Future of General Relativity

Abhay Ashtekar
Institute for Gravitation and the Cosmos, Penn State

It is difficult to make predictions, especially about the future
... Niels Bohr

Seven Pines Symposium, May 2015

1. Classical General Relativity

Advances on the conceptual/mathematical front: 1960s-1980s
Paradigm shifts within the field: Key examples

- Singularity Theorems
- Positive energy theorems
- Physics of gravitational waves in full non-linear GR
- Black hole uniqueness
- Black hole thermodynamics

Evolution of the field since then & Future

The field has matured and moved to new directions. General Relativity has extended its reach tremendously, but also moved away from the type of mathematical directions that dominated before the 1980s. Key examples:

- Geometric Analysis (much more emphasis on PDEs; emphasis on rigorous global results; existence, uniqueness)

- Numerical GR (huge leap, especially over the last decade; coalescence of BHs and neutron stars, magneto-hydrodynamics)

- Gravitational Waves and Astrophysics (e.g.: No mountains higher than 1M on the crab pulsar; new window on the universe will open in the next 5 years!) Looking at the universe using all 4 fundamental forces of Nature (AMON project: Gravitational messengers no longer ignored!). Black holes have become ubiquitous in astrophysics.

- Physical cosmology (CMB fluctuations at the time when the universe was 380,000 years young give rise to large scale structure we see today, 13B years later! In human terms, from the snapshot of a baby when it was a day old, predict what the person would look like when the person is 100 years old!)

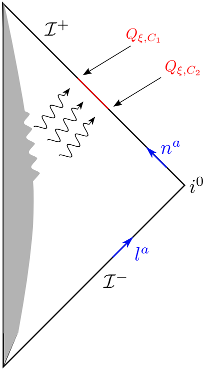
A few conceptual problems still remain

Example: Isolated systems and Gravitational radiation in presence of a positive cosmological constant Λ

- Recall from Cliff Will's talk yesterday: There was considerable confusion about the reality of gravitational waves before 1960.

- The Bondi-Sachs et al framework settled the issue. Penrose's construction of null infinity. The Bondi-Metzner-Sachs group. Admits a unique 4-d normal subgroup of asymptotic translations. Used heavily to extract physics.

- Gauge invariant notion of Bondi news-tensor: Unambiguous signal of gravitational radiation in full, non-linear GR.



- None of these constructions go through in the positive Λ case! We do not have even basic notions: Bondi news; Balance law; positive energy or flux; the 'no incoming radiation' condition. Don't know what gravitational waves mean in full, non-linear GR for positive Λ, however small !

2. Unifying GR and Quantum Physics

> *"Nevertheless, due to the inner-atomic movement of electrons, atoms would have to radiate not only electro-magnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in Nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."*
> *A. Einstein (1916)*

Almost a century has passed. Significant time and effort has been devoted to create a satisfactory unification. Major advances in quantum field theory in curved space-times: Black holes (Hawking effect) and the very early universe (particularly, the inflationary paradigm). But the more general problem is still open!

Why?

The standard answer: No experiments to guide us.

- But this cannot be the entire story. If this was the only reason, then having 'No experimental constraints' should be a theorist's haven! There should be a plethora of theories. We do not have a single, complete, consistent theory!

- My view point on the central difficulty: In GR, gravitational field is encoded in the very geometry of space-time. So a quantum theory of gravity should lead to quantum space-time geometry. How do we do physics without a metric in the background? What would one mean by 'dynamics'? How do we even formulate physical questions we have been discussing last two days?

- An early attempt to bypass this difficulty was to treat GR as any other field theory and use perturbative methods. Split the geometric and the gravitational roles: $g_{ab} = \eta_{ab} + h_{ab}$; use the geometry provided by the Minkowski metric η_{ab} and let h_{ab} encode the gravitational field. But this perturbative strategy fails by its own criteria: Theory is non-renormalizable; no predictive power.

- It is relatively recently that we learned to lift the anchor that tied us to background space-time continuum and sail the open seas. Had to develop new concepts and appropriate mathematical tools to first phrase and then analyze appropriate questions. It is only recently that we had sufficient tools to address physical issues.

Broad Strategies

- Similarity with the origin of General Relativity: Incompatibility between the principles underlying 'fundamental theories' of the day.

- Gauging progress: But important to keep in mind that the discovery of GR was more of an anomaly than a rule. Quantum theory is more representative of how paradigm shifts occur in science. A century has passed since its discovery, yet fundamental conceptual issues remain! On the mathematical side, we do not still have a single complete, coherent interacting QFT in 4-dimensions!

