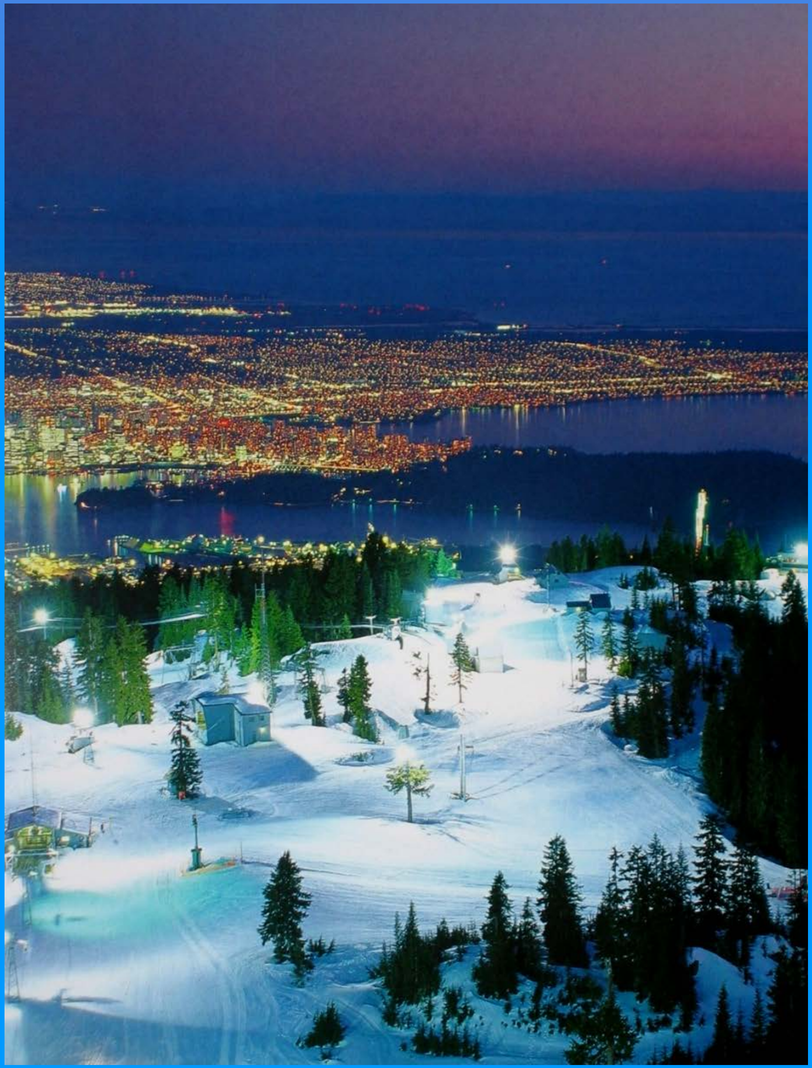
TIME EVOLUTION of PATH INTEGRALS

P.C.E. STAMP

7 Pines, May 17th 2014



Physics & Astronomy
UBC
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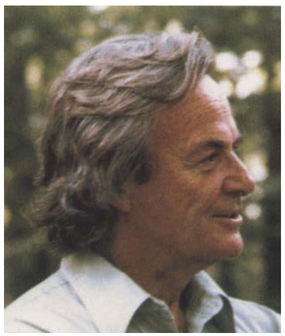
Theoretical Physics

The talk will address the following themes:

(i) History of the path integral – from Huyghens to Q Gravity
(ii) A Correlated Worldline Theory incorporating gravity



The original intent was to focus on the 2 characters shown at left. As you will see this was not how it turned out.

FURTHER INFORMATION:

Email: stamp@phas.ubc.ca
Web: http://www.physics.ubc.ca/~berciu/PHILIP/index.html

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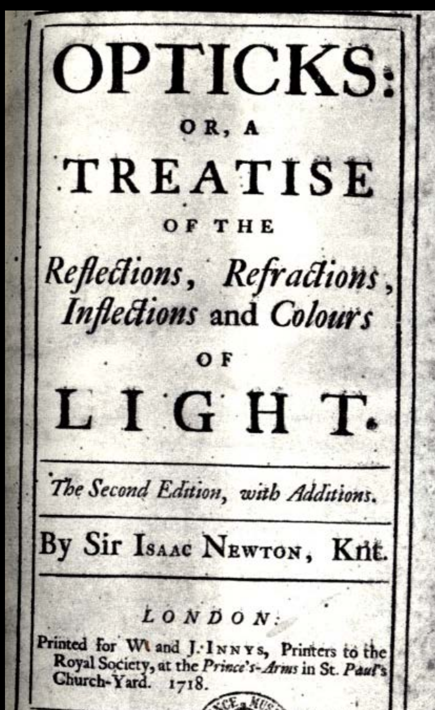
The NATURE of LIGHT: NEWTON vs HUYGHENS (PARTICLE THEORY vs WAVE THEORY)

In Newton's corpuscular theory, light propagated in straight lines, except at interfaces. Still, light particles were acted upon by an invisible aether. Newton did not publish his theory until 1704, after the death of Huyghens; he was by then the best-known scientist in Europe.

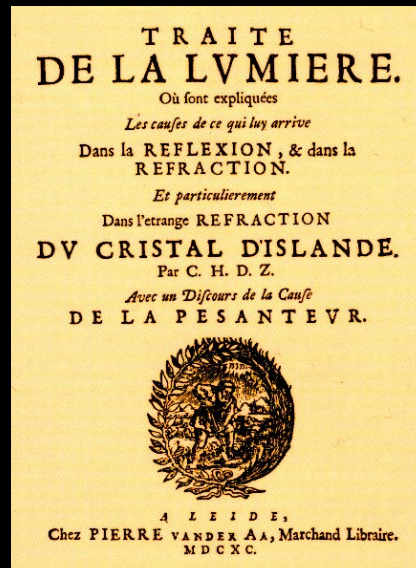

OPTICKS: OR, A TREATISE OF THE *Reflections, Refractions, Inflections* and *Colours* OF LIGHT.

