

The exigencies of war or the stink of a theoretical problem? Feynman's abandonment of the model of the quivering electron*

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Outline

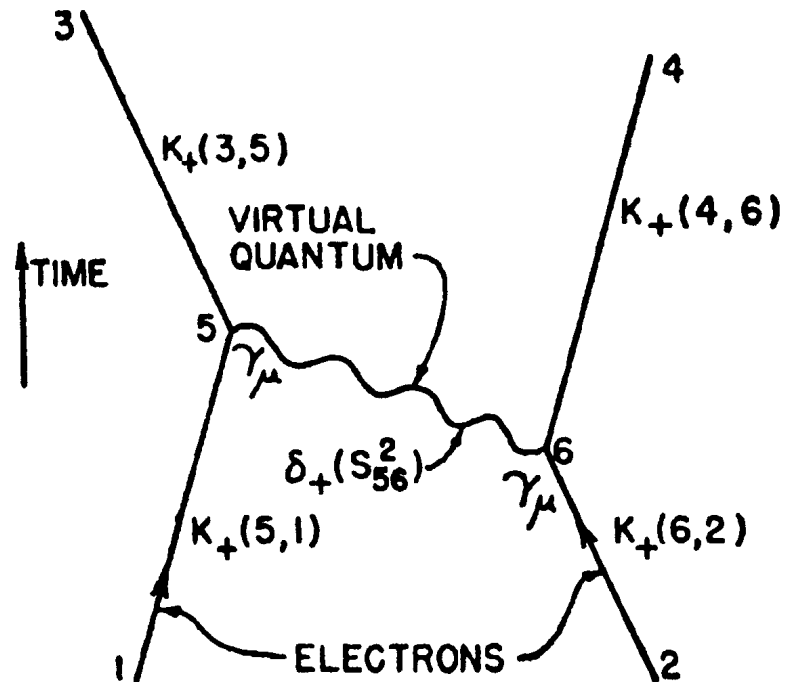
1. Feynman's PhD thesis (1942): Using the action function to determine the time evolution of a quantum version of a classical system with no Hamilton function
2. Space-time approach to non-relativistic quantum mechanics (1948): Elaborating and summarizing the results of the thesis and tentatively generalizing the method to spin and relativity
3. Feynman's (unpublished) struggle (ca. 1947): Using the model of the quivering electron to justify the correct mathematical formulas
4. Space-time approach to quantum electrodynamics (1949): The microscopic model abandoned
5. Conclusions and outlook

Richard P. Feynman



- 1918 Born in New York
- 1942 PhD thesis
- 1943 Manhattan Project (– ca. 1945)
- 1948 “Space-time approach to NRQM”
- 1949 “Space-time approach to QED”
- 1965 Nobel prize in physics
- 1988 Died in Los Angeles

The first "Feynman diagram"



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Feynman's main objective was the removal of infinite quantities from electrodynamics

To achieve this he wanted to make a quantum version of the infinity-free electrodynamics by Wheeler and himself

$$\psi(Q, T) \approx \iint \dots \int \exp \left\{ \frac{i}{\hbar} \sum_{i=0}^m \left[L \left(\frac{q_{i+1} - q_i}{t_{i+1} - t_i}, q_{i+1} \right) (t_{i+1} - t_i) \right] \right\}$$

$$\times \psi(q_0, t_0) \frac{\sqrt{g_0} dq_0 \sqrt{g_1} dq_1 \dots \sqrt{g_{[m]}} dq_m}{A(t_1 - t_0) A(t_2 - t_1) \dots A(T - t_m)}$$

Feynman 2005 (1942), eq. 47

In the limit as we take finer and finer subdivisions of the interval t_0 to T and thus make an ever increasing number of successive integrations, the expression on the right side of (47) becomes equal to $\psi(Q, T)$. The sum in the exponential resembles $\int_{t_0}^T L(\dot{q}, q) dt$ with the integral written as a Riemann sum.

