of Physics, University of*
*College Park, Maryland*

INTERSCIENCE PUBLISHERS, INC., NEW
Interscience Publishers Ltd., London

1916




Karl Schwarzschild in his academic robe in Göttingen, Germany. [Courtesy AIP

(1910 in US?

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2m}{r}(dr + dt)^2$$

(Eddington 1924)

# 169. From Karl Schwarzschild[1]

[at the Russian front,] 22. XII. 15.

Verehrter Herr Einstein!

Um mit Ihrer Gravitationstheorie vertraut zu werden, habe ich mich näher mit dem von Ihnen in der Arbeit über das Merkurperihel[2] gestellte und in 1. Näherung gelöste Problem beschäftigt.[3] Zunächst machte mich ein Umstand sehr konfus. Ich fand für die erste Näherung der Koeffizienten $g_{\mu\nu}$ außer ihrer Lösung noch folgende zweite:[4]

$$g_{\rho\sigma} = -\frac{\beta x_\rho x_\sigma}{r^5} + \delta_{\rho\sigma}\left[\frac{\beta}{3r^3}\right] \quad g_{44} = 1$$

Danach hätte es außer Ihrem $\alpha$ noch eine zweite gegeben und das Problem wäre physikalisch unbestimmt. Daraufhin machte ich einmal auf gut Glück den Versuch einer vollständigen Lösung.[5] Eine nicht zu große Rechnerei ergab folgendes Resultat: Es gibt nur ein Linienelement, das Ihre Bedingungen 1) bis 4)[6] nebst Feld- und Determinantengl. erfüllt[7] und im Nullpunkt und nur im Nullpunkt singulär ist.

Sei: $\quad x_1 = r\cos\varphi\cos\vartheta \quad x_2 = r\sin\varphi\cos\vartheta \quad x_3 = r\sin\vartheta$

$$R = (r^3+\alpha^3)^{1/3} = r\left(1+\frac{1}{3}\frac{\alpha^3}{r^3}+\ldots\right)$$

dann lautet das Linienelement:[8]

$$ds^2 = \left(1-\frac{\gamma}{R}\right)dt^2 - \frac{dR^2}{1-\frac{\gamma}{R}} - R^2(d\vartheta^2+\sin^2\vartheta\, d\varphi^2)$$

$R$, $\vartheta$, $\varphi$ sind keine „erlaubten" Koordinaten, mit denen man die Feldgleichungen bilden dürfte, weil sie nicht die Determinante 1 haben, aber das Linienelement schreibt sich in ihnen am schönsten.

Die Gleichung der Bahnkurve bleibt genau die von Ihnen in erster Näherung erhaltene (11),[9] nur muß man unter $x$ nicht $\frac{1}{r}$, sondern $\frac{1}{R}$ verstehen, was ein Unterschied von der Ordnung $10^{-12}$ ist, also praktisch absolut gleichgültig.

Die Schwierigkeit mit den zwei willkürlichen Konstanten $\alpha$ und $\beta$, welche die erste Näherung gab, löst sich dahin, daß $\beta$ einen bestimmten Wert von der Ordnung $\alpha^4$ haben muß,[10] so wie $\alpha$ gegeben ist, sonst würde die Lösung bei Fortsetzung der Näherungen divergent.

Es ist also auch die Eindeutigkeit Ihres Problems in schönster Ordnung.[11]

Es ist eine ganz wunderbare Sache, daß von einer so abstrakten Idee aus die Erklärung der Merkuranomalie so zwingend herauskommt.

Wie Sie sehen, meint es der Krieg freundlich mit mir, indem er mir trotz heftigen Geschützfeuers in der durchaus terrestrischer Entfernung diesen Spaziergang in dem von Ihrem Ideenlande erlaubte.

ADft (GyGöU, Cod. Ms. K. Schwarzschild 2:2, 2–3). *Schwarzschild 1992*, pp. 36–39. [81 806]. The presentation here departs from that in the original, where both interlineations and undeleted portions of the text that the interlineations have superseded appear.

[1]Schwarzschild (1873–1916) was Director of the Astrophysical Observatory in Potsdam.

[2]*Einstein 1915h* (Vol. 6, Doc. 24).

[3]The results reported in this document were published in *Schwarzschild 1916a*, which was submitted to the Prussian Academy by Einstein on 13 January 1916.

[4]The first-order solution for the field of a point mass given in *Einstein 1915h* (Vol. 6, Doc. 24), eq. (4b), is $g_{\rho\sigma} = -\delta_{\rho\sigma} - \alpha\frac{x_\rho x_\sigma}{r^3}$. The square brackets in the equation below are in the original.

[5]The undeleted but superseded portions of the text that precede this point and have not been incorporated are set out in the following:

"Um mit Ihrer Gravitationstheorie vertraut zu werden, habe ich mir die Aufgabe gestellt, das von Ihnen in der Arbeit über das Merkurperihel gestellte und näherungsweise gelöste Problem womöglich vollständig zu lösen. Man kann ja leicht das allgemeinste Linienelement angeben, das die nötigen Symmetrieeigenschaften hat. Von diesen ausgehend bestimmte ich zunächst die erste Näherung in Ihrer Weise und fand:

$$g_{\rho\sigma} = -\alpha\frac{x_\rho x_\sigma}{r^3} - \beta\frac{x_\rho x_\sigma}{r^5} + \delta_{\rho\sigma}\left[\frac{\beta}{3r^3}\right] \quad g_{44} = 1$$

also *zwei* willkürliche Größen $\alpha$ und $\beta$, was sonderbar war. Dann ging ich zur vollständigen Lösung über."

[6]The conditions are: (i) time independence; (ii) spherical symmetry; (iii) $g_{i4} = 0$ (i = 1, 2, 3); (iv) the metric is Minkowskian at infinity (see *Einstein 1915h* [Vol. 6, Doc. 24], p. 833).

[7]Following *Einstein 1915h* (Vol. 6, Doc. 24), Schwarzschild used the field equations in coordinates satisfying the condition $\sqrt{-g} = 1$.

[8]The line element given here became known as the Schwarzschild solution. See *Eisenstaedt 1982, 1987, 1989* for discussions of its origin and of the history of its interpretation.

[9]The reference is to eq. (11) in *Einstein 1915h* (Vol. 6, Doc. 24).

[10]$\alpha^4$ should be $\alpha^3$ (see *Schwarzschild 1916a*, p. 194).

[11]In deriving an approximate solution for the field of the sun, Einstein had pointed out that he had no proof that his solution was unique (*Einstein 1915h* [Vol. 6, Doc. 24], p. 832). For a historical discussion, see *Earman and Janssen 1993*, pp. 140–141.

## 176. To Karl Schwarzschild

[Berlin,] 29 December 1915

Highly esteemed Colleague,

Your calculation providing the uniqueness proof for the problem[1] is extremely interesting. I hope you publish it soon![2] I would not have thought that the strict treatment of the [mass-]point problem was so simple.

That your particular solution is of the 3rd order is immediately apparent for reasons of dimension. For $\frac{\kappa m}{r}$ is a dimensionless number. As your $\beta$ must depend on $m$, your expression

$$g_{\rho\sigma} = -\beta \frac{x_\rho x_\sigma}{r^5} + \delta_{\rho\sigma} \frac{\beta}{3r^3},$$

in which $\frac{\beta}{r^3}$ is a (dimensionless) number, thus requires that $\beta$ be equal, apart from a numerical factor, to $\left(\frac{\kappa m}{r}\right)^3$.[3]

I am very satisfied with the theory. It is not self-evident that it already yields Newton's approximation; it is all the more gratifying that it also provides the perihelion motion and line shift, although it is not yet sufficiently secure.[4] Now the question of light deflection is of most importance.

With my best regards and wishes for the New Year, yours,

Einstein.

## 181. To Karl Schwarzschild

Sunday

[Berlin,] 9 January 1916

4

Highly esteemed Colleague,

I examined your paper with great interest.[1] I would not have expected that the exact solution to the problem could be formulated so simply. The mathematical treatment of the subject appeals to me exceedingly. Next Thursday I am going to deliver the paper before the Academy with a few words of explanation.[2]

Meanwhile, I received another letter from you yesterday evening, which I would also like to answer right away.

