# Emergent Relativistic Fermions

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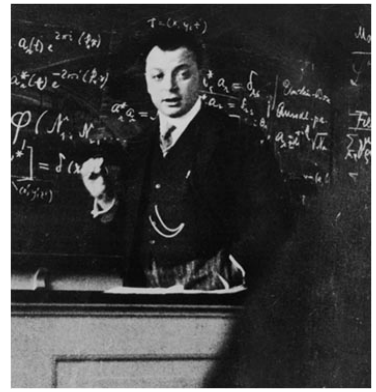
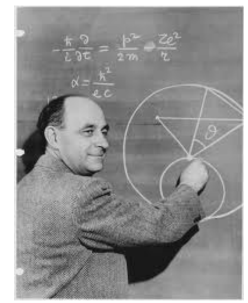
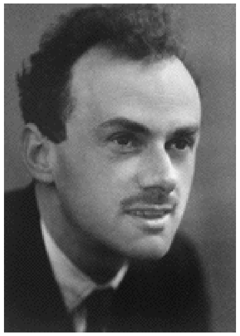
7 Pines, Stillwater, Minnesota, May, 2012



**Drude-Sommerfeld model of metals** = **Free electrons in a box + Schrodinger equation + Pauli exclusion principle** results in a **Fermi Surface** and many of the properties of metals.

## Dirac equation

The Dirac equation 

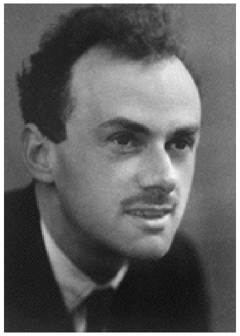
$$\left[i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\cdot\vec{\nabla} + \beta mc^2\right]\psi(t,\vec{x}) = 0$$

describes the quantum physics of a relativistic electron traveling at speeds comparable to light, $c = 299,792,849$ m/s.

**Has negative energy states.** Stability?

**Dirac Hole Theory:** Use Pauli exclusion principle, "ground state" has all negative energy states filled, in exact analogy with Drude-Sommerfeld theory of metals.

