From Experiments to Theory: Emergence and Analogy

presented by
Leo P. Kadanoff
The University of Chicago, USA
Perimeter Institute, Canada and
e-mail: lkadanoff@gmail.com

drawn in part from a paper by

Mogens Jensen* and Leo Kadanoff
*Copenhagen University, Denmark

abstract

This talk is about the role of experiment* in condensed matter physics. Specifically, it focuses upon two points:

- Emergence, that is how unexpected result may arise in science, and
- Analogy, how the results of a few experimental observations may be generalized to give an overall picture of natural behavior.

Here I shall focus upon the qualitatively different kinds of motions observed in classical mechanics, and how we learn about them.

*Here, computer simulations are viewed as parts of experimental science

Nagel Group

University of Chicago

"Experiment - where theory comes to die"

So says the sign on the door of Sidney Nagel's laboratory at the University of Chicago. Sid, a condensed matter experimentalist, and son of the late philosopher of science Earnest Nagel, clearly has a strong view of the role of experiment in physics.

Despite my high respect for Sid's views, and the earlier representation of such views in the work of Popper* and others, I offer here a different view, viz that the experimental laboratory is

Where Theories Come to Life

You recall that Popper said that one crucial experiment can eliminate a physics theory*. So I start with one candidate for a crucial experiment:

* John Gray, Straw Dogs, p.22 Granta Books, London, 2002

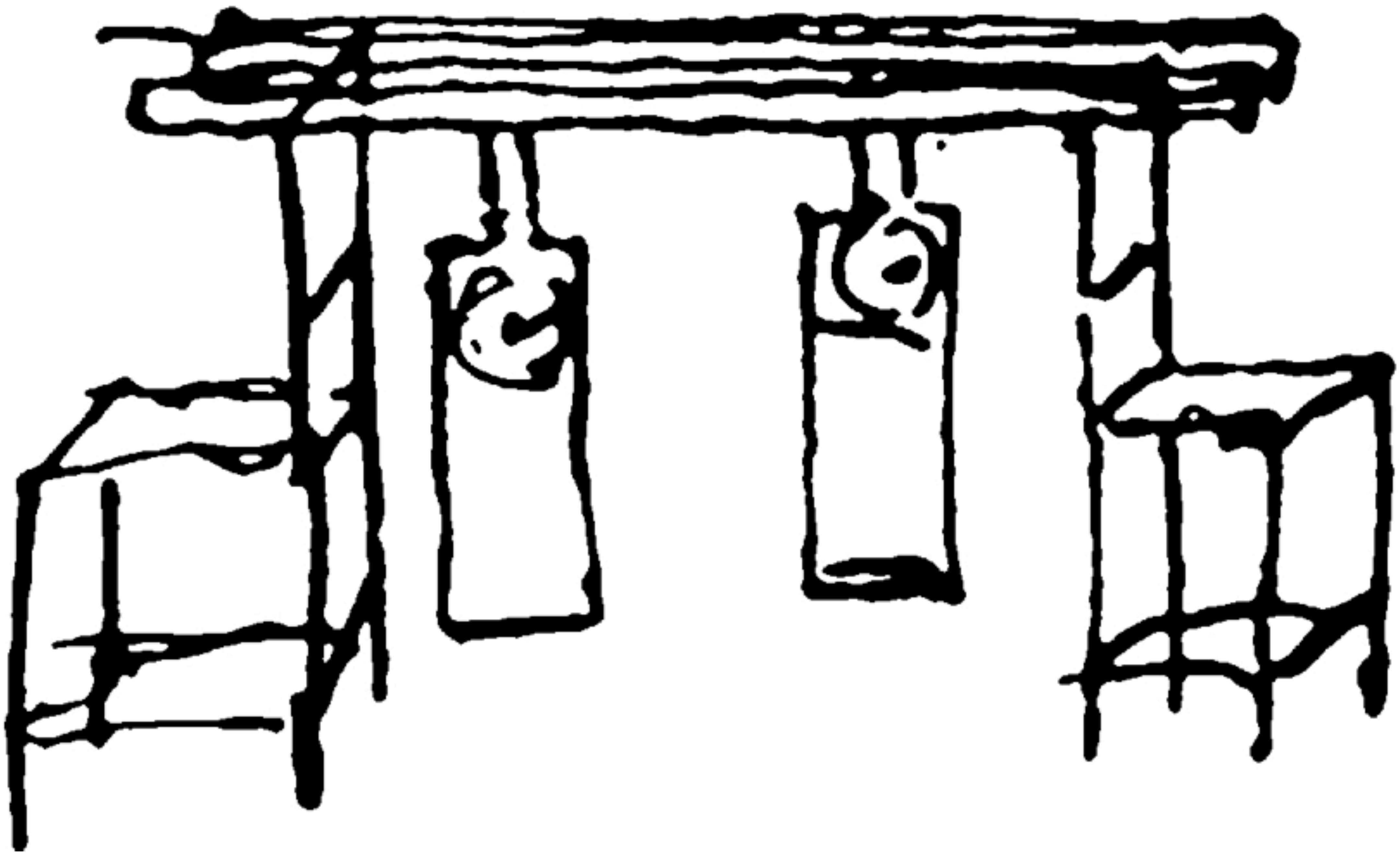
In 1665, the clockmaker Christiaan Huygens noticed that two pendulum clocks hanging on a wall tend to synchronize the motion of their pendulums. A similar scenario occurs with two metronomes placed on a piano: they interact through vibrations in the wood and will eventually coordinate their motion.