The Second Edition, with Additions.

By Sir ISAAC NEWTON, Knt.

LONDON: Printed for W. and J. INNYS, Printers to the Royal Society, at the *Prince's-Arms* in St. *Paul's* Church-Yard. 1718.

Isaac Newton, Opticks (1704)


TRAITE DE LA LVMIERE.

Où font expliquées

Les caufes de ce qui luy arrive

Dans la REFLEXION, & dans la REFRACTION.

Et particulierement

Dans l'etrange REFRACTION DV CRISTAL D'ISLANDE.

Par C. H. D. Z.

Avec un Difcours de la Caufe DE LA PESANTEVR.

A LEIDE, Chez PIERRE VANDER AA, Marchand Libraire. MDCXC.

Christiaan Huyghens 'Traite de la Lumiere' (1690)

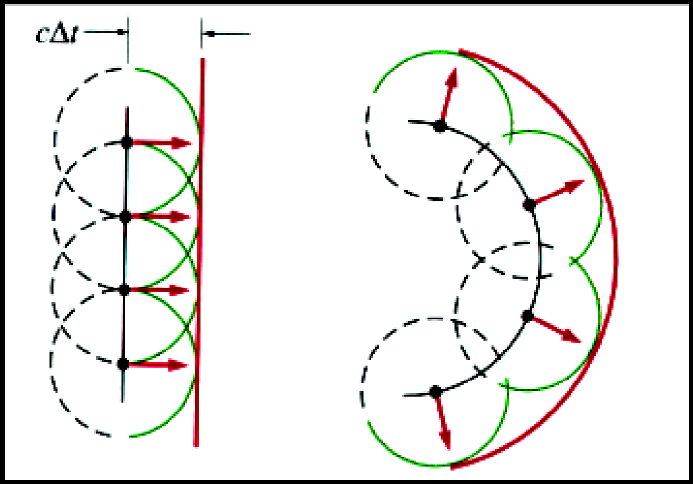
Huyghens argued that the known properties of light, such as refraction, reflection, & propagation in straight lines, could be understood if light was a wave in some invisible aether, analogous to waves in a fluid. Refraction could be understood if the waves traveled more slowly in a dense medium (like waves in shallow water). Wave propagation could be built up from 'elementary wavelets', radiated in circular patterns from multiple sources.


Christiaan Huyghens (1629-95)

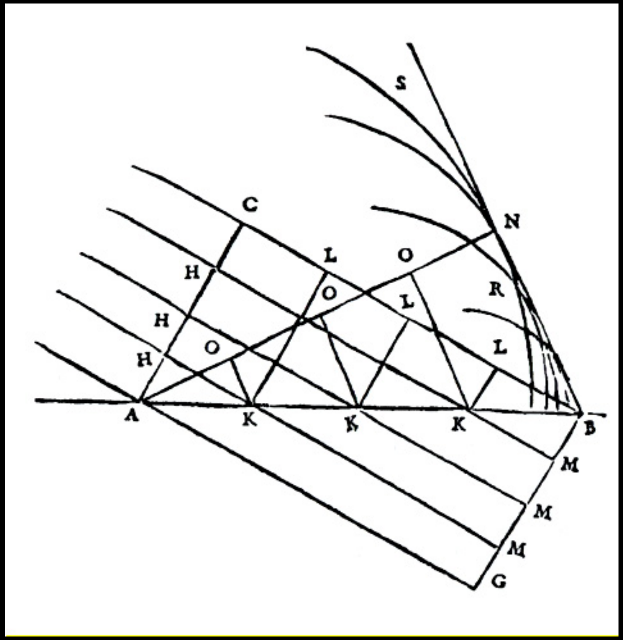
Although neither Newton nor Huyghens realised it, they had uncovered 2 key aspects of one of the most crucial questions in physics. It would need another 250 years before Quantum Mechanics would provide a resolution of this question. We have no particular reason to believe that we have the final resolution .

The WAVE THEORY of LIGHT
Wavelets, Reflection, & Refraction

The famous Huyghens construction is shown at right. At each point of a wave-front, another wavefront is emitted in all directions at equal velocity (unless it arrives in another medium where the velocity is different). In this way, by imagining the 're-emission' of wavefronts after successive short intervals of time, one can build up the dynamics of the wavefronts

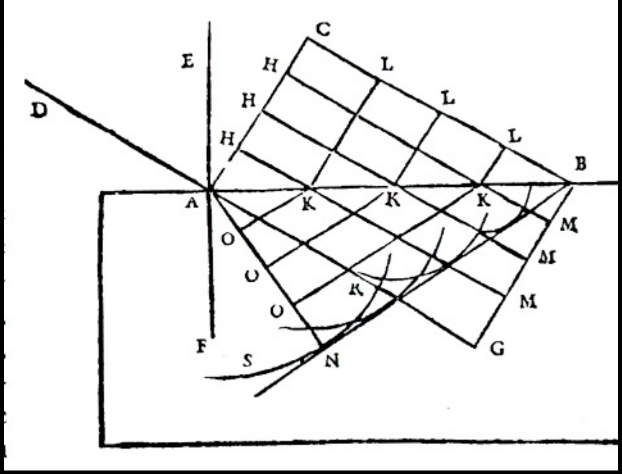


Reflection is easily understood as the radiation of the wave back into the medium – it is fairly obvious by symmetry that a wave incident at some angle on an interface must have lead to a wave moving out at the same angle to the interface.