**Graphene has an emergent Dirac equation**



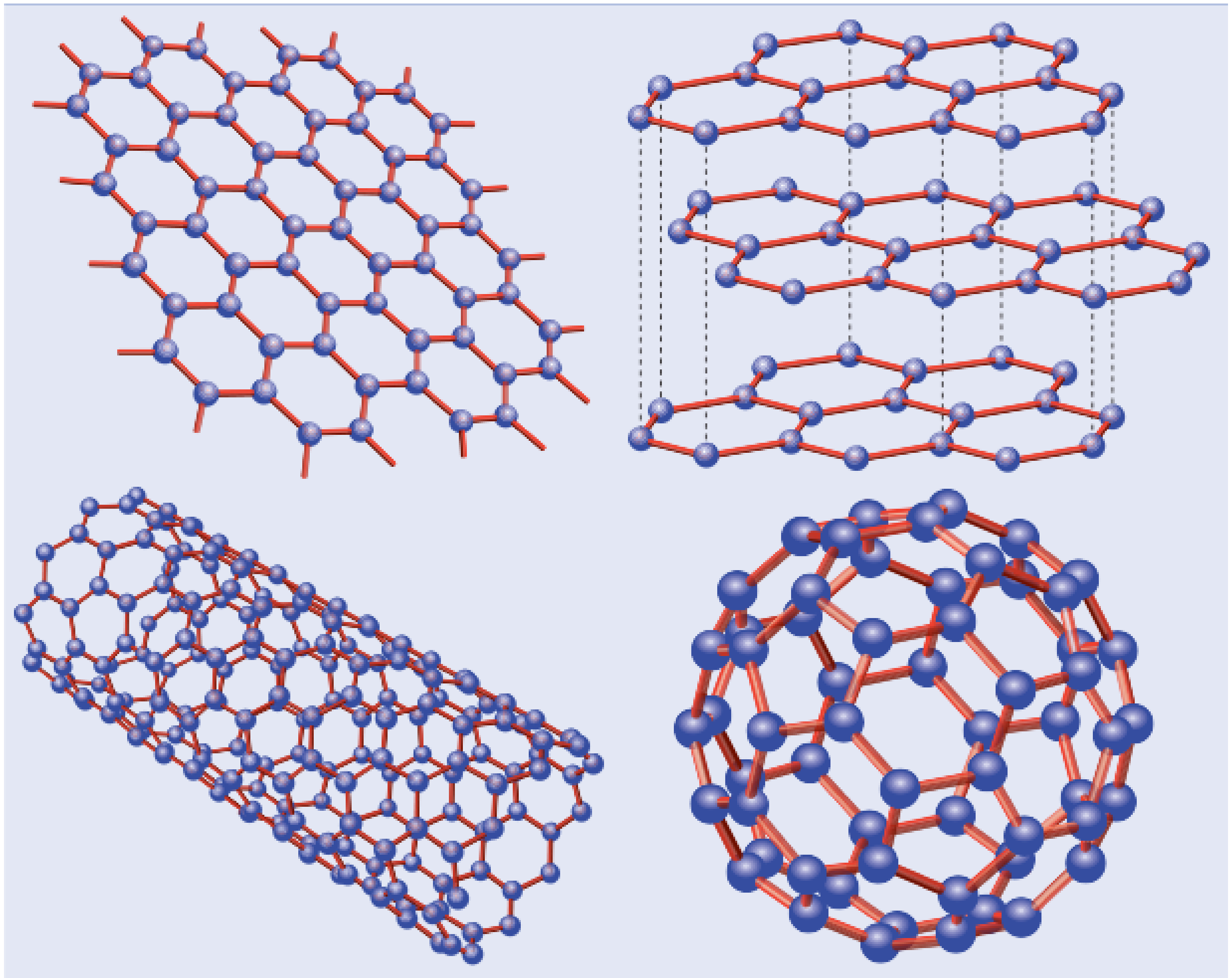
$$i\hbar \frac{\partial}{\partial t}\psi(x,y,z,t) = -\frac{\hbar^2\nabla^2}{2m}\psi(x,y,z,t) + V_{\text{lattice}}(\vec{x})\psi(x,y,z,t)$$

**results in**

$$\left[i\hbar\frac{\partial}{\partial t} + i\hbar v_F\left(\alpha_x\frac{\partial}{\partial x} + \alpha_y\frac{\partial}{\partial y}\right)\right]\psi(t,x,y) = 0$$

describes the quantum physics of the graphene electron which **always** has speed , $v_F \sim 1,000,000m/s \sim \frac{c}{300}$.

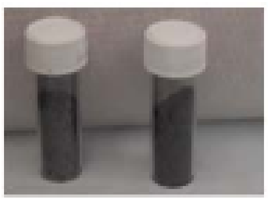
**Graphene is a 2-dimensional array of carbon atoms with a hexagonal lattice structure:**



## Graphene was produced and identified in the laboratory in 2004

- Micromechanical cleavage of bulk graphite up to 100 micrometer in size via adhesive tapes !

Novoselov et al, Science **306**, 666 (2004)