14 Jan

1) The theory is fully developed, as far as the fundamental formulas are concerned, so no other difficulties remain in the treatment of the individual problems aside from the computational ones, which are, however, inordinately large. But you will gather from the following reflection that no notable modification is made for the perturbation problem.

sufficient to produce *secular* effects accessible to observation. If $M$ is taken for planetary masses, then this relative quantity is diminished significantly. However, secular variations that are produced by the interaction of the planets only amount to at most 1,000″ in 100 years. According to the theory, they would be modified by the tiny fraction indicated. Thus a more exact development of the perturbation calculation to modify orbital motion theory cannot provide anything within the reach of observation.

2) The statement that "the fixed-star system" is rotation-free undoubtedly is meant in a relative sense, which is described by a comparison.

The Earth's surface is irregular as long as I envisage very small sections of it. But it approaches the flat basic form when I envisage larger sections of it, whose dimensions are still small against the length of the meridian. This basic form becomes a curved surface when I envisage even larger sections.

Likewise for the gravitational field. On a small scale the individual masses produce gravitational fields that even with the most simplifying choice of reference system reflect the character of a quite irregular small-scale distribution of matter. If I regard larger regions, as those available to us in astronomy, the Galilean reference system provides me with the analogue to the flat basic form of the Earth's surface in the previous comparison. But if I consider even larger regions, a continuation of the Galilean system providing the description of the universe in the same dimensions as on a smaller scale probably does not exist, that is, where throughout, a mass-point sufficiently removed from other masses moves uniformly in a straight line. Ultimately, according to my theory, inertia is simply an interaction between masses, not an effect in which "space" of itself were involved, separate from the observed mass. The essence of my theory is precisely that no independent properties are attributed to space on its own.

It can be put jokingly this way. If I allow all things to vanish from the world, then following Newton, the Galilean inertial space remains; following my interpretation, however, *nothing* remains.

3) As concerns Jupiter, I understand that it is a difficult proposition for astronomers.[3] However, in my view the importance of the matter supports only *one* standpoint, and that is: It *has* to work! Jupiter's moons could serve in studying closely the systematic errors of which you speak; for the apparent displacement of Jupiter's moons through light deflection is entirely negligible owing to the smallness of the moon–Jupiter distance. The angle to be confirmed amounts to $2 \cdot 0.02''$ and is thus within the order of currently attainable precision.

4) It never occurred to me to think of a clique against Freundlich. It is generally far from my mind to think of such things. Struve's attitude is understandable.[4]

5

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6

He is an old man and no longer has the flexibility needed to delve into new issues. That is why he assumes a negative stance on technical matters, and this negative attitude also extends to Freundlich, whom he envisions somewhat as an incarnation of these things. I readily believe that Freundlich on his side has little tact and lacks social skills, and in general has little psychological understanding for his fellows, which only makes the circumstances more disagreeable.

I do not take Freundlich for a very great talent, but for a person with a burning interest and a remarkable tenacity. He was the first astronomer to understand the significance of the general theory of relativity and to address enthusiastically the astronomical issues attached to it.[5] *That is why I would regret it deeply if he were deprived the possibility of working in this field.* I know from my own experience that the necessary technical skills can be acquired, if the requisite understanding and a great interest are combined. Should such deficiencies nevertheless hamper the enterprise, help from a well-meaning expert could lead to valuable results.

Best regards, yours truly,

A. Einstein.

## 188. From Karl Schwarzschild

[at the Russian front,] 6. II. 16

Verehrter Herr Einstein!

Vielen Dank für Ihren Brief vom 9.ten Januar.[1] Wegen Jupiter schrieb ich an Hertzsprung,[2] der mich gleich darauf aufmerksam machte, daß Jupiter die nächsten Jahre für uns zu südlich geht. Diese extreme Genauigkeit ist nur zwischen Zenith und vielleicht 50° Höhe erreichbar. Die Sache muß daher südlicheren Sternwarten überlassen bleiben. Herr Freundlich könnte sich ein Verdienst erwerben, wenn er Sternvorübergänge und Bedeckungen aussuchte. (Mir scheint nur, daß sich Herr Banachiewiz[3] schon für andere Zwecke damit beschäftigt hat). Über Freundlich werden wir uns im übrigen nicht zu leicht einigen und ich möchte nur noch sagen: Hin- und Herreden über ihn nützt nichts. Ich glaube nur, daß er es mit Struve schon so weit verdorben hat, daß Sie am besten Ihren Einfluß dafür verwenden würden, ihm eine andere Thätigkeit zu verschaffen.[4]

Was das Inertialsystem angeht, so sind wir einig. Sie sagen, daß jenseits des Milchstraßensystems sich Verhältnisse einstellen können, in dem das Galilei'sche System nicht mehr das einfachste ist. Ich behaupte nur, daß sich innerhalb des Milchstraßensystems solche Verhältnisse nicht einstellen. Was die ganz großen Räume angeht, hat Ihre Theorie eine ganz ähnliche Stellung, wie Riemann's Geometrie, und es ist Ihnen gewiß nicht unbekannt, daß man die elliptische Geometrie aus Ihrer Theorie herausbekommt, wenn man die ganze Welt unter einem gleichförmigen Druck stehen läßt (Energietensor $-p, -p, -p, 0$).[5]

Ich kann nicht leugnen, daß Sie die darüber hinausgehende Freiheit in der glücklichsten Weise ausgenutzt haben.

Um mich mit Ihrem Energietensor anzufreunden, habe ich inzwischen das Problem der flüssigen inkompressiblen Weltkugel behandelt (Energietensor $-p, -p, -p, +\rho_0$). Ich hätte es nicht gethan, wenn ich gewußt hätte, daß mich so viel Schererei kosten würde.[6]

Das Linienelement im Inneren lautet:

$$ds^2 = \left(\frac{3\cos\sigma_0 - \cos\sigma}{2}\right)^2 dt^2 - \frac{3}{\kappa\rho_0}[d\sigma^2 + \sin^2\sigma(d\vartheta^2 + \sin^2\vartheta d\varphi^2)]$$

$\vartheta$, $\varphi$ die üblichen Polarkoordinaten. $\sigma$ hänge mit dem Radiusvektor zusammen nach:

$$\left(\frac{\kappa\rho_0}{3}\right)^{\frac{3}{2}} r^3 = \frac{9}{4}\cos\sigma_0\left(\sigma - \frac{1}{2}\sin 2\sigma\right) - \frac{1}{2}\sin^3\sigma$$

wobei $\sigma_0$ durch den Kugelradius $r_0$ gemäß

$$\left(\frac{\kappa\rho_0}{3}\right)^{\frac{3}{2}} r_0^3 = \frac{9}{4}\cos\sigma_0\left(\sigma_0 - \frac{1}{2}\sin 2\sigma_0\right) - \frac{1}{2}\sin^3\sigma_0$$

bestimmt ist. Für den Druck $p$ gilt:

$$(\rho_0 + p)\left(\frac{3\cos\sigma_0 - \cos\sigma}{2}\right) = \text{const.}$$

Das Linienelement außen ist das alte, wobei nur in

$$R^3 = r^3 + \rho$$

nicht $\rho = \alpha^3$ ist, sondern statt dessen gilt:

$$\alpha = m\frac{2}{3Z-1} \qquad \rho = \frac{3}{2}\frac{\alpha^3}{\sin^6\sigma_0}(1-Z) \qquad m = \frac{\kappa\rho_0}{3}r_0^3$$

$$Z = \frac{3}{2}\frac{\cos\sigma_0}{\sin^3\sigma_0}\left(\sigma_0 - \frac{1}{2}\sin 2\sigma_0\right)$$

Das Sonderbare ist, daß für eine endliche Kugel mit $\cos\sigma_0 = \frac{1}{3}$ der Druck im Mittelpunkt ($\sigma = 0$) unendlich wird, kleinere Kugeln von gegebener Masse nicht möglich sind. Der Übergang zu unendlich kleinem Radius, wo dann $\rho = \alpha^3$ herauskommen muß, vollzieht sich auch vermittelst physikalisch bedeutungsloser Lösungen—ich glaubte mich immer verrechnet zu haben, bevor ich das erkannte. Sehen Sie anschaulich, wo die Grenze $\cos\sigma_0 = \frac{1}{3}$ herkommt?

Die eckige Klammer ist das Linienelement der sphärischen Geometrie! Im Inneren der Kugel herrscht also sphärische Geometrie. Das Kugelinnere ist nicht der ganze sphärische Raum, sondern nur eine Kugel vom Radius $\sigma_0$ in diesem Raum. Man merkt nur an der Zunahme der Lichtgeschwindigkeit, ob man sich näher beim Centrum oder näher der Oberfläche befindet.

