Gauge/gravity duality

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Duality

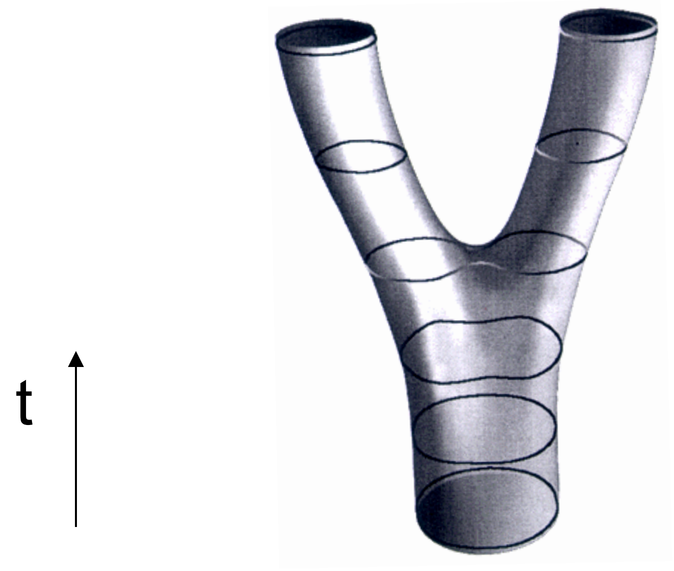
Two different descriptions with exactly the same physical predictions.

String Theory

This is a promising candidate for both

1) a complete quantum theory of gravity
2) a unified theory of all forces and particles

It is based on the idea that elementary particles are not pointlike, but excitations of a one dimensional string.



Strings interact with a simple splitting and joining interaction with strength g.

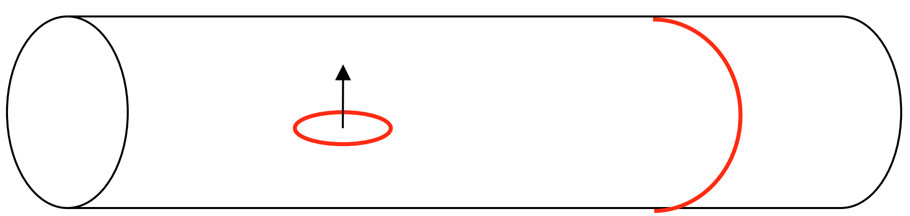
String theory reduces to general relativity (with certain matter) in a classical limit.

T – duality

Two geometrically different spacetimes can be physically equivalent in string theory.

E.g. flat spacetime with one direction a circle of radius R is equivalent to one with radius L_s^2/R

Physically this is a result of the fact that strings have two kinds of states: winding modes and momentum modes



Gauge Theories

These are generalizations of electromagnetism in which the U(1) gauge invariance is replaced by e.g. SU(N).

Our standard model of particle physics is based on a gauge theory.

QCD has SU(3) gauge symmetry. The interactions are weak at high energy but become strong at low energy causing quark confinement.

‘t Hooft argued in the 1970’s that a 1/N expansion of an SU(N) gauge theory would resemble a theory of strings.

It took more than 20 years for this idea to be made precise.