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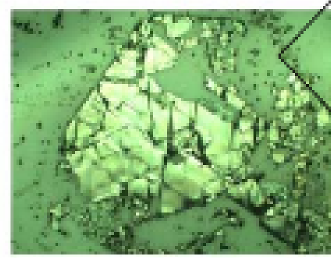
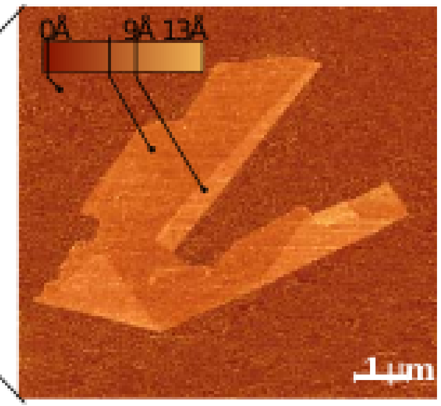
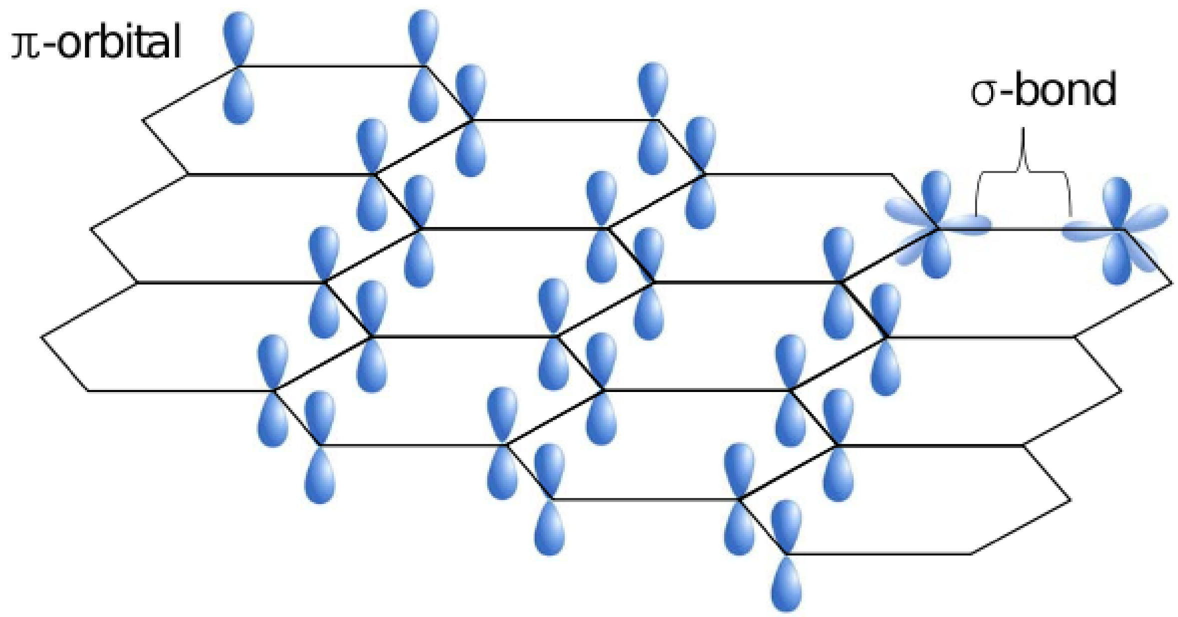





Kostya Novoselov

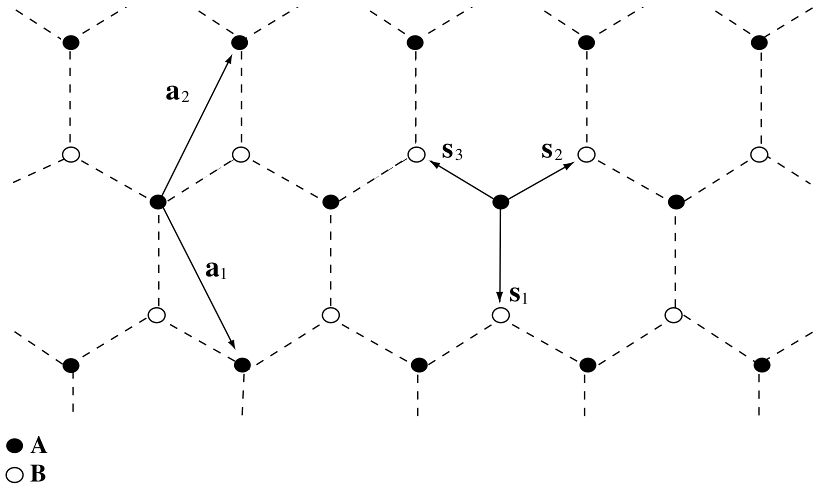
Andre Geim




A carbon atom has four valence electrons. Three of these electrons form strong covalent $\sigma$-bonds with neighboring atoms. The fourth, $\pi$-orbital is un-paired.