Es fällt hier noch mehr auf, wie hier das äussere Feld, daß $r$ gar keine physikalische Bedeutung hat, sondern nur eine Variable ist, die $|g_{\mu\nu}|$ gleich 1 macht. Dies sieht so aus, als ob sich eine andere Bedingung, als $|g_{\mu\nu}| = 1$ finden lassen müßte, welche die Feldgleichungen noch einfacher macht.

ADft (GyGöU, Cod. Ms. K. Schwarzschild Briefe 193, 7–8). [65 943]. The presentation here departs from that in the original where the third paragraph begins with two sentence fragments: "daß die Erfahrungen innerhalb des Milchstraßensystems erstreckt," and interlineated above it: "seine Bedeutung sich auf das ganze."

[1]Doc. 181.

[2]Ejnar Hertzsprung (1873–1967) was *Observator* at the Astrophysical Observatory in Potsdam.

[3]Tadeusz Banachiewicz (1882–1954) was *Assistent* at the Observatory of the University of Göttingen.

[4]There had been bad blood between Erwin Freundlich and the Director of the Royal Prussian Observatory in Neubabelsberg, Hermann Struve, at least since Einstein tried to extricate Freundlich from routine astronomical tasks a year earlier (see Docs. 53 and 54).

[5]As Felix Klein noted two years later, the interior Schwarzschild solution contains the De Sitter solution as a special case (see Doc. 566).

[6]The following considerations were published in much greater detail in *Schwarzschild 1916b*, which was submitted to the Prussian Academy at the end of February. The square brackets in the first equation are in the original.

# 194. To Karl Schwarzschild

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[Berlin, 19 February 1916]

Esteemed Colleague,

Unfortunately because of a lot of work I was not yet able to answer your earlier letter.[1] Also, the special cases discussed there awoke my interest to a lesser degree. But I find your new communication very interesting. I have found your calculation confirmed. My comment in this regard in the paper of November 4 no longer applies according to the new determination of $\sqrt{-g} = 1$, as I was already aware.[2] The choice of coordinate system according to the condition $\sum \frac{\partial g^{\mu\nu}}{\partial x_\nu} = 0$ is not consistent with $\sqrt{-g} = 1$. Since then, I have handled Newton's case differently, of course, according to the final theory.[3]—Thus there are no gravitational waves analogous to light waves. This probably is also related to the one-sidedness of the sign of scalar $T$, incidentally. (Nonexistence of the "dipole".)[4]

Cordial greetings and many thanks for the interesting communication. Yours,

A. Einstein.

## 195. To Max Born

[Berlin,] Sunday [27 February 1916][1]

Dear Mr. Born,

This morning I received the correction to your paper for the *Physikalische Zeitschrift*, which I read, not without embarrassment, but with the happy feeling of having been understood thoroughly and acknowledged by one of my most highly qualified colleagues.[2] But aside from the objective content, I was also filled with the happy sensation of cheerful goodwill that emanates from the paper and that otherwise so rarely lingers undiluted in the pale light of the study lamp.[3] I thank you from my heart for granting me the privilege of this fine joy.

With best regards, yours,

A. Einstein.

können auch alle speziellen Bildungsgesetze von Tensoren auf (22) in Verbindung mit Tensor-Multiplikationen auffassen.

## §11. Einige Spezialfälle von besonderer Bedeutung.

Einige den Fundamentaltensor betreffende Hilfssätze. Wir leiten zunächst einige im Folgenden viel gebrauchte Hilfsgleichungen ab. Nach der Regel von der Differentiation der Determinanten ist

$$dg = g^{\mu\nu} g \, dg_{\mu\nu} = - g_{\mu\nu} g \, dg^{\mu\nu} \quad \ldots \quad (28)$$

Die letzte Gleichsetzung rechtfertigt sich durch die vorletzte, wenn man bedenkt, dass $g_{\mu\nu} g^{\mu'\nu} = \delta_{\mu}^{\mu'}$, dass also $g_{\mu\nu} g^{\mu\nu} = 4$, folglich

$$g_{\mu\nu} dg^{\mu\nu} + g^{\mu\nu} dg_{\mu\nu} = 0.$$

Aus (28) folgt

$$\frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x_\sigma} = \frac{1}{2} \frac{\partial \lg(-g)}{\partial x_\sigma} = \frac{1}{2} g^{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial x_\sigma} = - \frac{1}{2} g_{\mu\nu} \frac{\partial g^{\mu\nu}}{\partial x_\sigma} \quad \ldots (29)$$

Aus

$$g_{\mu\sigma} g^{\nu\sigma} = \delta_{\mu}^{\nu}$$

folgt ferner durch Differentiation

~~$g_{\mu\sigma} \frac{\partial g^{\nu\sigma}}{\partial x_\lambda} = - g^{\nu\sigma} \frac{\partial g_{\mu\sigma}}{\partial x_\lambda} \quad \ldots (30)$~~

$$\left.\begin{aligned} g_{\mu\sigma} dg^{\nu\sigma} &= - g^{\nu\sigma} dg_{\mu\sigma} \\ \text{bezw.} \quad g_{\mu\sigma} \frac{\partial g^{\nu\sigma}}{\partial x_\lambda} &= - g^{\nu\sigma} \frac{\partial g_{\mu\sigma}}{\partial x_\lambda} \end{aligned}\right\} (30)$$

Durch gemischte Multiplikation mit $g^{\sigma\tau}$ bezw. $g_{\nu\lambda}$ erhält man hieraus (bei geänderter Bezeichnungsweise der Indizes

$$\left.\begin{aligned} dg^{\mu\nu} &= -g^{\mu\alpha}g^{\nu\beta}dg_{\alpha\beta} \\ \frac{\partial g^{\mu\nu}}{\partial x_\sigma} &= -g^{\mu\alpha}g^{\nu\beta}\frac{\partial g_{\alpha\beta}}{\partial x_\sigma} \end{aligned}\right\} (31)$$

bezw.

$$\left.\begin{aligned} dg_{\mu\nu} &= -g_{\mu\alpha}g_{\nu\beta}dg^{\alpha\beta} \\ \frac{\partial g_{\mu\nu}}{\partial x_\sigma} &= -g_{\mu\alpha}g_{\nu\beta}\frac{\partial g^{\alpha\beta}}{\partial x_\sigma} \end{aligned}\right\} \ldots (32)$$



Die Beziehung (31) erlaubt eine Umformung, von der wir ebenfalls öfter Gebrauch zu machen haben. Gemäss ( ) ist

. ~~(23a)~~

$$\frac{\partial g_{\alpha\beta}}{\partial x_\sigma} = \left[{\alpha\sigma \atop \beta}\right] + \left[{\beta\sigma \atop \alpha}\right] \ldots\ldots (33)$$

Setzt man dies in die zweite der Formeln 31 ein, so erhält man mit Rücksicht auf ( )

$$\frac{\partial g^{\mu\nu}}{\partial x_\sigma} = -\left(g^{\mu\tau}\left\{{\tau\sigma \atop \nu}\right\} + g^{\nu\tau}\left\{{\tau\sigma \atop \mu}\right\}\right) \ldots (34)$$

unsere Substitution der Formel ... Seite von (34) in (29) ~~erhält~~ ergibt sich

$$\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}}{\partial x_\sigma} = \left\{{\mu\sigma \atop \mu}\right\} \ldots\ldots (29a)$$

I hope you're enjoying the lecture course.[6] I still recall very well that a hefty push is needed to overcome an initial aversion and that one always thinks everything one has to say is obvious. But this is an optical illusion. Do you remember how in Berne you always used to attend mine so nicely?[7] And now I can't reciprocate. I have another quite amusing expert opinion to give for a patent case.[8] When we see each other again I'll tell you about it.

At the moment I'm working quite moderately, so I'm feeling nicely well and am living peacefully along without any discord. In gravitation I'm now looking for the boundary conditions at infinity; it certainly is interesting to consider to what extent a *finite* world exists, that is, a world of naturally measured finite extension in which all inertia is truly relative.[9] The funeral for Schwarzschild, director of the observatory in Potsdam, was held today.[10] Surely I have already told you about him; he is a real loss. He would have been a gem, had he been as decent as he was clever.[11] Of the photographs, keep one, give one to Maja, one to Zangger, and save the remainder. You may also give them away, though, if you believe you could make someone happy with it. I discovered a neat simplification of the thermodynamic derivation of the photochemical $h\nu$ law, somewhat in the manner of Van't Hoff.[12] I'm glad that my boys are in good spirits and that you are concerning yourself with them. Now I can soon send you the detailed paper on gravitation in which everything is calculated explicitly.[13]

Affectionate greetings also to Anna and Vero, yours,

Albert.