This result is somewhat surprising since the coupling between two clocks or two metronomes is likely to be very weak. A result that is surprising and that we cannot explain immediately using the natural laws at our fingertips is said to be "emergent." Huygens then looked further into his accidental discovery by setting up an experiment to demonstrate the synchronization phenomenon.

Christiaan Huygens by Bernard Vaillant, Museum Hofwijck, Voorburg

Huygens' Experiment

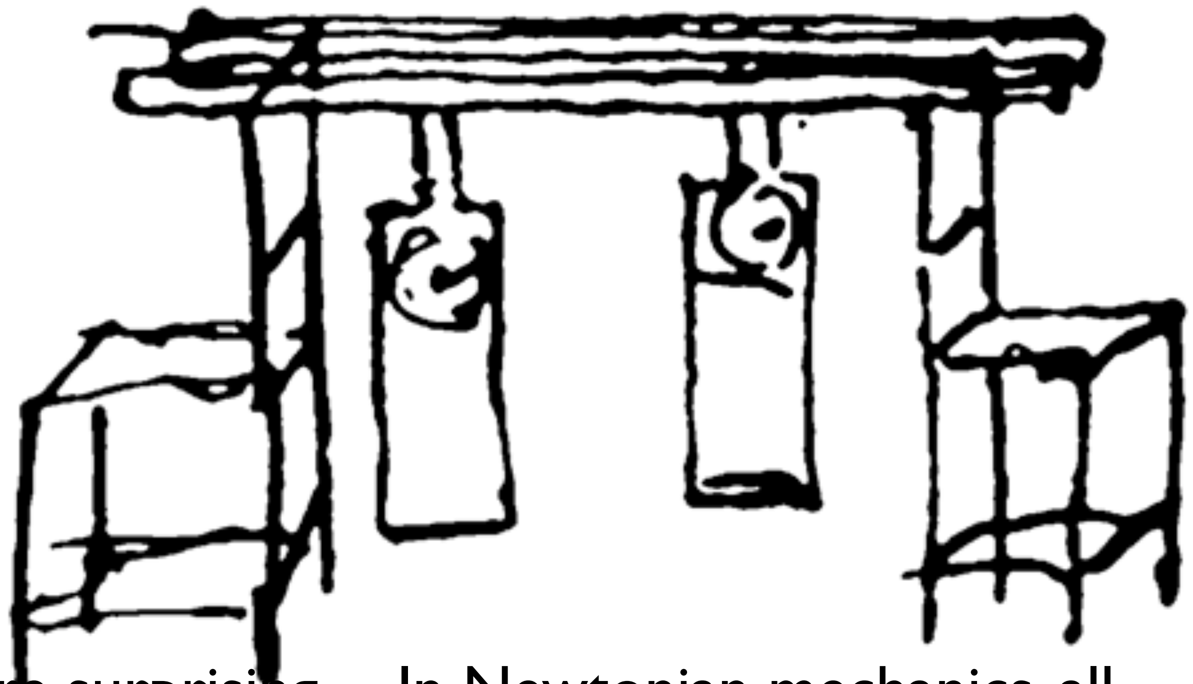
and the observed synchronization was duly reported to the Royal Society in London. The report included the illustration



I wonder whether Huygen's crude picture* was intended to remind us that this phenomenon can be observed using only the simplest of experimental setups.

* Drawn in the time and country of Rembrandt and Vermeer.

In modern language, Huygens has set up a dynamical system, and observed that the synchronized state of this system is an attractor*.



However, further study makes this result more surprising. In Newtonian mechanics, all regions of phase space can be equally occupied. The dynamical flows conserve volumes in phase space. Having an attractor[&] arise from the Newtonian mechanics of a few bodies is quite impossible. Therefore, surprisingly enough, the Huygens experiment demonstrates a qualitative failure of Newtonian mechanics.

* An attractor is a region of phase space into which orbits are drawn. The concept of "attactor" is, I think, a product of the 20th Century.

[&] We might be more surprised than was Huygens, since his experiment was done before Newton's *Principia* was published. Philosophiæ Naturalis Principia Mathematica, published in 1687

Huygen's experiment remains unrefuted to this day. Nonetheless we accept Galilian-Newtonian mechanics as a frictionless limit of a frictional world. In fact, we may accord a higher degree of reality to the frictionless theory than to the more "realistic" frictional one.

What do we want from our experimental observations? and from our theory?

Answer: We want them to suggest the kinds of things we may see in the world.

Thus, Newtonian mechanics gives us closed of planets and suggests that the solar system might be a collection of different orbits each with its own characteristic frequency. This quasiperiodic behavior (i.e. superposition of different periodic behaviors) is a natural extrapolation by analogy from two body Newtonian mechanics to the description of additional bodies. However, this extrapolation by analogy is wrong in two regards:

• As Huygens showed, mechanical motion often includes attractors. These attractos are results of friction and therefore fall outside of few-body Newtonian mechanics.

• In addition to periodic and quasiperiodic motion, to be defined soon, Newtonian mechanics generically gives rise to chaotic motion. In fact, Newton had considered that the solar system might be unstable, and hence probably chaotic, but he believed that God intervened from time to time and put things right.