- Yet no one would deny that quantum theory has been extremely successful. Indeed, much more so than GR!

- In quantum theory, progress was made by focusing on physical problems and applying 'quantum principles' to them. A good strategy for quantum gravity!

3. Three leading approaches

Asymptotic Safety

• Idea: To select, among theories –that are already 'quantum'– 'islands' of physically acceptable ones in the 'sea' of unacceptable theories which are plagued by short distance pathologies. 'Non-perturbative renormalization'.

• In principle: Consider the infinite dimensional theory space $\mathcal{T}$, that are compatible with pre-specified symmetries (diffeorphism group). Wilson's renormalization group (RG) equations provide a flow. Does a 'physically acceptable' trajectory exist? Or, is there an UV fixed point (corresponding to to the removal of the UV cut-off)? The coupling constants (in the Lagrangian) flow, i.e. become scale dependent along any trajectory. Thus each trajecgtory provides an 'effective theory' at each scale in which the degrees of freedom at shorter scale are 'integrated out'.

• In practice: Truncation to a few number of terms in the Lagrangian. But impressive technical progress. A non-trivial fixed point does exist for all truncations so far (10-20 terms in the Lagrangian). First for pure gravity. More recently, matter couplings. The non-trivial fixed point survives. Candidates for UV completions of the low energy effective theory of gravity.

String Theory

- Original Idea: Bad UV behavior in QFT comes from use of point particles and could be cured by using instead extended objects. First just strings (but later membranes had also to be added). Particles to be thought of as excitation-modes of a string. Hope: Not 'just' quantum gravity, but also a natural unification of gravity with other forces of nature.

- Coherent theory needs higher dimensions, supersymmetry and an infinite tower of fields/particles. Initially perturbation theory. But found not to converge.

- A non-perturbative definition via AdS/CFT correspondence: Quantum gravity in the bulk in asymptotically space-time defined as a conformal field theory on the boundary $\mathcal{I}$. Interestingly, this non-perturbative definition brings us back to a local quantum field theory!

- Limitations: Quantum gravity in 10d; negative cosmological constant; and large extra dimensions. Has not yet led to solutions of the major conceptual and physical problems in quantum gravity proper.

- Major achievement: Non-gravitational systems (fluids, superconductivity, quark-gluon plasma) described using (super-)gravity. Even though systems used are idealized, brings out the underlying unity of physics at least at a 'meta level'.

Loop Quantum Gravity

• Central Ideas: Shift the emphasis from metrics to connections to import techniques from gauge theories. Holonomies and spin networks at forefront. They lead to a specific theory of quantum geometry in which geometric operators such as areas of physically defined surfaces have discrete eigenvalues. The 'area gap' plays an important role in dynamics.

• Proposals for dynamics of the full theory —particularly via 'spin forms'. Interesting results on UV finiteness because there are no DOF at arbitrarily small scales, and IR finiteness in presence a positive cosmological constant. But theory still incomplete because it is still formulated using a simplicial decomposition of the space-time manifold, and because conceptual issues remain.

• Strategy: Truncate GR according to the physical problem of interest and apply LQG techniques, especially quantum geometry.

• The strategy has led to resolution of a few long standing problems: Resolution of singularities in the cosmological models; a derivation of the graviton propagator in the non-perturbative context; an approach to black hole entropy based on isolated horizons.

4. Interplay between theory and observations: Example

As discussed in yesterday's talks, the inflationary paradigm has had very significant successes. However, two sets of limitations were also discussed.

Particle Physics Issues: Discussed by George Ellis

• Where from the inflaton? A single inflaton or multi-inflatons? Interactions between them? How are particles/fields of the standard model created during 'reheating'? ...

Quantum Gravity Issues: Focus of this part (Brandenberger, Martin, Starobinsky, ..)

• Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped since the 1970's? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

• Is the pre-inflationary dynamics compatible with the standard assumptions underlying inflation? Does it leave an observational imprint?

Answers from Loop Quantum Cosmology: Status

- The Big Bang singularity is naturally resolved in LQG; replaced by a Big Bounce. Goals laid out by Wheeler, Misner and others were achieved, thanks to the quantum geometry effects that lie at the heart of LQG.

  In FLRW Models: $a(t), \phi(t) \quad \rightarrow \quad \Psi_o(a, \phi)$.