Hoffentlich hast Du mit dem Kolleg Freude.[6] Ich erinnere mich noch sehr gut, dass man dabei sich einen gehörigen Stoss geben und eine erste Avversion überwinden muss, und dass man immer glaubt, es sei alles selbstverständlich, was man zu sagen hat. Dies ist aber eine optische Täuschung. Weisst Du noch wie Du in Bern immer so hübsch zu mir kamst?[7] Und nun kann ich mich nicht revanchieren. Ich habe wieder eine recht amüsante Expertise in einem Patentprozess.[8] Wenn wir uns wiedersehen, erzähle ich Dir davon.

May 1916 to Besso

Ich arbeite gegenwärtig recht mässig, sodass es mir hübsch wohl ist, und lebe beschaulich dahin, ohne Misston. In der Gravitation suche ich nun nach den Grenzbedingungen im Unendlichen; es ist doch interessant, sich zu überlegen, inwiefern es eine *endliche* Welt gibt, d. h. eine Welt von natürlich gemessenen endlicher Ausdehnung, in der wirklich alle Trägheit relativ ist.[9] Heute war die Leichenfeier für Schwarzschild, den Leiter der Sternwarte in Potsdam.[10] Gewiss habe ich Dir schon von ihm erzählt; es ist schade um ihn. Er wäre eine Perle gewesen, wenn er so anständig wie gescheit gewesen wäre.[11] Von den Photographien behalte eine, gib eine Maja, eine Zangger und hebe die übrigen auf. Du kannst aber auch abgeben, wenn Du jemand eine Freude damit machen zu können glaubst. Ich fand eine hübsche Vereinfachung der thermodynamischen Ableitung des photochemischen $h\nu$-Gesetzes, so nach Vant Hoff'scher Manier.[12] Es freut mich, dass meine Buben vergnügt sind, und dass Du Dich ihrer annimmst. Nun kann ich Dir bald die ausführliche Arbeit über Gravitation senden, in der alles explizite gerechnet ist.[13]

Sei mit Anna und Vero herzlich gegrüsst von Deinem

Albert.

ALS (SzGB). *Einstein/Besso 1972*, 16 (E. 13). [7 277].

[1]The year is provided by the reference to Schwarzschild's funeral.

[2]Einstein's return to Berlin from Switzerland.

[3]A reference to Laurence Sterne's novel, *Tristram Shandy*. See Doc. 245, note 3.

"and then there was a war"

# On Continued Gravitational Contraction

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(Received July 10, 1939)

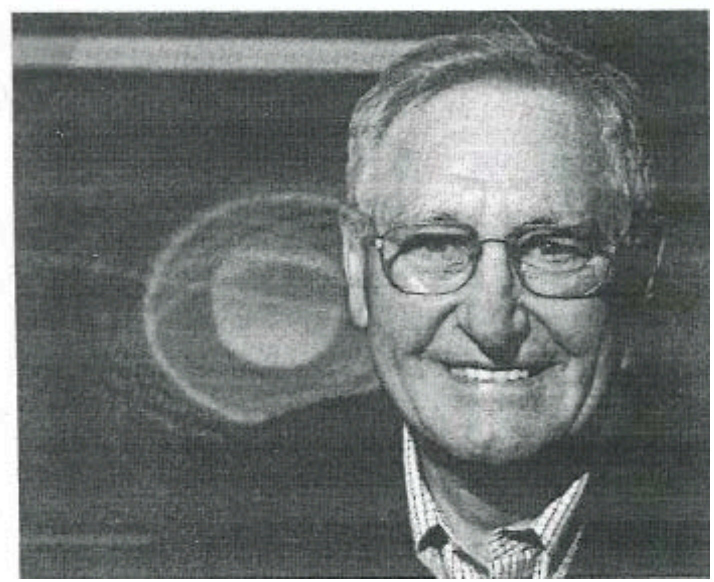
When all thermonuclear sources of energy are exhausted a sufficiently heavy star will collapse. Unless fission due to rotation, the radiation of mass, or the blowing off of mass by radiation, reduce the star's mass to the order of that of the sun, this contraction will continue indefinitely. In the present paper we study the solutions of the gravitational field equations which describe this process. In I, general and qualitative arguments are given on the behavior of the metrical tensor as the contraction progresses: the radius of the star approaches asymptotically its gravitational radius; light from the surface of the star is progressively reddened, and can escape over a progressively narrower range of angles. In II, an analytic solution of the field equations confirming these general arguments is obtained for the case that the pressure within the star can be neglected. The total time of collapse for an observer comoving with the stellar matter is finite, and for this idealized case and typical stellar masses, of the order of a day; an external observer sees the star asymptotically shrinking to its gravitational radius.

1939 Oct. A.E. Ann. Math. (US) 40, 922–36 with angular momenta in preventing collapse/singularity

No

de Laplace, Pierre-Simon. Courtesy of History of Science Collections, University of Oklahoma Libraries

*Left:* Roy Kerr ca. 1975. *R*



# *chapter 9* GRAVITATIONAL COLLAPSE AND ROTATION

R. P. KERR

In the past all exact solutions of collapsing gravitational systems have been spherically symmetric and have been based on the exterior Schwarzschild solution. This solution may be written in a form first given by Eddington (1924):

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2m}{r}(dr + dt)^2, \tag{1}$$

where $r^2 = x^2 + y^2 + z^2$, and units have been chosen so that the velocity of light $c = 1$ and the gravitational constant $G = 1$.

This metric has a true singularity at the origin $r = 0$. However, it has peculiar physical properties inside and on the Schwarzschild sphere, $S$,

$$r = 2m\,. \tag{2}$$

$S$ is a null surface, outside $(r > 2m)$ of which the metric is static, the $t$-axis being timelike. Inside $S(r < 2m)$ the metric is *not* static since the $t$-axis is spacelike.

For matter collapsing all the way to and beyond the Schwarzschild sphere we have the following behavior: matter and energy can pass from the exterior to the interior of $S$, but can never move out again, and so a spherically symmetric system collapsing beyond the Schwarzschild sphere can no longer radiate energy to the outside. It cannot be seen by an outside observer; only its gravitational field can be felt.

A collapsing particle will reach the Schwarzschild sphere and pass into its interior in a finite proper time, i.e., in a finite time as measured by a comoving clock. However, for a distant observer the time of collapse, measured by his clocks, is infinite. He will never observe the stage where the collapsing matter reaches the Schwarzschild sphere and passes to the inside.

This behavior causes difficulties in theories which attempt to explain the large energies emitted by quasi-stellar sources in terms of the gravitational collapse of large masses into the Schwarzschild sphere $S$.

In this paper we wish to show that the topological and physical properties of $S$ may

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R. P. KERR, The University of Texas.

change radically when rotation is taken into account. This suggests that it would be worthwhile to re-examine the problem of gravitational collapse for a mass whose external gravitational field is the stationary field of a rotating body.

An exact solution of Einstein's gravitational field equation for empty space is given by the metric (Kerr 1963):[1]

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2m\rho^3}{\rho^4 + a^2 z^2}(k_\mu dx^\mu)^2, \tag{3}$$

where $k_\mu$ is a null vector field given by

$$k_\mu dx^\mu = dt + \frac{z}{\rho}dz + \frac{\rho}{\rho^2 + a^2}(x dx + y dy) + \frac{a}{\rho^2 + a^2}(x dy - y dx), \tag{4}$$

$m$ and $a$ are arbitrary constants, and $\rho$ is given by

$$\frac{x^2 + y^2}{\rho^2 + a^2} + \frac{z^2}{\rho^2} = 1. \tag{5}$$

The surfaces of constant $\rho$ are confocal ellipsoids of revolution.

For large spatial distances, $\rho$ is given asymptotically by

$$\rho = r + 0(r^{-1}). \tag{6}$$

The metric (3), expanded in powers of $r^{-1}$ becomes

$$\begin{aligned} ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2m}{r}(dt + dr)^2 \\ + \frac{4ma}{r^3}(x dy - y dx)(dt + dr) + 0(r^{-3}). \end{aligned} \tag{7}$$

The term of order $r^{-1}$ shows, e.g., by comparing with equation (1), that $m$ is the mass of the body producing the gravitational field. The term of order $r^{-2}$ shows, by comparing with the solution of the linearized field equations, that $am$ is the angular momentum about the $z$-axis of the rotating body.