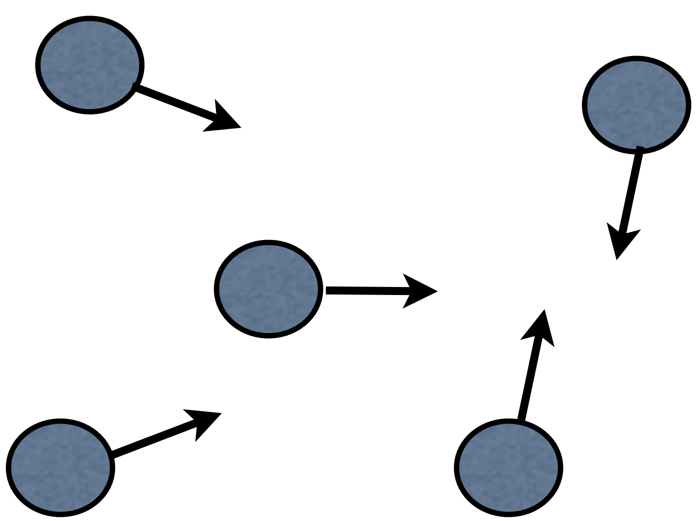
We construct these different kinds of motion in our mind in analogy to and as a generalization and extrapolation of experimental experience and/or of the results of simple models. This extrapolation and process of generalization forms our view of the world.

More frictional effects: inelastic collapse

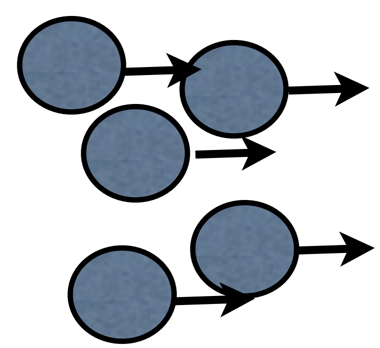
theoretical model: balls which collide conserving momentum but disssipating some energy

before:
approach with speed v

after:
depart with speed γ v,
γ <1

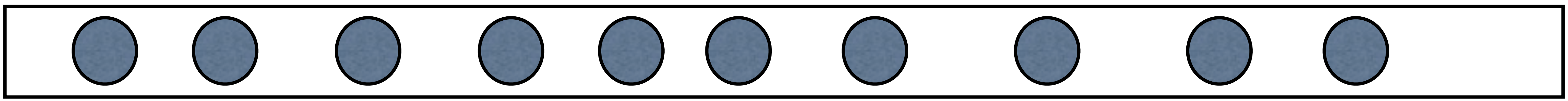


before:
several balls collide together



after many collisions:
they can all move together:
An unexpected attractor

One-dimensional motion:
computer model of inelastic balls



before: balls in random motion in a container with reflecting ends

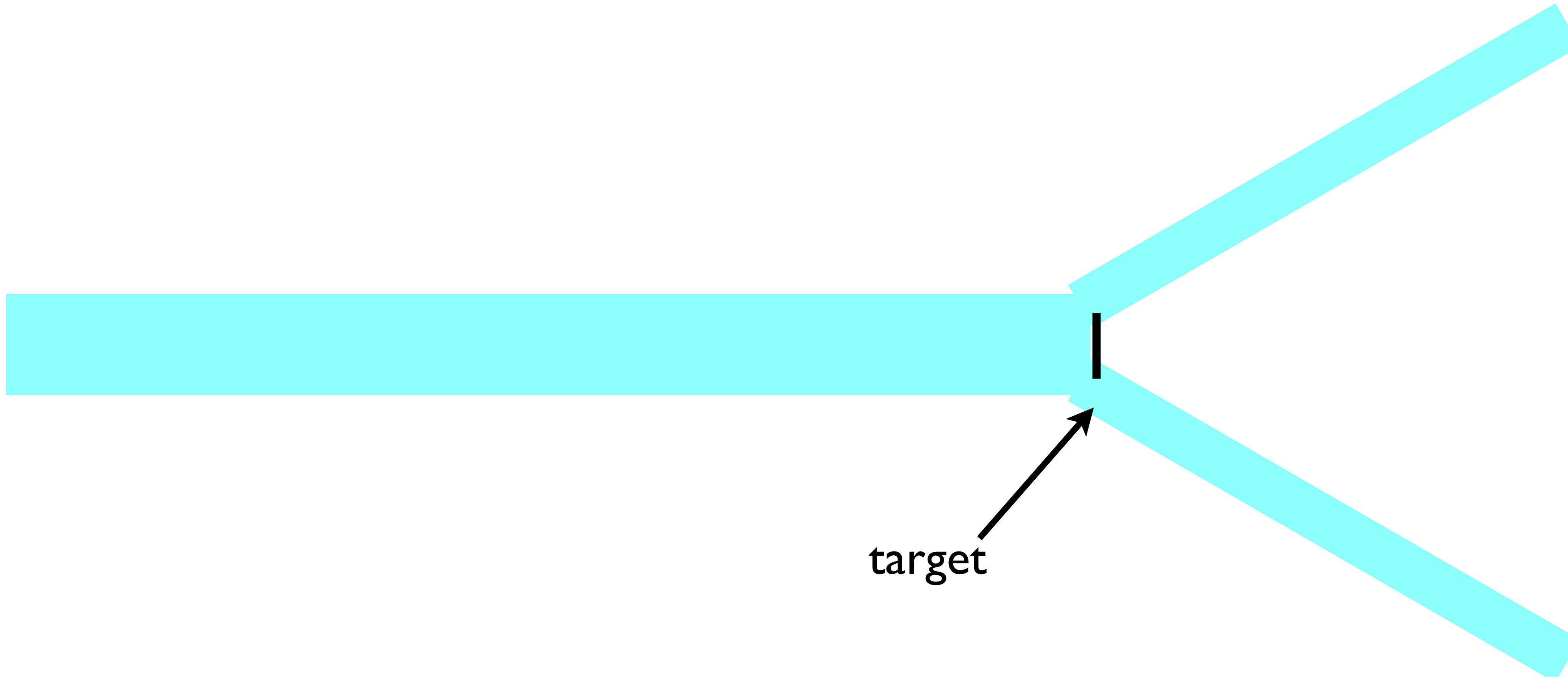
after many collisions: balls at ends almost stop. Ball in center in rapid motion.
when center ball hits end balls, they act as a spring and send center
ball back at high speed.

