PCE STAMP

# DECOHERENCE in REAL SYSTEMS: MECHANISMS of DECOHERENCE

(7 PINES, May 08, 2010)

**Physics & Astronomy**
**UBC**
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**Pacific Institute**
**for**
**Theoretical Physics**

# SOME HISTORICAL PERSPECTIVE

## 1: OLD-STYLE DECOHERENCE IDEAS

| | |
|---|---|
| Ludwig | (1953-56) |
| Green | (1956) |
| | |
| Zeh | (1970) |
| Simonius | (1978) |
| Zurek | (1981) |
| Van Kampen | (1988) |

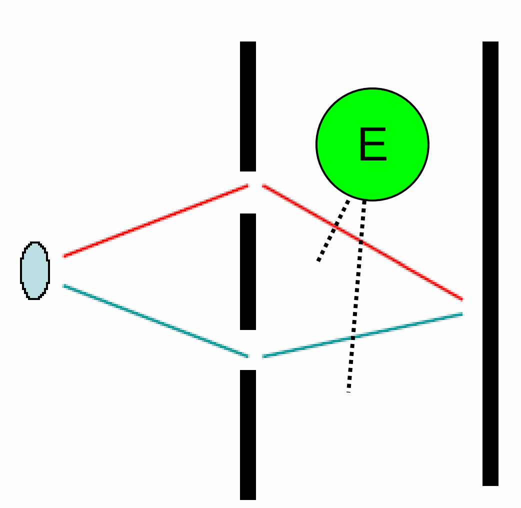
These papers formulated the idea of phase decoherence by entanglement with the environment, using toy models or no models at all. The idea that this solves the measurement problem, and that macroscopic systems (including measuring systems) thereby behaved classically, was proposed right from the beginning in these papers.

## 2: OLDER MODELS for ENVIRONMENTAL DECOHERENCE MECHANISMS

| | |
|---|---|
| Ullersma | (1956) |
| Feynman & Vernon | (1963) |
| Senitzky | (1960 - 63) |
| Caldeira-Leggett | (1981 - 85) |

All of these models describe the environment as a set of oscillators. Their roots go all the way back to Rayleigh. They are intended as a serious (experimentally testable) description of a large class of real systems. They produce equations of motion for the reduced density matrix of the system. One finds that decoherence leads to long-time “classical behaviour” (diffusion, Brownian motion, etc..).

# ENVIRONMENTAL DECOHERENCE 101



**When some quantum system with coordinate Q interacts with any other system (with coordinate x) , the result is typically that they form a combined state in which there is some entanglement between the two systems.**

**Example: In a 2-slit expt., the particle coordinate Q couples to photon coordinates, so that we have the following possibility:**

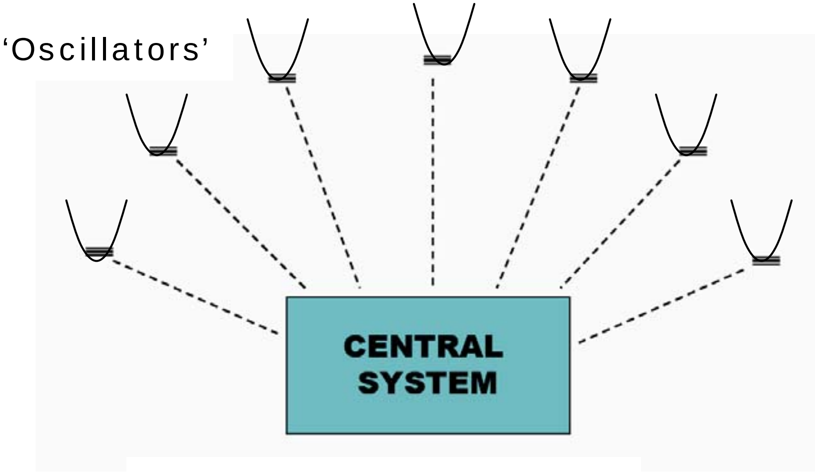
$$\Psi_o(Q)\ \Pi_q \phi_q^{in} \rightarrow [a_1 \Psi_1(Q)\, \Pi_q\, \phi_q^{(1)} + a_2 \Psi_2(Q)\, \Pi_q\, \phi_q^{(2)}]$$

**But now suppose we do not have any knowledge of, or control over, the photon states- we must then average over these states, in a way consistent with the experimental constraints. In the extreme case this means that we lose all information about the PHASES of the coefficients $a_1$ & $a_2$ (and in particular the relative phase between them). This process is called DECOHERENCE**

**NB 1: In this interaction between the system and its "Environment" E (which is in effect performing a measurement on the particle state), there is no requirement for energy to be exchanged between the system and the environment- only a communication of phase information.**

**NB 2: Nor is it the case that the destruction of the phase interference between the 2 paths must be associated with a noise coming from the environment- what matters is that the state of the environment be CHANGED according to the what is the state of the system.**

# MODELS of ENVIRONMENTAL DECOHERENCE



$$H_{\rm eff}^{\rm osc} = H_0 + H_{\rm int} + H_{\rm env}^{\rm osc}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left( \frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2 \right)$$

Int: $$H_{\rm int}^{\rm osc} = \sum_{q=1}^{N} [F_q(Q) x_q + G_q(P) p_q]$$

Phonons, photons, magnons, spinons, Holons, Electron-hole pairs, gravitons,..