The metric of equation (3) has a true singularity on the circle $\Gamma$,

$$z = 0, \qquad R \equiv (x^2 + y^2)^{1/2} = a. \tag{8}$$

This is the analogue of the true Schwarzschild singularity at $r = 0$ in equation (1).

The analogue of the Schwarzschild sphere is now the null surface $S$ given by

$$\rho^4 + a^2 z^2 = 2m\rho^3. \tag{9}$$

In Figure 1 $S$ is plotted in the $(R, z)$-plane for the two cases, $m < a$ and $m > a$. It will be observed that $S$ has a cusp on $\Gamma$. This is not significant, since the points of $\Gamma$ are singularities. For $m > a$, $S$ splits into two disjoint parts, $S_1$ and $S_2$. As $a \to 0$, the outer surface, $S_1$, becomes the Schwarzschild sphere, while $S_2$ collapses into the origin. When $a = m$ the two components, $S_1$ and $S_2$, touch on the $z$-axis. For $a > m$ the surfaces are as in

[1] More general vacuum solutions are given by Kerr and Schild (1964).

Figure 1, *a*. When $m \rightarrow 0$ ($a \neq 0$), $S$ shrinks to the ring $\Gamma$. In most physical situations $a \gg m$, and so neither $S_1$ nor $S_2$ has the topology of a sphere.

There is a further complication of this metric, which is not present when $a = 0$. Suppose we define $D$ as the disk bounded by $\Gamma$ given by

$$R < a, \quad z = 0. \tag{10}$$

It is represented in Figure 1 by a solid line. We shall now show that the metric in equation (3) is not even differentiable, let alone analytic, on $D$. To see this we first observe that equation (3) has two distinct real roots, $\rho_+ > 0$ and $\rho_- < 0$, for all points except $D$. In order for the metric to be continuous, we choose the root $\rho_+$ for all points. From equation (5) this gives

$$\rho_+ = \frac{a\,|\,z\,|}{\sqrt{a^2 - r^2}}, \qquad \text{near } D, \tag{11}$$

and so $\rho_+$ is not differentiable on $D$. Substituting equation (11) into equation (3) we can easily see that the metric itself is not differentiable on $D$.

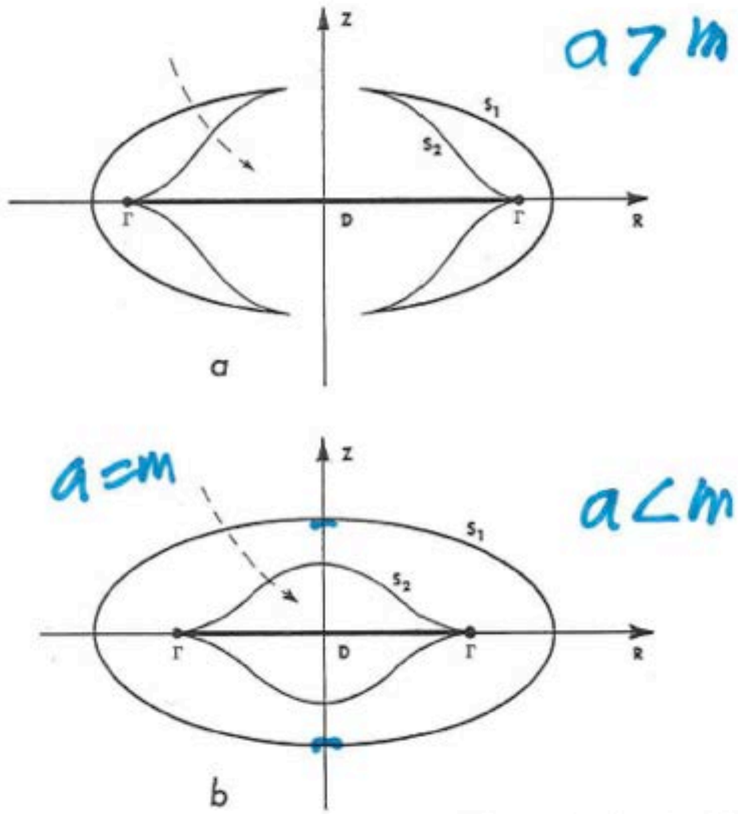


FIG. 1.—*a*, the "Schwarzschild surface" for $a > m > 0$. The solid disk, $D$, is a branch cut, bounded by the ring, $\Gamma$. *b*, the "Schwarzschild surface" for $m > a > 0$.

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ZUND, J., Southwest Center for Advanced Studies
ZUND, J. D., University of Texas


"IT'S BLACK, AND IT LOOKS LIKE A HOLE. I'D SAY IT'S A BLACK HOLE."

≲1963 RH Dicke
(-Calcutta)
1963-64 AAAS (tx??)
Hong Yee Chiu
Anne Ewing
1967-68 JA Wheeler

See Bartusiak
Black Hole
2015

How do you
look for Them

# Dr. Oktay Hüseyin'in Özgeçmişi
(1938-2009)

To my dear friends
Virginia Trimble
and Joe Weber!

I am respectful on
your scientific results,
on the impact done for
new research
But even more I am
valuating our personal
contacts, our human
relations! My late wife Angelica
also loved both of you!
22/XI 85

43

# КОЛЛАПСИРОВАВШИЕ ЗВЕЗДЫ В СОСТАВЕ ДВОЙНЫХ*

*(совместно с О. Х. Гусейновым)*

В работе предлагается способ обнаружения коллапсированных звезд в составе спектрально-двойных звезд. Среди спектрально-двойных систем с невидимыми спутниками выбраны несколько пар, в которых можно предполагать, что их компоненты являются коллапсировавшими звездами.

Коллапсировавшие звезды (КЗ), существование которых предсказывает общая теория относительности, могут быть обнаружены лишь по их гравитационному полю (см. обзор и ссылки на оригинальную литературу в [1]). Сравнение общей массы скопления с массой видимых звезд с целью определения массы КЗ весьма неубедительно ввиду ненадежности определения масс. Желательно найти индивидуальную КЗ. Это, в принципе, можно сделать, если КЗ входит в состав двойной в паре с обычной звездой (ОЗ).

Просмотр литературы [2] позволил выбрать среди спектрально-двойных звезд с невидимыми спутниками несколько пар, в которых можно предполагать, что спутник является КЗ. Приводим данные для этих пар (см. таблицу). В первых двух столбцах помещены справочные данные — название звезды и ее координаты. Затем следуют наблюдательные данные — видимая величина $m$ и спектральный класс наблюдаемой звезды, период двойной $p$, половина амплитуды периодического изменения скорости $K_1$, и вычисленные с помощью небесной механики $a_1 \sin i$ — проекции большой полуоси орбиты на картинную плоскость и $MF = \mathfrak{M}_2^3 \sin^3 i/(\mathfrak{M}_1 + \mathfrak{M}_2)^2$ в $\mathfrak{M}_\odot$, индекс 1 относится к ОЗ, индекс 2 — к КЗ. Следующие величины определяются весьма ненадежно: по спектральному классу и классу светимости находим массу ОЗ $\mathfrak{M}_1$. Предполагая наклонение $i = 90°$, $\sin^3 i = 1$, находим массу КЗ $\mathfrak{M}_2$. Величины $\mathfrak{M}_2'$ вычислены для среднего значения $\sin^3 i = 2/3$. И, наконец, приводится абсолютный параллакс $\pi$. Отметим, что предположение $i = 90°$ при данных $MF$ и $\mathfrak{M}_1$ дает минимальное значение массы второй звезды. В парах, приведенных в таблице, невидимый спутник имеет массу $\mathfrak{M}_2$ больше, чем масса наблюдаемой обычной звезды $\mathfrak{M}_1$. Можно высказать гипотезу, что невидимые компоненты указанных спектрально-двойных звезд являются КЗ. Можно считать, что невидимая компонента последней звезды таблицы является нейтронной.

Однако сделанный вывод отнюдь не является категорическим. По замечанию И. Д. Новикова, невидимая компонента спектрально-двойной системы может давать спектр без заметных на фотографии спектральных линий и «невидима» лишь в этом смысле. Л. И. Снежко отмечает, что в состав двойных часто входят звезды, прошедшие стадию гиганта, сбросившие водородную оболочку и сжигающие гелий. В этом случае светимость ОЗ гораздо больше, чем на главной последовательности[1], и вторая — невидимая — звезда может находиться на главной последовательности и тем не менее быть невидимой по

* Астрономический журнал, 1966, т. 43, № 2, с. 313—315.

