The interior of dynamical black holes and the strong cosmic censorship conjecture

Mihalis Dafermos

Princeton University/ University of Cambridge

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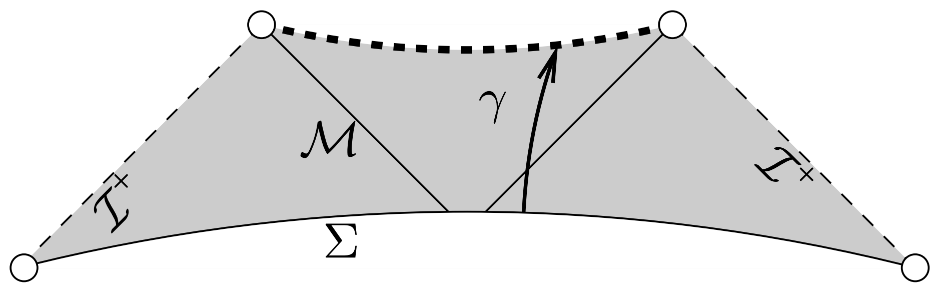
Outline

1. Schwarzschild, Kerr and the strong cosmic censorship conjecture
2. Linearised and nonlinear toy models
3. *Leaving toys behind:* Generic dynamical black hole interiors
4. What is left to be done?

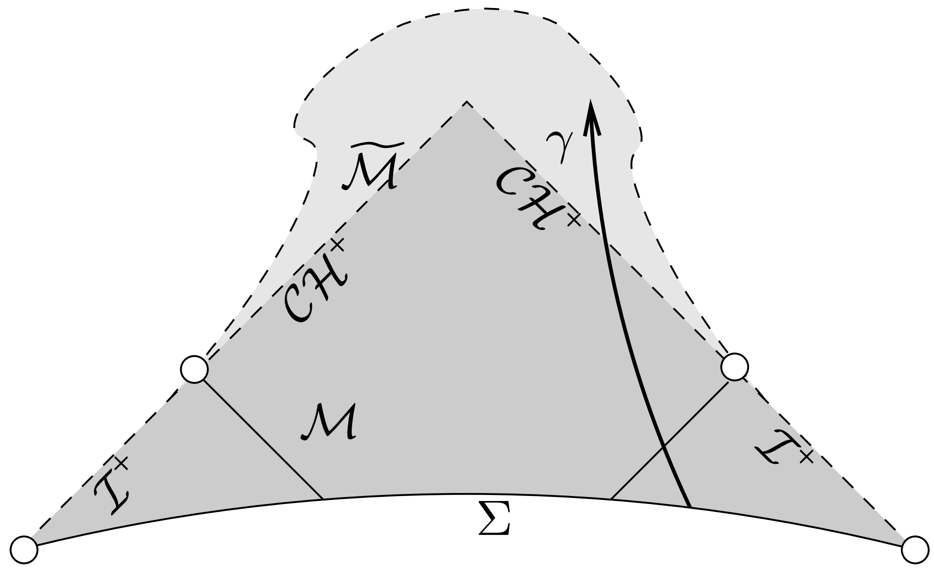
1. Schwarzschild, Kerr and the strong cosmic censorship conjecture

The Schwarzschild spacetime $(\mathcal{M}, g)$ is geodesically <u>in</u>complete: All obsevers γ entering the black hole region live only for finite time:



It turns out that all such observers γ above are **torn apart** by infinite tidal forces as they approach $r = 0$. A related statement is that the spacetime is *inextendible* as a Lorentzian manifold **with** C^0 **metric**. Moreover, $\{r = 0\}$ can be represented as a singular boundary hypersurface which is in a natural sense **spacelike**.

In the Kerr case on the other hand (for parameters $0 < |a| < M$)



the part of spacetime $(\mathcal{M}, g)$ determined by initial data is *smoothly extendible* beyond a null boundary $\mathcal{CH}^+$ (known as a *Cauchy horizon*) to a larger spacetime $(\widetilde{\mathcal{M}}, \widetilde{g})$ into which **all** such finite-living observers γ enter in finite proper time.