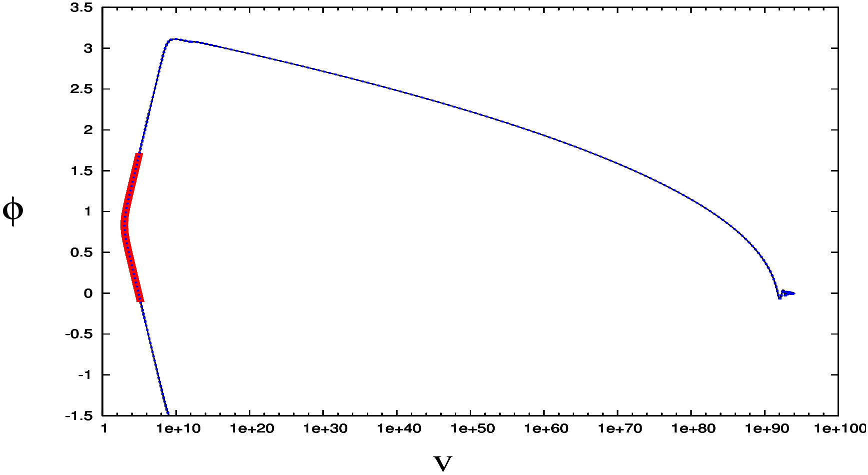
- Quantum states $\psi_{\text{pert}}(T^{(1)}, T^{(2)}, \mathcal{R})$ of perturbations propagate on the quantum FLRW geometry $\Psi_o(a, \phi)$. Planck scale issues faced squarely.

- Natural initial conditions for $\Psi_o \otimes \psi_{\text{pert}}$ at the bounce such that: i) The desired slow roll is achieved; and ii) back reaction of perturbations in ψ_{pert} on the quantum background Ψ_o remains negligible from the bounce to the onset of slow roll (evolution over 11 orders of magnitude in curvature).

- Agreement with the standard inflation for $\ell \gtrsim 30$. But the pre-inflationary dynamics can leave imprints on large wave length modes. e.g., Power spectrum can be suppressed for $\ell \lesssim 30$. (Seems counter-intuitive at first. Will explain.)

Thus, LQG opens a window to complete the inflationary scenario in a direction that is complementary to those based in particle physics. Interesting interplay between theory and observations. Summary: (AA & Barrau arXiv:1504.07559)

Singularity Resolution: $(1/2)m^2\phi^2$ Potential



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh). The Big Bang is replaced by a Big Bounce. Similar resolution in a wide class of cosmological models.

What is behind this singularity resolution?

• LQG is based on a specific quantum Riemannian geometry. In FLRW models, quantum Einstein's equations dictate the (relational) evolution of $\Psi_o(a, \phi)$.

• The key modification well-captured in effective equations. For example, the effective Friedmann equation (AA, Pawlowski, Singh) :

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where} \quad \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

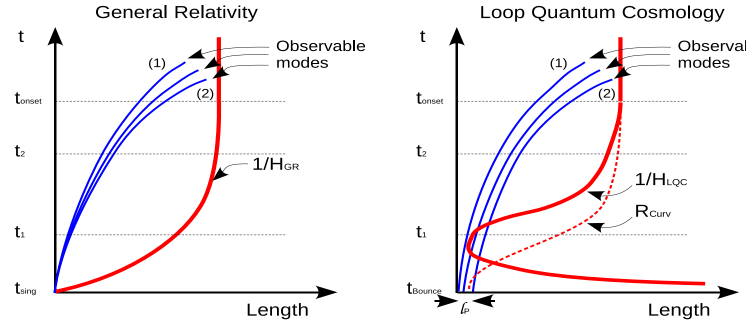
Big Bang replaced by a quantum bounce. Separation of scales: effects become negligible for $\rho << \rho_{\rm Pl}$. Eigenvalues of physical observables, such as matter density and curvature have an absolute upper bound on the physical Hilbert space. (AA, Corichi, Singh)

• Mechanism: No unphysical matter or new boundary conditions. Quantum geometry creates a brand new repulsive force in the Planck regime, overwhelming classical attraction. Understood in the Hamiltonian, Path integral and consistent histories frameworks. (AA, Campiglia, Henderson; Craig & Singh)

• Many generalizations: inclusion of spatial curvature, a cosmological constant Λ, anisotropies, ... (Bojowald; AA, Pawlowski, Singh, Vandersloot; Lewandowski; Corichi; Wilson-Ewing; Brezuela, Martin-Benito, Mena, ...). Qualitative summary: Every time a curvature scalar enters the Planck regime, the quantum geometry repulsive force dilutes it, preventing a blow up.