L. Pauling 1972 "*The Nature of the Chemical Bond*"

## Tight-binding model




hexagonal lattice = two triangular sub-lattices $\vec{A}$ and $\vec{B}$ connected by vectors $\vec{s}_1, \vec{s}_2, \vec{s}_3$.

$$H = \sum_{\vec{A},i} \left( t\ b^{\dagger}_{\vec{A}+\vec{s}_i} a_{\vec{A}} + t^*\ a^{\dagger}_{\vec{A}} b_{\vec{A}+\vec{s}_i} \right) \quad , \quad t \sim 2.7ev \quad |\vec{s}_i| \sim 1.4\mathring{A}$$

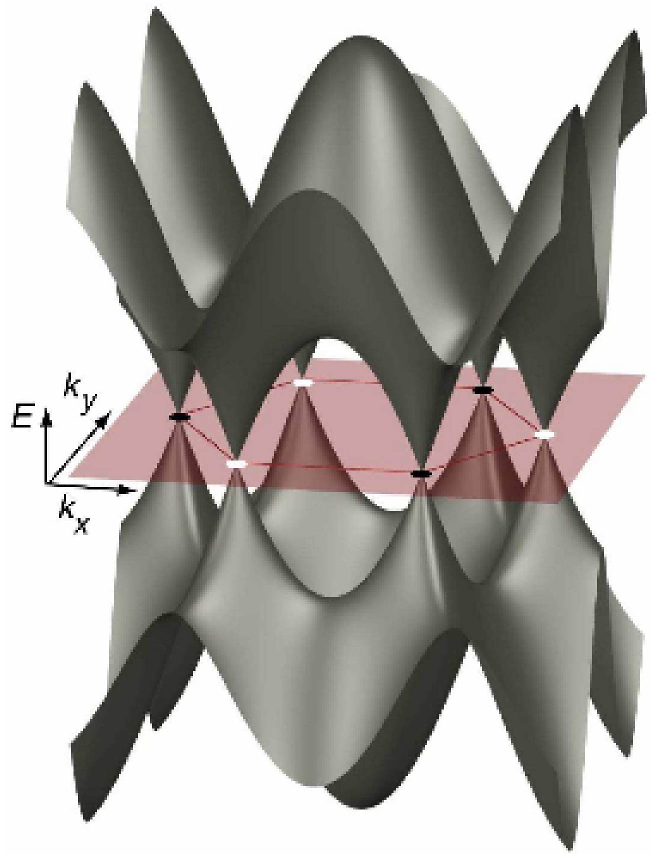
**P. R. Wallace,** *Phys. Rev. 71, 622 (1947)*

**J. C. Slonczewsi and P. R. Weiss,** *Phys. Rev. 109, 272 (1958).*

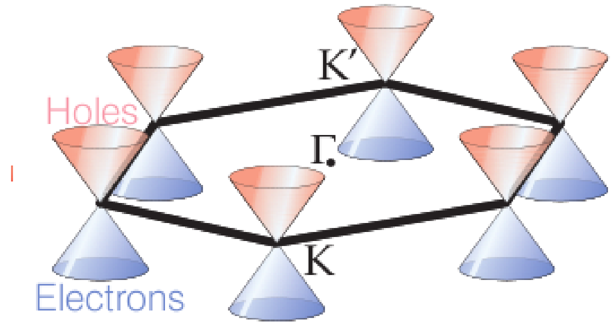
**G. W. S.,** *Phys. Rev. Lett. 53, 2449 (1984)*

## Band structure of graphene

$$E(k) = \pm|t|\sqrt{(1+2\cos(\frac{3k_y}{2})\cos(\frac{\sqrt{3}k_x}{2}))^2 + \sin^2(\frac{3k_y}{2})}$$



## Linearize spectrum near degeneracy points




$$E(k) = \hbar v_F |\vec{k}|$$

$v_F \sim 10^6 m/s \sim c/300$, good up to $\sim 1ev$

2 valleys $\times$ 2 spin states $=$ 4 2-component spinors $\psi$

$$H\psi = \hbar v_F \begin{bmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & k_x + ik_y \\ 0 & 0 & k_x - ik_y & 0 \end{bmatrix} \begin{bmatrix} \psi_A(k-K) \\ \psi_B(k-K) \\ \psi_A(k-K') \\ \psi_B(k-K') \end{bmatrix}$$

$$S = \int d^3x \sum_{\sigma=1}^{4} \bar{\psi}^\sigma \; i\gamma^\mu \partial_\mu \; \psi^\sigma + \text{interactions}$$