[1] Такая ситуация довольно часто отмечается среди двойных звезд [10].

| № | Звезда | $\alpha_{1950}$ | $\delta_{1950}$ | $m_1$ | Sp | P, дн | $K_1$, км/с | $a_1 \sin i$, $10^6$ км | MF | $\mathfrak{M}_1$ | $\mathfrak{M}_2$ | $\mathfrak{M}_2''$ | $\pi''$ | $m_2$ | Литературный источник |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | HD 187399 | $19^h46^m,6$ | $+29°17'$ | 7,7 | B9 | 27,97 | 104,5 | 37,6 | 2,72 | 4,4 | 7,1 | 9,6 | [illegible] | | [3] |
| 2 | +40°1196 | 05 03,7 | +40 53 | 8,1 | B3 | 3710 | 31,5 | 1528 | 10,4 | 10 | 22 | 29 | | | [4] |
| 3 | HD 30353 | 04 45,2 | +43 12 | 7,7 | $cAS_p$ | 359,7 | 51,3 | 244 | 4,5 | 12,6 | 15,1 | 19 | | | [5] |
| 4 | HD 193928 | 20 17,8 | +36 36 | 9,7 | WN6 | 21,64 | 130 | — | 4,94 | 10 | 14,2 | 18 | | | [6] |
| 5 | π Cep A | 23 06,0 | +75 07 | 4,56 | gG1 | 556,2 | 23,02 | 169 | 0,623 | 3 | 2,76 | 3,4 | 0,11 | 6,1 | [7] |
| 6 | ξ Pav | 18 18,6 | −61 31 | 4,25 | gM1 | 2214 | 17,92 | 526 | 1,188 | 7 | 5,9 | 7,2 | 015 | 3 | [8] |
| 7 | α Her B | 17 12,4 | +14 27 | 5,39 | F8 | 51,58 | 36,12 | 25,62 | 0,258 | 1,4 | 1,22 | 1,48 | 006 | 10,2 | [9] |

контрасту с более яркой (хотя и менее массивной) первой.

В спектре первой звезды — HD 187399 наблюдаются линии поглощения водорода, смещенные в сторону отрицательных скоростей, которые автор приписывает разлетающемуся облаку водорода, окружающему орбиту двойной звезды. В случае третьей звезды — HD 30353 авторы отмечают бедность звезды водородом и необычайно большое значение функции масс. Относительно звезды +40° 1196 известно, что движение, определяемое по водороду, кальцию и железу, сильно различается.

Главная задача предлагаемой заметки — привлечь внимание наблюдателей к указанным выше (и аналогичным) объектам. Наряду с тщательным изучением спектра было бы весьма важно определить параллакс рассматриваемых звезд или хотя бы его нижнюю границу. Зная расстояние до звезды, можно было бы определить ее абсолютную величину и выяснить, может ли вторая звезда быть невидимой по тривиальной причине, из-за аномально большой светимости первой. В последних случаях [5—7], где параллакс известен, мы нашли $m_2$ — видимую величину, которую имела бы звезда массы $\mathfrak{M}_2$, если бы она находилась на главной последовательности. Эта величина определена очень грубо, так как на возможные ошибки при определении массы $\mathfrak{M}_2$ здесь наложены еще возможные неточности значений параллаксов, например, из-за неправильного учета собственного движения ОЗ.

Можно пытаться обнаружить собственное движение звезды (порядка $0'',01$ — на грани точности наблюдения), а также с помощью интерферометра Майкельсона проверить, не является ли звезда визуально-двойной. Можно искать специфические явления, связанные с движением газа в поле КЗ [1]. Доказательство реального существования хотя бы одной коллапсировавшей звезды имело бы большое принципиальное значение для всей теории эволюции звезд [2]. Даже отрицательный результат, т. е. объяснение всех рассматриваемых пар без привлечения КЗ представляет определенный интерес: из статистических сопоставлений можно будет сделать вывод, что эволюция звезд большой массы не приводит к состоянию коллапсировавшей звезды, очевидно, вследствие закономерностей потери массы на определенных стадиях эволюции [3]; начало коллапса может затянуться также из-за вращения звезд, на что

[2] Возможность обнаружения коллапса по испусканию энергичных нейтрино рассмотрена в [11].

[3] Такую возможность обсуждал, например, А. Камерон [12].

уже указывает большое число видимых и затменных пар по сравнению с подозреваемыми нами КЗ среди звезд с массами больше $2M_{\odot}$. В работе [13] отмечается, что наличие белых карликов в молодых скоплениях (и, в частности, в составе двойных звезд) указывает на возможность сброса массы, протекающего за время порядка $10^4$—$10^5$ лет.

В заключение нужно снова подчеркнуть большую принципиальную важность обнаружения коллапсировавшей звезды. В силу этого каждый случай, в котором имеется вероятность доказать наличие такого необычного объекта, должен стать предметом тщательного и всестороннего критического изучения.

Выражаем благодарность за помощь и дискуссии П. Г. Куликовскому И. Д. Новикову и Л. И. Снежко.

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18 октября 1965 г.

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## КОММЕНТАРИЙ

Работа, вызвавшая в свое время громадный резонанс и способствовавшая постановке большого числа наблюдательных программ в ряде обсерваторий. Следуя идеям комментируемой статьи, В. Тримбл и К. Торн [1*] составили обширный список «кандидатов в черные дыры».

Следует тем не менее отметить, что благодаря успехам рентгеновской астрономии поиск черных дыр пошел иным путем.

Однако для того, чтобы отличить черную дыру от нейтронной звезды, и в настоящее время пользуются критерием массы, превышающей определенный предел, около 3 масс Солнца.

Вместе с замечанием о роли аккреции на нейтронные звезды (см. статью 44) комментируемая работа стимулировала теорию и наблюдения конечных продуктов эволюции.

[1*] *Trimble V. L., Thorne K. S.*— Astrophys. J., 1969, vol. 156, p. 1013.

THE ASTROPHYSICAL JOURNAL, Vol. 156, June 1969


# SPECTROSCOPIC BINARIES AND COLLAPSED STARS*

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*Received September 5, 1968*

## ABSTRACT

Lists are given of single-line, spectroscopic binaries with large mass functions. The absence of a secondary spectrum in these systems could, in principle, result from the secondary star's being either a collapsed star or a massive neutron star. For all these systems, however, other explanations are possible in the light of present observations. Statistical considerations suggest that few, if any, of the systems in these lists contain collapsed or neutron-star secondaries. None of these binary systems coincide with any published X-ray source position.

## I. NON-ECLIPSING, SINGLE-LINE BINARIES WITH LARGE MASS FUNCTIONS

Theoretical considerations suggest that some massive stars should terminate their evolution by gravitational collapse (see, e.g., Wheeler 1966; Thorne 1967). The end products of such collapse—"neutron stars" of $\mathfrak{M} \lesssim 1.5\,\mathfrak{M}_\odot$, and "collapsed stars" of $\mathfrak{M} \gtrsim 1.5\,\mathfrak{M}_\odot$—have never been observed and, indeed, should be virtually impossible to observe directly because of their small size.

Zel'dovich and Guseynov (1965) have proposed that collapsed stars and neutron stars might be found among the unseen companion stars of single-line spectroscopic binaries. Motivated by this suggestion, we have searched the *Sixth Catalogue of the Orbital Ele-*

OAP-146

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## ABSTRACT

Lists are given of single-line, spectroscopic binaries with large mass functions. The absence of a secondary spectrum in these systems could, in principle, result from the secondary star being a collapsed star or a massive neutron star. For all these systems, however, other explanations are possible in the light of present observations. Statistical considerations suggest that few, if any, of the systems in these lists contain collapsed or neutron-star secondaries. None of these binary systems coincides with any published X-ray source position.

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†National Science Foundation Predoctoral Fellow during part of the period of this research.

‡Alfred P. Sloan Foundation Research Fellow.