Thus, all γ live another day in an extension of spacetime which is severely non-unique. ***What happens to these observers?***

Penrose (c. 1972) suggested a way out:

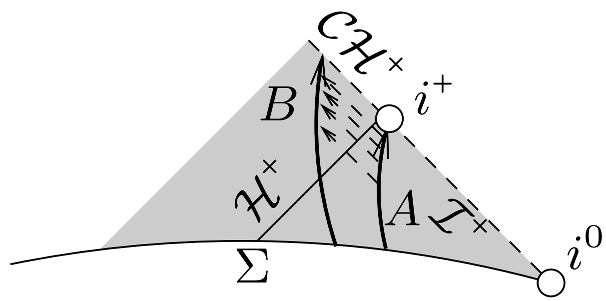
Conjecture (Strong cosmic censorship)**.** *For generic vacuum asymptotically flat initial data* $(\Sigma, \bar{g}, K)$*, the maximal Cauchy development* $(\mathcal{M}, g)$ *is future inextendible as a "suitably regular" Lorentzian manifold.*

One should think of this conjecture as a statement of *global uniqueness*, or, in more colloquial language:

> *"Generically, the future is uniquely determined by the present".*

"Suitably regular"? More on this later…

Penrose was not guided by philosophy or wishful thinking alone! A possible mechanism for instability is the celebrated **blue**-shift:

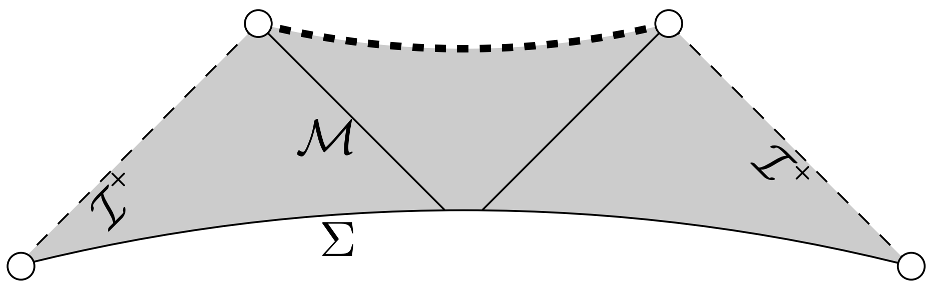


This is dual to the red-shift effect which is so important for the stability of the exterior.

Penrose argued that this blue-shift would cause linear perturbations to blow-up along $\mathcal{CH}^+$. This suggests Cauchy horizon formation is an unstable phenomenon *once a wave-like dynamic degree of freedom is allowed*, thus in particular, under the full nonlinear dynamics of the Einstein vacuum equations $\mathrm{Ric}(g) = 0$.

While <u>linear</u> perturbations as a matter of principle can at worst blow up *at the Cauchy horizon* $\mathcal{CH}^+$, in the **full non-linear theory** governed by the Einstein vacuum equations $\mathrm{Ric}(g) = 0$, one might expect that the non-linearities would kick in so as for blow-up to occur *before the Cauchy horizon has the chance to form*, making the singular boundary **spacelike**.

A natural working hypothesis was that for generic dynamic solutions of the Einstein equations, the causal picture would revert to Schwarzschild:

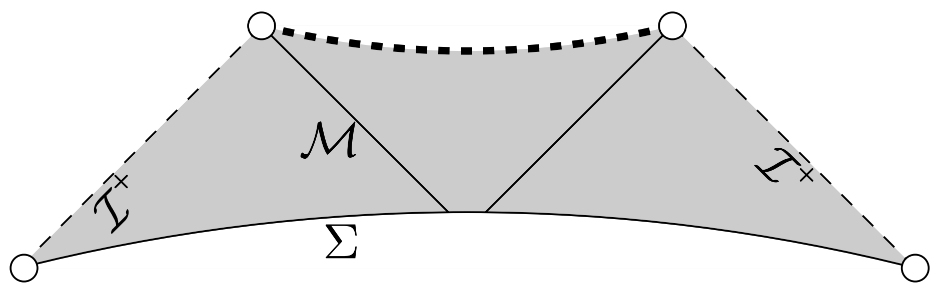


NOVIKOV, STAROBINSKI, etc.