## Consequence of the Dirac equation

Minimal coupling to magnetic field: $B = \vec{\partial} \times \vec{A}$

$$\vec{\partial} \;\rightarrow\; \vec{D} = \vec{\partial} + i\vec{A}$$

$$H_{\text{Dirac}} = \begin{bmatrix} 0 & -iD_x - D_y & 0 & 0 \\ -iD_x + D_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -iD_x + D_y \\ 0 & 0 & -iD_x - D_y & 0 \end{bmatrix}$$

## Atiyah-Singer Index Theorem

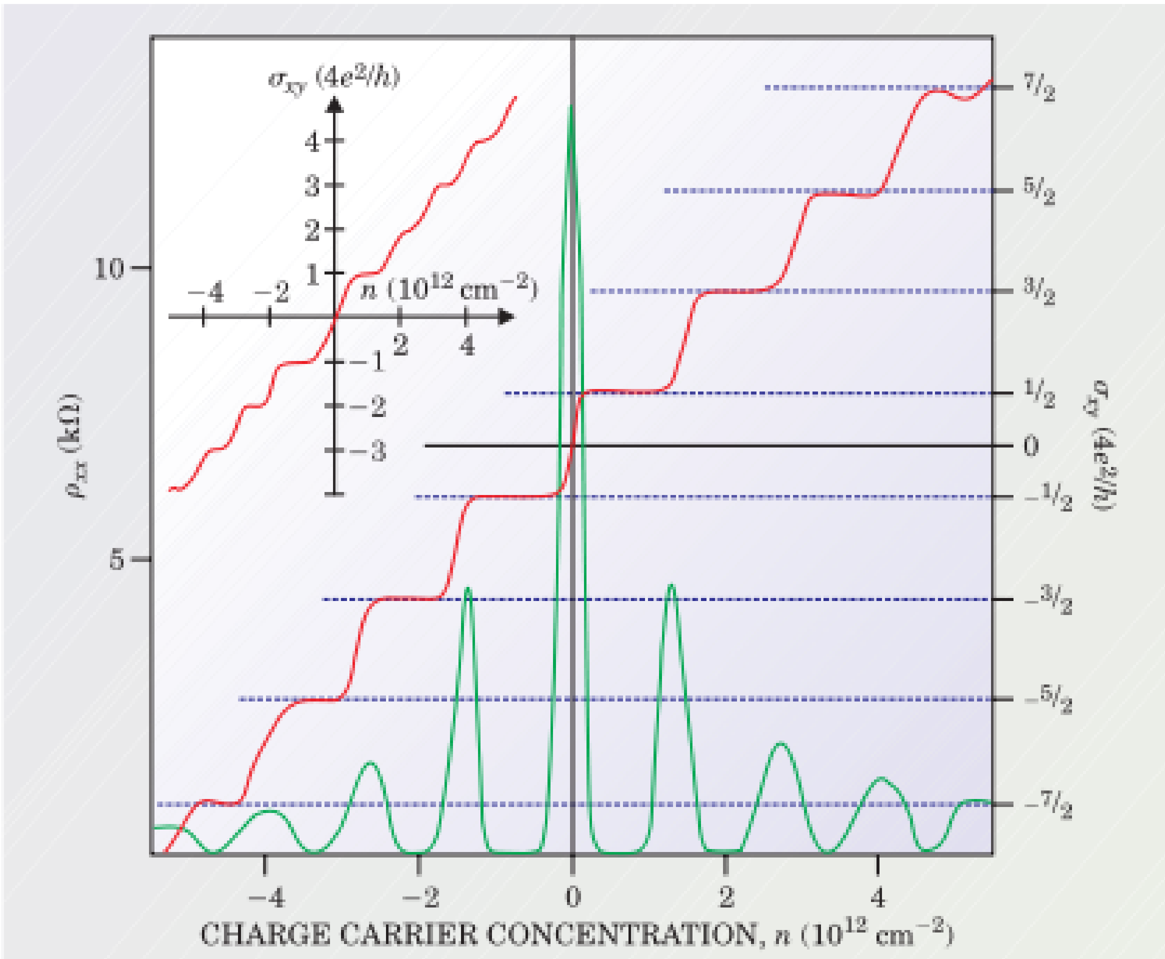
$\#$ zero modes $= 2(2)\left|\frac{1}{2\pi}\int d^2xB(x)\right|$ solutions of $H_{\text{Dirac}}\psi_0(x) = 0$

In neutral ground state, half of zero modes are filled.

