Black Holes and Thermodynamics

Robert M. Wald

Black Holes and Thermodynamic Systems

Black Holes: By definition, a *black hole* is a region of spacetime with the property that anything—including light—that enters that region cannot ever "escape" from it . In general relativity, sufficiently massive bodies are expected to undergo complete gravitational collapse, producing a black hole. The black hole contains a singularity in its deep interior into which all of the matter disappears. The outer boundary of a black hole is called its *event horizon*. For a large black hole, the spacetime curvature near the event horizon is small. Once formed, an isolated black hole would be expected to quickly "settle down" to a stationary final state.

Stationary black holes are characterized by their mass, M, angular momentum, J, and (if long range matter fields may be present) charges Q_i.

Thermodynamic Systems: For a dynamical system with many degrees of freedom—such as a gas of particles confined by a box—it is possible to give a statistical treatment of dynamical behavior. An isolated system with many degrees of freedom will spend an overwhelming fraction of its dynamical history in a *thermal equilibrium state*, whose macroscopic properties can be characterized by a small number of "state parameters." Insofar as macroscopic properties are concerned, an isolated system with many degrees of freedom would be expected to “settle down” to a thermal equilibrium state and remain in this apparently stationary state “forever.”

Black holes and thermodynamics would appear to have as little to do with each other as any two randomly chosen subjects in physics. Furthermore, the nature of the laws of black hole physics would appear to be very different from the nature of the laws of thermodynamics: The laws of black hole physics in classical general relativity are theorems in differential geometry that hold exactly for black holes of any size. The laws of thermodynamics applied to any system of finite size are statistical laws that hold only with high probability.

Black Holes and Thermodynamics

Despite the apparent lack of any physical relationship, a remarkable analogy between the mathematical form of the laws of black hole physics and the laws of thermodynamics was discovered in the early 1970's:

0th Law

Thermodynamics: The temperature, T, is constant over a body in thermal equilibrium.

Black holes: The surface gravity, κ, is constant over the horizon of a stationary black hole. (κ is the limit as one approaches the horizon of the acceleration needed to remain stationary times the "redshift factor".)

1st Law

Thermodynamics:

$$\delta E = T\delta S - P\delta V$$

Black holes:

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_H \delta J + \Phi_H \delta Q$$

2nd Law

Thermodynamics:

$$\delta S \geq 0$$

Black holes:

$$\delta A \geq 0$$

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T$

$\frac{1}{4}A \leftrightarrow S$

Particle Creation by Black Holes

Black holes are perfect black bodies! As a result of particle creation effects in quantum field theory, a distant observer will see an exactly thermal flux of all species of particles appearing to emanate from the black hole. The temperature of this radiation is

$$kT = \frac{\hbar\kappa}{2\pi}\,.$$

For a Schwarzshild black hole ($J = Q = 0$) we have $\kappa = c^3/4GM$, so

$$T \sim 10^{-7}\frac{M_\odot}{M}\,.$$

The mass loss of a black hole due to this process is

$$\frac{dM}{dt} \sim AT^4 \propto M^2 \frac{1}{M^4} = \frac{1}{M^2}\,.$$

Thus, an isolated black hole should “evaporate” completely in a time

$$\tau \sim 10^{73} (\frac{M}{M_\odot})^3 \text{sec}\,.$$

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S$

The Generalized Second Law

Ordinary 2nd law: $\delta S \geq 0$

Classical black hole area theorem: $\delta A \geq 0$

However, when a black hole is present, it really is physically meaningful to consider only the matter outside the black hole. But then, can decrease S by dropping matter into the black hole. So, can get $\delta S < 0$.

Although classically A never decreases, it *does* decrease during the quantum particle creation process. So, can get $\delta A < 0$.

However, as first suggested by Bekenstein, perhaps have

$$\delta S' \geq 0$$

where

$$S' \equiv S + \frac{1}{4}\frac{c^3}{G\hbar}A$$

where $S =$ entropy of matter outside black holes and $A =$ black hole area.

A careful analysis of gedanken experiments strongly suggests that the generalized 2nd law is valid!

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S \leftarrow$ Apparent validity of the generalized 2nd law strongly suggests that $A/4$ really is the physical entropy of a black hole!

Conclusions

This isn't an analogy! The laws of black hole mechanics *are* the laws of thermodynamics applied to black holes. We must expand our understanding of black holes to allow them to be thought of as thermal equilibrium states, and we must expand our understanding of thermodynamics to allow it to encompass black holes.