BELINSKII, KHALATNIKOV, LIPSCHITZ

In view of this, one often sees a "very strong" formulation of strong cosmic censorship

Conjecture (**Very** strong cosmic censorship)**.** *For generic vacuum asymptotically flat initial data* $(\Sigma, \bar{g}, K)$*, the maximal Cauchy development* $(\mathcal{M}, g)$ *is future inextendible as a Lorentzian manifold with continuous metric and the singularity can be naturally thought of as "spacelike".*



Linearised and nonlinear toy models

In the Reissner–Nordström setting, PENROSE's heuristic blow-up argument can indeed be turned into a theorem as follows:

Theorem (LUK–OH 2015). *On subextremal Reissner–Nordström with* $M > Q \neq 0$*, generic solutions of* $\Box_g \psi = 0$ *of intially compact support on* Σ *fail to have finite local energy at the Cauchy horizon* $\mathcal{CH}^+$*.*

See also MCNAMARA 1978.

The blow-up given by the above theorem however is in a sense weak!

In particular, the amplitude of the solution remains bounded.

Theorem (A. Franzen, 2014). *In subextremal Reissner–Nordström or Kerr with $M > Q \neq 0$ or $M > |a| \neq 0$, respectively, let ψ be a solution of the wave equation arising from compactly supported initial data. Then*

$$|\psi| \leq C$$

globally in the black hole interior and extends continuously to the Cauchy horizon $\mathcal{CH}^+$.

If one "naively" extrapolates the linear behaviour of $\Box_g \psi = 0$ to the non-linear $\mathrm{Ric}(g) = 0$, where we identify $\psi \sim g_\mu$ and $\partial\psi \sim \Gamma^\lambda_{\mu\nu}$, this suggests that the metric may extend continuously to the Cauchy horizon whereas the Christoffel symbols blow up, failing to be square integrable. This would make the boundary of spacetime a **weak null singularity**. Evidence for this was further suggested by the analysis of a fully non-linear spherically symmetric toy model (HISCOCK, ISRAEL–POISSON, ORI, BRADY,. . . , M.D.).

On the other hand, if one believes the original intuition, or the general considerations motivated by BKL, then the non-linearities of the Einstein equations should induce blow-up earlier to form a **spacelike singularity**.

Which of the two scenarios holds?

Leaving toys behind:

Generic dynamical black hole interiors

If the first scenario has any hope of holding, then the first question one might ask is:

Can one even construct (locally) a single example of a vacuum spacetime bounded by a "weak null singularity" across which the metric extends continuously but the Christoffel symbols fail to be square integrable?

"Weak null singularities" exist!

Theorem (Luk, 2013). *Let us be given characteristic data for the Einstein vacuum equations* $\mathrm{Ric}(g) = 0$ *defined on a bifurcate null hypersurface* $\mathcal{N}^{\mathrm{out}} \cup \mathcal{N}^{\mathrm{in}}$*, where* $\mathcal{N}^{\mathrm{out}}$ *is parameterised by affine parameter* $\underline{u} \in [-\underline{u}^*, 0)$*, and the data are regular on* $\mathcal{N}^{\mathrm{in}}$ *while singular on* $\mathcal{N}^{\mathrm{out}}$*, according to*

$$|\hat{\chi}| \sim |\log(-\underline{u})|^{-p} |\underline{u}|^{-1}, \tag{1}$$

for appropriate $p > 1$*. Then the solution exists in a region foliated by a double null foliation with level sets* u*,* $\bar{u}$ *covering the region* $u^* \leq u < 0$*,* $\underline{u}^* \leq \underline{u} < 0$ *for* $\underline{u}^*$ *as above and sufficiently small* u^**, and the bound* (1) *propagates. The spacetime is continuously extendible beyond* $\underline{u} = 0$*, but the Christoffel symbols fail to be square integrable in this extension.*