This is interesting and unexpected. It is the result of sticking of inelastic balls sticking together, i.e. inelastic collapse. It is another emergent phenomenon, impossible in Newtonian mechanics. Good computer experiments can give emergent results.

Breakdown of Hydrodynamics in a One Dimensional System of Inelastic Particles, Y. Du, H. Li, L.P. Kadanoff, Phys. Rev. Letts. 74 (8) 1268-1271 (1995).

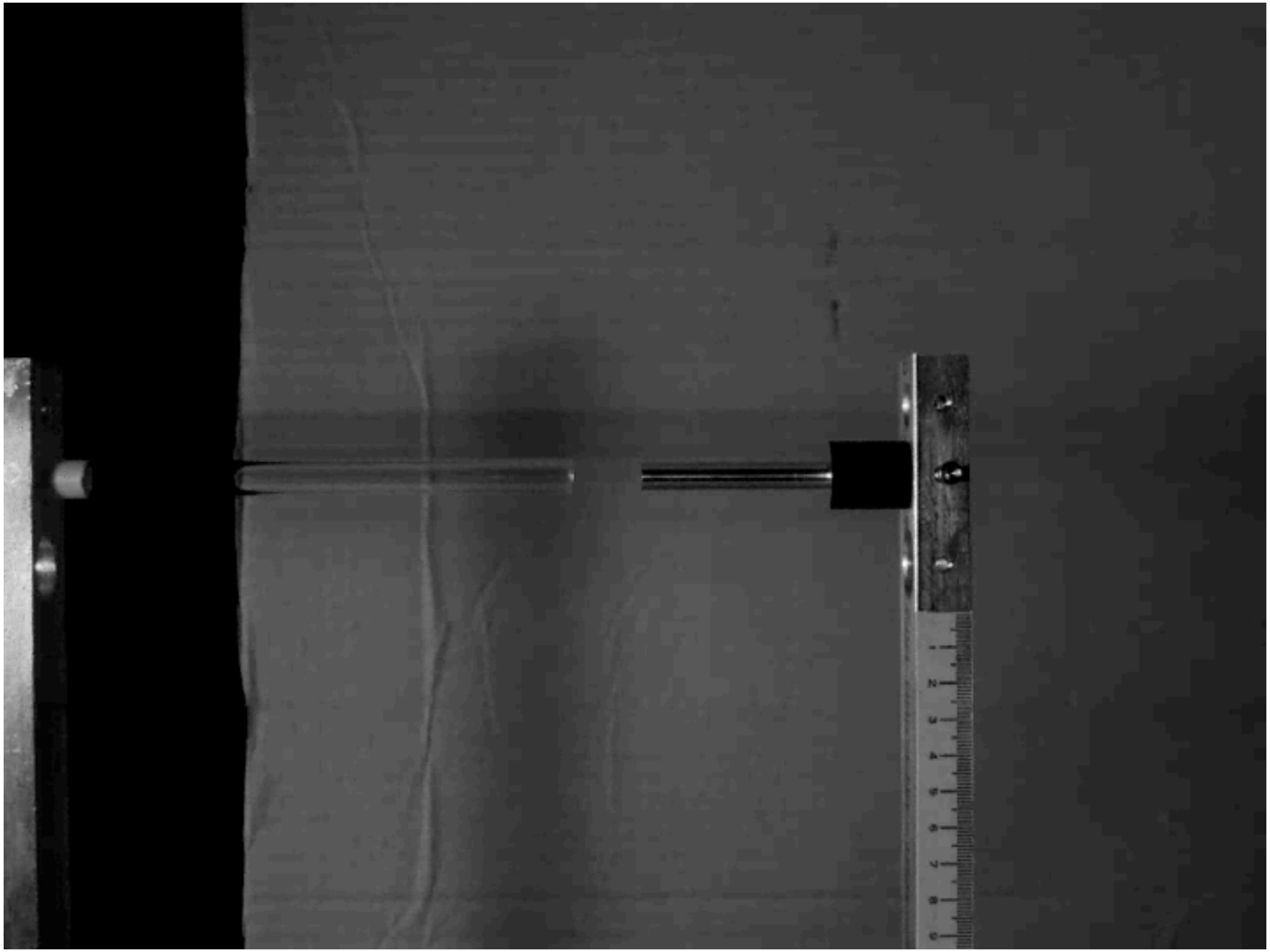
Real world: experiment brings collapse to life

Shoot a jet of water at a target. Water fans out, but surface tension keeps water together. See cross section below.



