Strong gravity & Numerical Relativity

L. Lehner

Perimeter Institute/CIFAR

NR goal: understand 'strongly gravitating' regimes

"Instead of: If we think hard enough, we don't need a computer,

With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with ***completely new and unexpected*** *things to think about" [M. Choptuik]*

- (M/L) ~ 1 ; (v/c) ~ 1 ; no `restrictive' symmetries solve Einstein equations in full generality

- Overarching questions:
  - Gravitational waves & connection with behaviour of source: gravitational wave astronomy
  - Understanding of spacetime structure in relevant/interest scenarios
  - Explore conjectures
  - Provide intuition for `analytical modeling', mathematical analysis
  - *uncover surprises…*

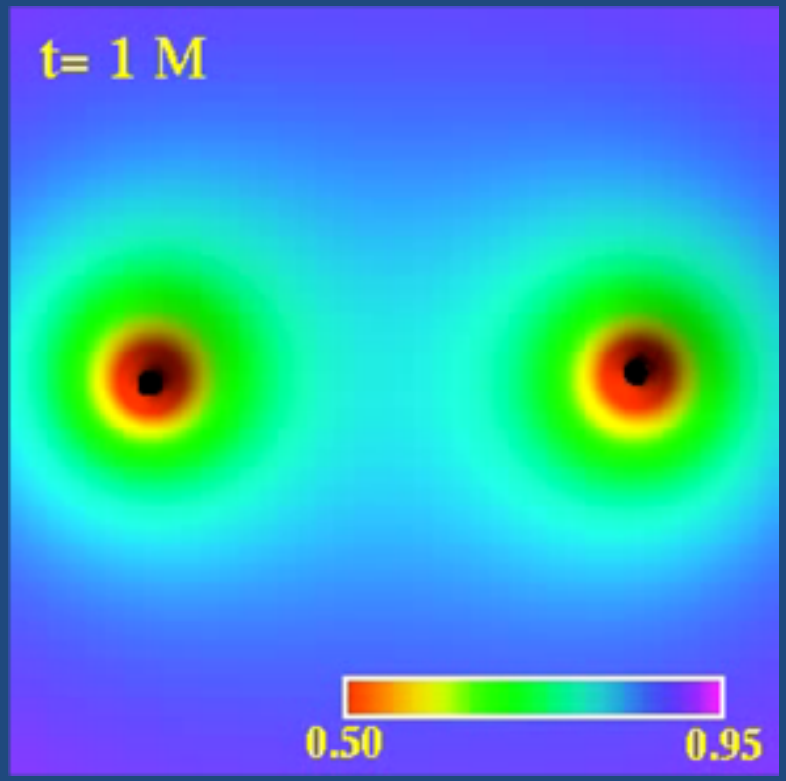
What's involved

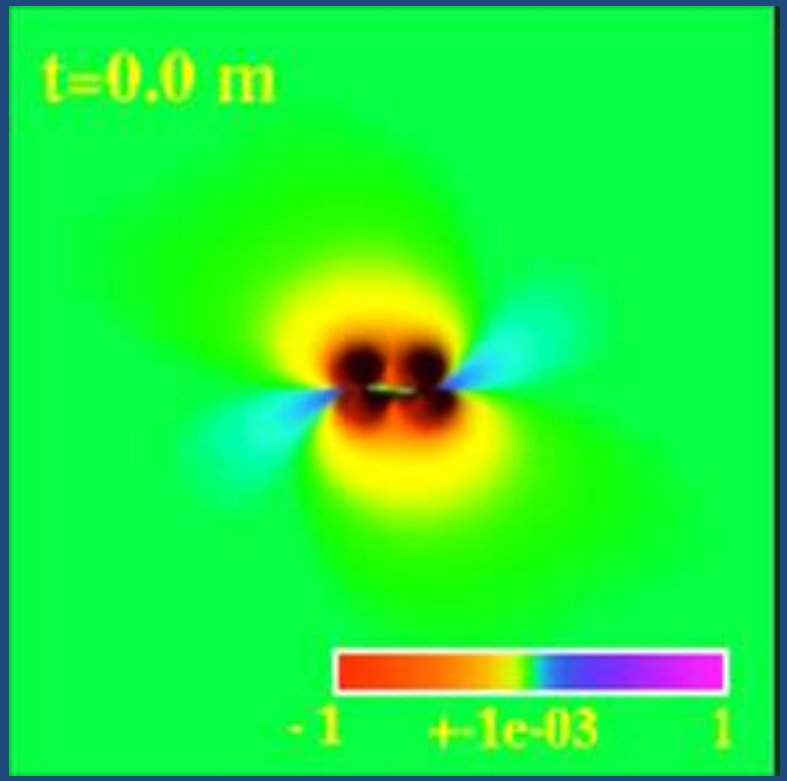
- express EEs in terms of an *initial boundary value problem (formulation)*
- ensure a well posed problem is defined (eqns & BCs) *adopt or rewrite eqns to ensure symmetric/strong hyperbolicity of evolution equations*
- discretize equations (to obtain an algebraic problem) which introduces a discrete length h (as h 0 one recovers continuum problem) *adopt particular approximation methods/algorithms/discretization strategies*
- recognize discrete equations do not necessarily behave well (in continuum terms, eqns & data off the constraint surface and physical conditions) *modify continuum eqns so that constraint surface is an attractor*
- *Avoid singular region (through excision or slicing)*
- Implement eqns on sufficiently powerful computational infrastructure (software & hardware)

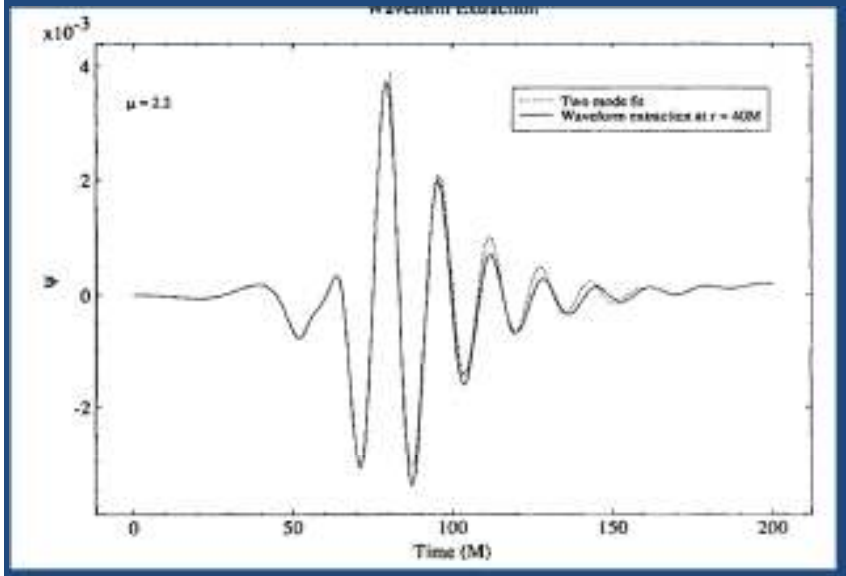
- Arguably < 1990s : head-on collision of black holes (Smarr'80) critical phenomena in GR (Choptuik 86-93). Things changed rapidly after that, especially from 2005 onwards