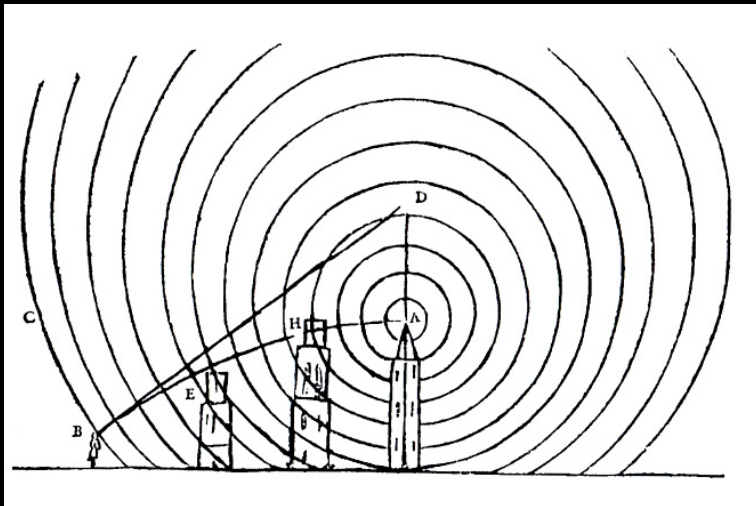


Reflection in the wavelet theory
(from the 'Traite de la Lumiere')

Refraction is produced by imagining the same wavelets now radiating INTO the new medium, but at a different velocity. One can actually show how all this works by purely geometric constructions, without elaborate mathematics. Note that simultaneous reflection & refraction is INEVITABLE in this theory.

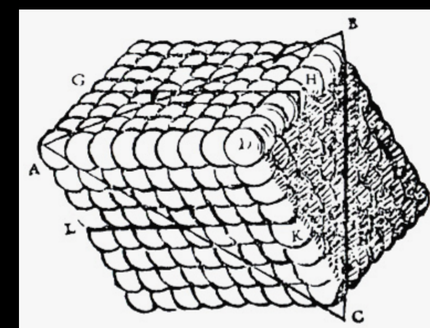


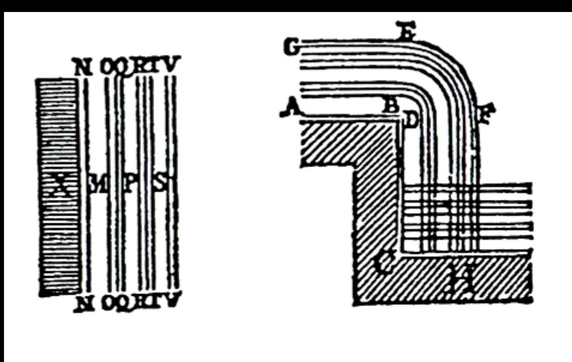
Refraction in the wavelet theory
(from the 'Traite de la Lumiere')



Using the wave theory Huyghens could also explain more complex phenomena – eg., the way sound and light can be slowly refracted downwards, because air density decreases with height; or the flickering light from a multiple light source like a candle flame, where the sources themselves changed in intensity with time.

Huyghens also understood that the way to understand wave dynamics in a crystal was to suppose that the crystal was made up of a lattice of particles. He assumed that the medium (the 'aether') via which light was transmitted was made up of tiny spherical particles, through which compression waves could pass. In remarkable work he treated the refraction of light through 'iceland spar' (calcite) which splits a light beam into 2 beams – he was able to partly understand this in terms of wavelets (but not what caused it).





Franges de diffraction au bord de l'ombre d'un obstacle rectiligne

Franges de diffraction au bord de l'ombre d'un obstacle non rectiligne

Huyghens also realised that phenomena like diffraction (see left) had a natural explanation in terms of his waves.

HAMILTON'S 'PRINCIPAL FUNCTION'

ON A GENERAL METHOD IN DYNAMICS

BY WHICH THE STUDY OF THE MOTIONS OF ALL FREE SYSTEMS OF ATTRACTING OR REPELLING POINTS IS REDUCED TO THE SEARCH AND DIFFERENTIATION OF ONE CENTRAL RELATION, OR CHARACTERISTIC FUNCTION*

Received April 1,—Read April 10, 1834.

[*Philosophical Transactions of the Royal Society*, Part II for 1834, pp. 247–308.]

Dunsink Observatory

Sir W.R. Hamilton (1805-65); lived and worked at the Dunsink Observatory from 1827-1865.

In the period 1833-39, most of our modern ideas on the principal of least action and its application to optical and mechanical systems were devised by Hamilton (NB: Hamilton's canonical eqtns were a by-product of this, not very important to him).

Etching, 1842

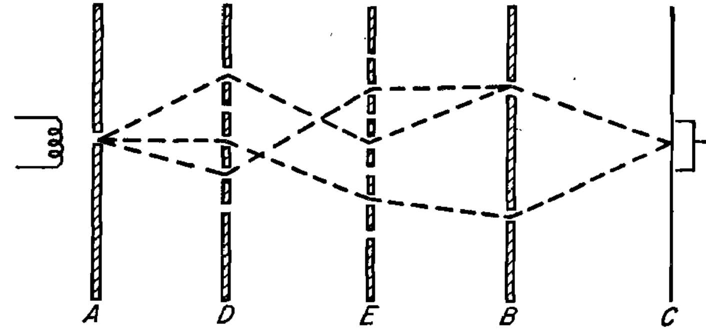
$$\delta S = \delta \int_{t_1}^{t_2} L(q, \dot{q}, t)\,\mathrm{d}t = 0$$

Photo, 1845

It may be mentioned here, that this dynamical principle is only another form of that idea which has already been applied to optics in the *Theory of systems of rays*, and that an intention of applying it to the motions of systems of bodies was announced† at the publication of that theory. And besides the idea itself, the manner of calculation also, which has been thus exemplified in the sciences of optics and dynamics, seems not confined to those two sciences, but capable of other applications; and the peculiar combination which it involves, of the principles of variations with those of partial differentials, for the determination and use of an important class of integrals, may constitute, when it shall be matured by the future labours of mathematicians, a separate branch of analysis.‡

PATH INTEGRAL: ORIGINAL FORMULATION

R.P. Feynman “Spacetime approach to non-relativistic Quantum Mechanics”, Rev. Mod. Phys. 20, 367 (1948)



I. If an ideal measurement is performed to determine whether a particle has a path lying in a region of space-time, then the probability that the result will be affirmative is the absolute square of a sum of complex contributions, one from each path in the region.