**Feynman & Vernon, Ann. Phys. 24, 118 (1963)**

**Caldeira & Leggett, Ann. Phys. 149, 374 (1983)**
**AJ Leggett et al, Rev Mod Phys 59, 1 (1987**

DELOCALIZED BATH MODES

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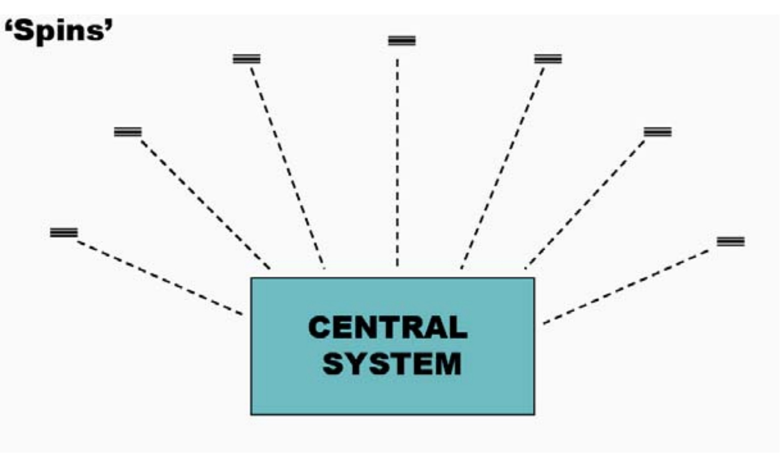
OSCILLATOR BATH



$$H_{\rm eff}^{\rm sp}(\Omega_0) = H_0 + H_{\rm int}^{\rm sp} + H_{\rm env}^{\rm sp}$$

Bath: $$H_{\rm env}^{\rm sp} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$

Interaction: $$H_{\rm int}^{\rm sp} = \sum_{k}^{N_s} \boldsymbol{F}_k(P,Q) \cdot \boldsymbol{\sigma}_k$$

Defects, dislocation modes, vibrons, Localized electrons, spin impurities, nuclear spins, ...

LOCALIZED BATH MODES

↓

SPIN BATH

**(1) P.C.E. Stamp, PRL 61, 2905 (1988)**
**(2) NV Prokof'ev, PCE Stamp, J Phys CM5, L663 (1993)**
**(3) NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)**

# A NOTE (for AFFICIONADOS ONLY ) on the FORMAL NATURE of the PROBLEM

We want the density matrix

$$K(Q_2, Q_2'; Q_1, Q_1'; t, t') = \int_{Q_1}^{Q_2} \mathscr{D}q \int_{Q_1'}^{Q_2'} \mathscr{D}q' \, \mathrm{e}^{-\mathrm{i}/\hbar(S_0[q]-S_0[q'])} \mathscr{F}[q, q'],$$

**with** $\mathcal{F}[Q, Q'] = \prod \langle \hat{U}_k(Q, t) \hat{U}_k^\dagger(Q', t) \rangle$

Here the unitary operator $\hat{U}_k(Q, t)$ describes the evolution of the $k$th environmental mode, given that the central system follows the path $Q(t)$ on its 'outward' voyage, and $Q'(t)$ on its 'return' voyage; and $\mathcal{F}[Q, Q']$ acts as a weighting function, over different possible paths $(Q(t), Q'(t'))$.

Easy for oscillator baths (it is how Feynman set up field theory). We write:

$$\mathcal{F}[Q,Q'] \;=\; \prod_q^{N_o} \int \mathcal{D}x_q(\tau) \int \mathcal{D}x_q(\tau') \; \exp\left[\frac{i}{\hbar}\int d\tau \frac{m_q}{2}[\dot{x}_q^2 - \dot{x}_q'^2 + \omega_q^2(x_q^2 - x_q'^2)] \;+\; [F_q(Q)x_q - F_q(Q')x_q']\right]$$

For the simplified bilinear 'Caldeira-Leggett' form, $\boldsymbol{F_q(P,Q)\, x_q \;= v_q x_q Q}$ we get

$$F[q,q'] \;=\; \exp\left[-\frac{1}{\hbar}\int_{t_o}^{t} d\tau_1 \int_{t_o}^{\tau_1} d\tau_2 [q(\tau_1) - q'(\tau_2)][\mathcal{D}(\tau_1 - \tau_2)q(\tau_2) - \mathcal{D}^*((\tau_1 - \tau_2)q'(\tau_2)]\right]$$

where $$\mathcal{D}(\tau) \;=\; \sum_q \frac{|v_q|^2}{2m_q\omega_q}\left[e^{i\omega_q\tau} \;+\; 2\frac{\cos\omega_q\tau}{e^{\beta\omega_q} - 1}\right]$$