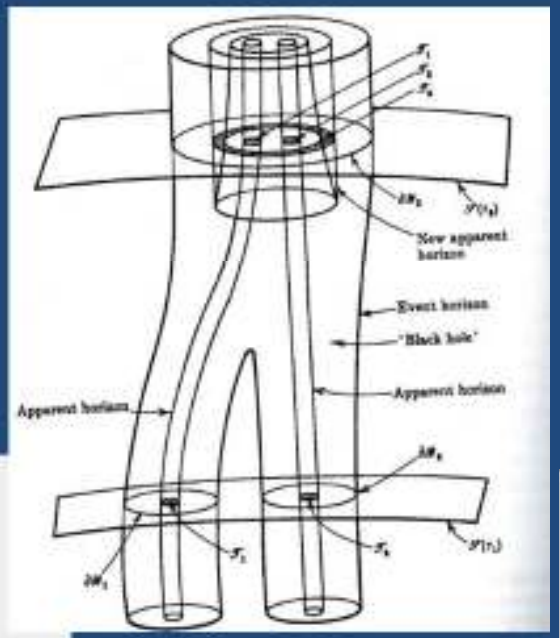
2-body problem in GR (NS/BH)

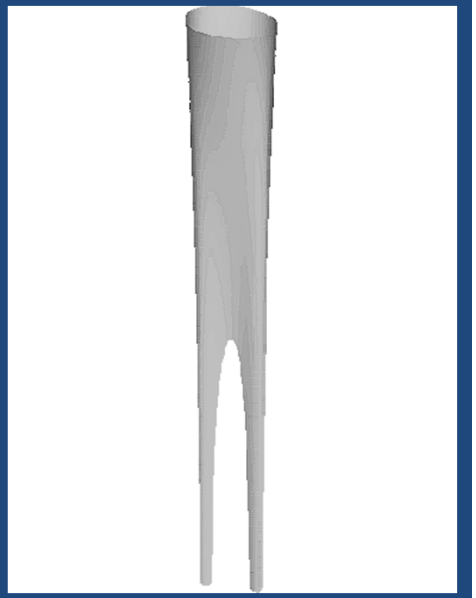
- Cauchy formulation of EEs: (generalized) Harmonic or: BSSN (ADM-augmented eqns for well posedness). Both employing a rather rigid set of gauge conditions. [100s of procs, ~ 1month timescale]
- "Quasi-circular" initial configuration (zero-eccentricity)

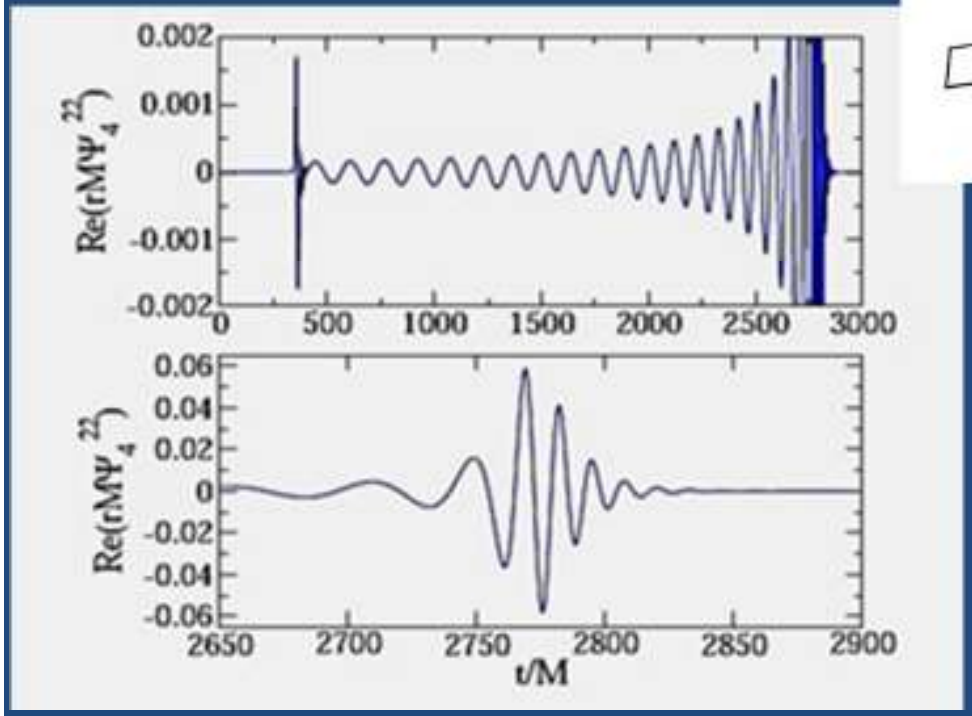





x10^-3
μ = 2.5
Two mode fit
Waveform extraction at r = 40M
ψ
Time (M)


New apparent horizon
Event horizon
"Black hole"
Apparent horizon
Apparent horizon




Re(rMΨ4 22)
t/M


t
y
x

Main take home messages…

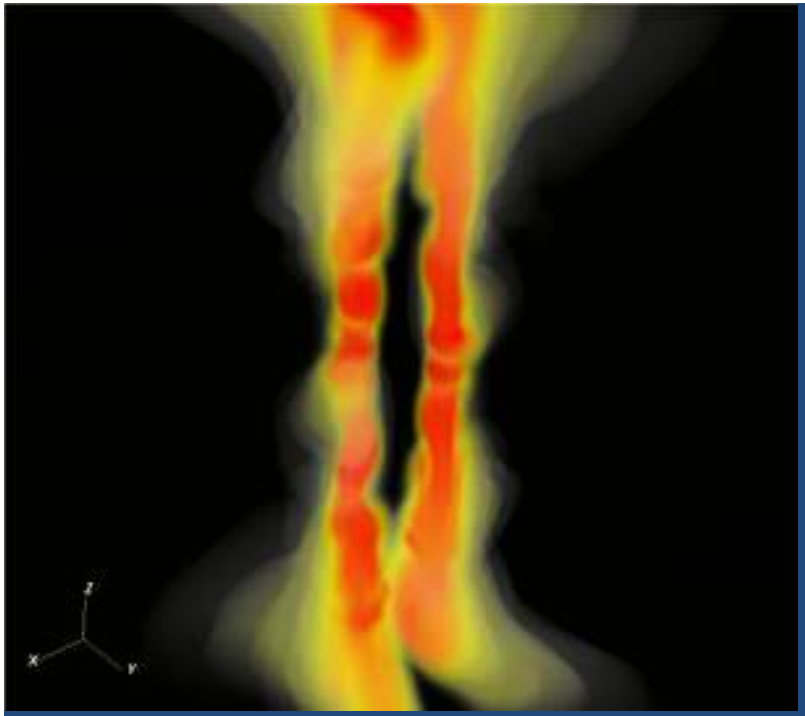
- Effective one body approach can capture quite a bit of observed behavior and be used for data analysis efforts

