The quantum mechanics that no one remembers.

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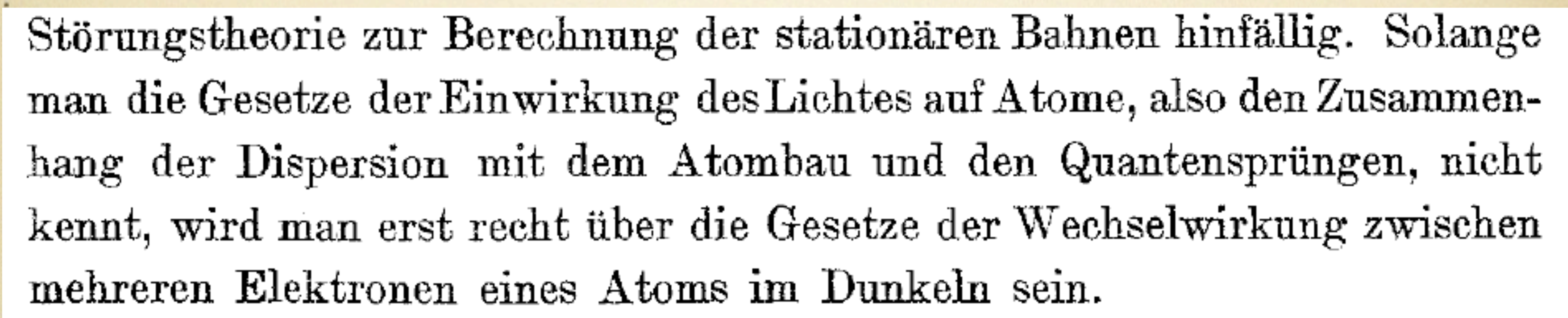
QUANTUM
HISTORY
PROJECT

No quantum mechanics without a quantum theory of light



Störungstheorie zur Berechnung der stationären Bahnen hinfällig. Solange man die Gesetze der Einwirkung des Lichtes auf Atome, also den Zusammenhang der Dispersion mit dem Atombau und den Quantensprüngen, nicht kennt, wird man erst recht über die Gesetze der Wechselwirkung zwischen mehreren Elektronen eines Atoms im Dunkeln sein.

Max Born 1924: As long as one does not know the laws of the action of light on atoms ... one will even more be in the dark about the laws of the interaction between several electrons in an atom.

Three approaches to quantum physics

After the establishment of quantum mechanics (1925–27), different ideas of what "quantization" meant implied different visions for a more general quantum theory that also encompassed electrodynamics.

Waves and particles

1924/25 Einstein‘s quantum theory of the ideal gas based on Bose statistics: Close analogy between statistics of light and ideal gas. New “quantum” statistics for both.

Einstein showed that Bose statistics leads to the dual fluctuation term he had found in 1909 for the energy fluctuations of black body radiation. Referred in 1925 to de Broglie‘s idea of matter waves as a possible explanation.

Schrödinger on waves and particles

Einstein's reference to de Broglie led Schrödinger to wave mechanics: To him, the new statistics implied that matter should not be seen as consisting of individual independent particles, but as consisting of waves.

"This means nothing else but to follow through with the de Broglie-Einstein undulatory theory of the moving corpuscle, according to which the corpuscle is nothing more than a sort of 'foam crest' on a radiating wave which forms the substratum of the world."

Jordan on waves and particles

In 1925 (still before Heisenbergs ‘Umdeutung’), Jordan drew a different conclusion: Treating the equilibrium of light and B-E gas, he stayed much closer to Einstein’s own formulation (“a far-reaching formal similarity between radiation and gas”).

“The elementary acts of dispersion [between radiation and matter] can be viewed not only as dispersion of light radiation on material corpuscles but also as dispersion of matter radiation on corpuscular light quanta; therefore, the probabilistic law will be symmetric. . . [between the densities of radiation and matter].” (Jordan 1925)

The principle of symmetry of representations.