---

For a spin bath it is not so simple. We have:

$$\mathcal{F}[Q,Q'] \;=\; \prod_k^{N_s} \int \mathcal{D}\boldsymbol{\sigma}_k(\tau) \int \mathcal{D}\boldsymbol{\sigma}_k(\tau') \; \exp\left[\frac{i}{\hbar}(S_{int}[Q,\boldsymbol{\sigma}_k] - S_{int}[Q',\boldsymbol{\sigma}_k'] \;+\; S_E[\boldsymbol{\sigma}_k] - S_E[\boldsymbol{\sigma}_k'])\right]$$

where

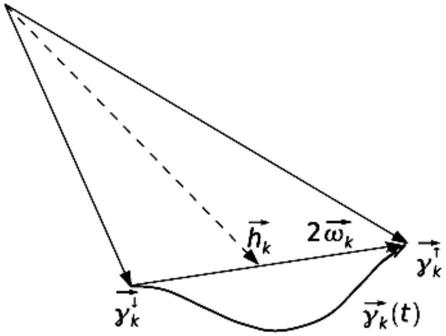
$$S_{int}^{sp}(Q,\boldsymbol{\sigma}_k) \;=\; -\int d\tau \sum_k^{N_s} \boldsymbol{F}_k(P,Q)\cdot\boldsymbol{\sigma}_k \qquad \& \qquad S_{env}^{sp} \;=\; \int d\tau \left[\sum_k^{N_s}(\mathcal{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{dt} \;-\; \mathbf{h}_k\cdot\boldsymbol{\sigma}_k) \;-\; \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta}\sigma_k^\alpha\sigma_{k'}^\beta\right]$$

Integrating out the spin bath is non-trivial, but has been done for important cases.

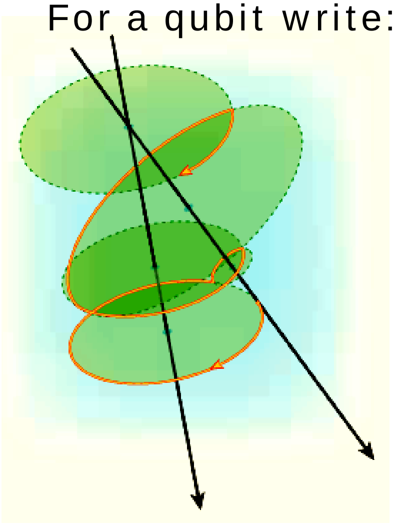
# Example 1: QUBIT DECOHERENCE

Considerable success has been achieved for some problems – eg., a qubit coupled to a spin bath, or a set of dipolar interacting qubits coupled to a spin bath.

A general feature of the results is that one can have extremely strong decoherence with almost no dissipation – the spin bath is almost invisible in energy relaxation, but causes massive PRECESSIONAL DECOHERENCE)



For a qubit write:



Precessional decoherence

$$\hat{H}_{QB} = H^0_{QB}(\vec{\tau}) + \sum (\vec{\gamma}_k + \xi_k) \cdot \vec{\sigma}_k$$

where

$$\gamma_k^\alpha = h_k^\alpha + \sum_\beta \omega_k^{\beta\alpha} \tau_\beta$$

$$\xi_k^\alpha = \sum_{k'} \sum_\beta V_{kk'}^{\alpha\beta} \sigma_{k'}^\beta$$

Then decoherence rates are

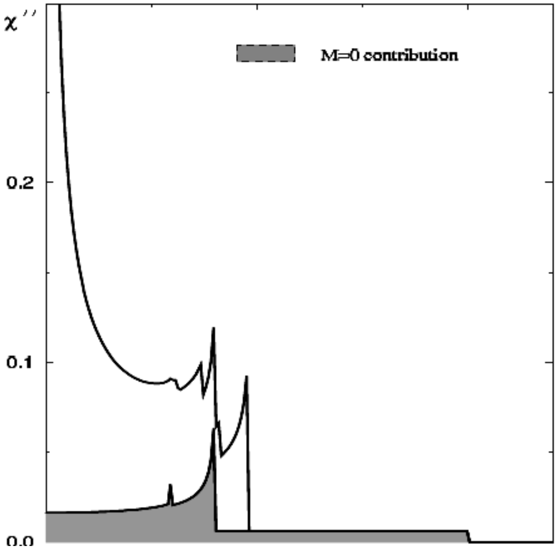
$$\Gamma_\phi^P \sim 1/2 \sum_k (\omega_k / h_k)^2 ; \omega_k \ll h_k$$