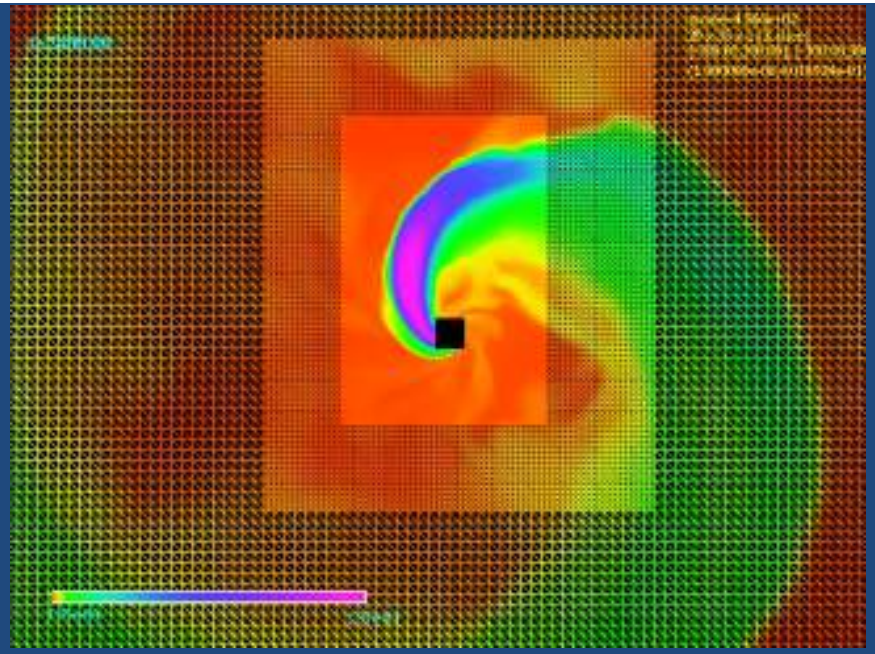
- Radiation: converts ~ 5% of total intial mass and angular momentum. (can be higher for 'tuned' collisions).
  - $E_{GW} \sim 10^{58}$ ergs $(M_T/10^6\, M_{sun})$ in ~ 100 $(M_T/10^6\, M_{sun})$ s
  - $L_{GW} \sim 10^{23}\, L_{sun}$
  - Asymmetric scenarios give rise to 'kicks', which can be as large as 3-8 10^3 km/s!
    - Yet… these need some tweaking.
    - A few 100s km/s more typical. (Mech Energy~ 10^{53} ergs $(M_T/10^6\, M_{sun})$ >> SN !)

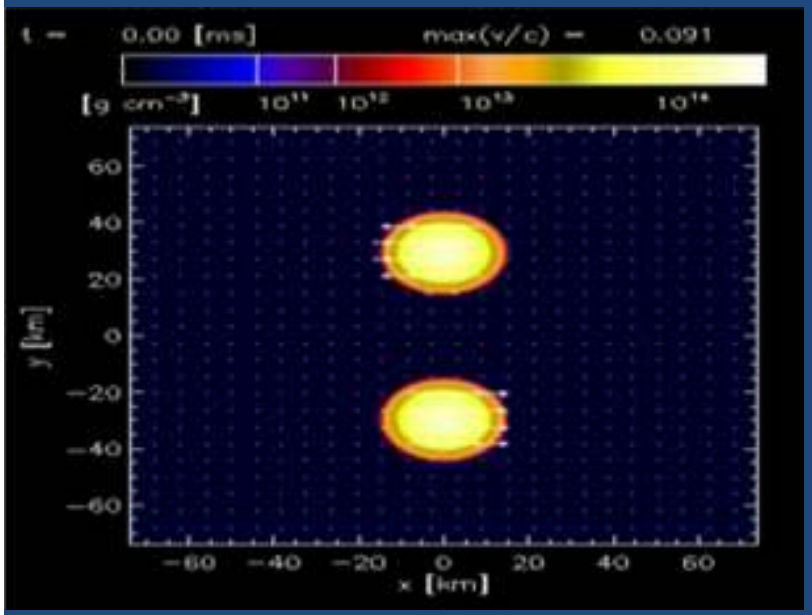
- Just a fraction of this into surrounding gas/matter/fields can trigger an observable counterpart. e.g. GRBs, etc.
  - Obviously exciting prospects for multimessenger astronomy
  - Models must include suitable ingredients…