Gauge/gravity duality

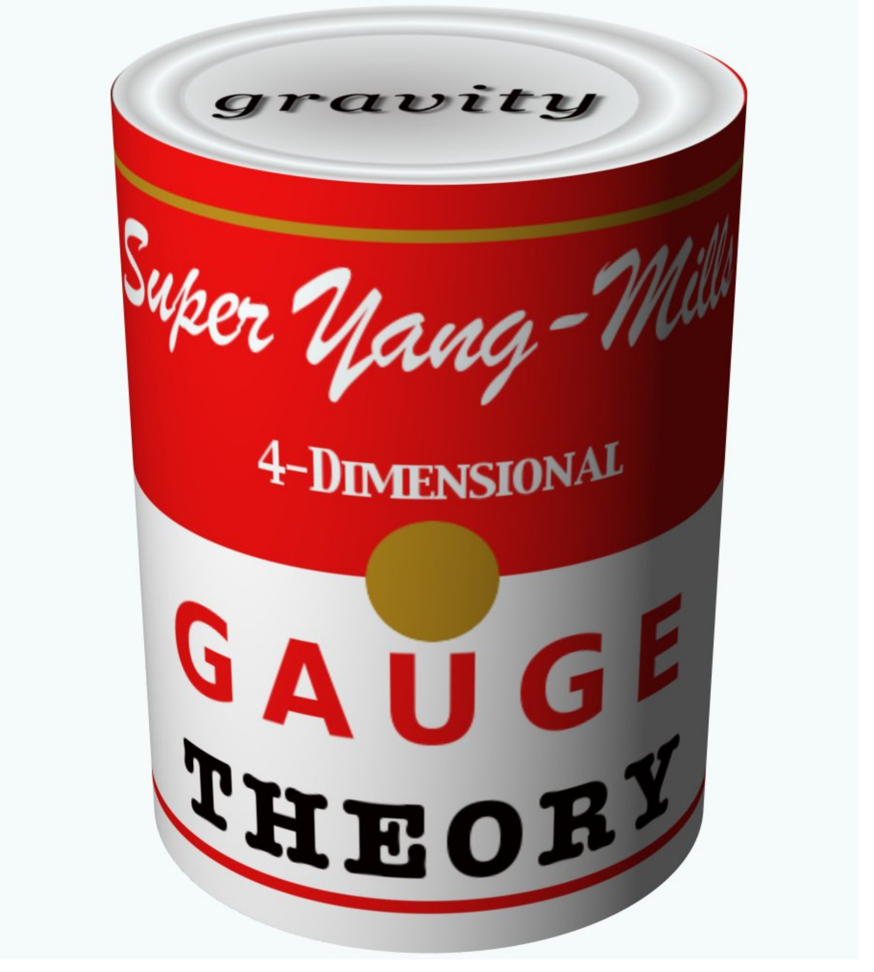
(Maldacena, 1997)

Under certain boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

Shows that quantum gravity is holographic

('tHooft, Susskind)


gravity
Super Yang-Mills
4-Dimensional
GAUGE
THEORY

The boundary condition that is required is that the spacetime must approach constant negative curvature. This is called anti-de Sitter (AdS) spacetime. The metric looks like

$$ds^2 = r^2 \underbrace{(-dt^2 + d\vec{x}^2)}_{\text{Metric of special relativity}} + \frac{dr^2}{r^2}$$

Metric of special relativity

Rescaling r $\longrightarrow$ a r, (t, x) $\longrightarrow$ (t/a, x/a) leaves the metric invariant. Small radius corresponds to large distance (low energy) in gauge theory.

Early Evidence

- Symmetries agree
- Gauge theory analogs of all massless string modes are known
- Gauge theory analogs of many massive string modes have been found (strings arise as long chains of operators)
- Many interactions have been shown to agree

Many calculations agree

- Microscopic derivation of black hole entropy
- Partition functions
- Expectation value of Wilson loops
- Renormalization group flow
- …

Renormalization group (RG) flow in a QFT corresponds to obtaining an effective low energy action by integrating out high energy modes.

This corresponds to radial dependence on the gravity side:

Gauge theory: add mass terms and follow RG flow to low energies to obtain a new field theory

Gravity theory: modify the boundary conditions for certain matter fields and solve Einstein's equation

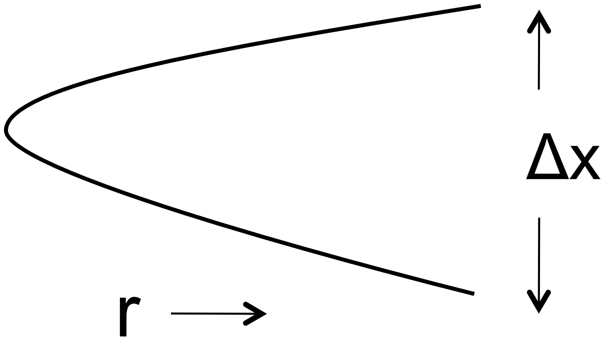
One finds detailed numerical agreement between the small r behavior of the gravity solution and the endpoint of the RG flow.

Traditional applications of gauge/gravity duality

Gain new insight into strongly coupled gauge theories, e.g., geometric picture of confinement.