What will happen if you replace water by glass beads. There is no surface tension, but there are inelastic collisions

Granular jet



labs of Heinrich Jaeger and Sidney Nagel

Xiang Cheng, German Varas, Daniel Citron, Heinrich M. Jaeger and Sidney R. Nagel, “Collective Behavior in a Granular Jet: Emergence of a Liquid with Zero Surface Tension”, Phys. Rev. Lett. 99, 188001 (2007).

The glass beads stick together also, despite the lack of surface tension. Note that we are here using a natural system, a jet of water, as an anlog to understand the flow of a more artificial system made up of glass beads.

Unexpectedly, sand jet behaves like water jet.

Compacted sand behaves like a solid.
Low density sand grains behave like a gas.
Intermediate densities show a liquid-like behavior.

Work with granular materials started with (over- ??) simplified models due to theorists and then continued with exemplifications of these models in simulations.

Experiment explored and discovered the rich behavior of granular materials, soon got ahead of theory and the simulations. Since then, theory has been gradually working to catch up.

Physicists love phenomena to be exemplified by many different realizations

• They tend to ignore experimental observations unless there is a theoretical model or (second-best) a computer simulation model to describe the observations.

• They love best those phenomena that appear in many natural guises. For example, a fluid with a particularly low viscosity coefficient is beloved in part because it appears in field theory, theoretical astrophysics, relativity, (AdS-CFT) heavy ion collisions, and low temperature atomic systems.

• Important physicists, including Einstein and Uhlenbeck, looked for an illusory universal behavior in the phase diagram of fluids. Later, when such a universality was discovered near the critical point of the liquid-gas phase transition, this universal behavior became, for a time, an important subject of study for physicists and chemists. (Interestingly, important metallurgists never believed in this universality.)

Conversely, biologists tend to be wary of theory. Instead of a theory of a disease they look for an animal that exhibits the disease and then speak of this analog result as "an animal-model" for the disease.

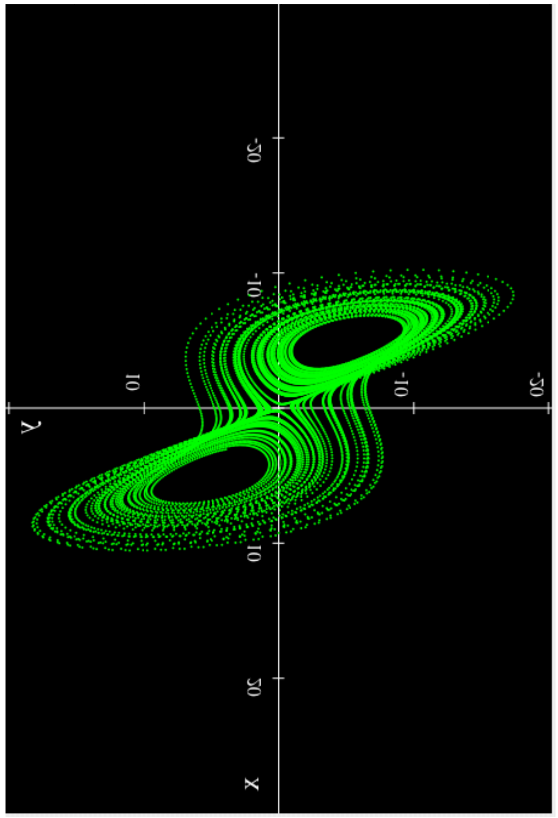
Back to the fundamentals of motion in classical mechanics:

Ed. Lorenz discovered another kind of motion by studying a very primitive model of a weather system in a computer simulation.

Chaotic behavior, now characterized as sensitive dependence upon initial conditions, or “the butterfly effect”, was discovered “accidentally” by Edward Lorenz working with an early and very primitive program for solving linked sets of ordinary differential equations.

The crucial element is exponential separation of initially close orbits.

-A study of weather brought about an important advance in pure science.



$$dx/dt = 3\ (y-x)$$

$$dy/dt = 26.5\ x - y - xz$$

$$dz/dt = xy - z$$

Back to Huygens

The analysis of Galileo and Newton would give us pendulum equations

$$m \ell^2 d^2\theta_1/dt^2 = - mg\ell\ \theta_1 - k(\theta_1 - \theta_2)$$

$$m \ell^2 d^2\theta_2/dt^2 = - mg\ell\ \theta_2 + k(\theta_1 - \theta_2)$$

but Huygens result clearly showed the need for another term, a friction term. With friction, we need forcing terms to keep the system in motion. Thus our analysis requires equations

$$d^2\theta_1/dt^2 = - \gamma\, d\theta_1/dt - \omega_1^2\ \theta_1 - k(\theta_1 - \theta_2) + F_1 \sin(\omega t)$$

$$d^2\theta_2/dt^2 = - \gamma\, d\theta_2/dt - \omega_2^2\ \theta_2 + k(\theta_1 - \theta_2) + F_2 \sin(\omega t)$$

The friction term is not accessible through any analysis that describes the coupled pendulums as a Newtonian system. Indeed, to bring in friction, one requires a quite different point of view, admitting effects not contained in simple Newtonian mechanics.