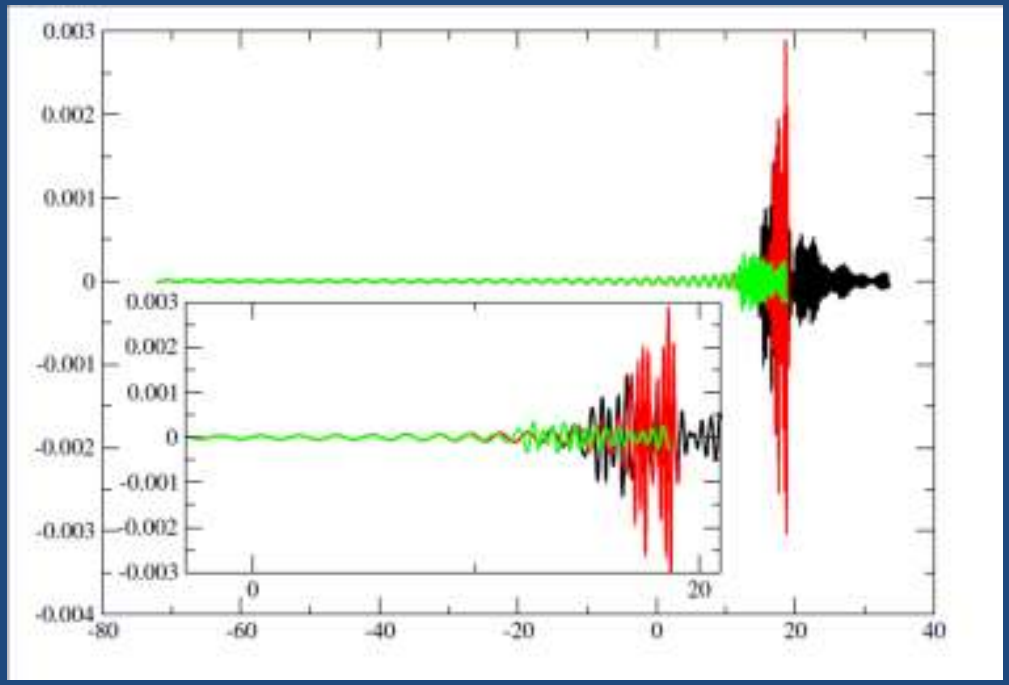
Current efforts

- Construct accurate waveform templates for usage in data analysis [PN + NR + BH Perturbations]. Cover parameter space
- Astrophysical implications in supermassive BH growth by mergers
- Non-vacuum binaries (BH-NS; NS-NS). Further physics in BH-BH systems
- Gravitational waves in alternative gravity theories
- Multimessenger astronomy (for astrophysics & fundamental qns)






t = 0.00 [ms]
max(v/c) = 0.091
[g cm⁻³]
10¹¹
10¹²
10¹³
10¹⁴
y [km]
x [km]


0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
-80
-60
-40
-20
0
20
40

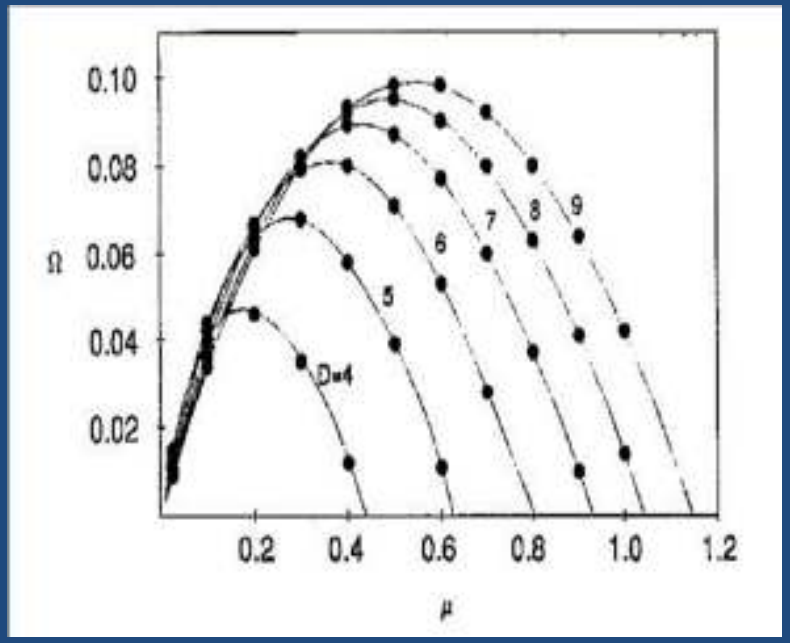
- Qualitative features of waveforms in non-vacuum cases also understood in simple terms
- no ‘extremal’ scenario arises (i.e. cosmic censorship seems to hold)
- Regions ‘violating’ Kerr bound loose angular momentum before a trapped surface arises
- Late time behavior consistent with Kerr solution (QNMs, horizon behavior, asymptotic structure)

Additional front: higher dimensional gravity

- At the gravity level alone, D=4 is special, but how special?
  - No stable circular orbits and no "Kerr-bound" . There are richer geometries in higher dimensional Ricci-flat Lorentzian manifolds, in particular the zoo of "black objects" – black spheres, rings, strings, saturns, …
- If string theory is providing the correct path to a consistent theory of nature valid at Planck scales, the universe is fundamentally higher dimensional
- Lots of examples on (holographic) supposedly describing many aspects of conventional non-gravitational 4D physical processes in terms of 5-dimensional gravity
  - interestingly, the gravitational dual to many relevant processes involves *black holes*

Black strings

1.- Contain singularities
2.- Ruled by null-rays
3.- Non-unique even in spherical symm

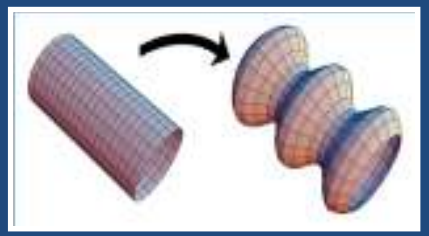


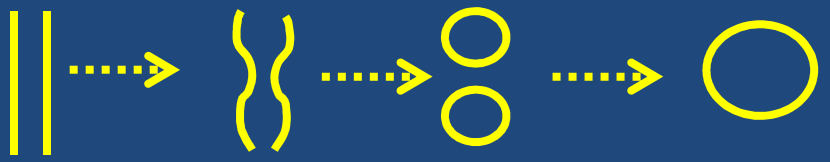
Stability?

- Black string perturbations admit exponential growth for $L > L_c$ (Gregory-Laflamme)

- Entropy $S_{BS} < S_{BH}$ (for a given M) [bs ~ M^2/L ; bh ~ $M^{3/2}$]

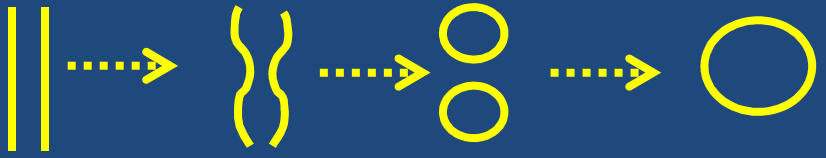
Conjecture: Black strings will bifurcate