$$\Gamma_\phi^P \sim 1/2 \sum_k (h_k / \omega_k)^2 ; h_k \ll \omega_k$$

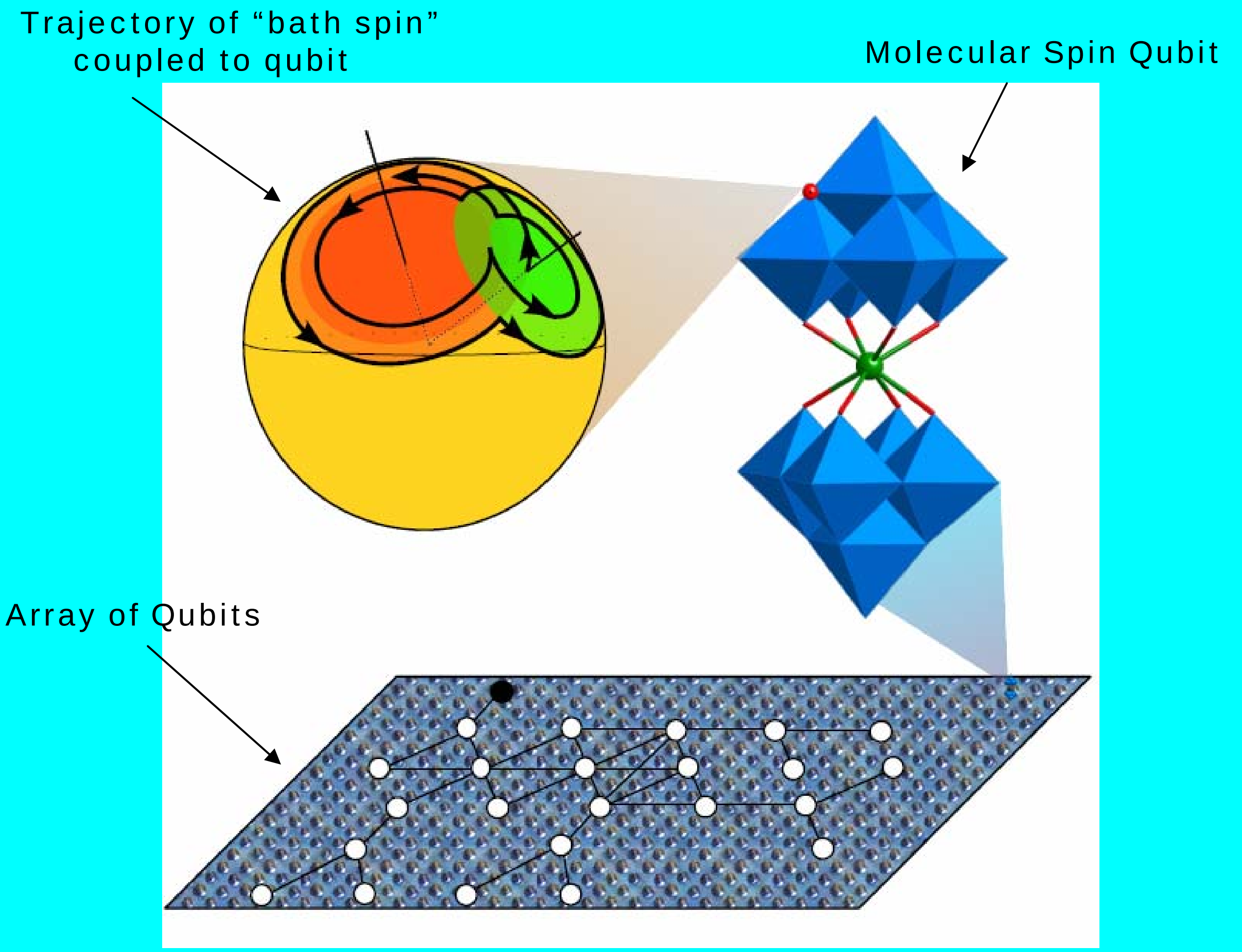
PCE Stamp, Nature 453, 167 (2008)

B Bertaina et al., Nature 453, 203 (2008)

However the lineshape is not conventional at all:



# ANOTHER VIEW of the QUBIT PROBLEM



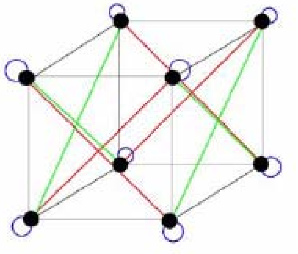
Trajectory of “bath spin” coupled to qubit

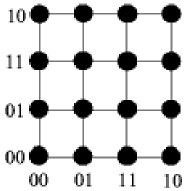
Molecular Spin Qubit

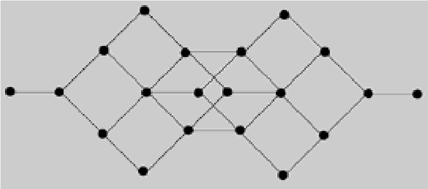
Array of Qubits

# DECOHERENCE DYNAMICS Example 2: QUANTUM WALKS

In a quantum walk,
a particle hops around
some graph







No BATH: Return probability: $P_0^0(t) \propto 1/t^d$ Variance: $\langle R^2 \rangle \propto t^2$

OSCILLATOR BATH: Return probability: $P_0^{(cl)}(t) \propto 1/t^{d/2}$

Variance: $\langle R^2 \rangle \propto t$

Classical diffusion

SPIN BATH:

Consider following example:

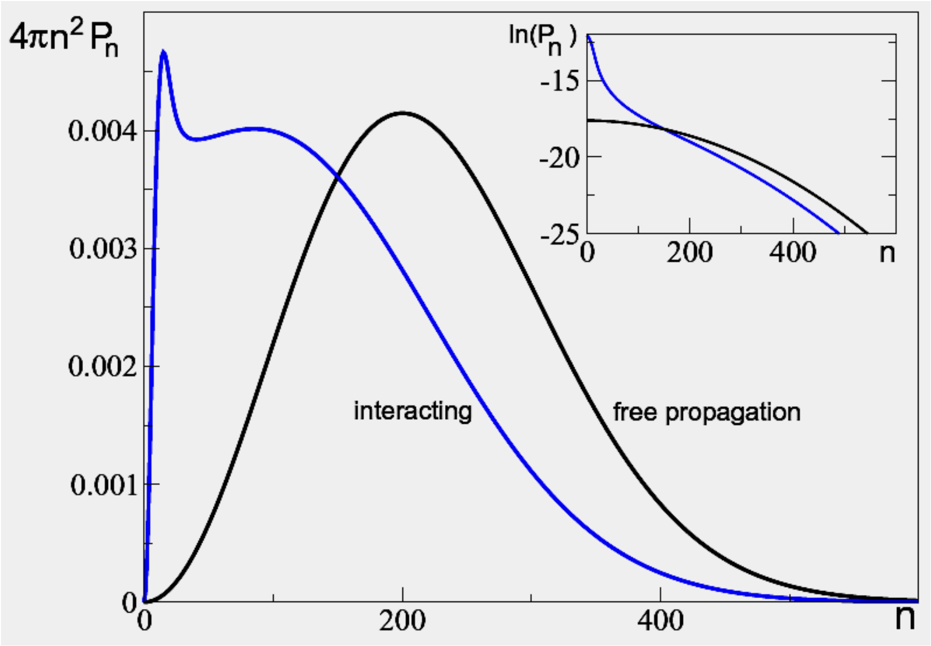
$$\mathcal{H} = \Delta_o \sum_{<ij>} \left\{ c_i^\dagger c_j \cos\left( \sum_k \alpha_k \sigma_k^x \right) + H.c. \right\}$$