Different Kinds of Pendulum Motion

Nothing very interesting happens to the pendulums described in this way for short periods of time, but over long periods, we can see three different kinds of motion:

periodic motion: the motion gains some period $2\pi/\Omega$. It simply repeats itself. This is the synchrony seen by Huygens. The frequency Ω can be different from the forcing grequency, ω, and from either of the "natural" periods of the pendulums, ω_1 or ω_2. This periodic motion was detected by Huygens in his synchronization study and much studied after his work.

but, as you know, there are more possibilities

Different Kinds of Motion…..

periodic motion: ….

quasiperiodic motion: This motion can be described as an infinite sum of terms in the expansion of the θ's of the form A sin ($n_1 \Omega_1 t + n_2 \Omega_2 t + \ldots$) with the n's being integers, the Ω's being incommensurate. The amplitudes of these terms decreases rapidly for the larger values of the n's. An understanding of this case is a product of our times produced by Lev Landau, David Ruelle, and Floris Takens.

chaotic motion: The motion is of the form $\int d\Omega \; A(\Omega) \sin(\Omega t)$ with a smooth $A(\Omega)$. The understanding of this form of motion is also a product of our own era. Edward Lorenz and Lev Landau made us aware of this possibility toard the middle of the 20th Century.

A simplified Model

An undamped pendulum model well-served Galileo and Huygens' needs. To analyze the different kinds of motion actually realized with the addition of damping and forcing, it is useful to have a more compact description than that provided by the two ODE's. The circle map, introduced by A.N.Kolmogorov and extensively analyzed by V. I. Arnold, serves this purpose. When expressed as an equation, this model looks simpler than pendulum equations. It is an equation for a phase angle, θ in a system that is kicked at regular intervals to produce a new values of the angle, θj. The new angle depends upon the old as

$$\theta_{j+1} = \theta_j + \omega + k/(2\pi) \sin(2 \pi \theta_j)$$

This equation exemplifies an approach that goes back to the work of Poincaré at the end of the Nineteenth Century in which one replaces a problem with continuous changes in time, like the pendulum, with a strobed description that catches the essence of the continuous problem. This approach is described as replacing a flow by a map.

The point that I wish to make is that the circle map gives all the qualitative information of the two pendulum equations, and hence serves as a good starting point for developing analogies that describe the nature of mechanical motion.

The circle map is

$$\theta_{j+1} = \theta_j + \omega + k/(2\pi) \sin(2 \pi \theta_j)$$

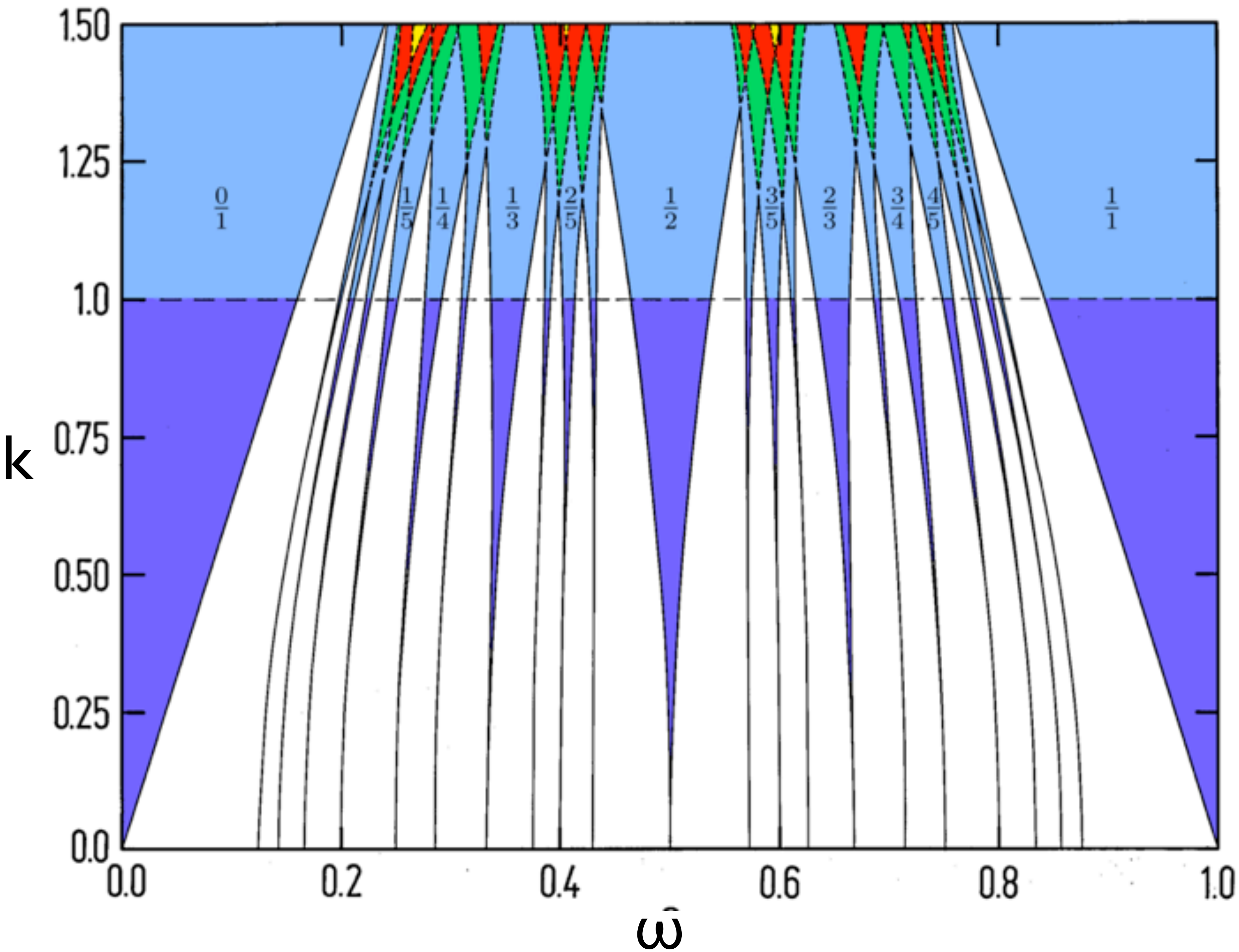
It looks simple but it is not.