II. The paths contribute equally in magnitude, but the phase of their contribution is the classical action (in units of $\hbar$); i.e., the time integral of the Lagrangian taken along the path.

We have $\quad P(b,a) = |K(b,a)|^2$

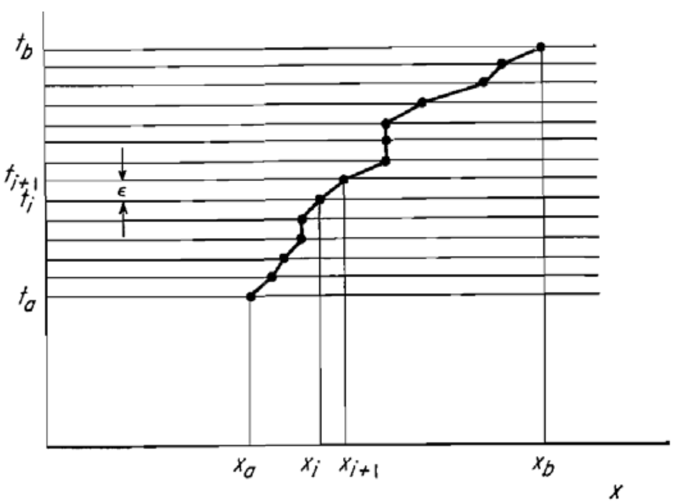
where $\quad K(b,a) = \sum_{\substack{\text{over all paths}\\ \text{from } a \text{ to } b}} \phi[x(t)]$

and $\quad \phi[x(t)] = \text{const}\; e^{(i/\hbar)S[x(t)]}$

ie., $\quad K(b,a) = \int_a^b e^{(i/\hbar)S[b,a]}\, \mathcal{D}x(t)$

Feynman’s initially tried: $\quad \phi[x(t)] = \lim_{\epsilon \to 0} \prod_{i=0}^{N-1} K(i+1,\, i)$

$$K(i+1,\, i) = \frac{1}{A} \exp\left[\frac{i\epsilon}{\hbar} L\left(\frac{x_{i+1} - x_i}{\epsilon},\, \frac{x_{i+1} + x_i}{2},\, \frac{t_{i+1} + t_i}{2}\right)\right]$$



Later he tried: $\quad K(2,1) = \Omega_o(2,1)\, e^{\frac{i}{\hbar} S_{cl}[x(t)]}$

with a ‘fluctuation determinant’: $\quad \Omega_o(2,1) = \int_0^0 \mathcal{D}y(t)\; e^{\frac{i}{\hbar}\int_{t_1}^{t_2} dt\, \frac{m}{2}(\dot{y}^2 - \omega^2 y^2)}$

where we write the deviation from the classical path as: $\quad y(t) = \sum_n y_n \sin\frac{n\pi(t - t_1)}{t_2 - t_1}$

TWO CURRENT WAYS of FORMULATING QUANTUM MECHANICS

(1) WAVE-FUNCTION & PROPAGATOR

Start with Schrodinger eqtn: $\hat{\mathcal{H}}\,\psi(\mathbf{r},t) = i\hbar\,\partial_t\psi(\mathbf{r},t)$ (plus B.C.s)

Equivalently we have:

$$\begin{aligned}(\hat{\mathcal{H}} - i\hbar\hat{\mathbf{1}}\partial_t)\,\hat{G}_o^+(t,t') &= -i\hbar\,\hat{\mathbf{1}}\,\delta(t-t') \\ \hat{G}_o^+(t,t') &= e^{\frac{i}{\hbar}\hat{\mathcal{H}}(t-t')}\,\theta(t-t')\end{aligned}$$

B.C.s included

where G_o is the propagator $\psi(\mathbf{r}_2,t_2) = \int d\mathbf{r}_1\, G_o(2,1)\,\psi(\mathbf{r}_1,t_1)$

(2) PATH INTEGRAL & PROPAGATOR

We have a propagator:

$$\begin{aligned}\mathbf{G}_o(2,1) &= \int_1^2 \mathcal{D}\mathbf{r}(\tau)\, e^{\frac{i}{\hbar}S_{21}[\mathbf{r}(\tau)]} \\ &= \sum_\alpha \chi(\alpha) G_o^\alpha(2,1)\end{aligned}$$

The 2nd form of this equation was not written down until 1977 – it recognizes a fundamental new aspect of QM that is only easy to formulate in path integral language. The basic point was already made earlier:

Thus, Feynman's formalism gives directly an unambiguous answer to global problems. Other formalisms use ad hoc, extraneous conditions to deal with global problems, such as boundary conditions on wave functions, symmetry or antisymmetry property of the wave function, etc. ... and their answers are not necessarily identical with Feynman's.

C. Morette-DeWitt, Comm. Math. Phys. 28, 47 (1972)

In what follows we will unpack some of the implications of this remark!