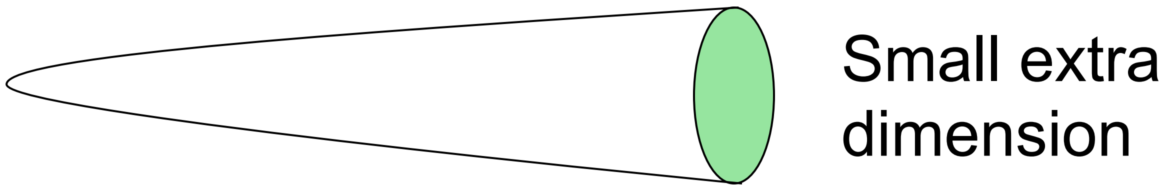
Gain new insight into quantum gravity, e.g., quantum properties of black holes

The potential between two quarks is obtained from the length of a string in the bulk:

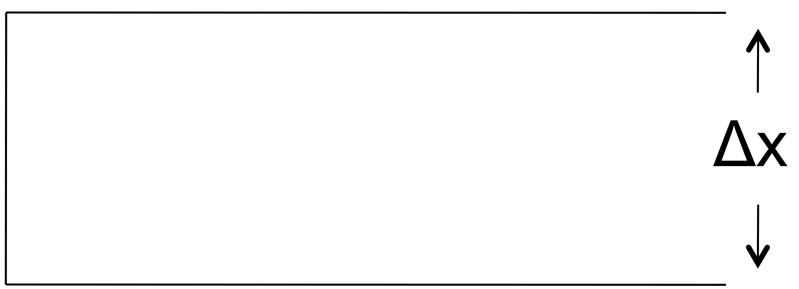


Due to the scaling symmetry of AdS, the length of this curve is independent of Δx. AdS describes the vacuum of a scale invariant theory.

The gravity dual of a confining vacuum differs from AdS in that the spacetime ends at nonzero radius e.g.



In this case, the length of the string grows linearly with Δx when it becomes large:



Fundamental BH questions

- What is the origin of black hole entropy?
- Does black hole evaporation lose information? Does it violate quantum mechanics?

Answers from Gauge/Gravity Duality

The gauge theory has enough microstates to reproduce the entropy of black holes.

The formation and evaporation of small black holes can be described by ordinary Hamiltonian evolution in the gauge theory. It does not violate quantum mechanics.

After thirty years, Hawking finally conceded this point in 2004.

In a certain limit, all stringy and quantum effects are suppressed and gravity theory is just general relativity

(in higher dimensions, with asymptotically anti-de Sitter boundary conditions).

Basic Ingredients of the Duality

A state of thermal equilibrium at temperature T is dual to a black hole with temperature T.

Fields in spacetime are dual to operators in the boundary theory.

Local properties of the gauge theory are related to the asymptotic behavior of the gravity solution.

This duality allows us to compute dynamical transport properties of strongly coupled systems at nonzero temperature.

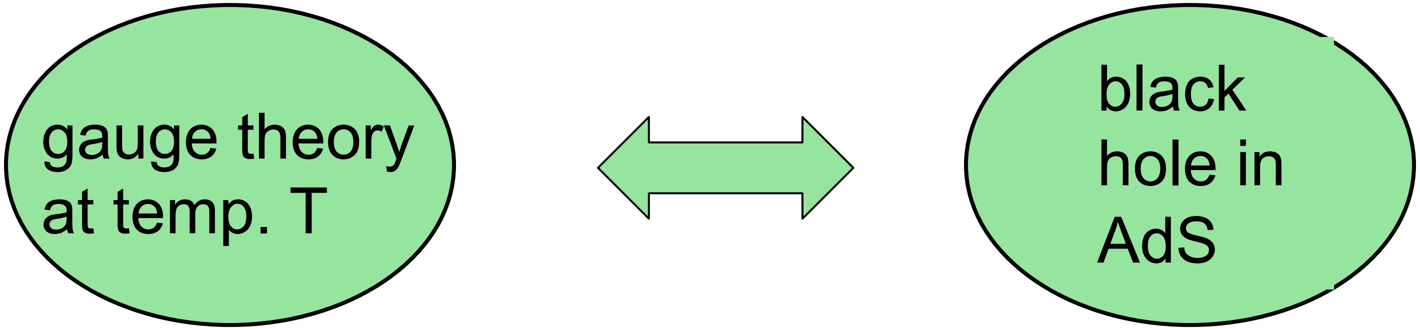
Theoretical physicists have very few other tools to do this.

“Real world” applications

Hydrodynamics from gravity

One expects that the long distance dynamics of any strongly interacting field theory is described by (relativistic) hydrodynamics.

