Physics and Complexity
A Tale of Two Disorders

Rémi Monasson
Institute for Advanced Study, Princeton
CNRS,Ecole Normale Superieure, Paris

Seven Pines Symposium, May 2011

What is complexity?

- Well defined notion in computer science

  Undecidability (Turing)
  Computational complexity (Cook)
  Algorithmic complexity (Kolmogorov, Chaitin)

- In biology? ... next talk ...

- In physics?

Chaos, Out-of-equilibrium dynamics (hard to predict future evolution)
Emergent collective phenomena (involving various time and space scales)
Heterogeneities, individuality of interactions
(≈ glass or material with impurities,
≠ crystal or liquid, quantum statistical physics)

- This talk: some of the concepts and tools developed over the past 50 years to understand 'complex' physical systems (many interacting and heterogeneous elements)

- Hereafter, (almost) no quantum physics!

extremely active and important field: condensed matter → quantum computing

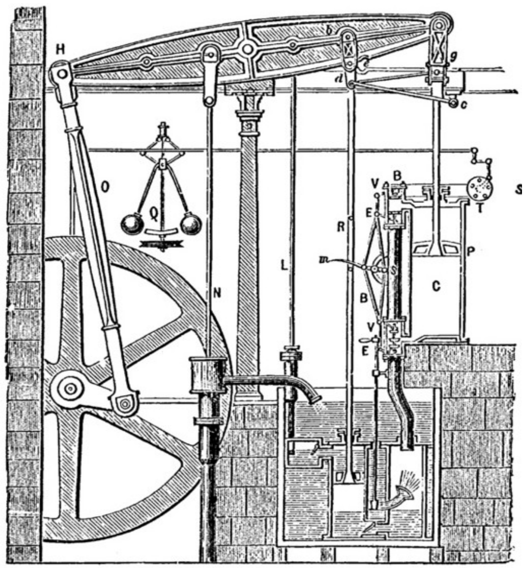
but does not matter for biology in first approximation:

« The essence of biology is fundamentally properties of molecular physics in **non-equilibrium circumstances** and on a **large scale**. By and large, quantum mechanics is not relevant.
Of course, the quantum mechanics of chemical bonds is essential, but while the making and breaking of these bonds by the enzyme catalysts in cells is of paramount importance, it is the rates of these processes as expressed in a **network** of chemical reactions and the quantum-mechanical details that matter.»

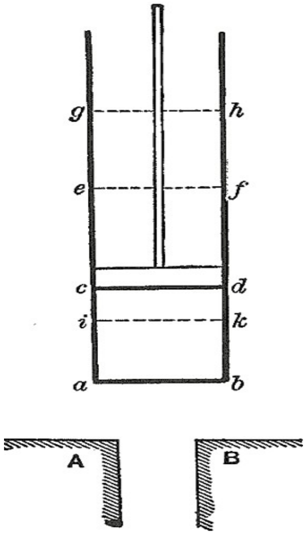
J.J. Hopfield, Physics, Computation and Why Biology Looks so Different, 1994

From thermodynamics to statistical mechanics

A « complex system » in the 19th century: heat engines



Boulton, Watt (1784)



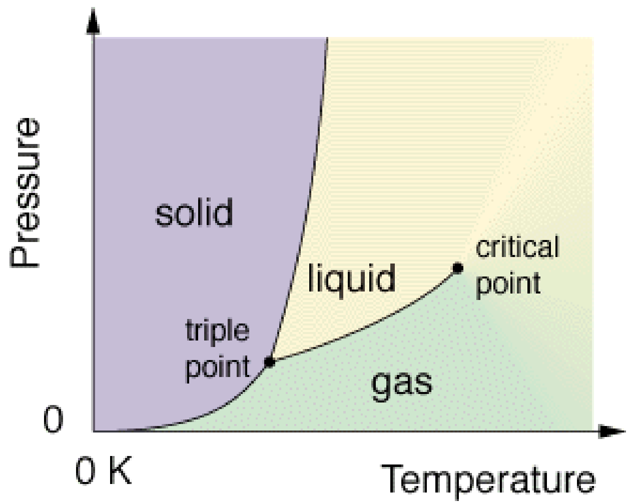
S. Carnot (1824)

Second law of thermodynamics, Notion of entropy (Clausius)

Microscopic foundations? (Boltzmann, Gibbs, ...)