This means that **both** matter and radiation can be described equivalently **either** as waves **or** as particles. Unlike for Schrödinger, no physical model, just a formal symmetry. (Preexistent hostility to 19th century materialism as a possible motivation.)

Jordan referred to William Duane's 1923 quantum model of interference as inspiration: Interference on a grid can be described as scattering of light quanta on a periodic system (standing wave representing grid).

Jordan wants to find a theory representing this symmetry. (In a later interview, Jordan claims that he already here thought about a quantum field theory.)

Jordan's Quantum Field Theory

After Heisenberg's 'Umdeutung' paper, Born and Jordan developed matrix formulation.

In the 'Dreimännerarbeit' (November 1925), Jordan applied matrix mechanics to a one-dimensional field theory (vibrating string). He derived the existence of "light quanta" obeying Bose-Einstein statistics and Einstein's dual fluctuation formula.

Saw this as a realization of a quantum wave field and envisioned a theory of matter and radiation on this basis (claiming later that he had already thought so **before** matrix mechanics).

Wave mechanics

Spring 1926: Schrödinger’s wave mechanics

Jordan‘s reaction (letter to Schrödinger, summer 1927): „Then your hydrogen paper gave hope that by following up this correspondence also the non-ideal gas could be represented by quantized waves—that therefore a complete theory of light and matter could be derived in which, as an essential ingredient, this wave field itself operates in a quantum, non-classical way.“

Schrödinger's wave mechanics

In the context of his wave mechanics, Schrödinger envisioned a relativistic theory of interacting matter and electromagnetic waves. He explicitly formulated such a theory in December 1926, combining the Klein-Gordon and Maxwell equations. The result was a consistent classical field theory. However, Schrödinger was aware of its two fundamental difficulties:

- fine structure not reproduced—electron spin missing.
- even electrostatic term comes out wrong—self-interaction of the electron wave needs to be ignored.

Against visualisation

Jordan explicitly rejects an underlying physical picture.

He offends Schrödinger with his aggressive attack on Schrödinger‘s physical wave interpretation.

Substantial argument: Jordan agrees that light and matter show analogous behavior and should be treated analogously in quantum theory.

However, since the light quantum effects show that classical wave optics fails, therefore wave mechanics can also not be the complete story for matter. Otherwise, there would be a disanalogy.

Wave mechanics is only a wave theory for the one-particle case. Schrödinger's visualization is misleading.

Dirac's theory of radiation

Paul Dirac: "The quantum theory of the emission and absorption of radiation." (Feb. 1927) presented as a provisional (not fully relativistic) theory of the interaction of radiation and matter.

"Second Quantization" as a method of going from one-particle Schrödinger equation (understood as describing a Boltzmann ensemble) to many-particle Schrödinger equation (describing a Bose-Einstein ensemble).

Hamiltonian no longer expressed in terms of one-particle observables, but in terms of occupation numbers N.

Allows for construction of a "light quantum Hamiltonian" (by relating N to energy density of electromagnetic field) without a classical mechanics of light quanta.

Dirac’s warnings

Dirac does not see his approach as a field quantization in the modern sense:

- He points out that the “wave function of the light quanta” is not the same as the electromagnetic field. (Amplitudes have a different meaning.)
- Therefore, the quantization procedure is not an explanation of the quantum nature of radiation. It is only an elegant way to take into account the Bose statistics of light quanta.
- The procedure is not applicable to electrons, since they do not obey Bose statistics. Electrons only have a wave function, but not a physical wave field.

Dirac’s theory is successful in treating various problems of radiation in quantum mechanics (dispersion, line widths, scattering, etc.). Dirac still looks for a fully relativistic theory.

Three approaches to quantum physics

- Dirac: Quantum mechanics is Hamiltonian mechanics with non-commutative algebra of the dynamical variables (q-numbers). A corresponding (relativistic) mechanics of light quanta needs to be found.
- Schrödinger: Wave mechanics is a theory of matter waves obeying a continuous equation of motion. Matter waves and light waves interact continuously.
- Jordan: Both matter and radiation are to be described through a union of classical particle and wave theories, formulated as a theory of "quantum wave fields."

