Analogy, Symmetry and Duality

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0: Introduction

1: Analogy

2: Symmetry

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0: Introduction:

As so often, the notions are more useful if left a bit vague. But one might distinguish them as:

Analogy: *Similarity between distinct things, even theories*;

Symmetry: *Sameness between distinct situations (e.g. states), according to a theory*;

Duality: *Sameness or similarity between distinct theories.*

The venerable schema,

"A is to B as A' is to B'"

applies to all three.

Usually, there is a family of such analogies: one for each row of the positive analogy in Hesse's schema.

1: **Analogy**:

1.1: Models?:

A spectrum of meanings for:
'a model interprets uninterpreted formalism'.

1) *Hardly a model*:
Assign a physical quantity, in particular dimension, to the symbols.

The more you know or believe about such quantities, eg about how they figure in other problems, or even other theories, the richer (and harder to state!) is the interpretation.

2) *'Same' equation(s)*:
Different problems, or even theories, with the same equations, albeit with different quantities for the symbols, e.g. area and entropy. At least: apparently different quantities.

3) *Inference*:

The model/analogy prompts an inference to a property of the 'target' problem/system/theory, from the corresponding property of the 'source'.
Sub-themes:—
the model is mechanical;
the model is visualizable;
the model makes calculation tractable;
the model is fertile ...

4) *Clustering*:

The more two objects are similar in examined respects, the more likely that they are similar in unexamined respects.

In Hesse's jargon: The more extensive the known positive analogy, the more confidently we should infer that the negative analogy is small.

1.2: Australia vs. the rest of the world?: Hume's problem of induction is 'the scandal of philosophy':—
We cannot give a general account of 'good reason to believe' that the unexamined will be 'like' the examined.

But there are rules of thumb about what we count as good evidence, and for what. Here, all agree that the 'warrant' that 'licenses'/'powers' the inference is fallible.

There are various codifications: Mill's methods, Bayesian confirmation theory ...
which partly agree, are all incomplete—and whose advocates surely admit they will never be exceptionless.

Some wise words:

(1): ‘Our account . . . will be adequate if it achieves such clarity as the subject-matter allows; for the same degree of precision is not to be expected in all discussions, any more than in all products of handicraft.’

(Aristotle: Ethics, Book I, Chap 3).

(2): ‘Better to be vaguely right than exactly wrong’ (Keynes).

1.3: From analogy to identity?:
Recall: '*apparently* different quantities'.
But there are surely no general rules about 'when' we leap from asserting analogy to asserting identity.
Various examples (also of unifications and identifications) ...

1) Newton's unification of terrestrial and heavenly mechanics: he leaps 'only when' he can prove the gravitational field of a spherically symmetric mass M is the same as for a point-mass M at the centre.

2) Maxwell's unification of optics and electromagnetism: he leaps 'already when' he deduces that the speed of conjectured electromagnetic waves is the observed speed of light (against the background acceptance of a wave theory of light...).

3) Black hole thermodynamics: we leap 'when' Hawking shows evaporation, with a temperature.

And there are surely no general rules about 'when' we take confirming a theory of one system/problem to also confirm an analogous theory of another (inaccessible) system.

Thus contrast:

(i) Bill Unruh's schema: A black hole is to Hawking radiation as a dumb (better: loud-mouth) hole is to 'Unruh' waves.

(ii) Isaac Unruh, in cloudy Cambridge, postulates planets, unobservable above the clouds, gravitationally attracting; and deduces elliptical orbits.

In a table-top electrostatic system (Rutherford's hydrogen atom), he confirms elliptical orbits. Does this confirm his inverse-square law of gravitation?

Surely not as well as Unruh waves confirm Hawking radiation ...

2: Symmetry:

2.1: Symmetry as a map on states:

A physical theory has a set of states $\mathbb{S}$ and a set of quantities $\mathbb{Q}$. I write $\langle Q; s\rangle$ for the value of Q in s.

Classically, the values are "possessed", "intrinsic". But in quantum physics, the values are expectations, conceived instrumentalistically.

I take $\mathbb{S}$ as primary, so that $\mathbb{Q}$ is given by mathematical structures on $\mathbb{S}$.

A *symmetry* is a map $a : \mathbb{S} \to \mathbb{S}$ the value of the quantities, i.e.

$$\langle Q; a(s)\rangle = \langle Q; s\rangle \ . \tag{0.1}$$

The Analogy schema, "A is to B as A' is to B'" applies.

We take A and A' as the states s and $a(s)$ respectively, and $B = B'$ as the quantity Q, or as the set of quantities.

Various comments:
(i) The special case where $a(s) = s$: *symmmetric states*.

(ii) Similarly for a symmetry as a map on quantities.