Return Prob: $P_0(z \to \infty) \approx \dfrac{A_d R^{2-d}}{\Delta_o t}$

Variance:

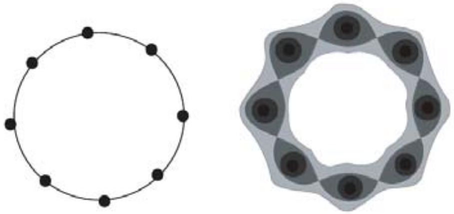
$$< ((\mathbf{n}(t) - \mathbf{n}(0))^2 >= 12 \sum_n \mathbf{n}^2 P_\mathbf{n}(z) = \frac{d}{2}(\Delta_o t)^2$$

Combination of weak localization and ballistic propagation! Very non-classical (non-diffusive) dynamics.



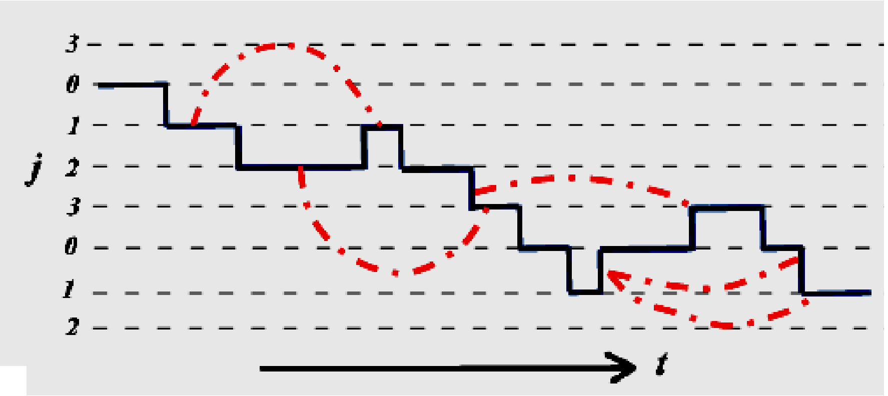
NV Prokof'ev, PCE Stamp
Phys Rev **A74**, 020102 (2006);

# DECOHERENCE DYNAMICS - Example 3: RINGS



Consider a particle moving around a ring – typical paths are shown at right

Z Zhu, A Aharony, O Entin-Wohlman, PCE Stamp
Phys Rev A (in press); /condmat 0909.4092



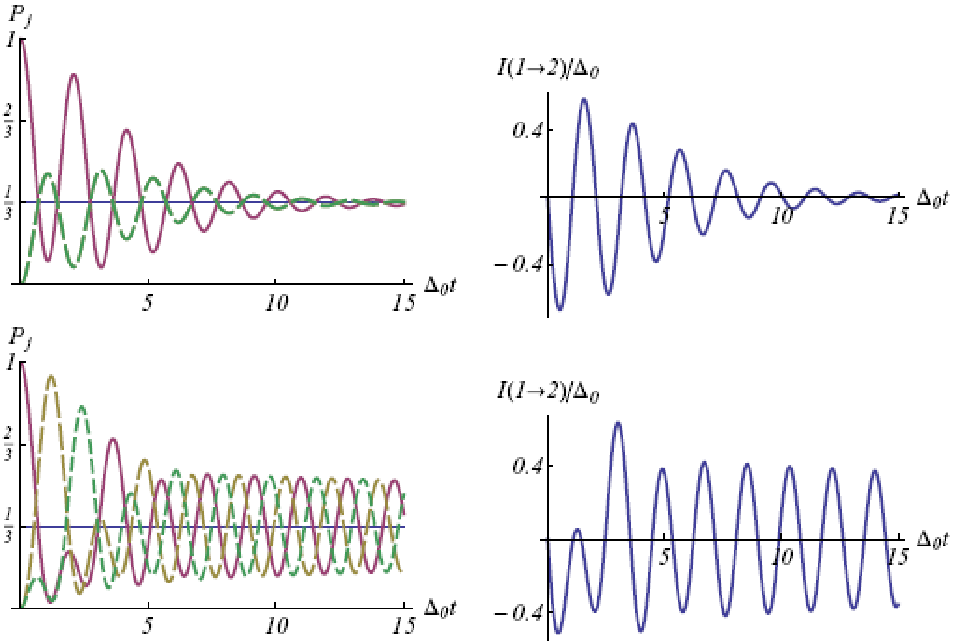


FIG. 4: Plot of $P_{j0}(t)$ for a 3-site ring, for a particle initially on site 1, in the intermediate decoherence limit, with $\lambda = .02$. Left: The probability to occupy site 0 (full line), 1 (large dashes), and 2 (small dashes). Right: the current from site 0 to site 1. Top: $\Phi = 0$. Bottom: $\Phi = \pi/2$ .