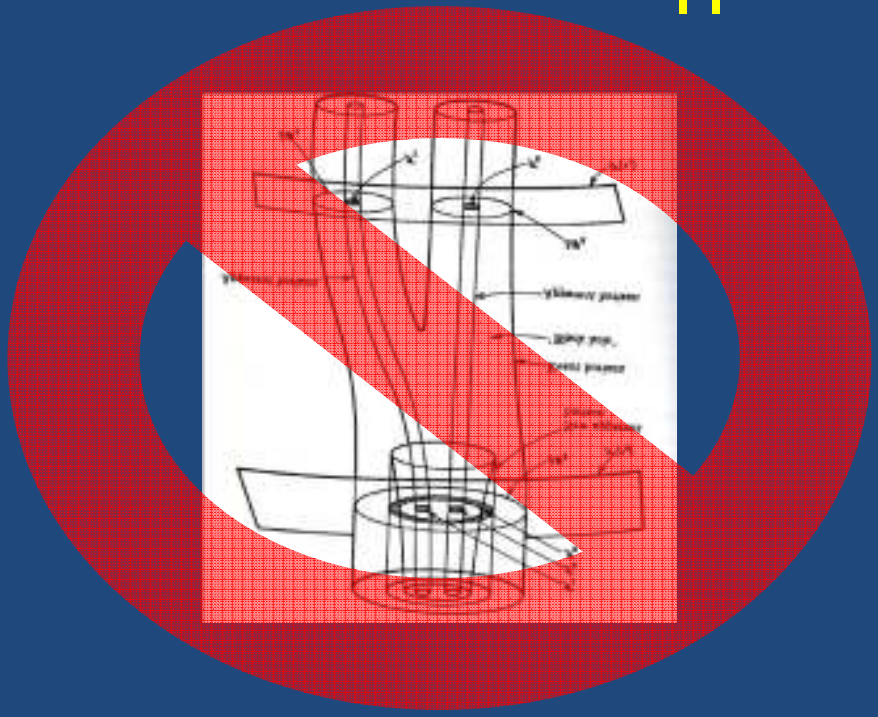




End-state of the instability?

Conjecture: Black strings will bifurcate





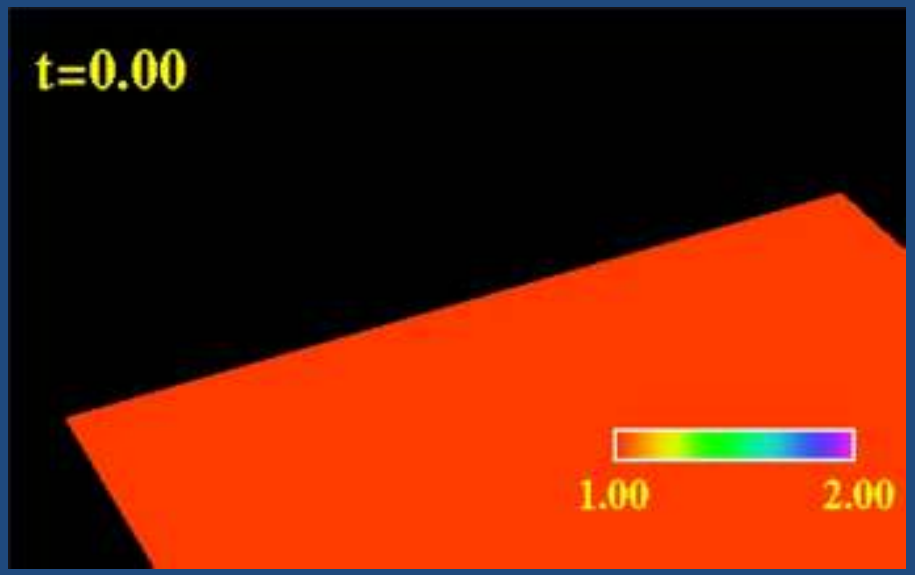
- This cannot happen [Hawking-Ellis '73] if spacetime is asymptotically predictable.
- A naked singularity must form for this to happen $\rightarrow$ a *generic* example of cosmic censorship violation in higher dimensional gravity

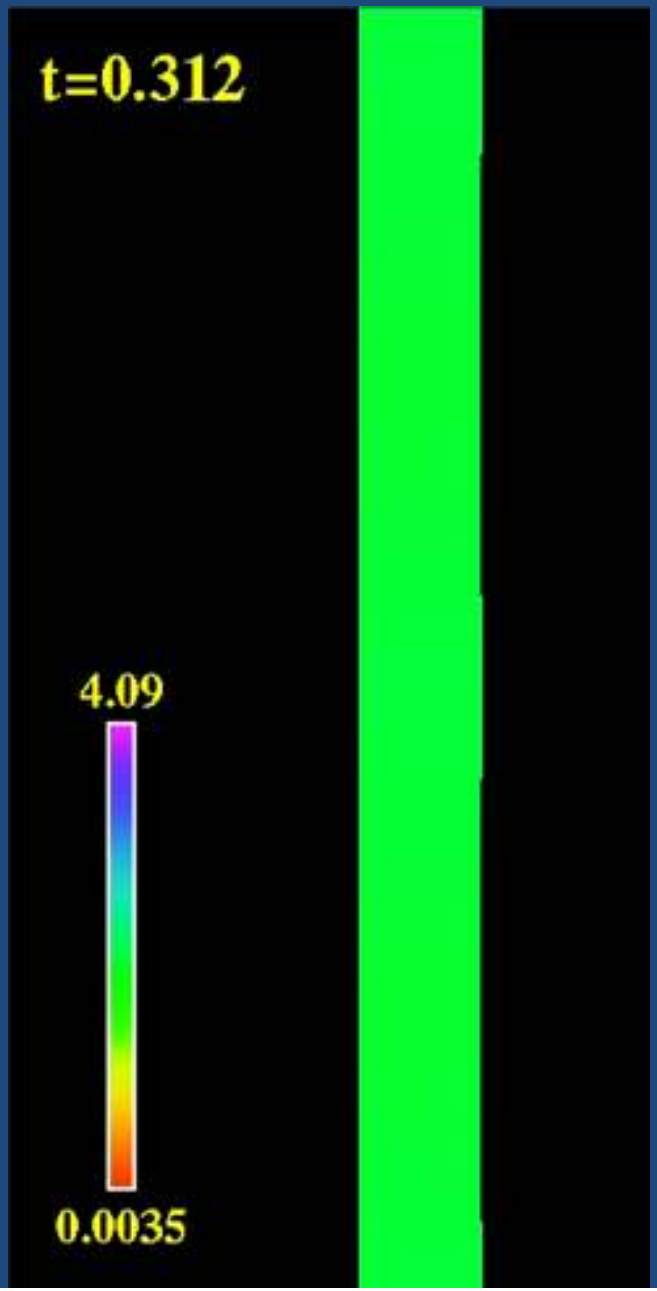
Dynamics?