Many many-particle theories

Correspondingly, in the summer of 1926 there are three different ideas to tackle the many-body problem:

- Heisenberg and Dirac: phenomenological arguments for symmetrical/antisymmetrical states.
- Schrödinger: self-interaction of continuous charge distribution (Heitler/London).
- Jordan: quantized waves.

Heisenberg/Dirac approach is quickly successful, which changes the landscape: no need for Jordan's much more abstract line of argumentation.

Jordan’s program has problems: Fermi-Dirac statistics, general skepticism from colleagues.

Not the answer yet

However, everyone agreed that a nonrelativistic quantum mechanics—including Dirac's quantized theory of emission and absorption—could only be a preliminary solution and not the full theory they looked for.

Jordan’s synthesis

Adopts Schrödinger’s wave mechanics as a theory in which matter and radiation are treated on the same footing—but sees it as a classical theory in need of quantization.

Adopts Dirac’s method of second quantization—but sees it as a method for going from a classical field theory to a quantum field theory, which explains the existence of particles.

Jordan’s vision

“Zur Quantenmechanik der Gasentartung” (July 1927) treats matter as a wave on exactly the same basis as light. Jordan uses generalized quantum mechanical commutation relations and is able to formulate a quantum field theory for (non-interacting) fermions.

Paper is flawed and rewritten half a year later with Eugene Wigner. Only then formulation of anticommutation rules. But already the first paper defines Jordan’s program: a unified quantum field theory for matter and radiation. Particles and waves are only complementary aspects of the underlying quantum field.

Electrons explained

“Despite the validity of the Pauli instead of Bose statistics for electrons, the results achieved so far leave hardly a doubt that […] the natural formulation of the quantum theory of the electron will have to be achieved by comprehending light and matter on equal footing as interacting waves in three-dimensional space. The fundamental fact of electron theory, the existence of discrete electrical particles, thus manifests itself as a characteristic quantum phenomenon, namely as equivalent to the fact that matter waves only appear in discrete quantized states.”

Pauli and functional calculus

Up until Summer 1927: Field (or second) quantization means quantizing the Fourier amplitudes corresponding to solutions of the free classical field equations.

In the spring of 1927, Pauli had studied functional calculus, hoping that this approach would give a technique for quantizing field variables directly. In Summer 1927, Jordan visits Pauli in Hamburg. Together, they use this technique to perform a manifestly Lorentz invariant quantization of the free electromagnetic field. They introduce the concept of q-function, a q-number that is a function of the spacetime coordinates to describe the electromagnetic field. The wave function of quantum mechanics is replaced by a functional, defined on the space of field configurations.

Addressing the self-energy problem

Oskar Klein was pursuing a program similar to Jordan's, attempting to quantize Schrödinger's wave theory with Dirac's technique. In the attempt to include (Coulomb) interaction, he encountered infinite self-energy terms.

Jordan, returning to Copenhagen in fall 1927, recognized that this problem could be solved using the non-commutativity of the field operators. By a correct ordering of the field operators in the Hamiltonian, the self-interaction term could be excluded, without changing the corresponding classical theory.

The Heisenberg-Pauli Program for Quantum Electrodynamics

In November 1927, Heisenberg and Pauli commence a cooperation on a full quantum electrodynamics, combining

- field quantization using the method of q-functions and wave functionals,
- a generalization of the Klein-Jordan normal ordering procedure to remove the self-energies which were known to exist in classical electron theory,
- fermionic quantization of electron waves,
- a “classical” theory of matter waves, initially of Klein-Gordon type, soon to be replaced by Dirac’s relativistic electron theory (January 1928).