One KEY Development beginning in the 1950's: STRONG-COUPLING PROBLEMS in CONDENSED MATTER

Feynman very quickly decided at the beginning of the 1950's that the big challenge for physics was not a weak-coupling problem like QED, but the strong-coupling problems which abounded at that time. Following Landau's lead, he attacked 2 of them:

(1) POLARON PROBLEM: Take, eg., the Frohlich model:

$$\mathcal{H} = \frac{\mathbf{p}^2}{2M} + \sum_{\mathbf{q}} b_q^\dagger b_q + i(\pi\alpha)^{1/2} \sum_q \frac{1}{q}\left(b_q^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}} - b_q e^{i\mathbf{q}\cdot\mathbf{r}}\right)$$

couples an electron to optical phonons

Then integrating out the phonons gives an electron propagator

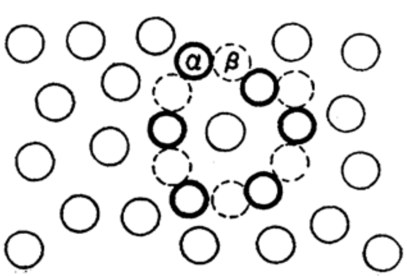
$$G(2,1) = \int_2^1 \mathcal{D}\mathbf{r}(t)\, e^{\frac{i}{\hbar} S_{21}[\mathbf{r}]} \quad \text{with} \quad S_{21} = \frac{m}{2}\int dt\, \dot{\mathbf{r}}^2 + i\frac{\alpha}{4}\int dt \int dt' \frac{e^{-i|t-t'|}}{|\mathbf{r}(t) - \mathbf{r}(t')|}$$

This formulation gave results impossible to obtain in any perturbative expansion.

(2) SUPERFLUID He-4: Now we look at the partition function of a many-particle system, rotating to imaginary time.

Again, one can obtain results inaccessible to perturbative expansions. Most important are the large scale 'ring exchange' processes, which are involved in vortices and in the phase transition.

$$Q = N!^{-1} \sum_P \int d^N \mathbf{z}_i \int_{trP} \exp\left\{ -\int_0^\beta \left[\frac{m}{2\hbar^2} \sum_i \left(\frac{d\mathbf{x}_i}{du}\right)^2 + \sum_{ij} V(\mathbf{x}_i - \mathbf{x}_j) \right] du \right\} \mathfrak{D}^N \mathbf{x}_i(u).$$



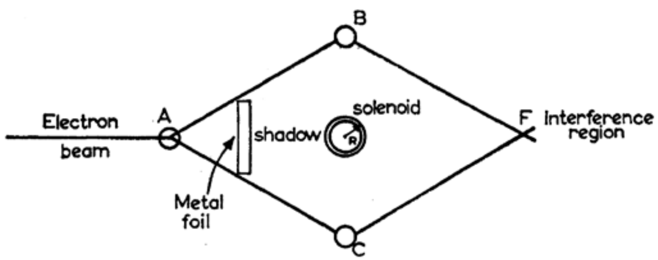
LATER EXAMPLES: Kondo problem (Anderson, Nozieres, etc; 1970...)
Renormalization Group (Wilson, etc., 1971...)
Feynman-Vernon/Caldeira-Leggett (1963, 1983) & other Q Environment models

Another KEY Development beginning in the 1950's: The ROLE of TOPOLOGICAL PHASE

(1) VORTICES & OTHER Q. SOLITONS: The vortices in He-4, first postulated by Onsager in 1950, discussed in detail by Feynman (1952-56), rejected by Landau, and predicted for superconductors by Abrikosov (1957), were the first topological solitons in QM; many others followed, from Quantum fluids to string theory.

Clearly they are non-perturbative entities – but we see that they also reveal the key role of QUANTUM PHASE .

(2) The AHARONOV-BOHM Effect: The 1959 prediction of this effect (anticipated by Ehrenberg & Siday, and rapidly confirmed experimentally by RG Chambers), was a big shock to the community, and initially widely disbelieved.



It is remarkable that, in spite of the remarks at right, the authors solved this as a scattering problem! The reasons for this are interesting....

The phase difference, $(S_1-S_2)/\hbar$, can also be expressed as the integral $(e/\hbar)\oint \varphi dt$ around a closed circuit in space-time, where φ is evaluated at the place of the center of the wave packet. The relativistic generalization of the above integral is

$$\frac{e}{\hbar}\oint\left(\varphi dt-\frac{\mathbf{A}}{c}\cdot d\mathbf{x}\right),$$

where the path of integration now goes over any closed circuit in space-time.

In non-relativistic QM the subsequent history, involving the Berry phase, is well known. But the real story lies in the impact on field theory ....

The PATH INTEGRAL REVOLUTION in PHYSICS (1967-NOW)

Despite the very obvious successes of both the path integral theory and the related functional formulation of Schwinger, physicists were reluctant to use it until the late 1960's. It is very interesting to see how this situation changed.

(1) The ASSAULT on GRAVITY: This began in style:

..... I started with the Lagrangian of Einstein for the interacting field of gravity .....

I can only do one example at a time; I took spin zero matter; then, since I'm going to make a perturbation theory, just as we do in quantum electrodynamics, where it is allowed (it is especially more allowed in gravity where the coupling constant is smaller), $g_{\mu\nu}$ is written as flat space as if there were no gravity plus $\varkappa$ times $h_{\mu\nu}$, where $\varkappa$ is the square root of the gravitational constant. Then, if this is substituted in the Lagrangian, one gets a big mess,

The algebraic complexity of the gravitational field equations is so great that it is not easy to do exploratory mathematical investigations and checks. Gell-Mann suggested to me that the Yang-Mills theory of vector particles with zero mass also is a nonlinear theory with a gauge group and might show the same difficulties, and yet be easier to handle algebraically. This proved to be the case,

R.P. Feynman,
Acta Phys. Pol. 24, 697 (1963)

R.P. Feynman, in
"Magic without Magic: J.A. Wheeler"
(ed. J.R. Klauder, 1971)