Why pre-inflationary dynamics matters

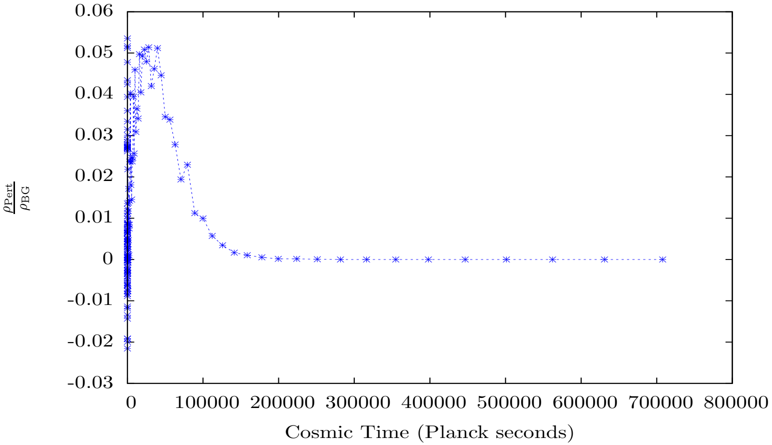
Contrary to a wide-spread belief, pre-inflationary dynamics does matter because modes with $\lambda_{\rm phys} > R_{\rm curv}$, the curvature radius, in the pre-inflationary era are excited and populated at the onset of inflation. They can leave imprints on CMB, naturally leading to 'anomalies' at low ℓs .



The UV LQG regularization tames the FLRW singularity. The modified Planck scale dynamics in turn affects the IR behavior of perturbations!

Illustrative Results

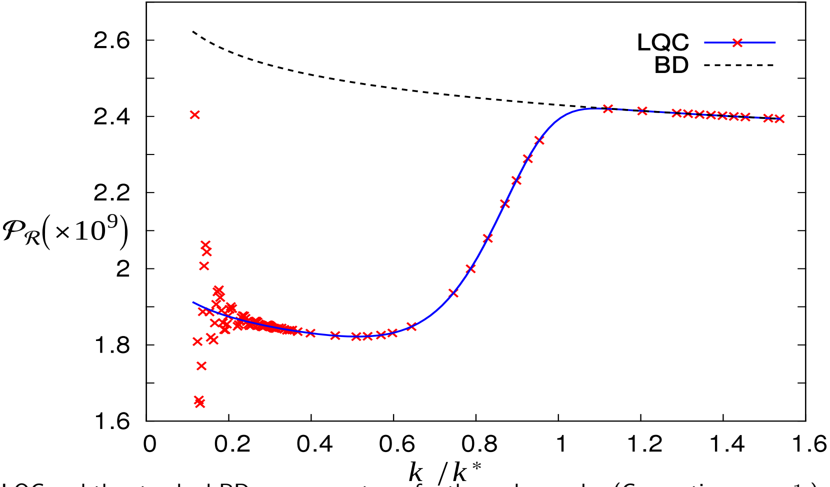
Facing trans-Planckian issues squarely: Is $\rho_{\rm Pert}/\rho_{\rm BG} \ll 1$ all the way from the bounce to the onset of slow roll? If so, self-consistency.



Yes!. Our initial conditions on ψ do ensure self-consistency of the test field approximation as hoped. Illustrative plot. (Agullo, AA, Nelson)

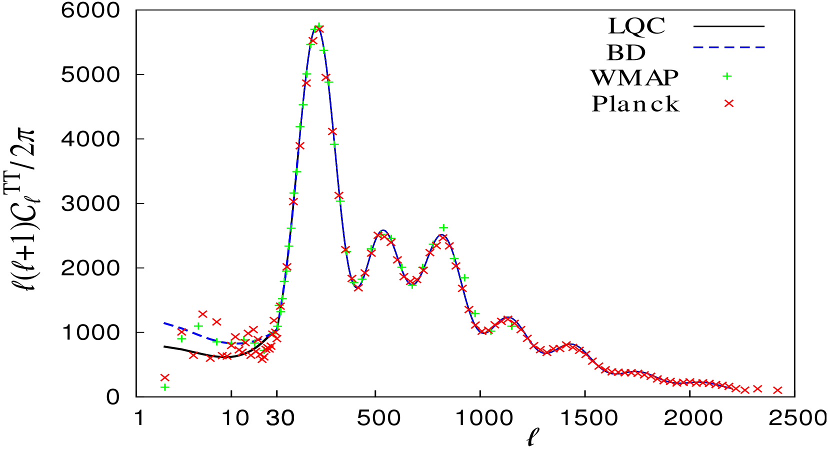
The Scalar Power spectrum: Ratio $(\mathcal{P}_{\rm LQG}/\mathcal{P}_{\rm BD})$

"Top-down approach"