L/m = 20; L_c ~ 14 m

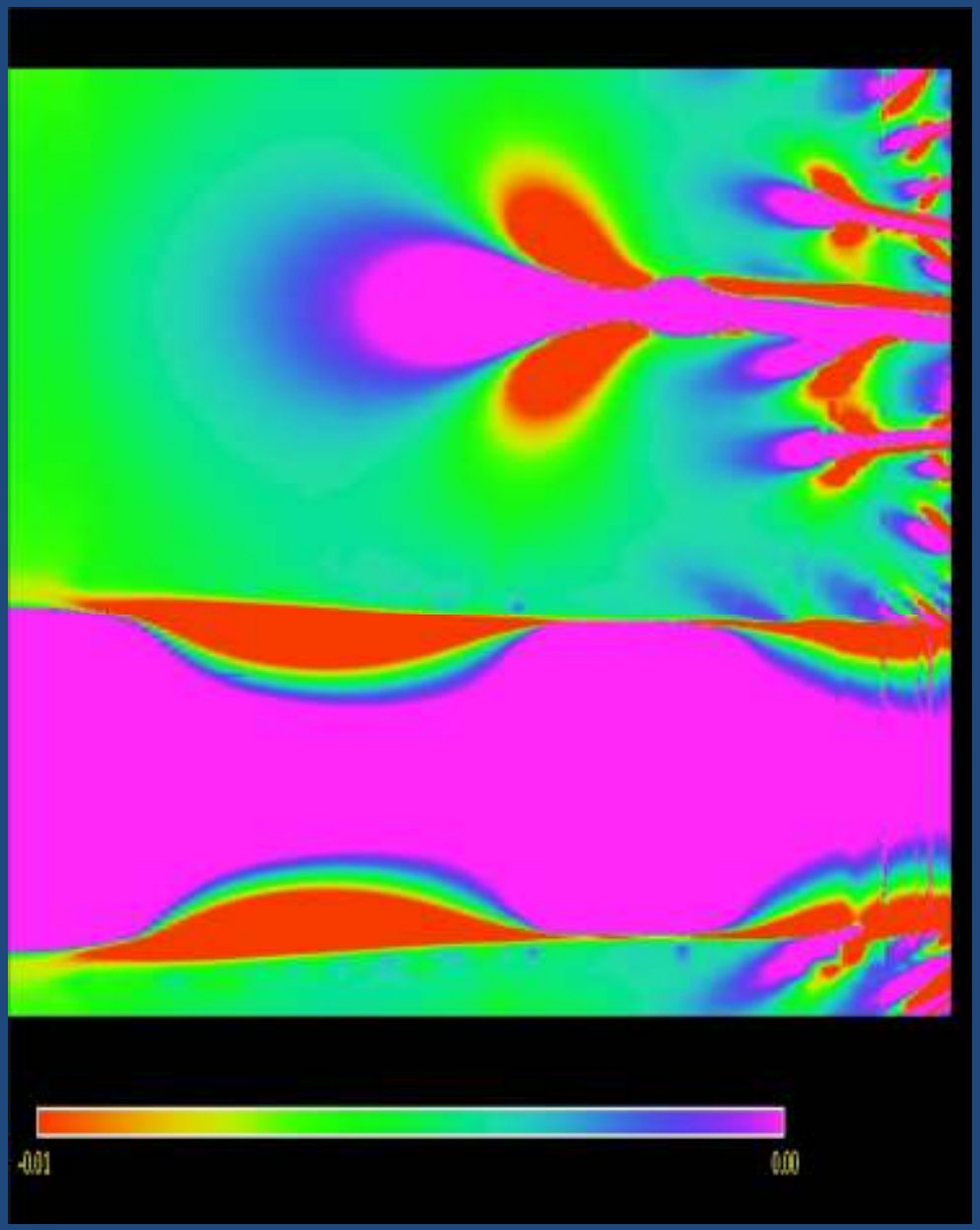
$M = 4\pi(2m)^2 L$

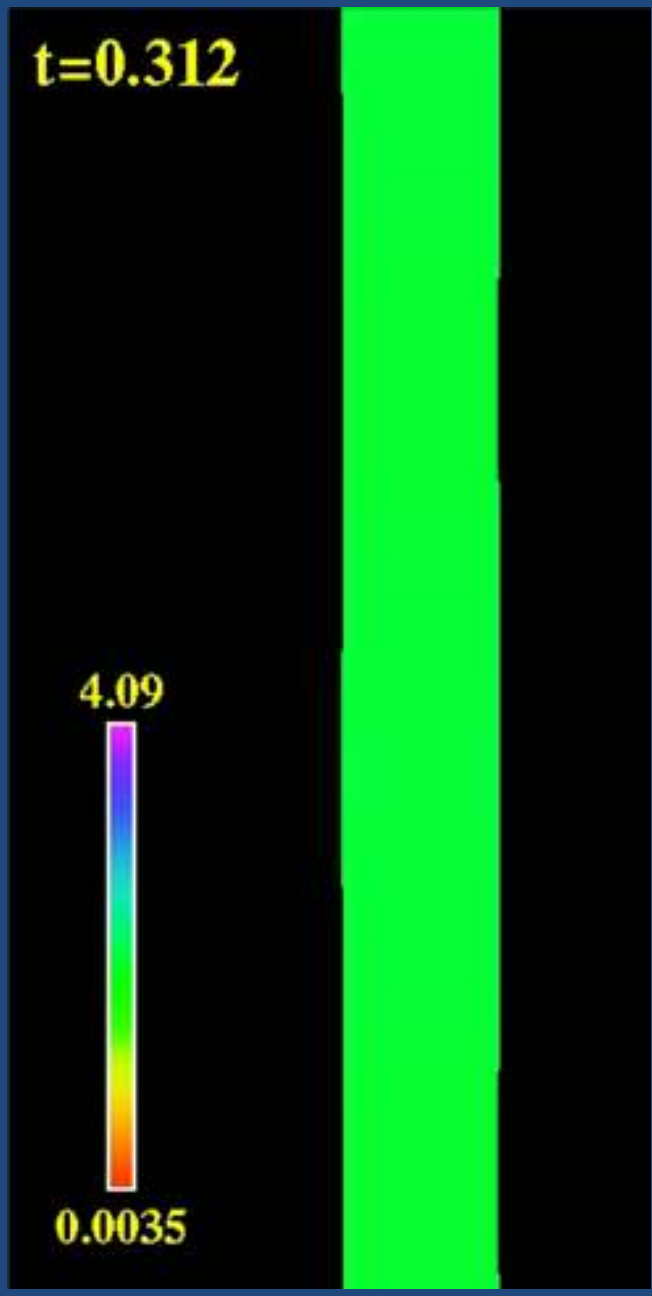
Btw…S_{BH}/S_{BS} = 1.374…



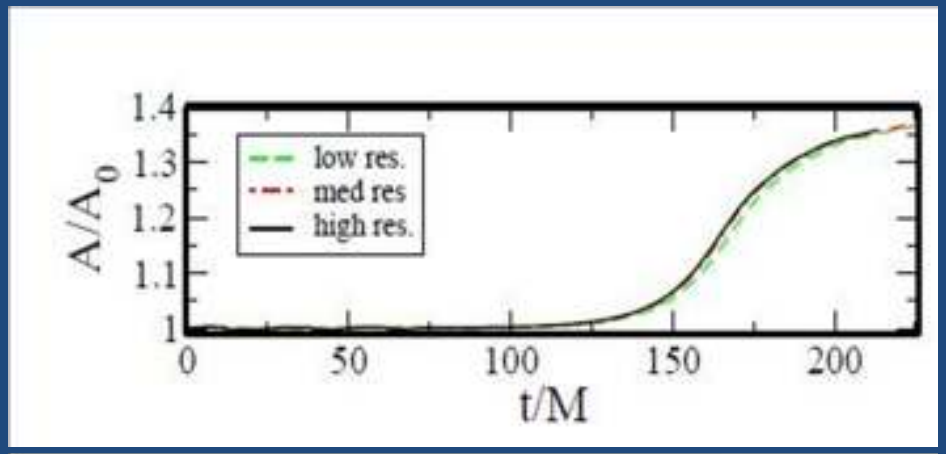


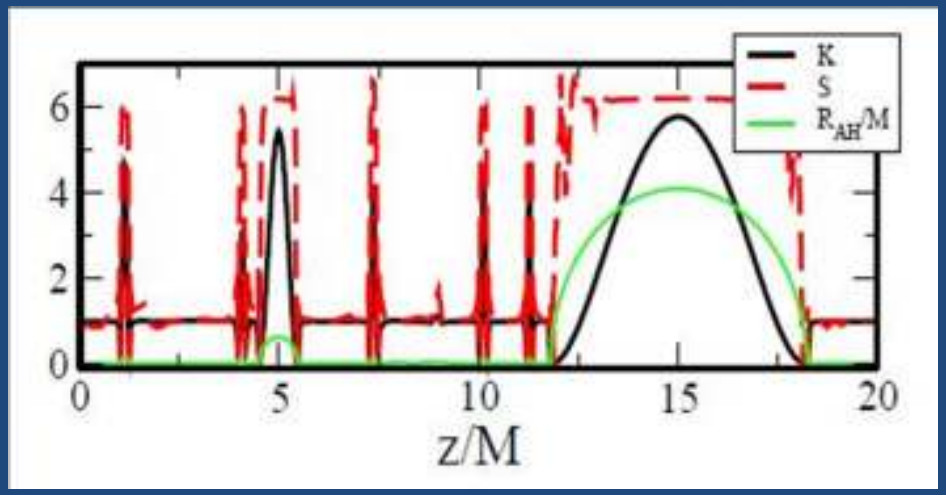
