Decoherence in Cosmology

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Introduction

Importance of gravity

Due to its geometric nature, the gravitational field couples to all forms of mass–energy. In a quantum theory of gravity, entanglement and decoherence should thus be of great importance.

- Quantum cosmology: Superpositions of states with different scale factors and matter configurations should occur
- Origin of structure in the Universe from quantum fluctuations: How come?

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (path integrals, . . .)
  - Canonical approaches (geometrodynamics, loop dynamics, . . .)
- String theory

All of these approaches preserve the superposition principle

Canonical quantum gravity

Central equations of canonical quantum gravity are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Oldest approach: Quantum geometrodynamics
(Wheeler–DeWitt equation):

- Apart from non-gravitational fields, Ψ depends only on the threedimensional metric, but is invariant under coordinate transformations
- no external time parameter is present
- Wheeler–DeWitt equation has the structure of a wave (Klein–Gordon type) equation and thereby defines an intrinsic time
- very conservative approach because this equation gives the correct Einstein equations in the semiclassical limit and should thus be valid at least away from the Planck scale

Quantization of a Friedmann Universe

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

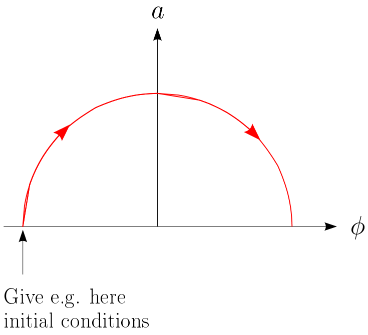
$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

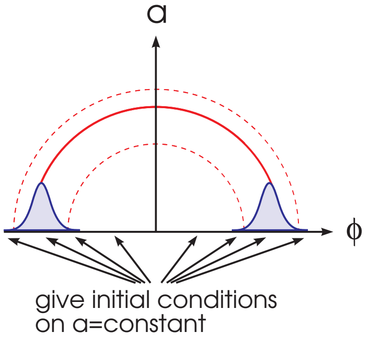
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory

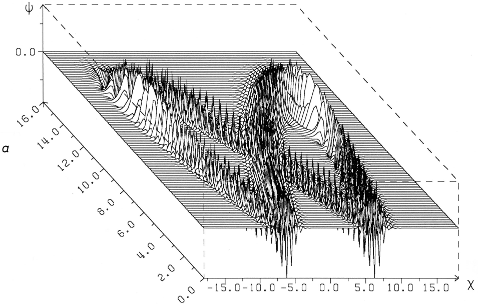


“Recollapsing” wave packet must be present “initially”

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



(C.K. 1990)

Introduction of inhomogeneities

Describe small inhomogeneities by multipoles $\{x_n\}$ around the minisuperspace variables (e.g. a and ϕ)

$$\left(H_0 + \sum_n H_n(a, \phi, x_n)\right) \psi_0(a, \phi) \prod_n \psi_n(a, \phi, x_n) = 0$$

(Halliwell and Hawking 1985)

If ψ_0 is of WKB form, $\psi_0 \approx C \exp(\mathrm{i} S_0/\hbar)$ (with a slowly varying prefactor C), one will get

$$\mathrm{i}\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n$$

with

$$\frac{\partial}{\partial t} \equiv \nabla S_0 \cdot \nabla$$

t: "WKB time" – controls the dynamics in this approximation

Decoherence in quantum cosmology

Decoherence?

- "System": Global degrees of freedom (radius of Universe, inflaton field, ...)
- "Environment": Density fluctuations, gravitational waves, other fields

(Zeh 1986, C.K. 1987)

Example: Scale factor a of de Sitter space ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) ('system') is decohered by gravitons ('environment') according to

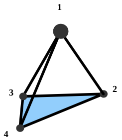
$$\rho_0(a,a') \to \rho_0(a,a')\exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right)\ ,\ C>0$$

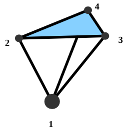
The Universe assumes classical properties at the "beginning" of the inflationary phase

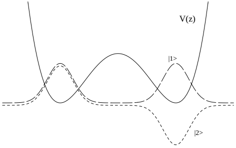
(Barvinsky, Kamenshchik, C.K. 1999)