**G. W. S.,** *Phys. Rev. Lett. 53, 2449 (1984)*

**K. Novoselov et. al.** *Nature 438, 197 (2005)*

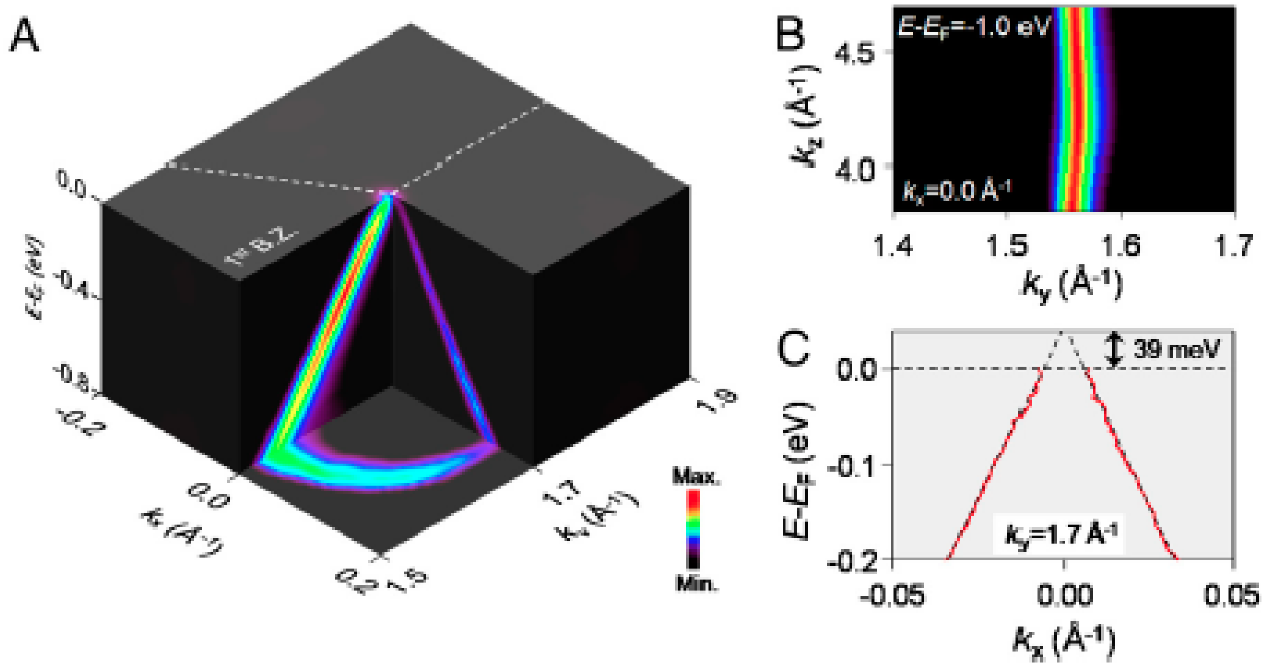
**Y. Zhang et. al.** *Nature 438, 201 (2005)*




$$\sigma_{xy} = 4\frac{e^2}{h}\left(n + \frac{1}{2}\right)$$

# Electron dispersion relation with ARPES

## D.A. Siegel et. al. PNAS,1100242108



## The Dirac equation in condensed matter

- unusual electronic properties:
  redo all of semiconductor physics with
  Schrödinger $\rightarrow$ Dirac
- nanotechnology using graphene
- explore issues in relativistic quantum mechanics which are otherwise inaccessible to experiment
  Zitterbewegung
  Klein paradox
- explore dynamical issues in graphene as an analog of those in quantum field theory, e.g. symmetry breaking, phase transitions, quantum critical behavior