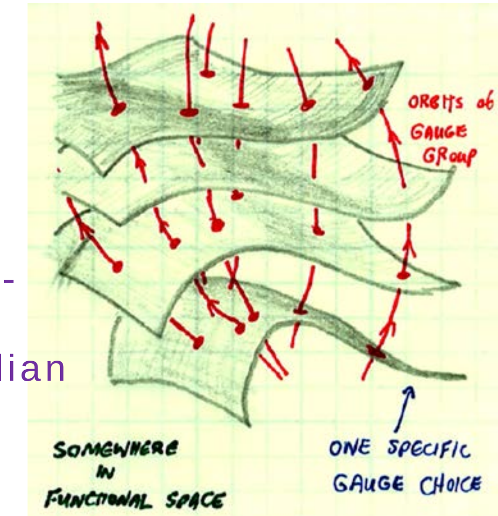
One has: $\mathcal{L} = \frac{1}{\varkappa^2}\int R\sqrt{g}\,d\tau + \frac{1}{2}\int\left(\sqrt{g}\,g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - m^2\sqrt{g}\,\varphi^2\right)d\tau$ with $g_{\mu\nu} = \delta_{\mu\nu} + \varkappa h_{\mu\nu}$.

and $L_{\text{matter}} = \bar{\psi}\gamma_\mu(i\nabla_\mu - \boldsymbol{\tau}\cdot\mathbf{A}_\mu)\psi + m\bar{\psi}\psi$

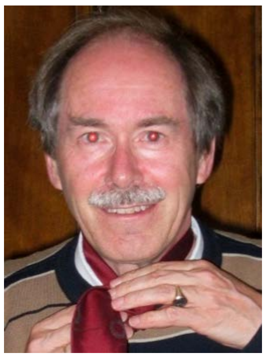
$L_{YM}(\mathbf{A}) = \frac{1}{4}\mathbf{E}_{\mu\nu}\cdot\mathbf{E}_{\mu\nu}$ with $\mathbf{E}_{\mu\nu} = \mathbf{A}_{\mu,\nu} - \mathbf{A}_{\nu,\mu} + \mathbf{A}_\mu \times \mathbf{A}_\nu$

This led to the discovery of 'ghosts'. Incredibly, Feynman never tried a path integral approach – and eventually he gave up, defeated by the complexity of the problem.

(2) SOLUTION for GAUGE THEORIES: Eventually Feynman's diagrammatic approach was worked out by DeWitt (1967). But it was overtaken by the remarkable solution to the 'ghost' problem given by Faddeev & Popov (1967); the key step was to stop thinking in terms of diagrams, and to write everything in path integral form. One defines Jacobian (the 'Faddeev-Popov determinant') in terms of a dummy ghost field, which eliminates the redundancy in the functional integration over all gauge field configurations.



In a series of key papers 't Hooft (partly with Veltman) then rewrote this determinant as a 'gauge-fixing' term. Then, employing 'dimensional regularization' techniques, he showed that non-Abelian gauge theories (including the electroweak model of Salam & Weinberg) were renormalizable, & derived their key properties.



More than anything else, it was these developments that converted physicists to the religion of the path integral....

The result of this in, eg., QED, is a generating functional for the combined coupled fields:

$$\mathcal{Z}[\bar{\eta},\eta;j^\mu] = \int_{in}^{out} \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A^\mu \quad \exp\frac{i}{\hbar}\int d^4x \; [(\bar{\psi}\eta+\bar{\eta}\psi+j_\mu A^\mu)]$$
$$\times \exp\frac{i}{\hbar}\int d^4x \left[L_A^o(A^\mu) - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \bar{\psi}(i\gamma_\mu\partial^\mu - m_o - e\gamma_\mu A^\mu)\psi\right]$$

Now in all this work, the Schwinger-Dyson generating functional/Path integral was being used to generate diagrammatic expansions. The inadequacies of such perturbative expansions were important in Yang-Mills theories, and fatal in quantum gravity.

L.D. Faddeev, V.N. Popov, "Feynman diagrams for Yang-Mills theories", Phys. Lett B25, 29 (1967)

L.D. Faddeev, V.N. Popov, "Covariant Quantization of the gravitational field", Usp Fiz Nauk 16, 777 (1974)

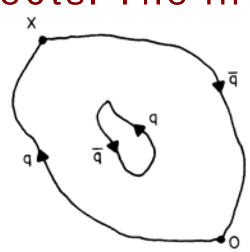
(3) TOPOLOGICAL FIELD THEORIES, & ALL THAT:

Since the heroic period just discussed, the path integral has played a central role in the development of quantum field theory. Here are a few key examples:

Fractional charge & fractional statistics: Using the path integral formalism, Laidlaw & Morette-DeWitt (1977), and then Leinaas & Myrheim (1977) showed, astonishingly, that one could have fractional statistics. The statistics are defined by the winding of paths around each other – in 2+1 dimensions, this allows for ANYONS. After this, the whole enterprise was generalized to field theory, notably by Jackiw & Semenoff. Thus was the field of topological field theory invented. Another result to come out of this was the possibility of fractionally charged particles/quasiparticles (Jackiw & Rebbi (1976), Schrieffer + al (1976)).

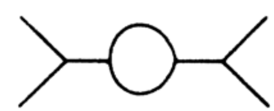
As is well-known, this work was amply confirmed in the discovery of the Fractional Quantum Hall fluid (1983), fractional charge in polyacetylene (1979), and the topological state in graphene (2005), predicted by Semenoff in 1984. It has led to the development of a whole new class of field theories, defined mainly via path integrals, such as Chern-Simons theory.

More sophisticated objects (loops, strings, branes, etc.): Once we have gone from wave-functions to paths, we can envisage more complicated non-local objects. The first to develop this was Wilson (1974), in the form of 'Wilson loops'



A vacuum loop &
a quark-antiquark pair

$$\langle W_P \rangle = e^{i \oint_P A_\mu dx^\mu}$$



From here it was a small step to early string theories, defined either on a lattice or in some higher-D continuum flat space (the 'background problem').