One can also construct bifurcate weak null singularities

Theorem (Luk, 2013)**.** *Now suppose both* $\mathcal{N}^{\text{in}} \cup \mathcal{N}^{\text{out}}$ *are parameterised by* $u \in [u^*, 0)$, $\underline{u} \in [\underline{u}^*, 0)$, *with* u^*, $\underline{u}^*$ *sufficiently small, and suppose initially that both*

$$|\hat{\chi}| \sim |\log(-\underline{u})|^{-p} |\underline{u}|^{-1}, \qquad |\hat{\underline{\chi}}| \sim |\log(-u)|^{-p} |u|^{-1}, \tag{2}$$

Then the solution exists in $[u^*, 0) \times [\underline{u}^*, 0)$ *and both bounds* (2) *propagate.*

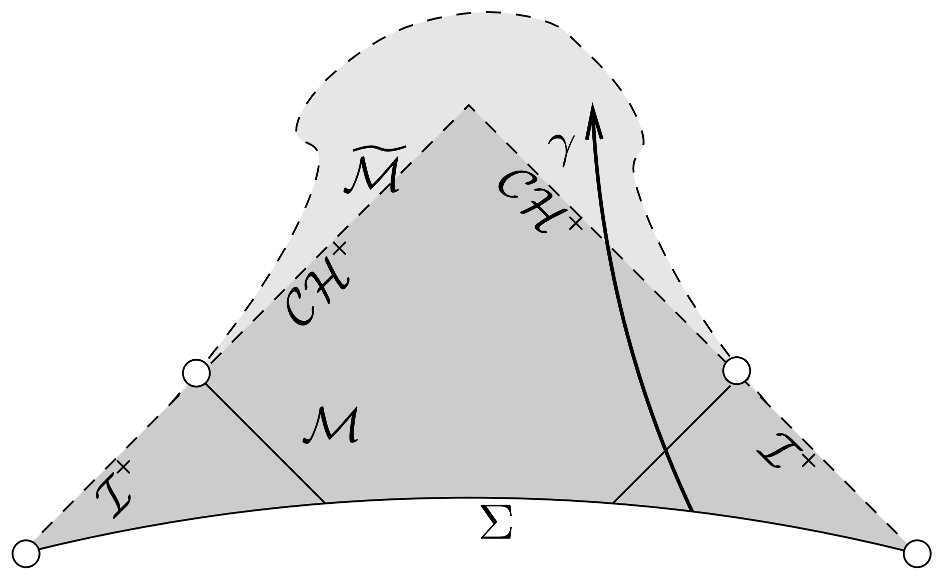
Of course, these purely local results *per se* say nothing about whether the boundaries that arise dynamically in black hole interiors are null and singular.

Nonetheless, putting together Luk's techniques with intuition gained from the analysis of the linear wave equation in the black hole interior with that from the nonlinear spherically symmetric model problem, we have proven in joint work the following result:

Theorem (M.D.–Luk, to appear). *Suppose we are given characteristic initial data for the Einstein vacuum equations on two intersecting null hypersurfaces* $\mathcal{H}^+_A \cup \mathcal{H}^+_B$*, such that, along each, the data are near to and in fact asymptote to (at a sufficiently fast inverse polynomial rate) event-horizon data of a subextremal Kerr with* $a \neq 0$*.*

Then the future evolution of the data is covered by a global double null foliation and there exists a future extension $(\widetilde{\mathcal{M}}, \widetilde{g})$ *of the solution* $(\mathcal{M}, g)$ *with* C^0 *metric* $\widetilde{g}$ *such that* $\partial\mathcal{M}$ *is a bifurcate null cone in* $\widetilde{\mathcal{M}}$ *and* **all** *future incomplete geodesics in* γ *pass into* $\widetilde{\mathcal{M}} \setminus \mathcal{M}$*.*

Corollary. *If the stability of the Kerr <u>exterior</u> conjecture is true, then the Penrose diagramme of Kerr is also globally stable, i.e. all solutions $(\mathcal{M}, g)$ arising from asymptotically flat data $(\Sigma, \bar{g}, K)$ sufficiently close to Kerr will have a global bifurcate Cauchy horizon $\mathcal{CH}^+$, and moreover, the metric is continuously extendible beyond $\mathcal{CH}^+$. In particular, vSCC is false.*



What is left to be done?