Zoom in..





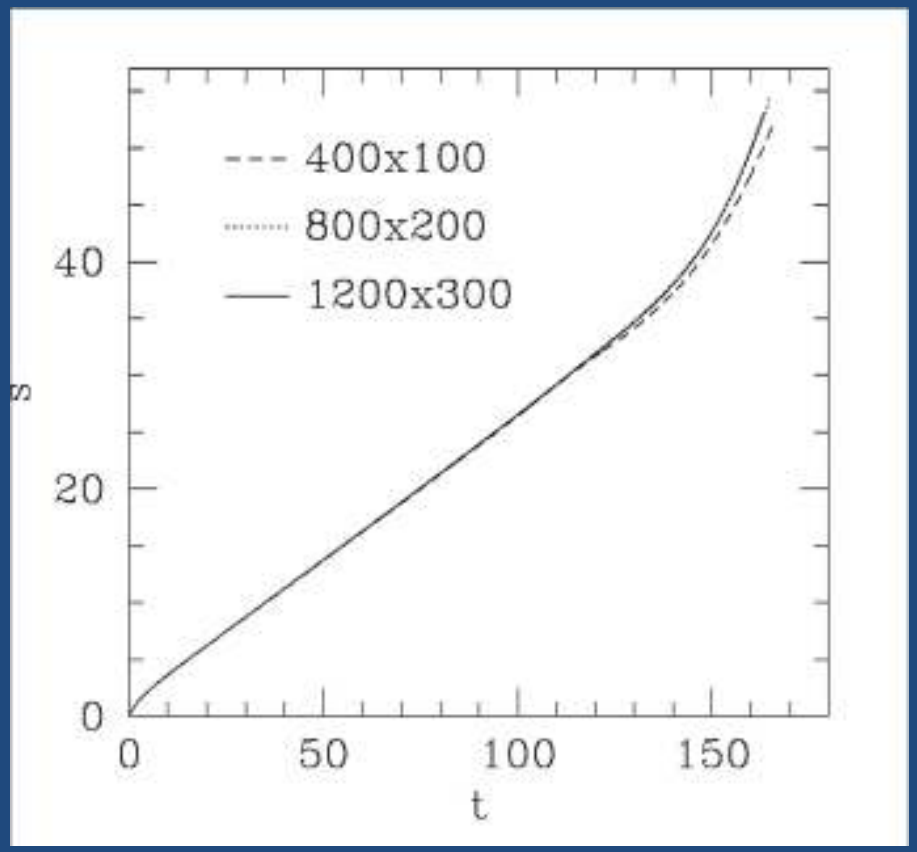
Extra details…



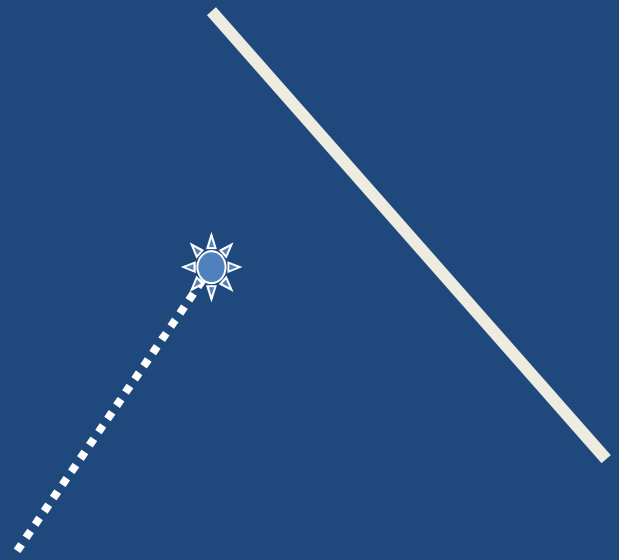


- S(late time) ~ 1.369 S(t=0)
- invariants.. bhs+ bss !
- Can calculate ‘thin-to-zero’ time T~ 231M
- finite observer time

Observers?