Difficulties of the Heisenberg-Pauli Program

- No generalization of Klein-Jordan procedure possible—self-energy cannot be removed (becomes central problem of QED).
- Functional methods not practicable (QED can only be expressed as a theory of quantized, discrete Fourier amplitudes—not a central problem for calculations).
- Negative energy states (Problem of the Dirac equation itself—not particular to QED. Solved by hole theory in 1930.).
- Once interaction with matter waves was included, the manifestly Lorentz invariant quantization of Jordan and Pauli no longer worked (solved by defining equal-time commutation relations and explicitly proving Lorentz invariance).
- To describe interaction, theory needs to be expressed using electromagnetic potentials—difficulty of quantizing redundant degrees of freedom (solved first by auxiliary terms, then by exploiting the newly discovered gauge invariance).
- Theory doesn't determine whether electrons are fermions or bosons (not a problem for calculations).
- Not a monistic field theory (not a problem for calculations).

Heisenberg-Pauli Quantum Electrodynamics

Solving some problems, while ignoring others for the time being, Heisenberg and Pauli present a full theory of quantum electrodynamics in two massive papers (March/September 1929).

They initially hoped that their theory might lead to predictions for radiative corrections going beyond Dirac's radiation theory (e.g., β-decay).

Soon, Heisenberg, Pauli, and Oppenheimer showed that

- the theory could be equivalently formulated without matter waves, using instead many-particle quantum mechanics.
- the components of the electromagnetic potential that did not belong to the transversal radiation field could be eliminated, leaving only the Coulomb interaction term.

The theory was equivalent to Dirac's radiation theory plus Coulomb interaction.

The Problem of the Infinite Self-Energy

In the following years, Pauli's students and guests in Zürich investigated the problem of self-energy.

- Oppenheimer (1930): Self-energy is not an additive constant, but also causes an infinite separation between energy levels and thus an infinite shift in spectral lines.
- Waller (1930): The same is true for the self-energy of free electrons.
- Rosenfeld (1930): Gravitational self-energy of light quanta is also infinite. Infinite self-energy also appears in theory without classical point singularity.

The Fundamental Flawedness of the Theory

Rosenfeld (1931) studied the simplest possible setup in radiation theory, a charged harmonic oscillator. He showed that this results in an infinitely strong coupling to the electromagnetic field if all frequencies (not just the harmonics of the oscillator) are considered.

Pauli concluded that the difficulties were already present in Dirac's radiation theory of 1927, in which matter was treated non-relativistically—the difficulties resulted from a coupling to all possible radiation modes in a theory of emission and absorption.

Pauli demanded of Dirac that "you sufficiently stress in all future publications the fundamental flawedness of the foundations of your theory of photons."

A parting of the ways

In the first years of quantum mechanics, its development was tightly intertwined with the project of finding in quantum electrodynamics a foundation for all of quantum physics. This changed when in 1929–1931 it became clear that a generalization of quantization procedures to electrodynamics led to a theory beset with fundamental problems. Only in reaction to that did the codification of quantum mechanics as a self-contained theory become attractive.

A parting of the ways

This is evident in von Neumann's “Mathematische Grundlagen der Quantenmechanik” (1932): quantum electrodynamics is only presented to derive the Einstein coefficients. The goals of a unified theory of matter and radiation or of a relativistic quantum theory are not even mentioned. Neither are the problems that result from an application of the Hilbert space formalism on systems with infinitely many degrees of freedom.

Outlook

The problems of quantum electrodynamics led to a general conviction that it would have to be replaced by a more fundamental theory. Different prognoses were made as to how that theory would have to be constructed:

- Modification of the classical theory to avoid the singularities of classical electron theory (Born/Infeld, Dirac, Wheeler/Feynman).
- Rejection of the idea that matter and radiation are to be treated equivalently—attempt to formulate a novel quantum theory of radiation (Dirac).
- Development of an autonomous quantum field theory, not based on the quantization of a classical field (Jordan).
- The concept of a quantum field is flawed, since field strengths at a point are not measurable due to the uncertainty relations (Landau/Peierls).

Against all expectations, quantum electrodynamics was not replaced. However, the division remained between a neatly formalized nonrelativistic quantum mechanics, and a relativistic quantum field theory without clearly defined foundations (both formally and ontologically).