Predict the macroscopic behaviour of a system from the knowledge of its microscopic components and of their interactions.

Phase transitions and symmetry breaking

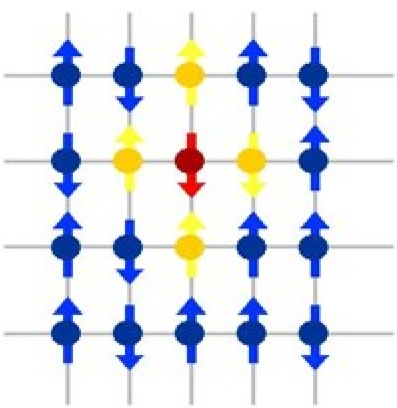


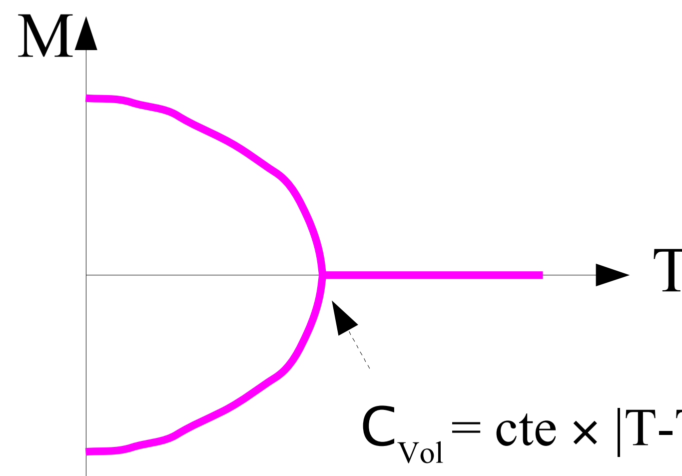
Critical point Liquid - Vapor

$$C_{Vol} = cte \times |T-T_c|^{-\alpha}$$

$$dV/dP = cte \times |T-T_c|^{-\gamma}$$

Paramagnetic-Ferromagnetic phase transition





$$C_{Vol} = cte \times |T-T_c|^{-\alpha}$$

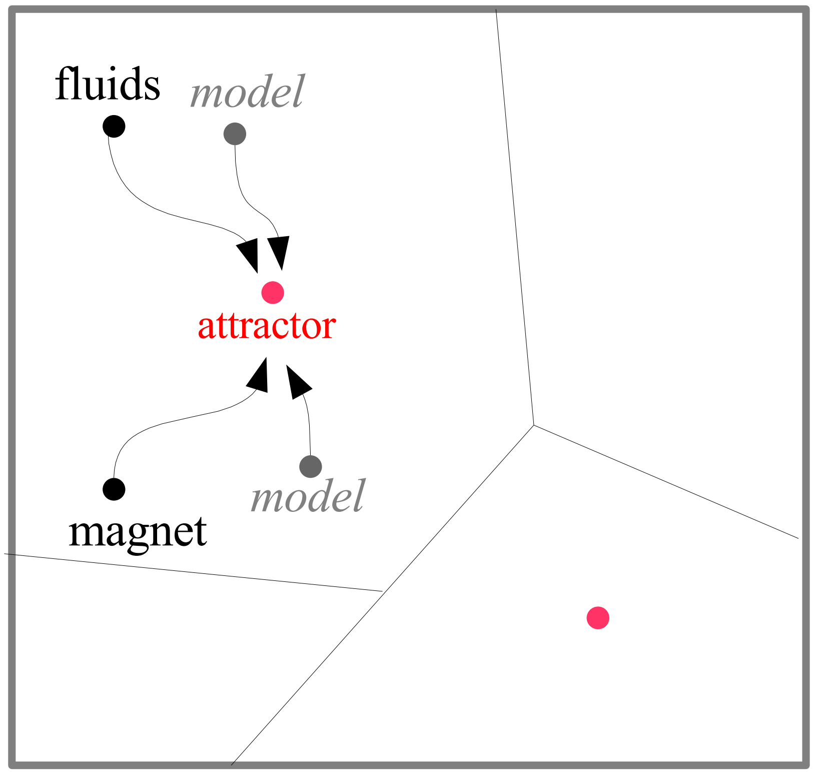
$$dm/dH = cte \times |T-T_c|^{-\gamma}$$

$\alpha = +0.1$ (magnetic alloys, fluids), -0.1 (Ni, Fe, ...),

$\gamma = 1.22$ ($CrBr_3$, fluids), 1.36 (Ni, Fe, ...)