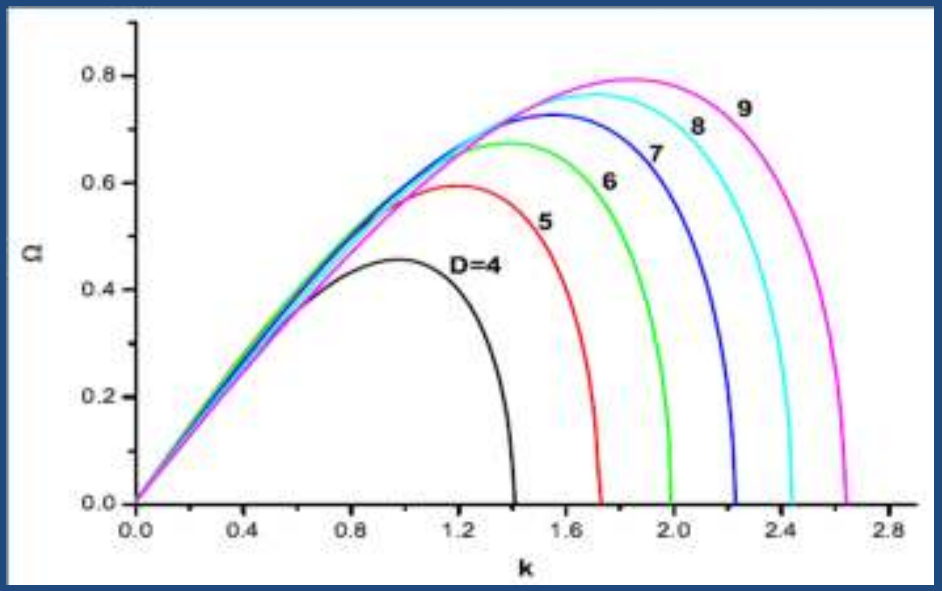


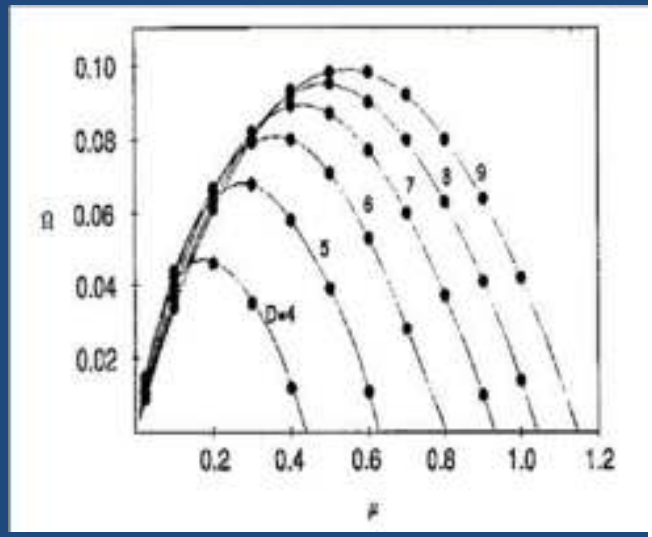
Affine time ~ $e^{t/M}$

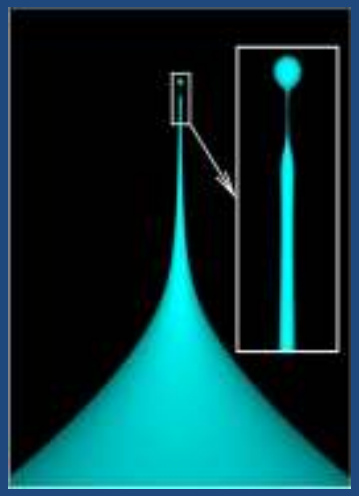


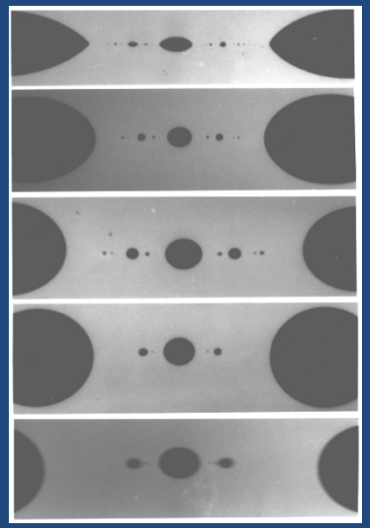
- local solution: M=0 BH (Choptuik critical phenomena). Without fine tuning

Fluid connection?







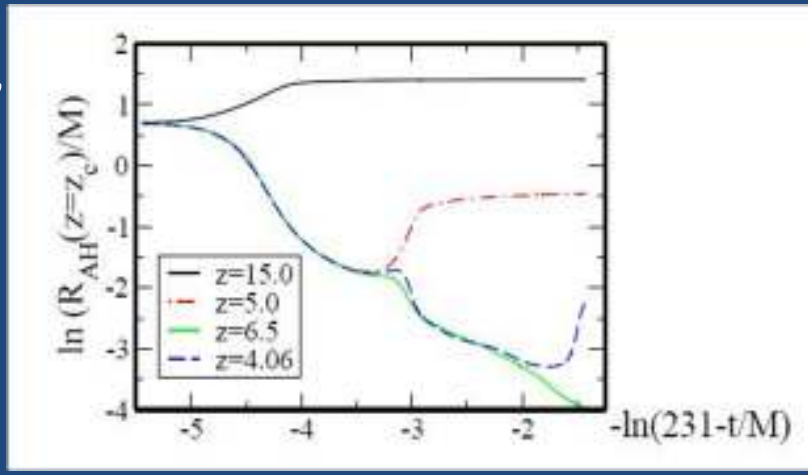


Rayleigh-Plateau instability: Satellite formation in fluids… for lower viscosity higher number of satellites

More than an analogy?

- Eggers, Miyamoto. In fluids, solution is self-similar, $r \sim (t_o - t)$
  $d \ln(r) / d(-\ln(t_o - t)) = -1$

Also, after breakup, scaling in 'recessing' bubble



- We see roughly such slope (10-20%)

for 'bifurcation' stage

Cosmic censorship is *violated 'generically' in higher dims (note: many BHs show this instability. Eg. Myers-Perry BHs, black rings, black saturns, etc.)*

For this case (and a number of others that can be mapped to it)

Classically: Naked singularity has 0 mass. Semi-classically: Local spacetime would behave as a Hawking evaporating BH (unless higher curvature corrections kick in). Need QG to tell us what happens

If QG effects do not drastically affect the picture, the solution should transition to a smooth behavior

Fluid analogue: nothing drastic takes place at pinch off, 2nd solution (bubbles) proceed smoothly

nothing drastic expected in the spacetime

A lot more is going on

- Astrophysical connection: grav waves, role in energetic phenomena (eg. GRBs), grav waves in alternative theories, black hold ‘balding’, etc.
- Black hole evaporation
- Cosmology: bubble collisions in multiverse, ekpyrotic scenarios,…
- Higher dimensional gravity: black rings, Myers-Perry bhs…
- AdS/CFT motivated work: instability of AdS and ‘equilibration’ questions
- Surprises in gravity: turbulence in the gravitational field, fractal behavior of horizons