## Graphene with Coulomb interaction

Relativistic electrons in 2 space dimensions

$$\left( i\hbar\gamma^0 \frac{\partial}{\partial t} + i\hbar v_F \alpha_x \frac{\partial}{\partial x} + i\hbar v_F \alpha_y \frac{\partial}{\partial y} \right) \psi_a(t,x,y) = 0 \quad , \quad a = 1,\ldots,4$$

Coulomb interaction

$$V(x-y) = \frac{e^2}{4\pi |\vec{x} - \vec{y}|}$$

Since $v_F \sim \frac{c}{300} << c$ approximately instantaneous.

Coulomb is a **strong** force: Graphene fine structure constant is the ratio of potential to kinetic energy

$$\alpha_{\text{graphene}} = \frac{e^2}{4\pi\lambda} \frac{1}{\hbar v_F / \lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c} \frac{c}{v_F} \approx \frac{300}{137} \quad \left( \cdot \frac{1}{\epsilon} \right)$$

Tight binding $t \sim 2.7ev$, $E_c \sim 10ev$.

## AC Conductivity of Neutral Graphene

$\omega >> k_B T$ RG improved two-loop correction

$$\sigma(\omega) = \frac{e^2}{4\hbar}\left(1 + \frac{11-3\pi}{6}\frac{e^2}{4\pi\hbar}\frac{1}{v_F\left(1+\frac{e^2}{4\pi\hbar v_F}\frac{1}{4}\ln(\Lambda/\omega)\right)}\right)$$

**V. Juricic et.al. Phys. Rev. B 82, 235402 (2010)**

Experiments $\sigma(\omega) \simeq \frac{e^2}{4\hbar}$, $\omega$-independent

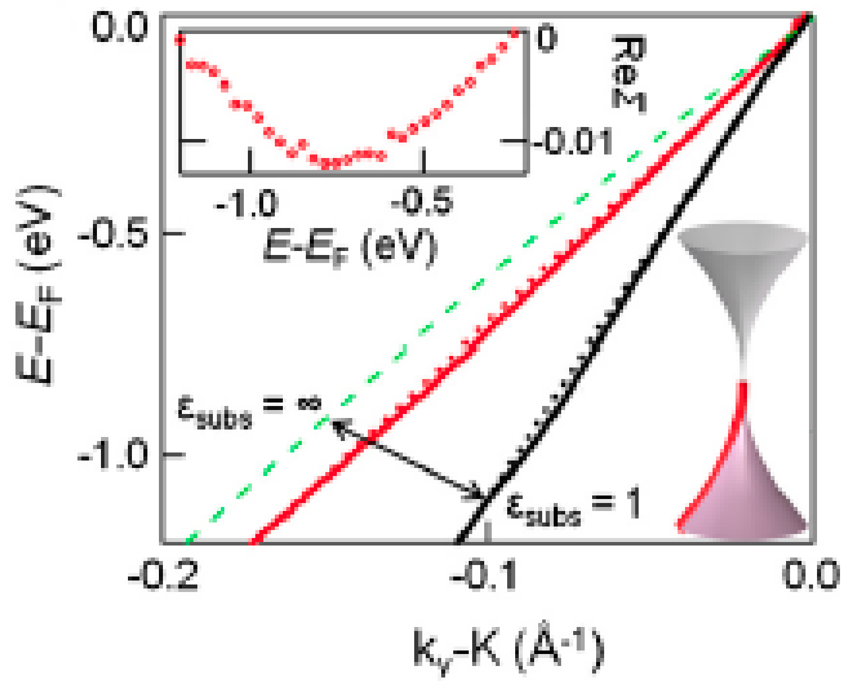
**R. Nair et.al., Science 320, 1308 2008.**

## Hypothesis

- Graphene is in a quantum fluid state which is described by a conformal field theory with fermion content.
- Perhaps strongly coupled.
- How does one analyze it?
- What are the consequences?