Phase transitions and universality

Mathematical theory depend on dimensions d (space) and n (order parameter) only
Many applications: liquid crystals, superconductivity, polymers, ...



Space of Energy Functions

K. Wilson

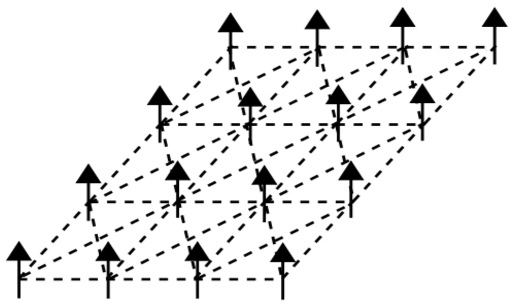
⇒ Models need not be very accurate to capture the right physics !

Complex and Disordered Systems

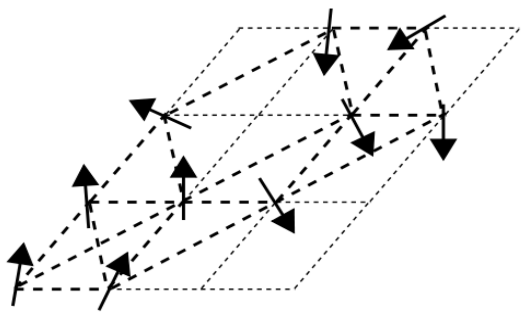
- « usual » meaning: configurations are disordered at high T, ordered at low T

- From now on: disorder means heterogeneous interactions

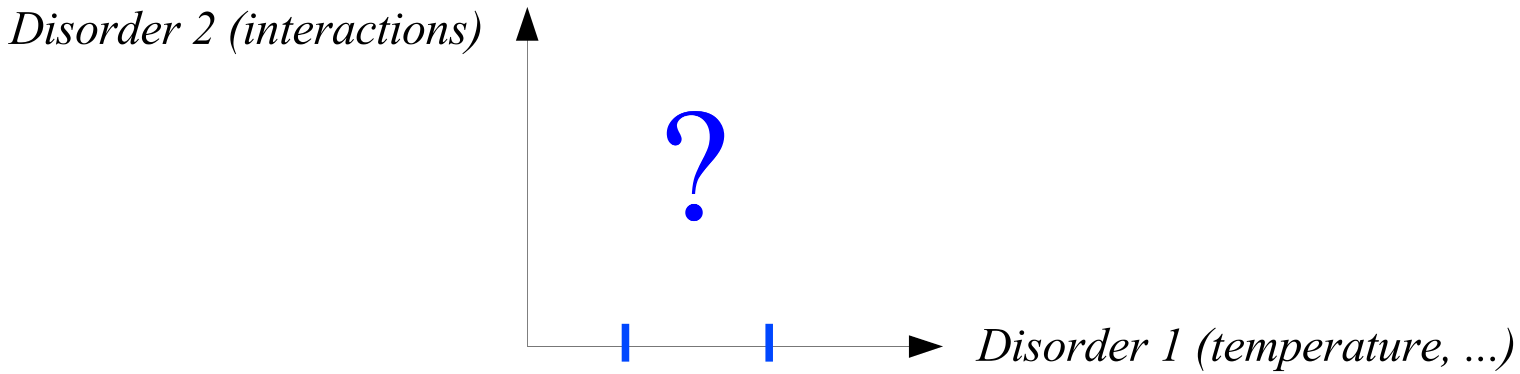
Ferromagnet



Disordered magnetic material



Heterogeneities can be dynamically induced (glass, turbulent flows, ...)



« A new kind of statistical mechanics »

The complete description of the interactions between the components is, in general, very hard, not to say impossible.