This equation leads to a very complex phase diagram that probes the difference between rational and irrational numbers. The complexities of this phase diagram have been observed within experimental observations of fluid flow, solid state devices, and non-linear electrical circuits. It has been analyzed by physicists, mathematicians, and engineers using, among other methods, computer simulations and renormalization analysis.

Philosophers of science will wish to notice that for 20th and 21st Century theory, the simplified model became an interest in itself, to a large extent replacing the analysis of pendulums.

That occurs because the map has some generic features realized by real pendulums and more broadly by a wide variety of analogous systems.

The phase diagram for the circle map



blue= periodic regions
red= chaos
white=mixed quasiperiodic and periodic

Periodic orbits are described by numbers, $\Omega = P/Q$. In Q steps a periodic orbit will advance θ by the integer amount P.

different kinds of orbits in mapping

After many iterations the system approaches a repeating orbit. The different kinds are

periodic: $\theta_1, \theta_2, \theta_3 \ldots\ \theta_Q = \theta_1 + P$ P,Q are integers $\Omega = P/Q$ is rational
The Fourier spectrum has Q lines

quasiperiodic: infinite orbit:
$\theta_j = \Omega + f(\theta_j)$ Ω is an irrational number, $f(x+1)=f(x)$ $0<k<1$ and $f(x)$ is smooth
The Fourier spectrum consists of an infinite number of lines, appearing at $n\Omega + m$ with n and m being integers

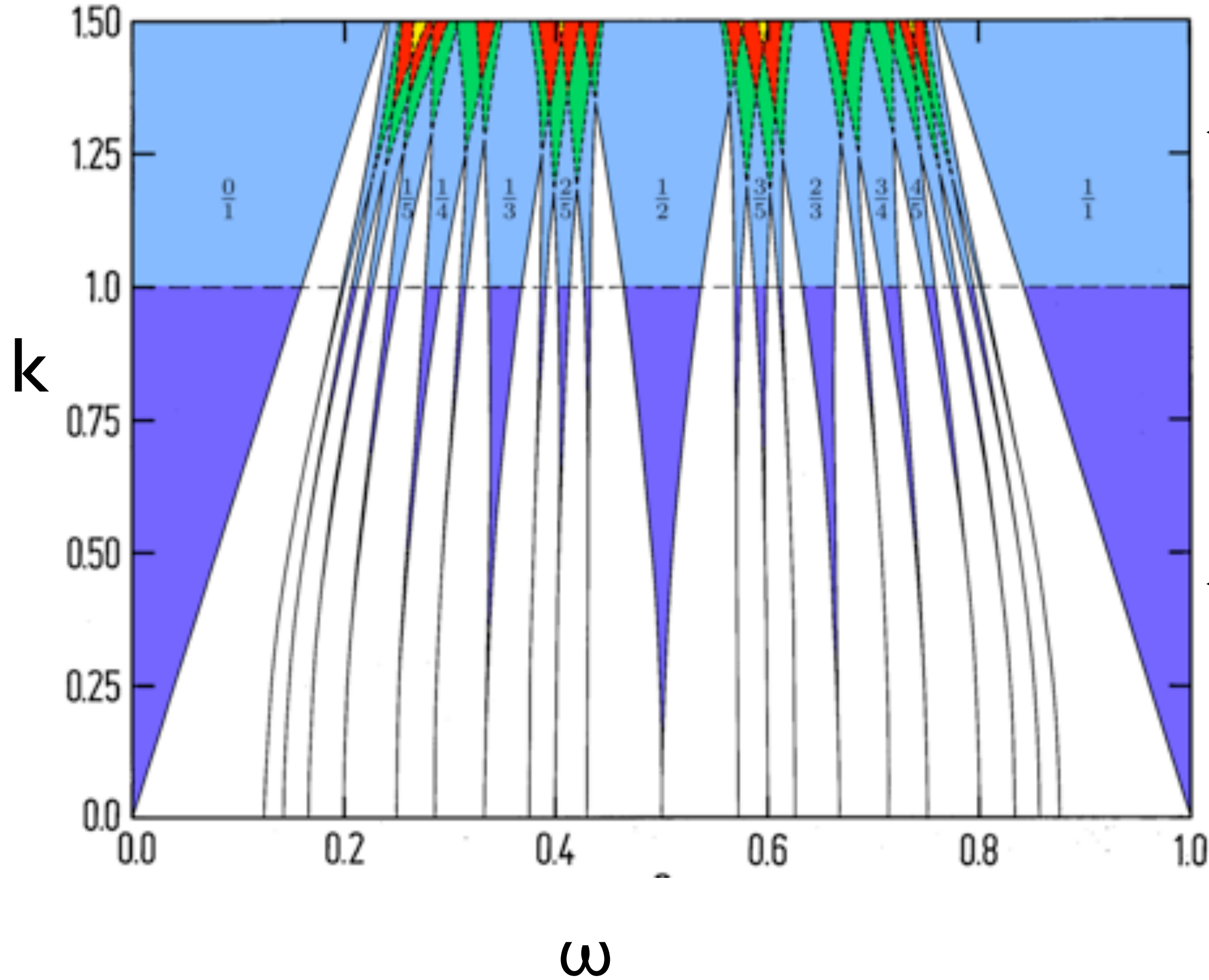
chaotic: infinite orbit, exponential separation of trajectories in long run:
The Fourier spectrum is smooth