(iii) 'Preserving (the values of) the quantities' means: 'Preserving a salient ("large") set of quantities'.

Besides:

(a) The larger this set, the smaller, in general, the set of symmetries.

(b) Austerity is alluring. Maybe we should accept only quantities that are preserved ...
we should quotient the state-space under the symmetry group ...
we should be a relationist.

2.2: Symmetries of solutions and theories:

(1): *Symmetries of solutions*:
A symmetry a lifts to a map on solutions (i.e. courses of history). Indeed: under two-way determinism, a state fixes a solution.

The Analogy schema applies again: with A, A' as solutions, and $B = B'$ as a quantity or quantities.

(2): *Symmetries of theories*:
A theory is symmetric, has a as a symmetry, if its solution space is closed under a.

One might say: 'theories are analogous if they have the same symmetries'.

Fair enough. One could even say this with the Analogy schema: with A, A' as two theories, and $B = B'$ as a symmetry.

But such shared symmetries do not prompt a leap from analogy to identity. That would require much more similarity of the theories.

3: Duality

3.1: A brutal summary:

A relation between theories, given by a 'dictionary' pairing some of the notions of one theory with some of the other, preserving theorems. I specialize to 'gravity/gauge'.

Some gauge QFTs in d spacetime dimensions are dual to some quantum theories of gravity in a $d+1$ dimensional spacetime that has the d-dimensional manifold of the QFT as its boundary (or part of it).

One hint of this: In a QFT, the beta-function is local in u (due to local couplings in the Lagrangian). This hints that u (better: its reciprocal) be taken as an extra spatial dimension, whose addition defines the bulk.

Example: AdS/CFT:
AdS_{d+1} 'is' a family of copies of d-dimensional Minkowski spacetime of varying size. The family is parameterized by $z \equiv L^2/u$. The boundary is the locus $z \to 0$, corresponding to the UV sector of the QFT.

The idea is that the AdS metric

$$ds^2 = \left(\frac{L}{z}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \; . \qquad (0.2)$$

solves the equations of motion of an action of the form

$$S_{\text{bulk}}[g, ...] = \frac{1}{16\pi G_N} \int d^{d+1}x \, \sqrt{g} \, (-2\Lambda + R + ...) \qquad (0.3)$$

where the dots represent various higher powers of curvature and-or other bulk fields.

Thus different QFTs on the boundary d-dimensional Minkowski spacetime correspond to different theories of gravity in the bulk, varying in their field content and bulk actions— and the main role of string theory is to provide consistent ways of filling in the dots.

The original case (Maldacena (1998):
The QFT is a strong coupling regime of maximally supersymmetric gauge theory in four spacetime dimensions ($d = 4$), with gauge group $SU(N)$ with large $N \equiv N_c$.

Here: 'large N' and 'strong coupling' means that we take the limit $N \to \infty$ and $g \to 0$ with $g^2_{YM} N$ large and fixed (the ''t Hooft limit').

For this theory the beta-function vanishes, and the theory is conformally invariant.

Some dictionary entries:

coordinate u in AdS $\leftrightarrow$ RG flow in QFT

fields in AdS $\leftrightarrow$ local operators in CFT

mass of a field in AdS $\leftrightarrow$ a scaling dimension of correlation functions (vacuum expectation values of products of local operators)

radius of curvature in AdS $\leftrightarrow$ number N of colours

Hawking temperature of black hole in AdS $\leftrightarrow$ temperature of the corresponding state of the boundary QFT

3.2: Emergence? Theories and calculations:
Many say that the gravity theory in the bulk emerges from the QFT on the boundary.

Agreed: with 'emergence' meaning roughly
(i) one theory T_{top} is obtained by taking a judicious limit or limits, and-or a judicious sector (i.e. range of parameter space), within another T_{bottom}; and
(ii) T_{top} exhibits features that from the perspective of T_{bottom} are novel or surprising.

This suggests, even implies, the theories are about the same 'hunk of reality'. But it does not exclude the converse relation *also* holding, i.e. the theories being equally 'fundamental'.

Various issues arise: I pursue one.

An analogy, symmetry or duality can have two uses:

(i) to guess a new theory,

(ii) to simplify an otherwise intractable problem or calculation.

Both aims are in evidence in gravity/gauge.

Aim (i) is pursued ?mostly 'gauge to gravity'.

The QFT helps one guess the quantum gravity theory.

This raises the mind-bending idea that one dimension of spacetime is emergent, while the others are fundamental/geometric ...

Aim (ii) is pursued mostly 'gravity to gauge'.

One uses the gravity theory to simplify calculations in the QFT.

We probe the strong-coupling regime of the boundary QFT using weak coupling or even classical regime of the gravity theory.

5: References

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