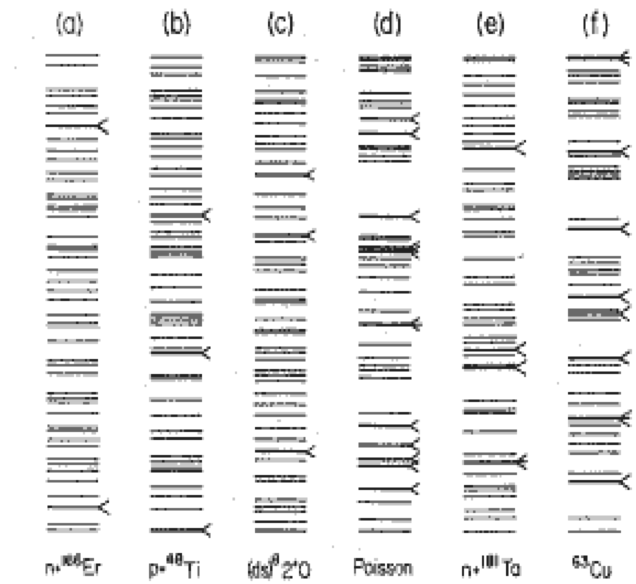
How, then, can one make predictions?

« What is here required is an new kind of statistical mechanics, in which we renounce exact knowledge not of the state of the system but of the system itself. We picture a complex **system** as a 'black box' in which a large number of particles are interacting according to unknown laws. The problem then is to define in a mathematically precise way an ensemble of systems in which all possible laws of interactions are equally possible. »

adapted from F.J. Dyson, Statistical theory of the energy levels of a complex system (1962)