Open problem 1 (Stability of the Kerr exterior)**.** *Small perturbations of Kerr initial data on a Cauchy hypersurface indeed form an event horizon* $\mathcal{H}^+$ *outside of which the solution settles down to a nearby Kerr solution at a sufficiently fast polynomial rate.*

As noted above, a definitive disproof of vSCC and thus of the usual understanding of BKL would be an immediate **corollary** of the above.

Concerning the prospects for resolution of the above problem, the natural progression is to understand

(a) the "poor man's" linearised problem

$$\Box_g \psi = 0$$

on fixed Kerr exteriors, then

(b) the problem of linearised gravity, then

(c) the full non-linear problem.

The “poor-man’s” linearised problem is now completely resolved in the full subextremal range $|a| < M$:

Theorem (M.D.–Rodnianski–Shlapentokh-Rothman, 2014). *Solutions of the linear scalar wave equation $\Box_g \psi = 0$ are bounded and decay (polynomially) on fixed subextremal Kerr exterior backgrounds.*

c.f. Wald and Kay–Wald for boundedness on Schwarzschild and Whiting for mode-stability on Kerr

$|a| = M$ Aretakis instability

Very recently, the true linearised problem is resolved *in the case of Schwarzschild*:

Theorem (M.D.–Holzegel–Rodnianski, to appear)**.**
Schwarzschild is <u>*linearly*</u> *stable: Solutions of the linearised Einstein equations around Schwarzschild remain bounded in the exterior and decay (polynomially) to a linearised Kerr solution.*

c.f. Chandrasekhar's book for mode-stability statements

This suggests that one should soon be able to understand the nonlinear dynamics of the $\mathrm{Ric}(g) = 0$.

Open problem 2 (Generic singularity of the Cauchy horizon). *For generic initial data as in Open problem 1, the resulting Cauchy horizon $\mathcal{CH}^+$ is indeed (globally) singular in the sense that any C^0 extension $\widetilde{\mathcal{M}}$ as above will fail to have L^2 Christoffel symbols in a neighbourhood of any point of $\partial\mathcal{M}$.*

This inextendibility notion, identified by CHRISTODOULOU, ensures that extensions $(\widetilde{\mathcal{M}}, \widetilde{g})$ cannot be interpreted as weak solutions of the Einstein vacuum equations. A corollary of the above would be

Corollary. *The Christodoulou formulation of strong cosmic censorship is true in a neighbourhood of the Kerr family.*

Finally, there is an interesting connection with the backwards scattering problem on the black hole exterior:

Open problem 3. *For smooth vacuum "scattering" data on $\mathcal{H}^+ \cup \mathcal{I}^+$ in the class of (suitably fast) polynomially decaying seed functions, there exists a vacuum spacetime $(\mathcal{M}, g)$ "bounded by" $\mathcal{H}^+$ and $\mathcal{I}^+$, attaining the data, regular away from $\mathcal{H}^+$. For* <u>*generic*</u> *such data, $\mathcal{H}^+$ is a weak null singularity.*

Note in contrast that

Theorem (M.D.–Holzegel–Rodnianski). *If scattering data above decay* <u>*exponentially*</u> *at a suitably fast rate, then a vacuum spacetime $(\mathcal{M}, g)$ "bounded by" $\mathcal{H}^+$ and $\mathcal{I}^+$ indeed exists with* <u>*regular*</u> *$\mathcal{H}^+$.*

Final remarks

In the case of small perturbations of two-ended Kerr, as we have seen, a successful resolution of Open problem 1 would imply that there is no spacelike singularity–*full stop*.

What happens more generally, in particular, in the case of black holes forming in gravitational collapse from asymptotically flat initial data with one end?

As long as the black hole asymptotes to Kerr, the above results still apply to show that part of the singular boundary is always **null**.

Is there also a spacelike portion of the singularity?

Or does this null piece close up before such a singularity can occur?