Feynman 2005 (1942), p. 31

Feynman used the action function to quantize the classical theory even though it had no Lagrangian

What we have been doing so far is no more than to reexpress ordinary quantum mechanics in a somewhat different language. In the next few pages we shall require this altered language in order to describe the generalization we are to make to systems without a simple Lagrangian function of coordinates and velocities. (Feynman 2005 (1942), p. 39)

REVIEWS OF
MODERN PHYSICS

VOLUME 20, NUMBER 2

APRIL, 1948

**Space-Time Approach to Non-Relativistic
Quantum Mechanics**

R. P. FEYNMAN

Cornell University, Ithaca, New York

Unsatisfactory results in RMP 1948

These results for spin and relativity are purely formal and add nothing to the understanding of these equations. There are other ways of obtaining the Dirac equation which offer some promise of giving a clearer physical interpretation to that important and beautiful equation. (Feynman 1948, p. 387)

The one dimensional Dirac Eqn.

$$\frac{\partial \psi_1}{\partial t} + \frac{\partial \psi_3}{\partial z} = -i\mu\psi_1$$

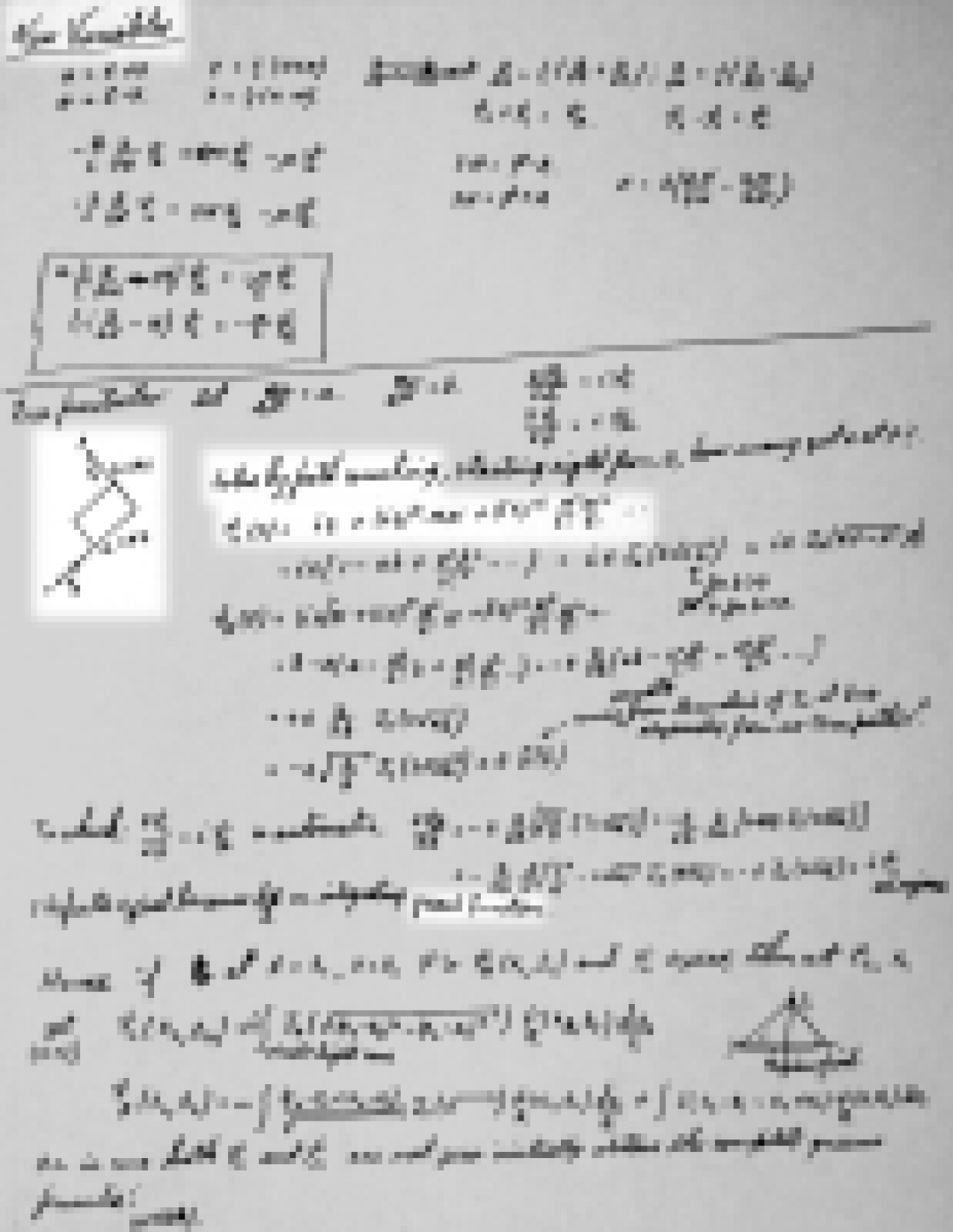
$$\frac{\partial \psi_3}{\partial t} + \frac{\partial \psi_1}{\partial z} = +i\mu\psi_3$$