Spectra of Nuclei

Results from neutron, proton scattering

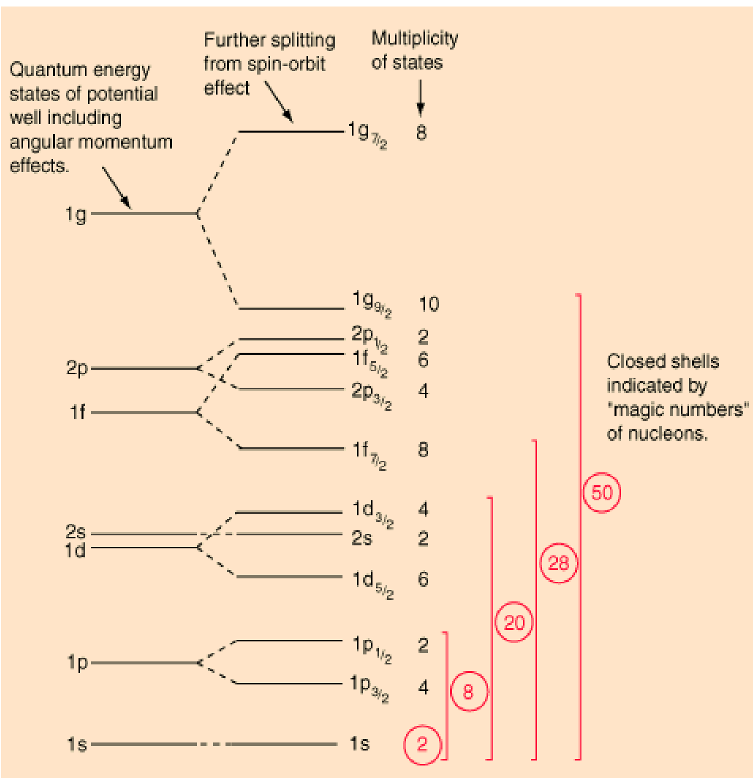


Experiments

Mixed

Poisson

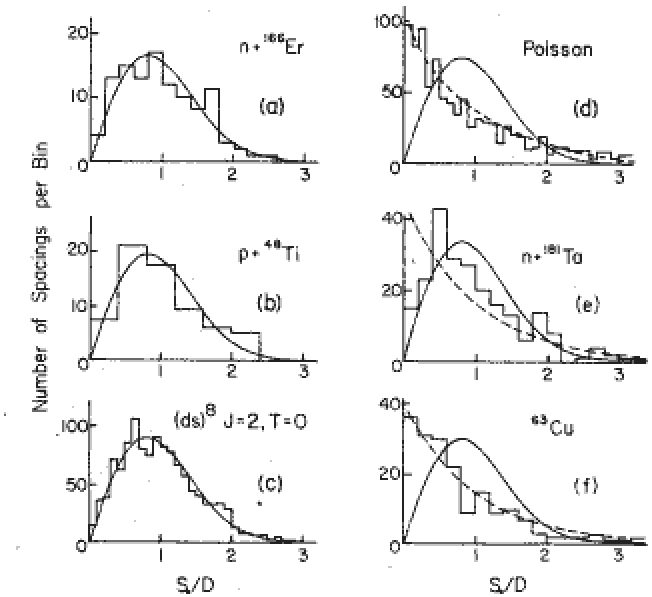
< = ¼ average spacing



Shell model

(similar to atomic spectra, J = total spin)

Random Matrix Theory



Distributions of level spacings

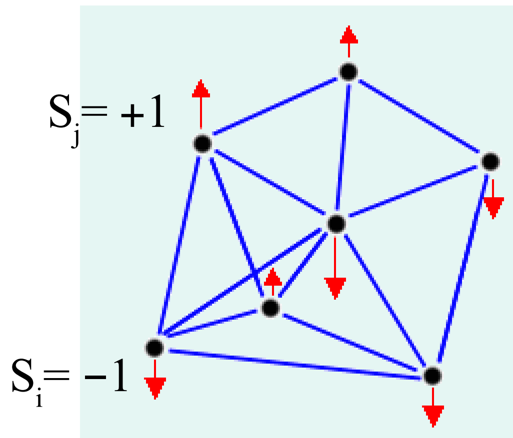
E. Wigner

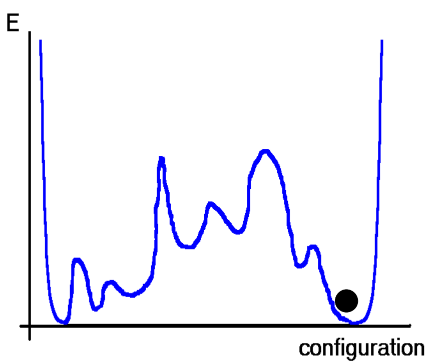
- Spectral features of nuclei are too complicated: statistical description ...
- Eigenvalues (energy levels) are not independent
- Try independent matrix elements (universality with respect to interactions)
- Predictions for level spacing distribution in very good agreement with experimental data

Complex landscapes and replica symmetry breaking

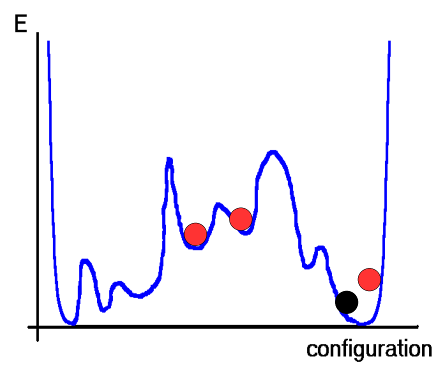
(Parisi, 1980 ...)

Spin glass
Cu Mn alloy

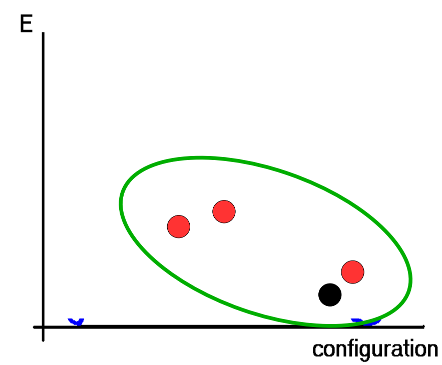




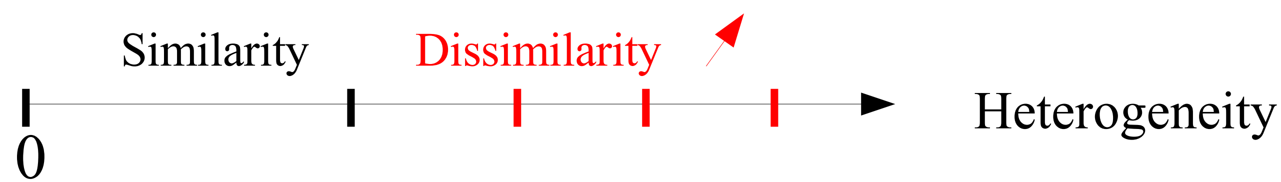
1 configuration
with disorder



1+**n** configurations
no interaction
with disorder



1 configuration
interacting with **n** other
configurations (**n**→ -1)