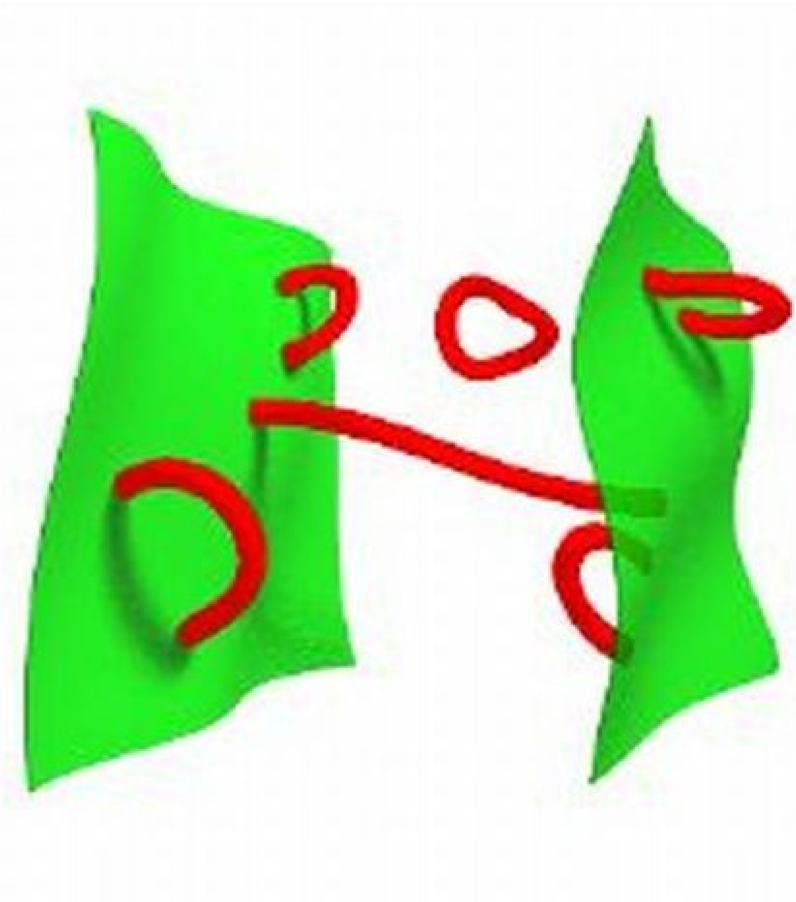
## Electron dispersion relation with ARPES

## D.A. Siegel et. al. PNAS,1100242108



## String theory holographic construction of graphene

solutions of superstring theory that resemble idealized low energy graphene



## String theory holographic construction of graphene

- D3-D7 system brane extends in directions $X$
  brane sits at single point in directions $O$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D3$ | $X$ | $X$ | $X$ | $X$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ |
| $D7$ | $X$ | $X$ | $X$ | $O$ | $X$ | $X$ | $X$ | $X$ | $X$ | $O$ |

- $\#ND = 6$ system – no supersymmetry – no tachyon – only zero modes of 3-7 strings are in R-sector and are 2-component fermions

- need flux on brane worldvolume for stability

$$S = \int d^3x \sum_{\sigma=1}^{N_7} \sum_{\alpha=1}^{N_3} \bar{\psi}_\alpha^\sigma \; i\gamma^\mu \partial_\mu \; \psi_\alpha^\sigma + \text{interactions}$$

## AC conductivity

$$\sigma(\omega) \simeq \frac{3e^2}{\pi^2 \hbar}$$

## Debye screeming length

$$\mu L_D \simeq e \quad , \quad e \simeq 5$$

## Diamagnetism

$$M \simeq -(0.24) e \sqrt{B}$$

Free fermions:

$$\sigma(\omega) = \frac{e^2}{4\hbar}$$
$$\mu L_D \simeq 1.6$$
$$M \simeq -0.28 \sqrt{B}\ \mathrm{sign}(B)$$

WIP: Plasmon frequency, Thermodynamics, Heat transport

## Conclusions

- Review of graphene
- Graphene as a strongly coupled quantum fluid
- D7-D3 system as strongly coupled 2+1-dimensional relativistic fermions
- Conformal field theory at strong coupling
  - AC conductivity
  - Screening length versus chemical potential
  - Magnetization density
  - Other computation in progress