Gauge/gravity duality predicts that hydrodynamics can be recovered from general relativity. Start with:



One can define a stress energy tensor $T_{\mu\nu}$ from the asymptotic metric which equals the expected value of the $T_{\mu\nu}$ of the dual field theory. This satisfies $\partial_\mu T^{\mu\nu} = 0$

Add long wavelength perturbations to the black hole and find that $T_{\mu\nu}$ takes the form of a fluid with viscosity.

(Bhattacharyya, Hubeny, Minwalla, Rangamani, 2007)

This viscosity is very low. The ratio of the (shear) viscosity η to entropy density s is

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

This is much smaller than most materials.

(Kovtun, Policastro, Son, Starinets)

Something close to this is seen in heavy ion collisions.

Relativistic Heavy Ion Collider at Brookhaven

Gold nuclei collide at 200GeV/nucleon

Gauge/gravity duality and RHIC

The quark/gluon plasma produced at RHIC is strongly coupled and thermalizes quickly. Surprisingly, it is well described by fluid dynamics with a very low viscosity - close to value calculated from gravity.

Gauge/gravity duality currently offers the best explanation of this fact.

Conductivity in strange metals

(Santos, Tong, G.H., 2012)

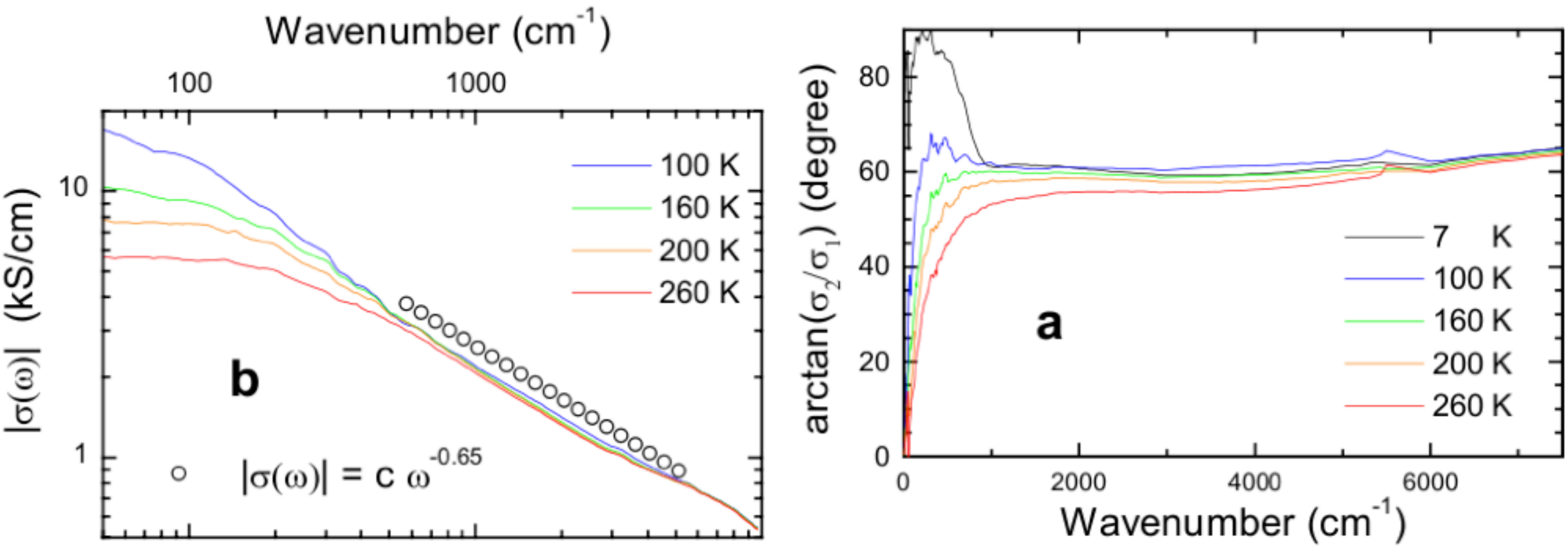
A simple model of the conductivity $\sigma(\omega)$ in normal metals was given by Drude

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

where K is a constant related to the number of electrons and τ is a relaxation time. In particular,

$$|\sigma(\omega)| \sim 1/\omega$$

Some materials behave very differently
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

To calculate the conductivity on the gravity side, we start with a charged black hole. At large radius, the vector potential behaves like

$$A_t = \mu - \frac{\rho}{r}$$

Gauge/gravity duality relates these constants to properties of the dual field theory:

μ = chemical potential, ρ = charge density

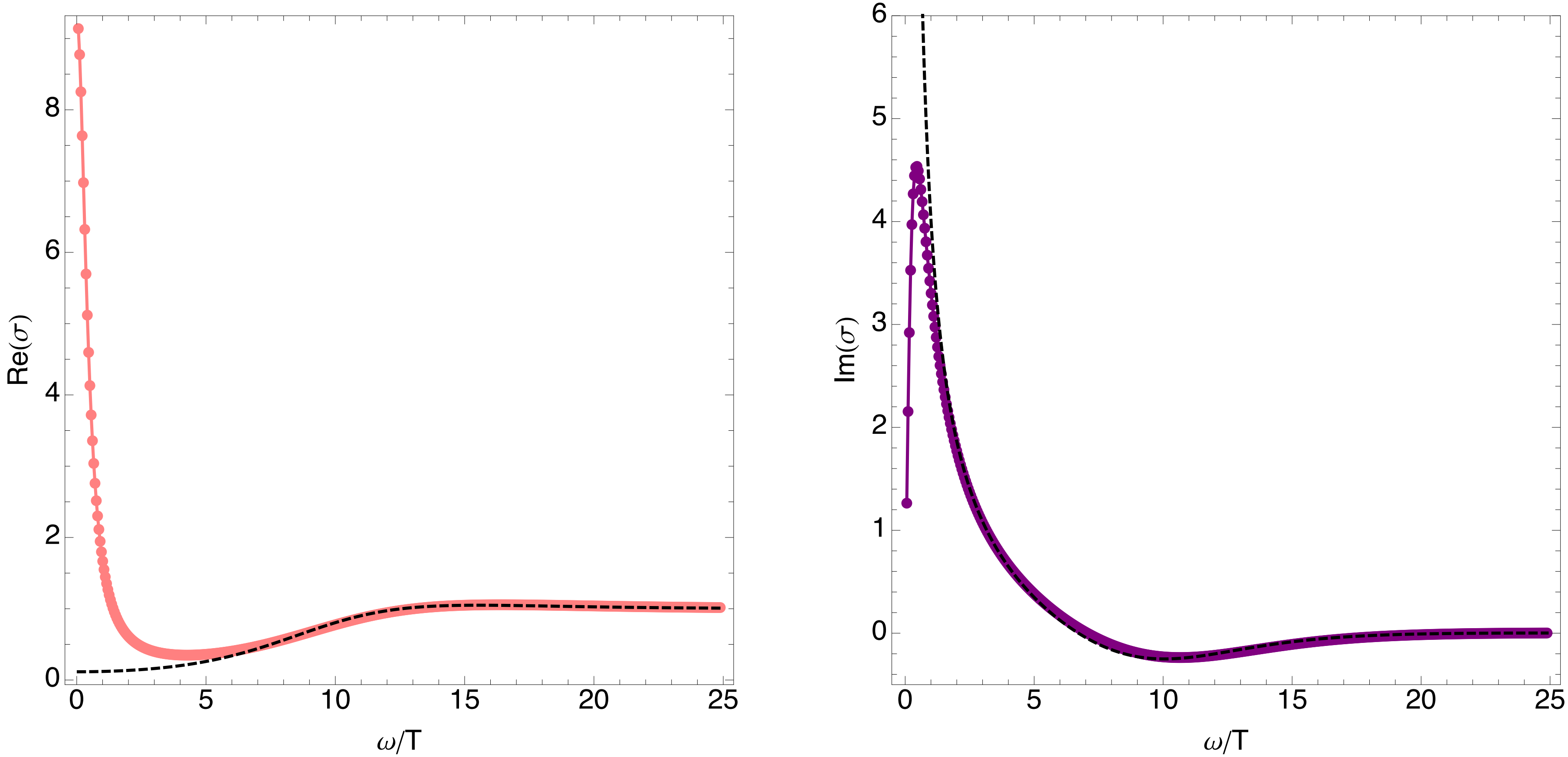
Black holes tend to be very symmetric. But a translationally invariant system at nonzero charge density always has infinite DC conductivity: a boost produces a current with no applied field.

We add a lattice, by adding a scalar field with periodic boundary condition at infinity.