$$H\psi = \phi\psi + \alpha(p - A)\psi - \beta\mu\psi$$

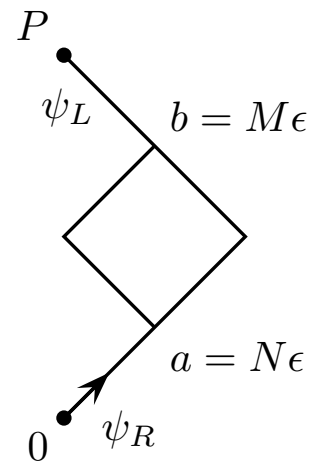
$$\dot{F} = i(HF - FH) + \frac{\partial F}{\partial t}$$

$$\dot{x} = \alpha$$

(Breit 1928, Schrödinger 1930,
Dirac 1933, 1935)



New Variables



Solve by path counting

$$\begin{aligned} \psi_L(P) &= i\epsilon \\ &+ (i\epsilon)^3 MN \\ &+ (i\epsilon)^5 \frac{M^2}{2!} \frac{N^2}{2!} \dots \end{aligned}$$

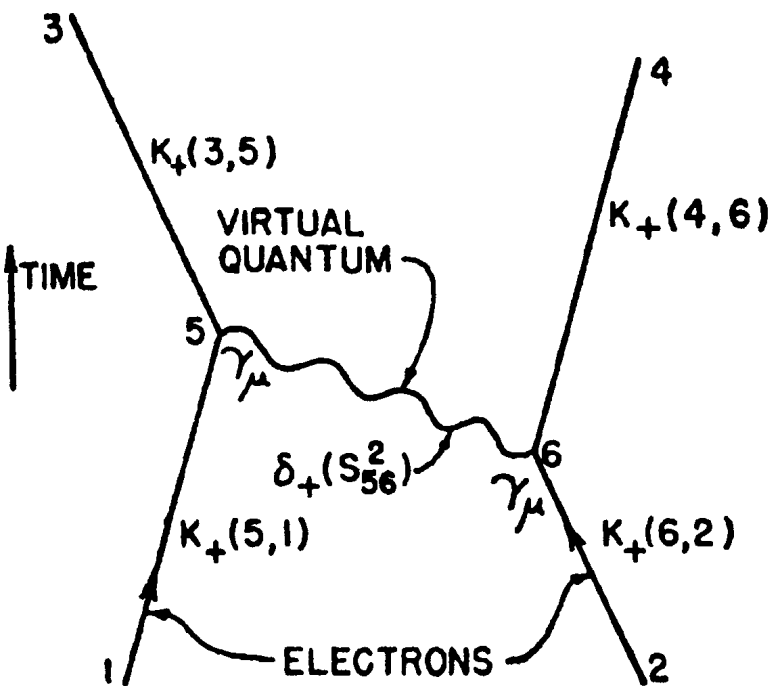
Green's function

Space-Time Approach to Quantum Electrodynamics

R. P. FEYNMAN

Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)



$$\begin{aligned}
 K^{(1)}(3, 4; 1, 2) = & -ie^2 \int \int K_{+a}(3, 5) K_{+b}(4, 6) \gamma_{a\mu} \gamma_{b\mu} \\
 & \times \delta_+(s_{56}^2) K_{+a}(5, 1) K_{+b}(6, 2) d\tau_5 d\tau_6, \quad (4)
 \end{aligned}$$