Time from Symmetry Breaking

Analogy from molecular physics: emergence of chirality







dynamical origin: decoherence due to scattering with light or air molecules

quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/\hbar)$- and $\exp(-\mathrm{i}S_0/\hbar)$-part of wave function through interaction with multipoles

one example for decoherence factor:

$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C. K. 1992)

Origin of time direction

Fundamental asymmetry with respect to “intrinsic time”:

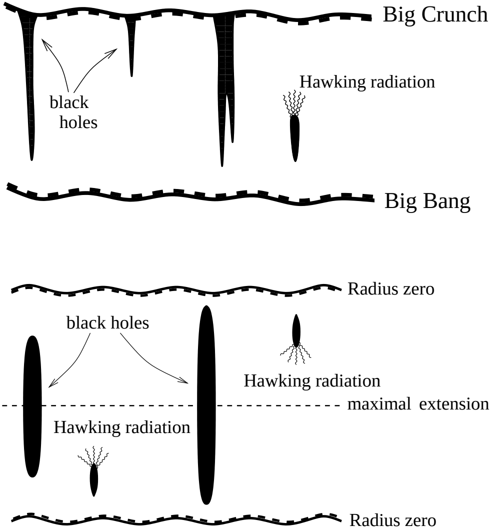
$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha\to-\infty}\right]\right)\Psi = 0$$

Is compatible with simple boundary condition:

$$\Psi \overset{\alpha\to-\infty}{\longrightarrow} \psi_0(\alpha)\prod_i \psi_i(x_i)$$

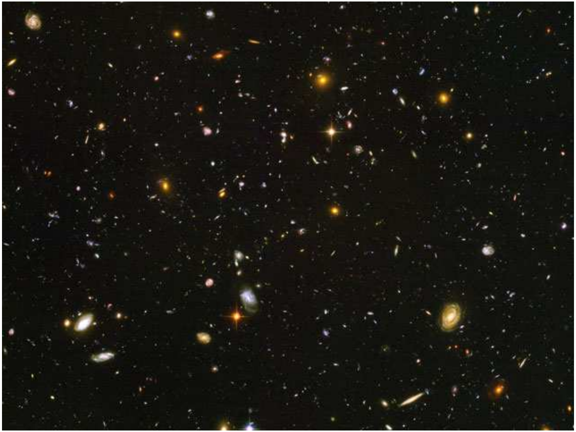
Entropy increases with increasing α, since entanglement with other degrees of freedom increases
$\longrightarrow$ defines time direction

Is the expansion of the Universe a tautology?



(C. K. and Zeh 1995)

Structure in the Universe



Where does it come from?

What is the problem?

Paradigm: Inflationary scenario (accelerated expansion of the Universe about 10^{-34} seconds after the Big Bang)

Quantum fluctuations of spacetime metric and inflaton

$\downarrow$

Density perturbations; gravitational waves

$\downarrow$

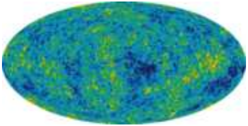
Temperature anisotropies, polarization in the Cosmic Background Radiation

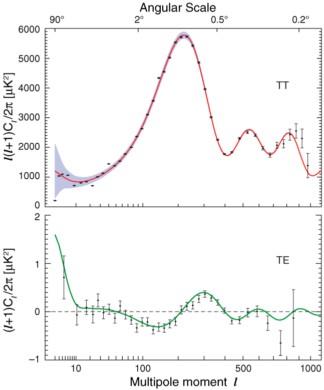
$\downarrow$

Galaxies, clusters of galaxies

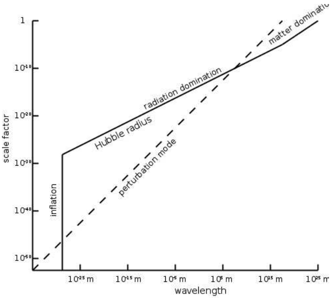
Quantum-to-classical transition?

Cosmic Background Radiation





Scales in cosmology



Physical scale: $\lambda(t) \equiv a(t)/k$ (a: scale factor)
Hubble radius: $H^{-1}(t)$

Modes far outside the Hubble radius:

$$\boxed{\frac{k}{aH} \ll 1}$$

Primordial fluctuations

What are they?