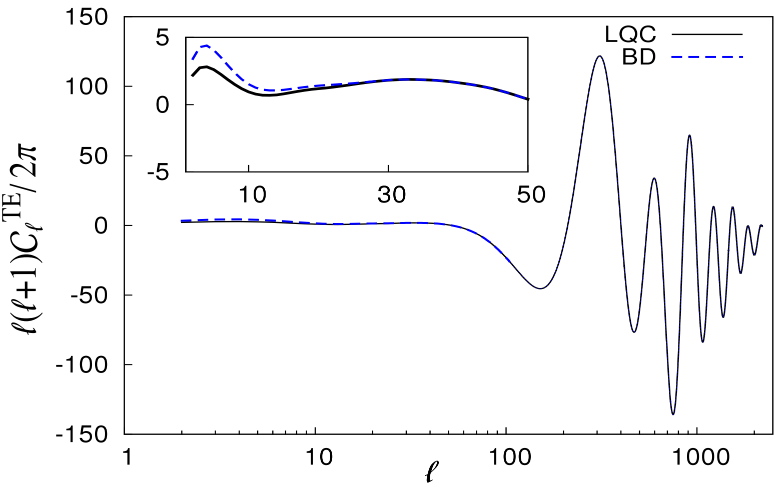
The LQC and the standard BD power spectrum for the scalar mode. (Convention $a_{\rm B} = 1$.) Red: Raw 'data' from LQC. blue: best fit curve. Here, the WMAP reference mode $k^\star_{\rm B}/a_{\rm B} = 54 m_{\rm Pl}$ and $k^{\rm min}_{\rm B}/a_{\rm B} = 6.3 m_{\rm Pl}$. (AA, Gupt)

LQC: Predicted TT-Power spectrum



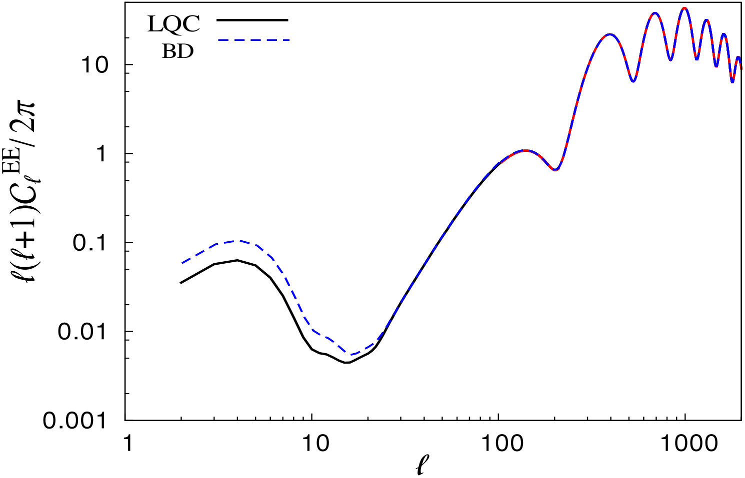
There exist permissible states $\Psi_o \otimes \psi$ such that the LQC power spectrum agrees with the standard BD power spectrum for $\ell \gtrsim 30$, but in LQC power is suppressed for $\ell \lesssim 30$. (AA, Gupt)

LQC: Predicted TE Correlations



The LQC prediction for the TE spectrum, for the initial state that gave the TT-spectrum in the last slide. Small suppression of power at small ℓ is a signature that the TT power suppression is of primordial origin. (AA, Gupt)

LQC: Predicted EE Correlations



The LQC prediction for the TE spectrum, for the initial state that gave the TT-spectrum in the last but one slide. The small suppression of power at small ℓ is a signature that the TT power suppression is of primordial origin. (AA, Gupt)

Epilog

Classical Gravity:

• The era Mathematical GR à la Penrose, Geroch, Hawking, ..., dominated more by geometry than by analysis is coming to an end. Replaced by geometric analysis and numerical relativity.

• Gravity has entered main-stream of astrophysics and cosmology. At least an equal parter.

Quantum Gravity:

• Several distinct approaches. Diversity is very healthy. Advances have extended the reach of gravitational methods to other areas of science. Although modest in terms of 'practical science', Einstein would have been astonished by these unforeseen applications of GR. Also, via cosmology, a healthy dialog between quantum gravity and observations has commenced.

• However, whereas in other areas of gravity, communities have come together, in quantum gravity they have steadily moved away. Main culprit: Bouts of irrational exuberance that "made practitioners mix what they know and what they believe, thereby creating a misplaced sense of certitude. For sustained progress, it is important that the the practitioners be aware of the limits of their science and thus their knowledge." And this can only come through dialog and not isolation.