Each line in a Feynman diagram represents a particle

Number of particles
 entering
 = number of particles
 leaving

any closed loop



∴ any completely closed loop cancels

One Dimensional Interaction of Particles

Classical $S = \int \sqrt{\dot{x}^2 - V(x)} d\alpha + \int \sqrt{\dot{\xi}^2 - V(\xi)} d\beta$

For two particles the interaction potential energy is given by
 $V(x, \xi) = \frac{k}{|x - \xi|^2}$
 $V(x, \xi) = \frac{k}{|x - \xi|^2}$
 $V(x, \xi) = \frac{k}{|x - \xi|^2}$



Hamilton's equations
 $\frac{\partial S}{\partial x} = p = \frac{\partial L}{\partial \dot{x}}$
 $\frac{\partial S}{\partial \xi} = P = \frac{\partial L}{\partial \dot{\xi}}$
 $\frac{\partial S}{\partial t} = -H = -\frac{\partial L}{\partial t}$
 $\frac{\partial S}{\partial \tau} = -H = -\frac{\partial L}{\partial t}$

$\dot{x} = \frac{\partial H}{\partial p}$
 $\dot{\xi} = \frac{\partial H}{\partial P}$
 $\dot{p} = -\frac{\partial H}{\partial x}$
 $\dot{P} = -\frac{\partial H}{\partial \xi}$
 $\dot{t} = -\frac{\partial H}{\partial H}$
 $\dot{\tau} = -\frac{\partial H}{\partial H}$

$$H = \frac{1}{2} \left(\frac{1}{p - k/r} \right) + \frac{1}{2} \left(\frac{1}{P - k/r} \right)$$

Conservation of energy
 $E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{\xi}^2 + \frac{k}{|x - \xi|^2}$
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One Dimensional Interaction of Particles

Classical

$$S = \int \sqrt{\dot{t}^2 - \dot{x}^2} d\alpha + \int \sqrt{\dot{\tau}^2 - \dot{\xi}^2} d\beta$$

$$+ k \iint \delta((t - \tau)^2 - (x - \xi)^2) \times (\dot{t}\dot{\tau} - \dot{x}\dot{\xi}) d\alpha d\beta.$$

$$S = \int \left(\sqrt{2\dot{v}} + \sqrt{2\dot{u}} + \frac{k(\dot{u} + \dot{v})}{|v - u|} \right) dw$$

$$H = \frac{1}{2} \left(\frac{1}{p - k/r} \right) + \frac{1}{2} \left(\frac{1}{P - k/r} \right)$$

One dimension 2 particles

Q. Mech:

assume

$$p = \frac{\hbar}{i} \frac{\partial}{\partial v}; \quad P = \frac{\hbar}{i} \frac{\partial}{\partial u}.$$

$$\frac{\hbar}{i} \frac{\partial \psi}{\partial w} = \frac{1}{2}(\phi + \chi),$$

$$\psi = \left(\frac{\hbar}{i} \frac{\partial}{\partial v} - \frac{k}{r} \right) \phi,$$

$$\psi = \left(\frac{\hbar}{i} \frac{\partial}{\partial u} - \frac{k}{r} \right) \chi.$$

are the differential equations requiring solution. For $k = 0$ they are Dirac's onedimensional equations.