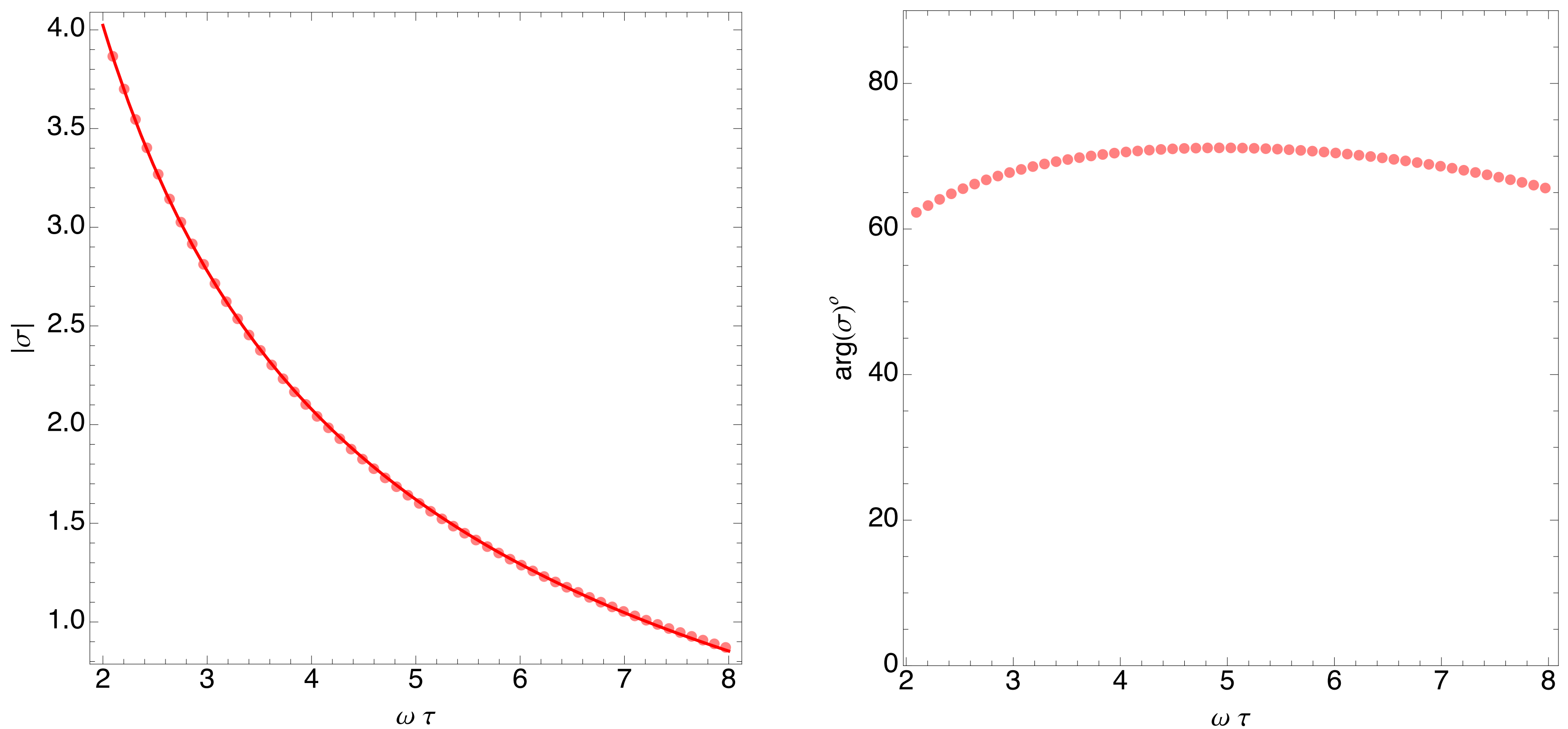
Conductivity

We want to compute the conductivity as a function of frequency. Start by perturbing the Maxwell field around the black hole. Assume time dependence $e^{-i\omega t}$ and impose ingoing wave boundary conditions at the horizon.