Bare Hamiltonian:

$$H_o = \sum_{ij}\left[t_{ij}c_i^\dagger c_j\, e^{iA^o_{ij}} + H.c.\right] + \sum_j \varepsilon_j c_j^\dagger c_j$$

Hamiltonian with bath:

$$H = H_{band} + H_{SB}$$

where $H_{band} =$

$$\textstyle\sum_{ij}[t_{ij}c_i^\dagger c_j e^{iA^o_{ij}+i\sum_k(\boldsymbol{\phi}_k^{ij}+\boldsymbol{\alpha}_k^{ij}\cdot\boldsymbol{\sigma}_k)} + H.c.]$$

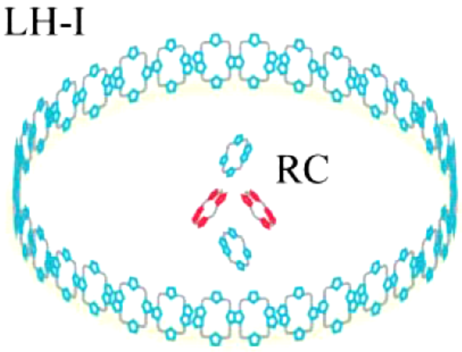
$$+ \textstyle\sum_j(\varepsilon_j + \sum_k \boldsymbol{\gamma}_k^j \cdot \boldsymbol{\sigma}_k)c_j^\dagger c_j$$

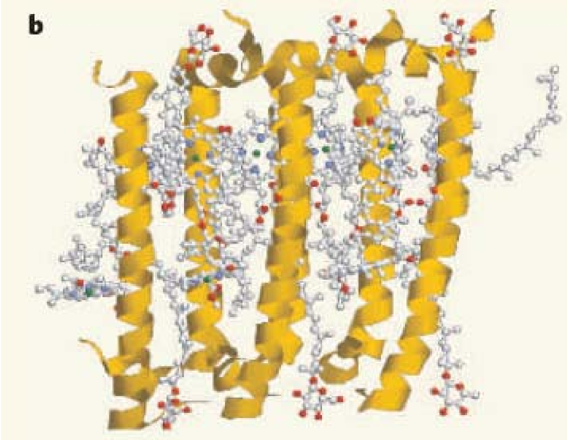
Combined influence of flux and spin bath is profound and counter-intuitive. The dynamics of decoherence is highly non-classical

# EXAMPLE of RINGS: LIGHT-HARVESTING MOLECULES

These molecules have coherent exciton propagation around large protein ring structures

It is quite astonishing to see coherence at such large length scales at ROOM TEMPERATURE

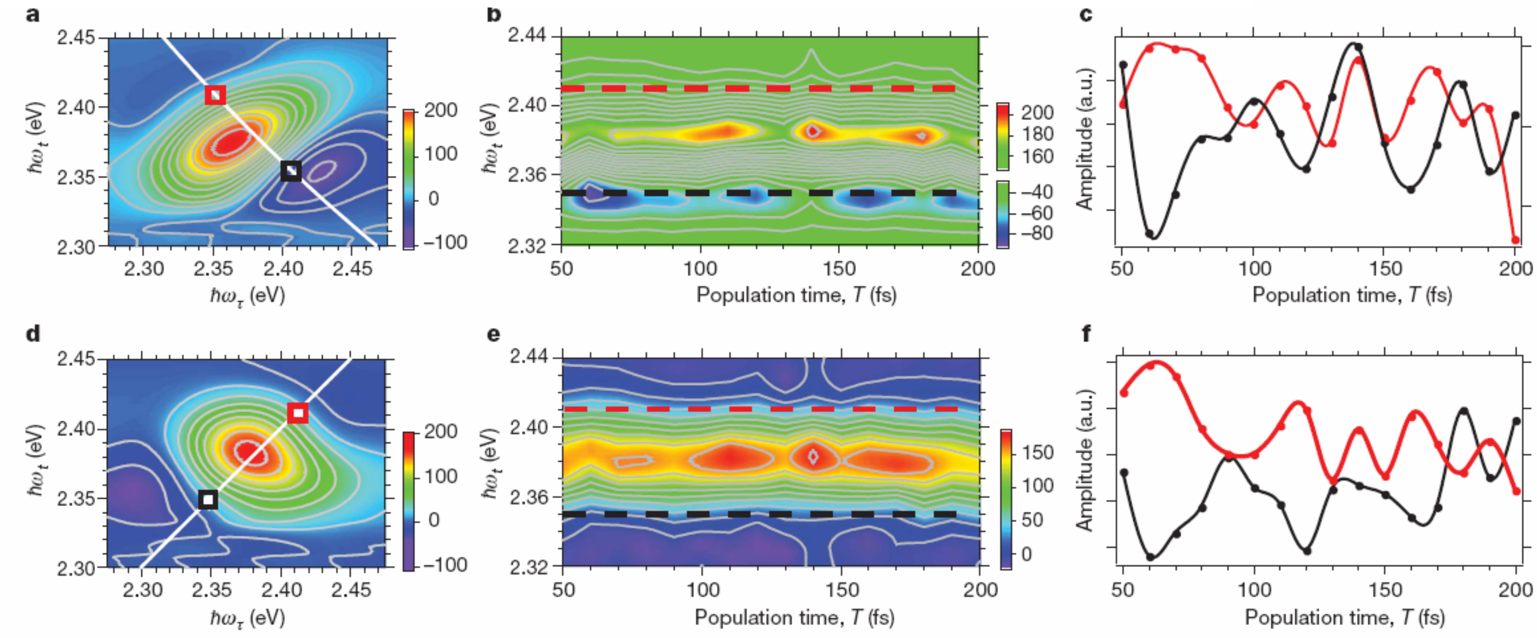




## Coherently wired light-harvesting in photosynthetic marine algae at ambient temperature

E Collini et al., Nature 463, 644 (2010)