$\frac{d^2x}{dt^2} = -\frac{GM}{r^3}x$
 $\frac{d^2y}{dt^2} = -\frac{GM}{r^3}y$
 $\frac{d^2z}{dt^2} = -\frac{GM}{r^3}z$

$\vec{r} = (x, y, z)$
 $\vec{v} = \dot{\vec{r}} = (\dot{x}, \dot{y}, \dot{z})$
 $\vec{a} = \ddot{\vec{r}} = (\ddot{x}, \ddot{y}, \ddot{z})$

$\vec{L} = \vec{r} \times \vec{p} = m \vec{r} \times \vec{v}$
 $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{r} \times (-\frac{GM}{r^3}\vec{r}) = 0$

$\vec{L} = \text{constant}$
 $\vec{L} \cdot \vec{r} = 0$
 $\vec{L} \cdot \vec{v} = 0$

$\vec{L} = L \hat{n}$
 $\vec{r} \cdot \hat{n} = 0$
 $\vec{v} \cdot \hat{n} = 0$

$\vec{r} = r \hat{e}_r$
 $\vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$
 $\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{e}_r + (2\dot{r} \dot{\theta} + r \ddot{\theta}) \hat{e}_\theta$

$\vec{F} = -\frac{GMm}{r^2} \hat{e}_r$
 $m \vec{a} = \vec{F}$

$m(\ddot{r} - r \dot{\theta}^2) = -\frac{GMm}{r^2}$
 $m(2\dot{r} \dot{\theta} + r \ddot{\theta}) = 0$

$\frac{d}{dt} (r^2 \dot{\theta}) = 0$
 $r^2 \dot{\theta} = \text{constant} = L/m$

$\ddot{r} - \frac{L^2}{m^2 r^3} = -\frac{GM}{r^2}$
 $\ddot{r} + \frac{L^2}{m^2 r^3} - \frac{GM}{r^2} = 0$

$u = 1/r$
 $\frac{d^2u}{d\theta^2} + u = \frac{GM}{L^2/m^2}$

$u = \frac{GM}{L^2/m^2} + A \cos(\theta - \theta_0)$
 $r = \frac{1}{\frac{GM}{L^2/m^2} + A \cos(\theta - \theta_0)}$

$r = \frac{a(1 - e^2)}{1 + e \cos(\theta - \theta_0)}$

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$\vec{L} = m \vec{r} \times \vec{v} = m r^2 \dot{\theta} \hat{e}_z$
 $\vec{L} \cdot \vec{r} = 0$
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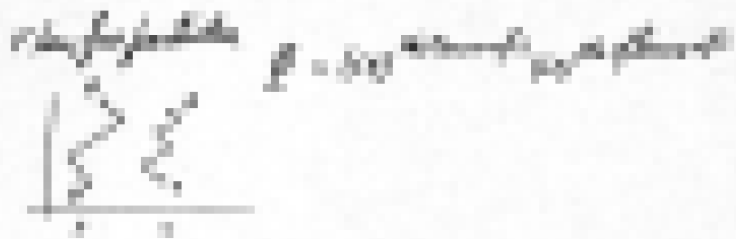
1 part[icle]

2 part.

N.G.[=no good?]



It is a bit hard to see how to
 define Φ for path pair AB and
 CD , since there are some terms
 from interaction at x from y
 which is unspecified. However
 if the interaction is zero be-
 yond P we are OK. Hence, at
 present, I can only specify Φ
 for paths which are long enough
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Assessment of the move

The move to modularity [...] was one that might have had roots in peace, but it flowered in the exigencies of war. (Galison 1998, 430)

or

The move to modularity [...] was one that might have had roots in the exigencies war, but it flowered in the stink of a theoretical problem.

?

Summary

In his PhD thesis, Feynman uses the action to extend the domain of applicability of the then known quantization procedures.

After the war, Feynman tries to justify the action functions, which he knew were correct, by a physical model.

Feynman abandons the search for a physical justification and uses Green's functions without further analysis in his proposal for a divergence-free quantum electrodynamics.

The reason for the abandonment is not (only) a pragmatic attitude adopted during the war but the problematic incorporation of interaction into the physical model.

Interpret Feynman's diagrams as representations of a model of QED processes, rather than only as a calculation tool.