Generic Features

These three kinds of motion can be observed in a very wide variety of situations, ones described by
Ordinary Differential Equations, ODE, e.g. electrical circuits or granular materials
Partial Differential Equations, PDE, e.g., fluids or electromagnetism
Maps, e.g Population Biology, one generation following upon another.

All these systems show a local ordering much like that in local regions of the circle map.

A phase diagram



strong coupling. Disorder lives here. Chaos, quasiperiodicity, periodicity all mixed together. Many chaotic orbits. Different initial θ's may produce different behavior.

Orderly. Orbits characterized by Ω value giving average change in θ in one step. Rational Ω gives periodic orbit. Irrational Ω is quasiperiodic. Ω increases monotonically with ω. Periodic and quasiperiodic orbit both have non-zero measure.

Number theory says that there are infinitely more irrational numbers than rational ones. Note that behavior of orbits is determined by number theory considerations.

Experiments

There are many experimental and computational studies informed by the circle map. These studies use fluids, electrical circuits, biological systems, … to explore circle map behavior. The point is that the different behaviors of the mapping all depend upon what happens after very large numbers of iterations. In contrast to properties based on few iterations, these "approaching infinity" properties are very robust. The technical word for this kind of robustness is "universality." The word "universality" implies that the behavior will persist under small changes in the parameters defining the map or flow or other mathematical basis of the problem.

Physicists like to study problems with universal features. Many problems in the theory of dynamical systems have been studied precisely because of their universality.

Predictability

Synchrony and quasiperiodicity can produce rather predictable forms of motion. In the synchronized cases, a small perturbation will generically give a small effect. In the quasiperiodic case, a small perturbation can, but will not always, give rise to a change that grows no more than algebraically in time. In contrast, the chaotic situation is one of exponentially rapid separation of orbits. Thus detailed predictions are possible in the periodic case:

The sun will rise tomorrow.

In the chaotic case, we cannot know what will happen.

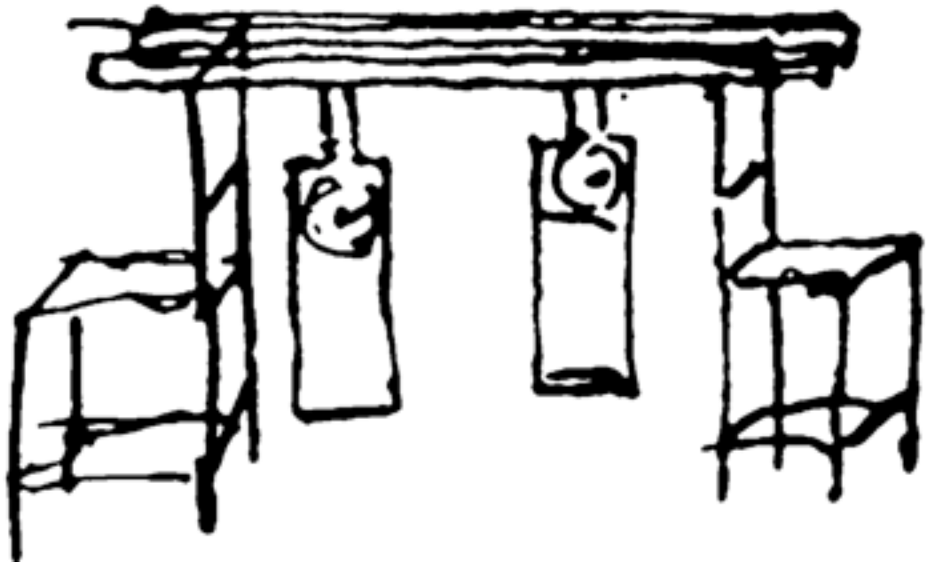
Predictability decays exponentially in time/

Will it be sunny in Chicago exactly one year from now?

In the quasiperiodic case, we will see an algebraic decay of predictability with time, rather than an exponential decay.

These possibilities have proven to be rather important in our qualitative view of the physical world. Any belief that important events are either entirely predictable (Marx's view of history) or quite unpredictable (the traditional conservative view of Donald Rumsfeld) has been eroded away, leaving us with both options as possibilities, depending upon the situation.

In contrast to the view of the philosopher Nancy Cartwright who says that only tiny portions of the real world are described by physical models…..



….I argue that the effects of physical models are observable everywhere, if one but has the eyes to look. No fancy apparatus is required.

Just recognize the synchronization of ones own sleep patterns with the rotation of the earth, and the incommensuration of the length of year, month, and day.

Our models and analogical thinking exist precisely to enable us to see better.