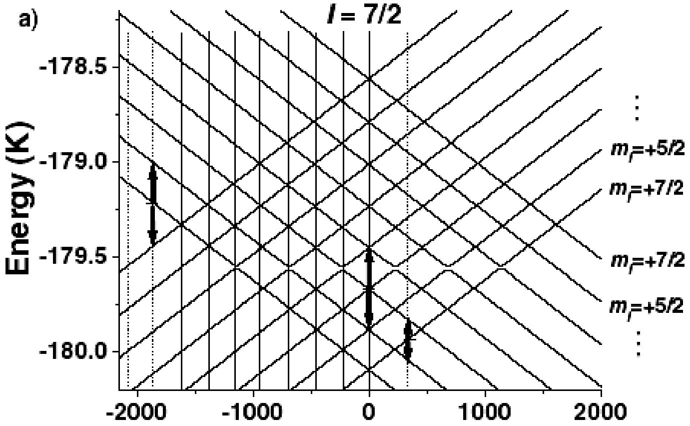
Elisabetta Collini[1]*†, Cathy Y. Wong[1]*, Krystyna E. Wilk[2], Paul M. G. Curmi[2], Paul Brumer[1] & Gregory D. Scholes[1]



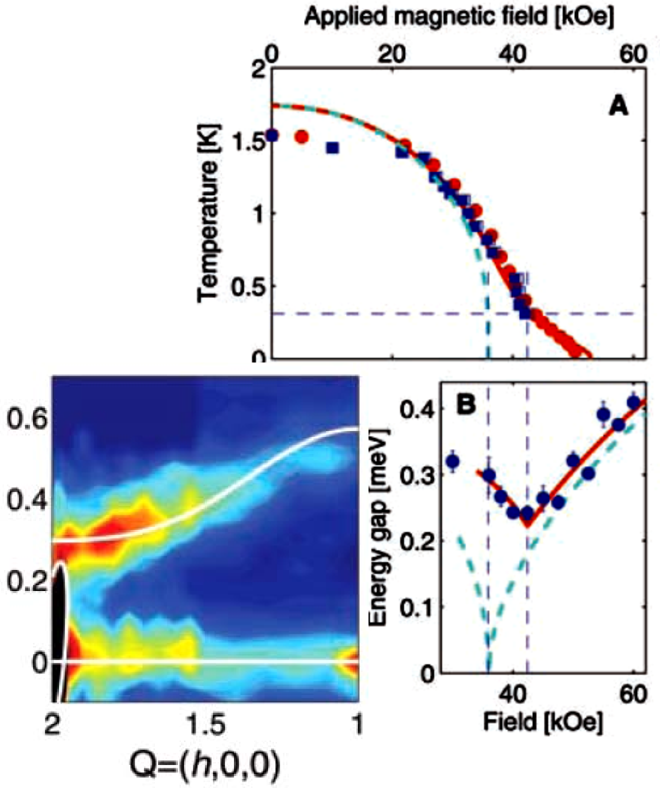
# QUANTUM PHASE TRANSITIONS

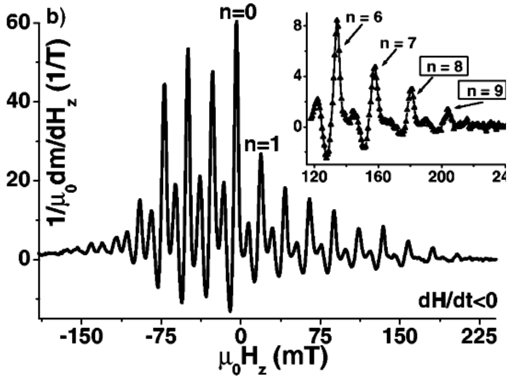
## Quantum Phase Transition of a Magnet in a Spin Bath

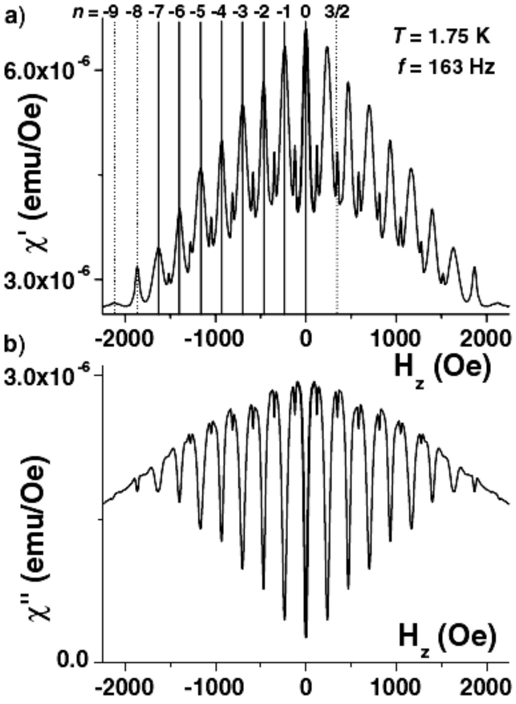
H. M. Rønnow,[1,2,3]* R. Parthasarathy,[2] J. Jensen,[4] G. Aeppli,[5]
T. F. Rosenbaum,[2] D. F. McMorrow[3,4,6]