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September 1968

## DO BLACK HOLES EXIST?

1. Definition 1: For AGNs etc, increasing probability from 1964 (Salpeter, Zeldovich....) to present.

   In spectroscopic binaries, Zeldovich & Guseinov,1965 Trimble & Thorne 1969;

   Cygnus X-1, the last wrong paper (T, Rose & Weber 1973 MN 162, pink page 1; DM Popper sad story)

   PBH and IMBH to be determined

2. Definition 2: Horizons: total absence of Type I X-ray bursts in nominal BHXRBx, range of a/m, 0.3-0.95 $\pm$ event horizon telescope

3. Definition 3: Radiatively inefficient accretion; ADA??? Penrose, Blandford-Znajek processes

4.5. Not my territory!


**Figure 1 | The globular cluster 47 Tucanae.** Kızıltan *et al.*[1] have discovered an intermediate-mass black hole that is about 2,200 times more massive than the Sun in the core of a bright cluster of stars called 47 Tucanae (centre). The authors' evidence comes from observations of spinning neutron stars called pulsars (not visible in this infrared image) in the globular cluster.

# LETTER

doi:10.1038/nature21361

# An intermediate-mass black hole in the centre of the globular cluster 47 Tucanae

Bülent Kızıltan[1], Holger Baumgardt[2] & Abraham Loeb[1]

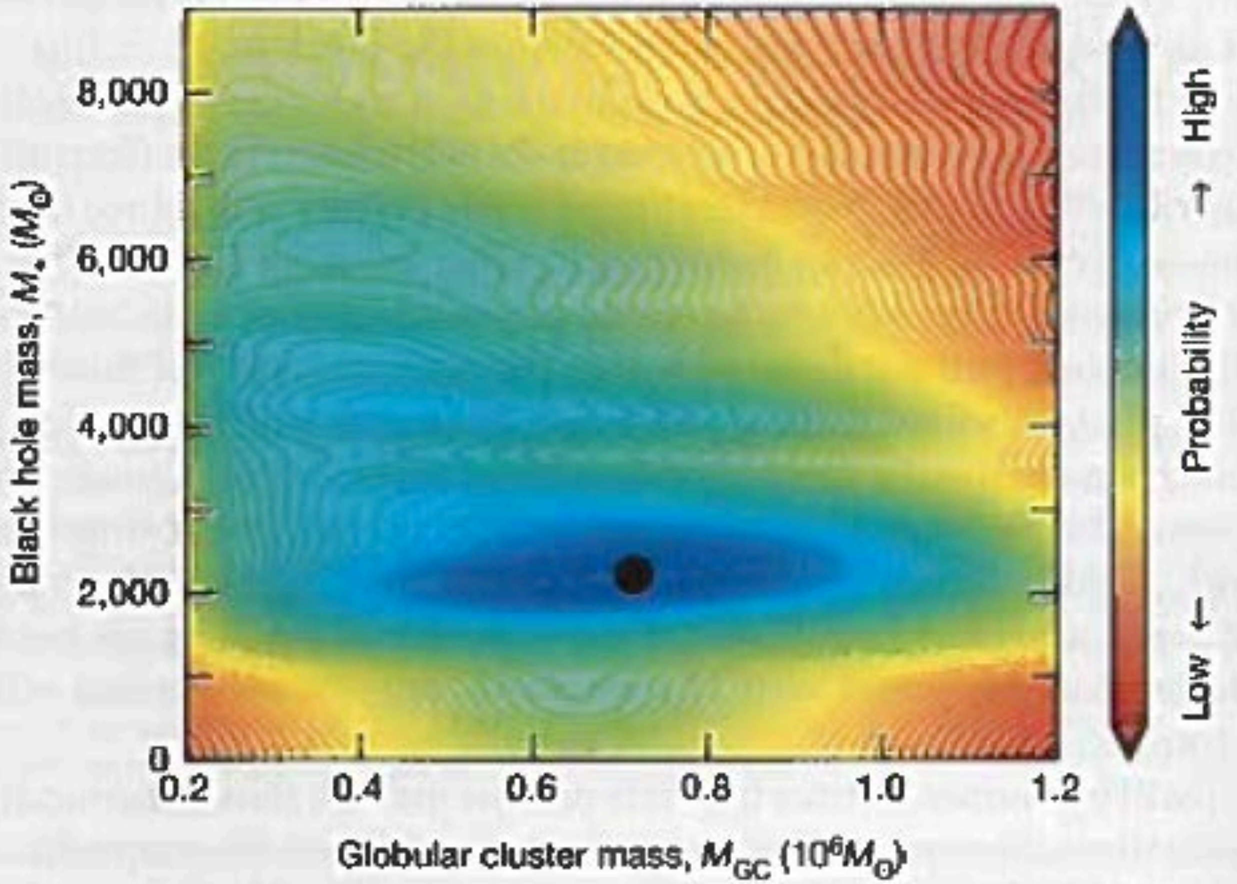
**Intermediate-mass black holes should help us to understand the evolutionary connection between stellar-mass and super-massive black holes[1]. However, the existence of intermediate-mass black holes is still uncertain, and their formation process is therefore unknown[2]. It has long been suspected that black holes with masses 100 to 10,000 times that of the Sun should form and reside in dense stellar systems[3–6]. Therefore, dedicated observational campaigns have targeted globular clusters for many decades, searching for signatures of these elusive objects. All candidate signatures appear radio-dim and do not have the X-ray to radio flux ratios required for accreting black holes[7]. Based on the lack of an electromagnetic counterpart, upper limits of 2,060 and 470 solar masses have been placed on the mass of a putative black hole in 47 Tucanae (NGC 104) from radio and X-ray observations, respectively[8,9]. Here we show there is evidence for a central black hole in 47 Tucanae with a mass of $2,200^{+1,500}_{-800}$ solar masses when the dynamical state of the globular cluster is probed with pulsars. The existence of an intermediate-mass black hole in the centre of one of the densest clusters with no detectable electromagnetic counterpart suggests that the black hole is not accreting at a sufficient rate to make it electromagnetically bright and therefore, contrary to expectations, is gas-starved. This intermediate-mass black hole might be a member of an electromagnetically invisible population of black holes that grow into supermassive black holes in galaxies.**

An intermediate-mass black hole (IMBH) strongly affects the spatial distribution of stars in globular clusters (GCs). Massive stars

and *N*-body models[14] imply that no clear distinction between globular cluster models can be made on the basis of available velocity dispersion measurements alone (Fig. 2). In order to constrain the dynamical effects of a black hole in a cluster, we take a fundamentally different approach. In addition to pulsar accelerations, we jointly use this spatial imprint that carries information about the black hole beyond the radius of influence. We quantify how likely it is that a range of observed pulsar accelerations are related to specific model distributions.

The dynamical *N*-body simulations of isolated star clusters evolve under the influence of stellar evolution and two-body relaxation. A grid of several hundred star clusters starting with different initial half-mass radii, density profiles and masses of their central black hole are run up to an age of $T = 11.75$ Gyr to match the age of 47 Tuc[14]. We select those clusters that best match the surface density and velocity dispersion profiles of this globular cluster. The presence of primordial binaries does not play a notable role in the final segregation profile of heavy stars if clusters with the same surface density profile are compared (see Extended Data Fig. 1 and Methods). No other *a priori* assumptions are made that limit the dynamics of the cluster.

The best-fitting models for the no-IMBH case and for IMBHs with 0.5% and 1% of the cluster mass are selected as a subset of viable replicas of 47 Tuc. Models with black hole masses larger than 1% of the cluster mass lead to fits which significantly deviate from the observed density and velocity dispersion profile of 47 Tuc. Hence such massive black holes are ruled out, while models with smaller black hole masses or without a black hole lead to comparable fits.




black hole (see key, numbers in parentheses in units of solar masses).
b, The joint inference for the total globular cluster mass and the central black hole mass in 47 Tuc. Relative probability is colour coded (colour bar at right). The black dot shows peak probability at $M_\bullet \approx 2{,}200 M_\odot$ and $M_{GC} \approx 0.75 \times 10^6 M_\odot$.