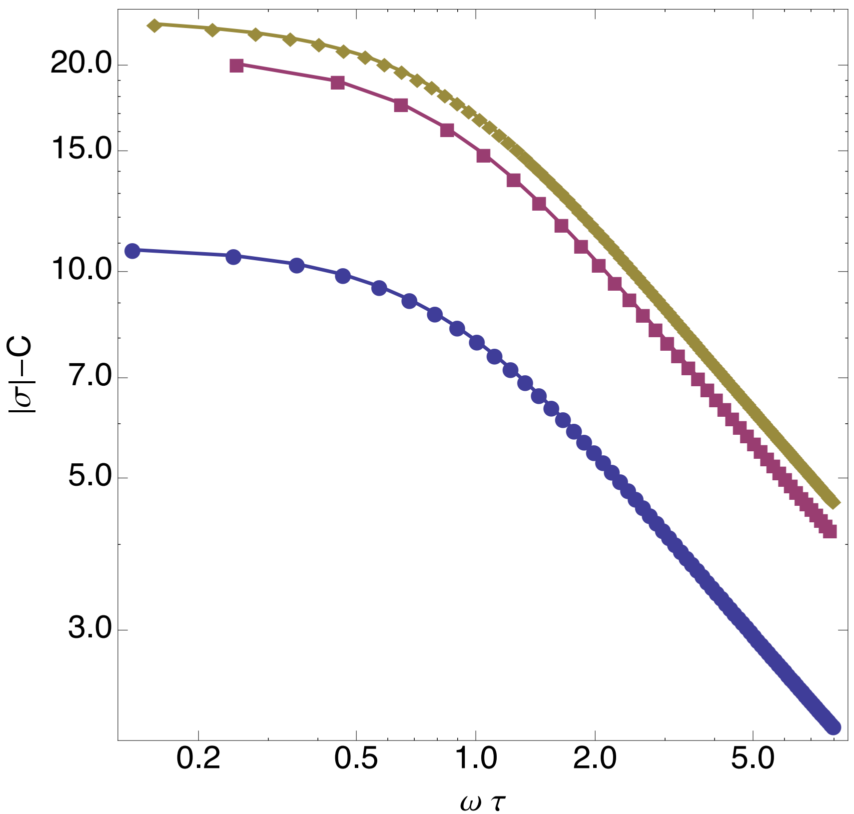
The asymptotic form of the perturbation determines the applied E field and induced current.