The primordial fluctuations in cosmology are small perturbations of the metric and the inflaton field; they can be described by a massless scalar field ϕ in a (here: flat) Friedmann–Lemaître Universe

Fourier transform: $y_k \equiv a\phi_k = y^*_{-k}$

In the following for simplicity: one real mode y

- Primordial fluctuations (if treated as isolated): Described by a squeezed state (squeezing in momentum); for $r \to \infty$ (r: squeezing parameter), we have a semiclassical quantum state; expectation values are indistinguishable from classical stochastic mean values
  But: This is unrealistic! Highly squeezed states are extremely sensitive to interactions
- Interaction with other fields (or non-linearities of the fluctuations) leads to decoherence

Simplest case of ideal interaction (only entanglement):

$$\rho_0(y, y') \longrightarrow \rho_\xi(y, y') = \rho_0(y, y') \exp\left(-\frac{\xi}{2}(y - y')^2\right)$$

ξ: Phenomenological parameter; contains details of interaction

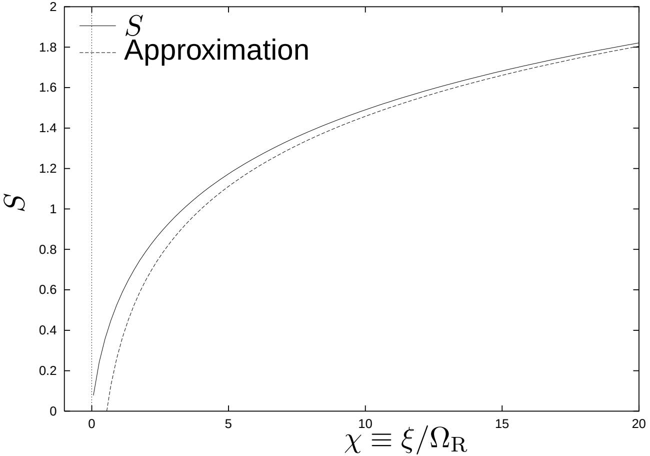
Main results

- Modes outside the Hubble scale assume classical properties through decoherence
- Pointer basis: (Approximate) field-amplitude basis
- This leads to classical stochastic Gaussian fluctuations which serve as the seeds for galaxies and clusters of galaxies
- Timescale $\sim H^{-1}$ for modes outside the Hubble scale during inflation

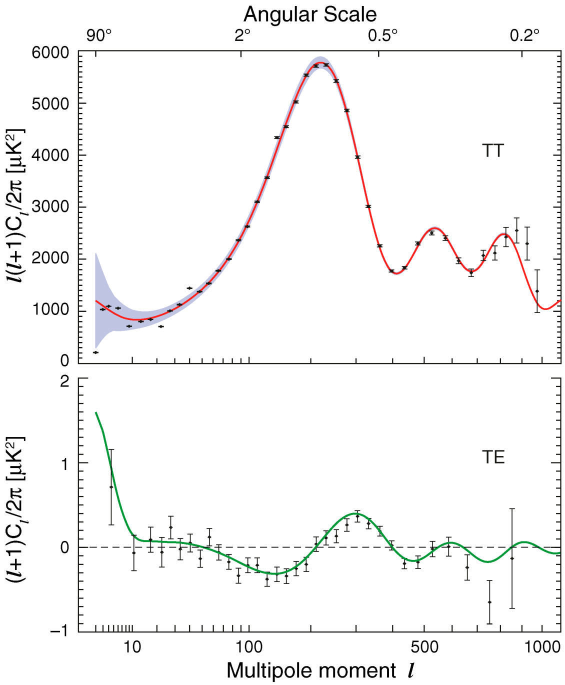
(C.K., D. Polarski, A. A. Starobinsky 1998, 2007)

Entropy

$$S = -\mathrm{tr}(\rho_\xi \ln \rho_\xi)$$



$$\boxed{S < r = S_{\max}/2}$$



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT
l(l+1)C_l/2π [μK²]
2
1
0
−1
TE
(l+1)C_l/2π [μK²]
10
100
500
1000
Multipole moment l

Hierarchy of classicality

Global gravitational degrees of freedom

$\downarrow$

Global matter degrees of freedom

$\downarrow$

Field modes outside the horizon

$\downarrow$

Galaxies, clusters of galaxies, ...