## WHAT ARE THEY GOOD FOR?

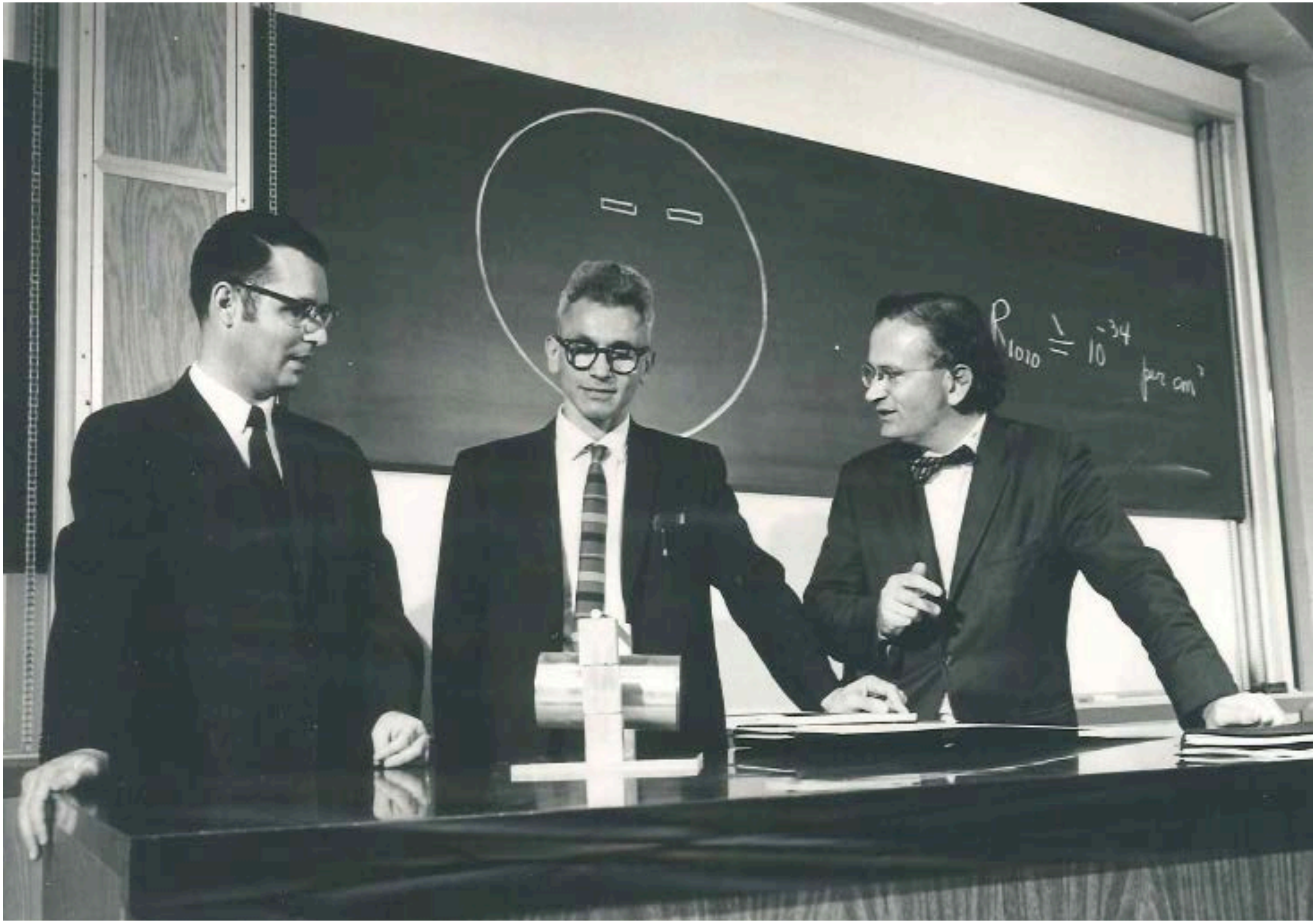
Gamma ray bursts as Hawking radiation? mostly not

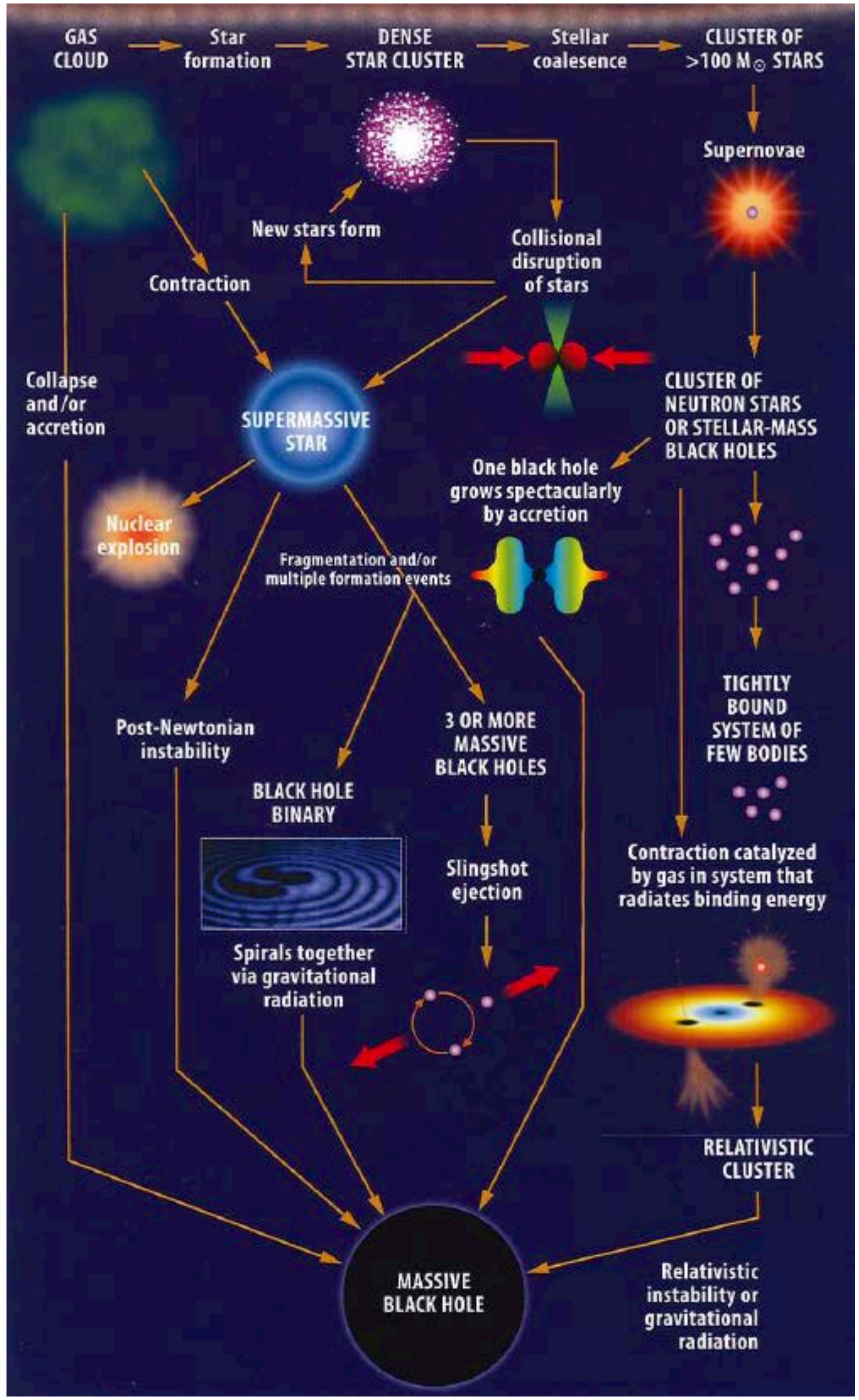
Dark matter: Not PBH (if H radiation exists)

some phase space between $10^{15}$ grams and Jupiter ($10^{30}$g)
MACHO limit

maybe some phase space for IMBH, too few for MACHO searches
not massive enough to mess up halo kinetmatics
not massive enough to lens QSOs

Burst of gravitational radiation (pre 1985 only)




GAS CLOUD
Star formation
DENSE STAR CLUSTER
Stellar coalesence
CLUSTER OF >100 M⊙ STARS
Supernovae
New stars form
Collisional disruption of stars
Contraction
Collapse and/or accretion
SUPERMASSIVE STAR
CLUSTER OF NEUTRON STARS OR STELLAR-MASS BLACK HOLES
One black hole grows spectacularly by accretion
Nuclear explosion
Fragmentation and/or multiple formation events
TIGHTLY BOUND SYSTEM OF FEW BODIES
Post-Newtonian instability
3 OR MORE MASSIVE BLACK HOLES
BLACK HOLE BINARY
Slingshot ejection
Contraction catalyzed by gas in system that radiates binding energy
Spirals together via gravitational radiation
RELATIVISTIC CLUSTER
MASSIVE BLACK HOLE
Relativistic instability or gravitational radiation