And this is another story ...

PROBLEMS with a PATH INTEGRAL TREATMENT of GRAVITY

Suppose we take the generating functional for quantum gravity seriously. We have

$$\mathcal{Z}[J_{\mu\nu}] = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x) \int \mathcal{D}\phi \, \Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)] \, \exp\{\frac{i}{\hbar}[\mathcal{S}_G + \mathcal{S}_\phi + \int d^4x \, \kappa^{-1} J_{\mu\nu}(x)\tilde{\mathfrak{g}}^{\mu\nu}(x)]\}$$

Does this work? The following problems arise:

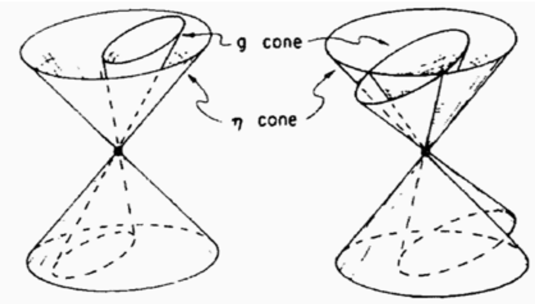
CAUSAL STRUCTURE: Suppose we expand in gravitons to get Feynman diagrams. Write

$$L_{EH}(h, \overline{\Gamma}) = (h\partial\overline{\Gamma} + \eta\overline{\Gamma}\,\overline{\Gamma}) + \kappa h \overline{\Gamma}\,\overline{\Gamma}$$

with $\overline{\Gamma} = \kappa^{-1}\Gamma$

and expand in *h*. Now this is a non-renormalizable theory.

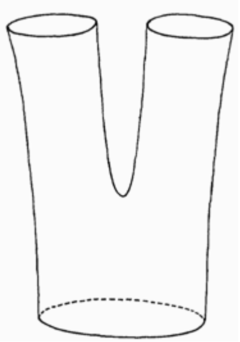
Also – adding gravitons *changes* the spacetime causal structure. The original gravitons can become superluminal.



Moreover, there is no distinction between *x* and *f(x)* where *f(M)* is a diffeomorphism on the manifold ***M***. What then is the meaning of the quantized "metric"? If it is a field, we can write correlators like $[\hat{g}_{\mu\nu}(x), \hat{g}^{\alpha\beta}(x')] = 0$ (for (x-x') spacelike). But – how do we decide if it is spacelike before knowing the metric?

So – the idea of causality seems to lose all meaning.

MEANING of the PATH INTEGRAL: Suppose we assume that diagrams are not meaningful but that the path integral is. But now we are faced with a new problem: we do not know the correct measure for the paths. They sum over spaces with different topologies, & this leads to terrible pathologies (infinite energies, etc.).



String theory, Loop gravity, etc., also have fundamental problems.

A NEW APPROACH – BEYOND QUANTUM MECHANICS

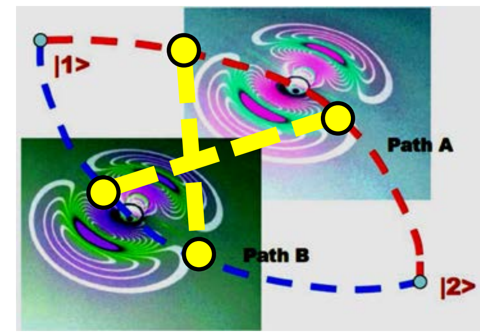
A key problem in quantizing gravity, is we have no way to <u>relate paths</u>. Consider the 2-path superposition

$$|\Psi\rangle = a_1|\Phi_1; \tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2; \tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$

where the mass drags its spacetime with it. How do we calculate $\langle\Phi_1|\Phi_2\rangle$? We can't write

$$\Phi(\mathbf{r},t) \equiv \langle\mathbf{r}|\Phi(t)\rangle = a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

because in the non-relativistic limit we actually have $\Phi_1(\mathbf{r}_1,t_1)$ & $\Phi_2(\mathbf{r}_2,t_2)$



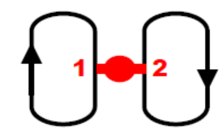
<u>CORRELATED WORLDLINE THEORY</u> In ordinary QM, we can define an object, related to the Wilson loop, given by

$$\mathcal{Q}_o = \oint \mathcal{D}q(s)e^{\frac{i}{\hbar}S_o[q]}$$

Now we want to go beyond QM, by writing $\mathbb{Q} = \mathcal{Q}_o + \Delta\mathbb{Q}$

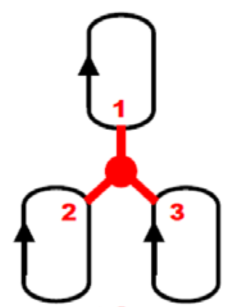
with $\Delta\mathbb{Q} = \sum_{n=2}^{\infty}\prod_{k=1}^{n}\oint \mathcal{D}q_k\ \kappa_n[q_1,...q_n]e^{\frac{i}{\hbar}\sum_k S[q_k]}$

Now the key assumption – that the correlation/relationship between worldliness is defined by gravitation itself:



ΔQ_2

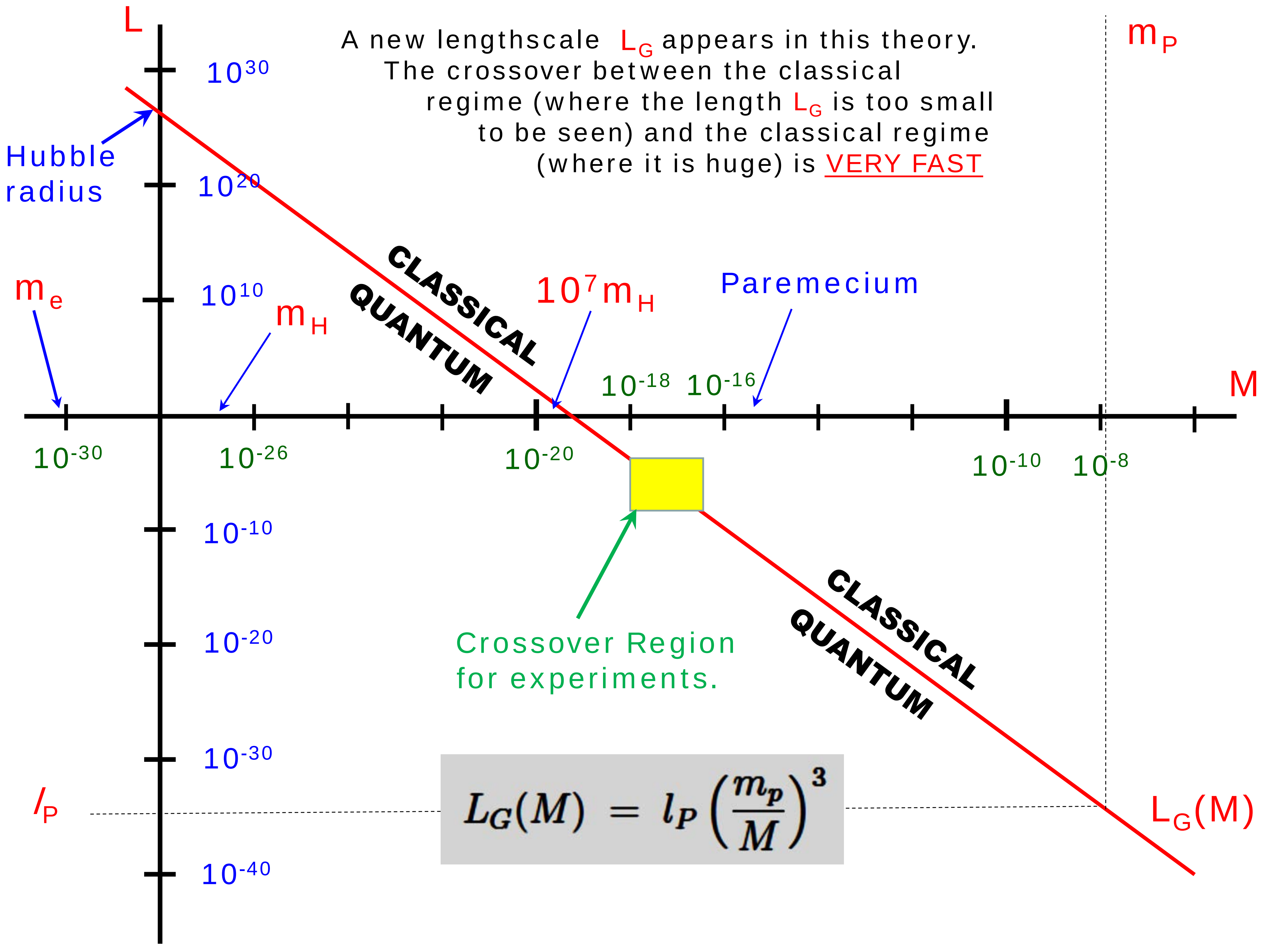
$$Q[j] = \oint \mathcal{D}\tilde{g}^{\mu\nu}(x)\ e^{iS_G/\hbar}\sum_{n=1}^{\infty}\prod_{k=1}^{n}\oint \mathcal{D}q_k\ e^{\frac{i}{\hbar}\sum_k[S[q_k]+j_k q_k]}$$



ΔQ_3

This can be generalized to field theory, and we can develop a perturbative expansion in gravitons. The theory can be used tp predict results for low-energy earth-based experiments with <u>no adjustable parameters.</u>

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
P.C.E Stamp: to be published
F Suzuki, P.C.E Stamp: to be published
F Queiser, G Semenoff, P.C.E Stamp: to be published



A new lengthscale L_G appears in this theory.
The crossover between the classical regime (where the length L_G is too small to be seen) and the classical regime (where it is huge) is VERY FAST
L
M
m_P
10^{30}
10^{20}
10^{10}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
Hubble radius
m_e
m_H
$10^7 m_H$
Paremecium
CLASSICAL
QUANTUM
10^{-30}
10^{-26}
10^{-20}
10^{-18}
10^{-16}
10^{-10}
10^{-8}
Crossover Region for experiments.
CLASSICAL
QUANTUM
l_P
$L_G(M)$
$$L_G(M) = l_P \left(\frac{m_p}{M}\right)^3$$

SOME REMARKS on PHILOSOPHICAL TOPICS

(1) What is a theory here? Comparing the wave-function and path integral formulations of QM, we see that they contain some elements that are FMPP equivalent, and others that are not (or may not be in the future). Is it not better to discuss theories as ever-shifting Families of ideas ?

(2) Have we found the right object yet? So far we have Worldlines, loops, worldsheets,...what else is coming? It seems to be premature to be arguing about what is 'real', or what is the 'fundamental ontology'.

(3) What is the proper definition of the path integral? Mathematicians like to ridicule the physicists' definition of path integrals, ignoring the crucial point that we are always dealing with effective theories. Should we really care?

(4) What are the implications for the interpretation of QM? There is a curious dearth of interpretations which focus on the path integral as a starting point. Are there any useful new interpretations?

(5) Why the extraordinary longevity of the action principle? We've discussed 2 things here which have endured a very long time – the particle/wave dichotomy, and the action principle – and they are closely related. This seems to be terribly important – why?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