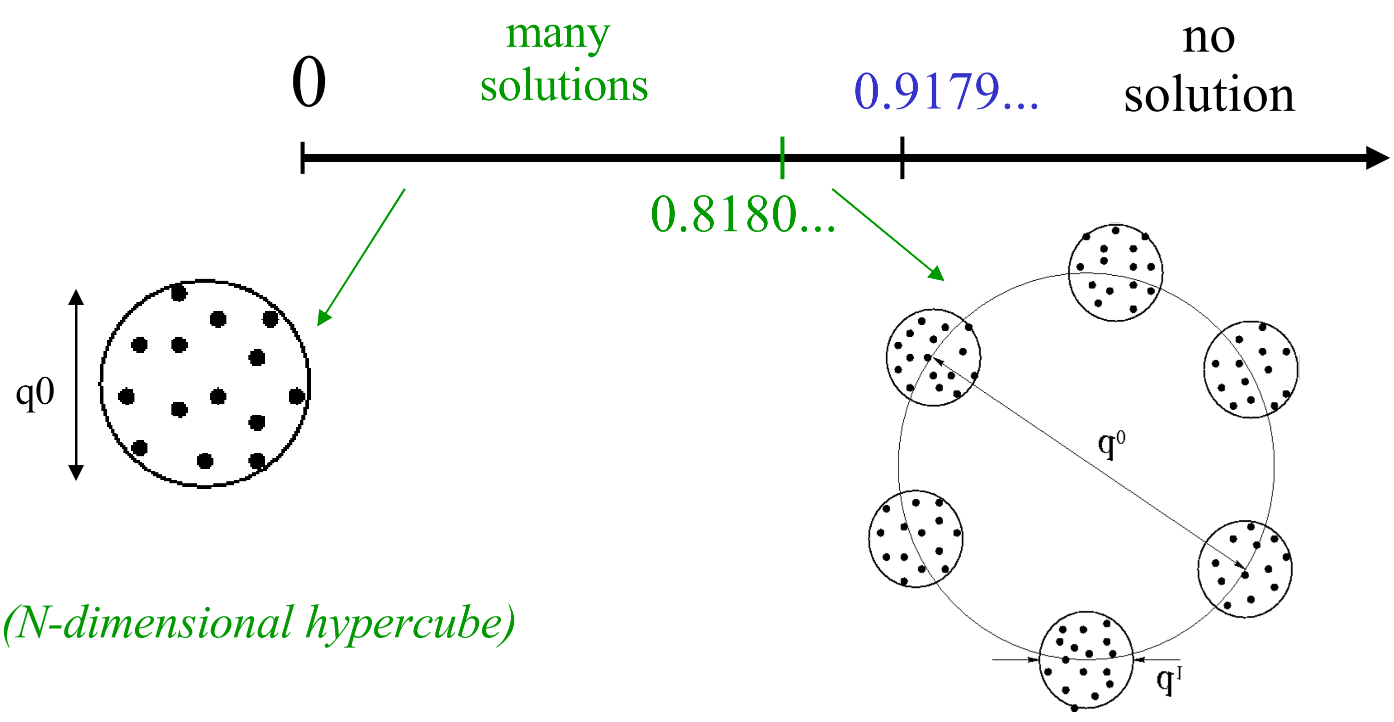
Application to constraint satisfaction problems

N boolean variables (True or False);
M equations on 3 variables;
second members = 0 or 1

$$x_1 + x_3 + x_4 = 0$$
$$x_1 + x_2 + x_7 = 1$$
$$\cdots$$

?

$\alpha = M/N$ = number of equations per variable



(N-dimensional hypercube)

Conclusions

Concepts and tools exist to describe complex systems of strongly interacting components in steady states (long time scales)

Ignoring details allows us to reach some understanding of the global properties of « complex » systems

There are major theoretical challenges ahead:

- dynamics at intermediate times?
- expanding configuration spaces?