Decoherence, particularly from a spin bath profoundly affects a Quantum Phase transition



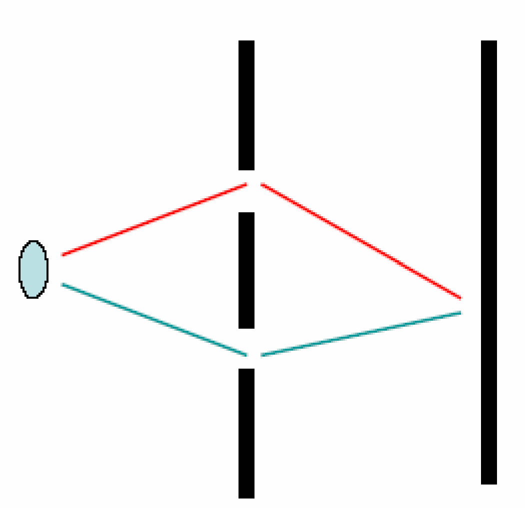


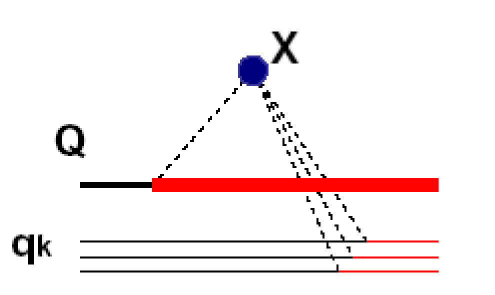


H.M. Ronnow et al., Science 308, 389 (2005)

R. Giraud et al., PRL 87, 057203 (2001)
“ PRL 91, 257204 (2003)

# OTHER KINDS of DECOHERENCE





(1) 3rd PARTY DECOHERENCE: **decoherence in the dynamics of a system A (coordinate Q) caused by *indirect* entanglement with an environment E- the entanglement is achieved via a 3rd party B (coordinate X).**

Ex: Buckyball decoherence

**Consider the 2-slit buckyballs. The COM coordinate Q of the buckyball does not couple directly to the vibrational $\{q_k\}$ of the buckyball- by definition. However BOTH couple to the slits in system, in a distinguishable way.**

**Note: the state of the 2 slits, described by a coordinate X, is irrelevant- it not need to change at all. We can think of it as a scattering potential, by a system with infinite mass. It is a PASSIVE 3rd party. We can also have 3rd parties**

PCE Stamp, Stud. Hist Phil Mod Phys **37**, 467 (2006)

(2) INTRINSIC DECOHERENCE: **This is a hypothetical decoherence in Nature that has nothing to do with environments at all. 2 examples are**

**(i) decoherence arising from spacetime distortion (gravitational decoherence)**

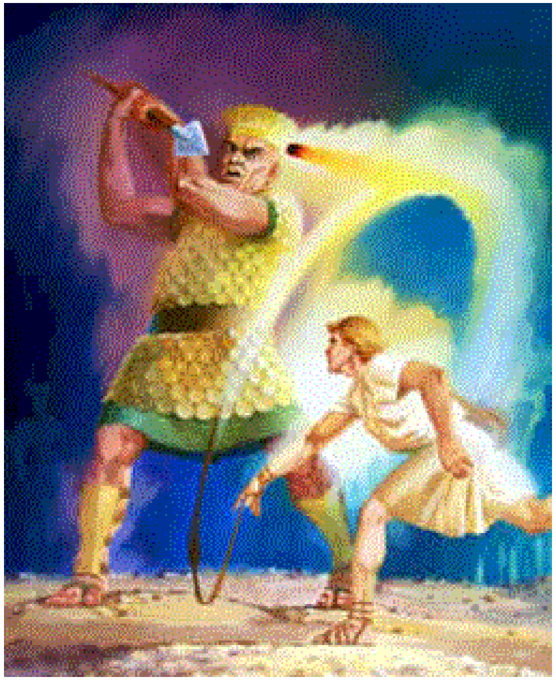
**(ii) decoherence suggested by the holographic principle, arising in all objects.**

L Diosi, Phys Rev A40, 1165 (1989)
R Penrose, “Shadows of the Mind” (OUP, 1994)

# SUMMARY

Decoherence is one of the most delicate processes ever investigated in Physics. A single nuclear spin, or a single photon, can upset the entire quantum dynamics of a macroscopic system.

This is a true case of David besting Goliath – at least for MOST systems



There are still many things we do not understand about how decoherence actually works, particularly in complex condensed matter systems (where it can have profound and non-intuitive effects). Tests of fundamental questions in physics, as well as the whole quantum information enterprise, will require this understanding. Theory and Expt really need to address the possibility of intrinsic decoherence, and of new ways of avoiding or suppressing decoherence.