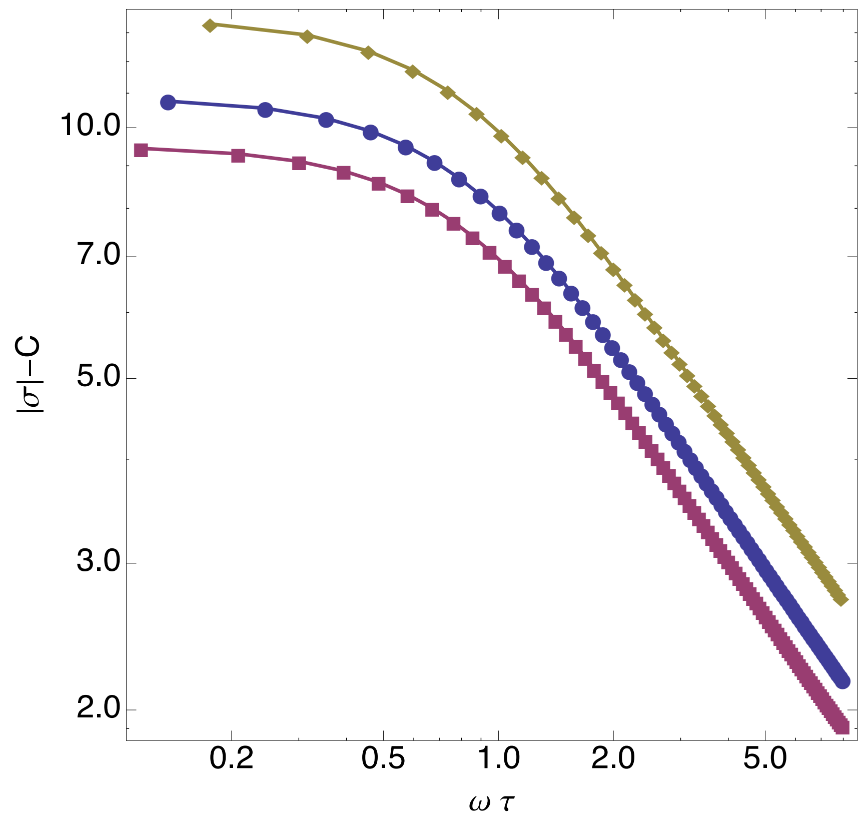
Conductivity computed from gravity



The data is very well fit by $|\sigma(\omega)| = \dfrac{B}{\omega^{2/3}} + C$

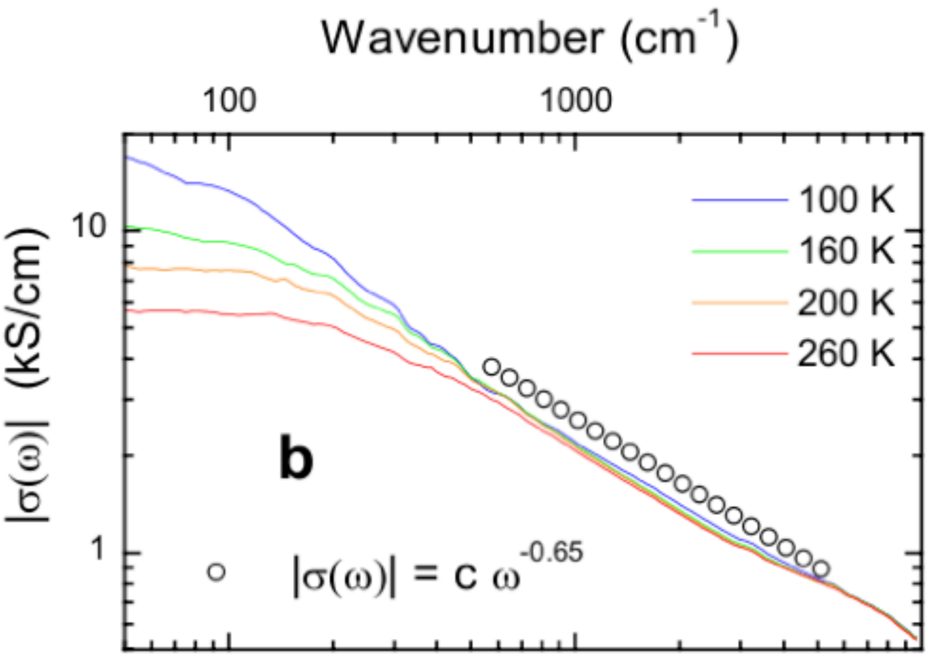


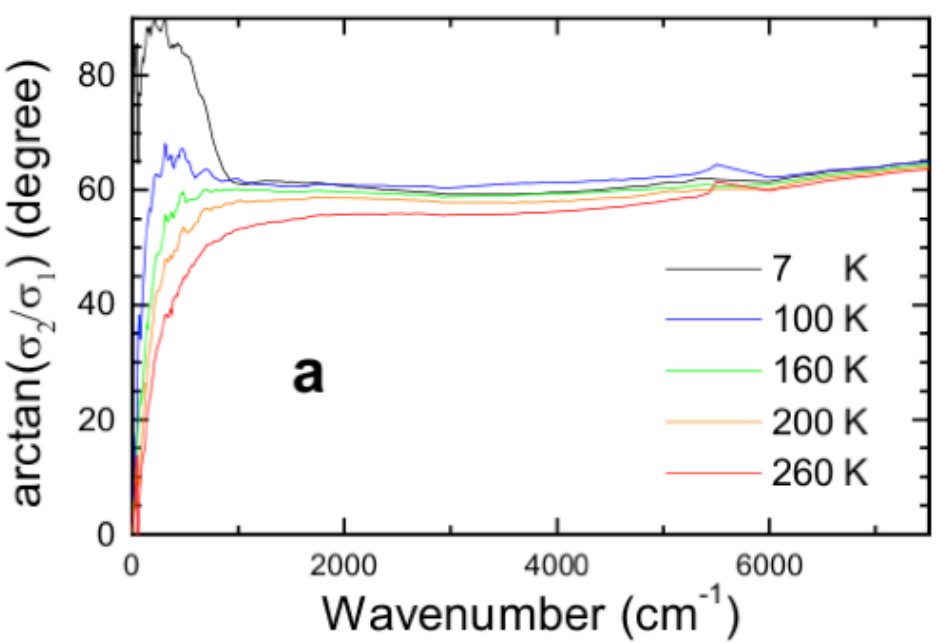
different lattice spacing



different temperature

This looks very similar to the experimental result:





$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Concluding remarks

Summary

- Gauge/gravity duality is an equivalence between string theory and a nongravitational theory
- There is lots of evidence, but no proof
- Provides answers to deep questions about quantum black holes
- Applications to heavy ion collisions and some condensed matter systems

One of the goals of physics is to find connections between seemingly different phenomena (unification).

Gravity, QCD, and condensed matter seem very different.

Recent work hints at a new and deep connection between them.