Danmarks Grundforskningsfond - Quantum Optics Center

QUANTOP

# Entanglement of simple macroscopic objects

Niels Bohr Institute - Copenhagen

---

*Quantum mechanical wonders (second wave)*

## Quantum objects

cannot be measured

cannot be copied

exist in superposition and entangled states

## Quantum Information Science

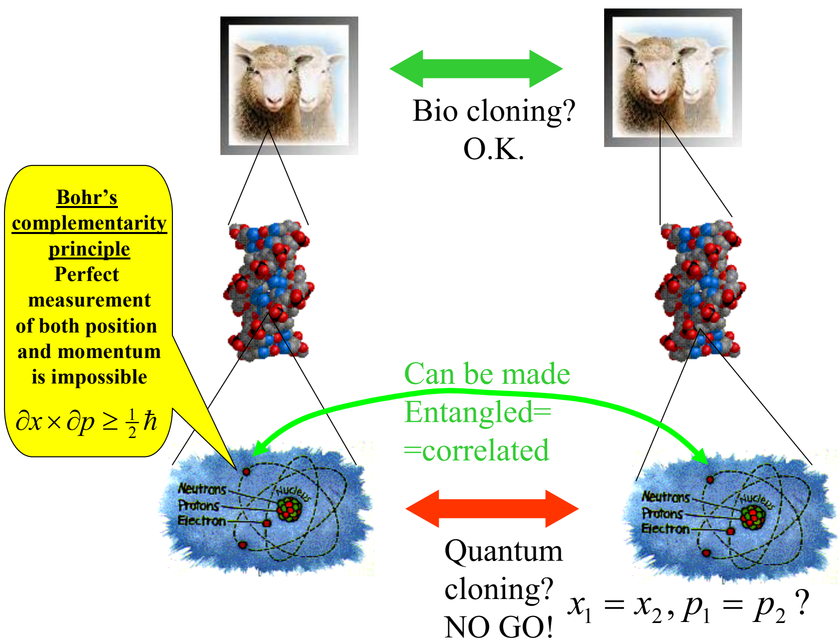
•Communications with absolute security

•Computing with unprecedented speed

•Teleportation of objects (or at least of their quantum states)

•Quantum memory

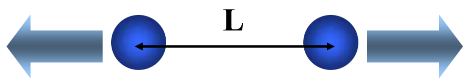
**Nothing is this world can be perfectly measured, or perfectly cloned…**




*"Entanglement is … perhaps the most fundamental issue in quantum mechanics"* – Erwin Schrödinger

- *Einstein-Podolsky-Rosen example 1935*

2 particles entangled in position/momentum




$\hat{X}_1 , \hat{P}_1$ $\qquad$ $\hat{X}_2 , \hat{P}_2 = mV$

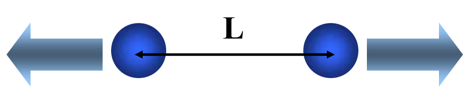
$$\hat{X}_1 - \hat{X}_2 = L \qquad \hat{P}_1 + \hat{P}_2 = 0$$

*Simon* (2000); *Duan, Giedke, Cirac, Zoller* (2000)
Necessary and sufficient condition for entanglement

$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2\hbar$$

---

2 particles entangled in position/momentum

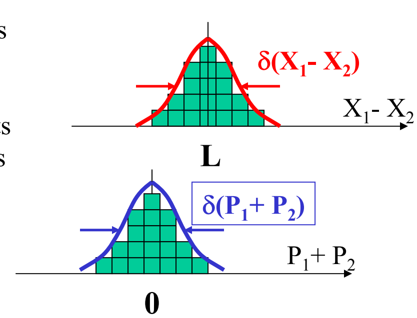



$\hat{X}_1 , \hat{P}_1$ $\qquad$ $\hat{X}_2 , \hat{P}_2$

**What does it mean in practice?**

$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2\hbar$$

1. Prepare many identical pairs of particles
2. Measure $X_1$- $X_2$ on some of those pairs
3. Measure $P_1$+$P_2$ on others
4. Plot statistical distributions of the results
5. Measure the width of these distributions



## Why does it make sense?

**Two independent particles**

$\hat{X}_1, \hat{P}_1$ $\hat{X}_2, \hat{P}_2$

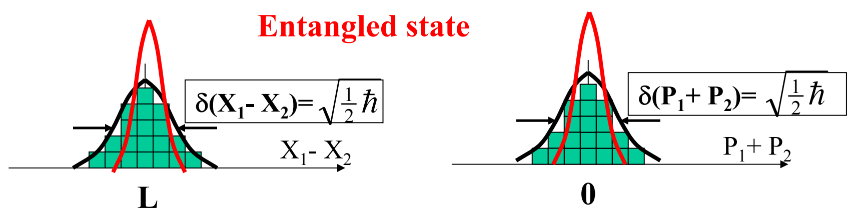
$$\delta x \times \delta p \geq \tfrac{1}{2}\hbar$$

**Minimal symmetric uncertainties**

$$\delta x_{1,2}{}^2 = \delta p_{1,2}{}^2 = \tfrac{1}{2}\hbar$$

$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2\hbar$$

**Entangled state**



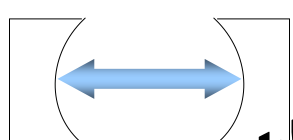

**Position – momentum uncertainty**

$$\delta x \times \delta p \geq \tfrac{1}{2}\hbar \approx 10^{-34}\, J \sec$$

## Macroscopic object – a mirror

**The best optical interferometry measurement:**

$$\delta x \approx 10^{-12}\, m$$



**Assume M=1mg**

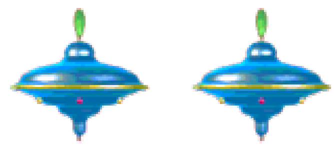
**tough!**

$$\delta p \approx 10^{-19}\, m / \sec$$

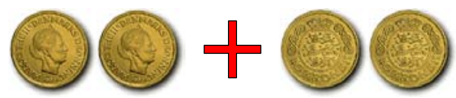
## Entanglement - spins

2 quantum coins

2 spins
( spin “up” or spin “down”)



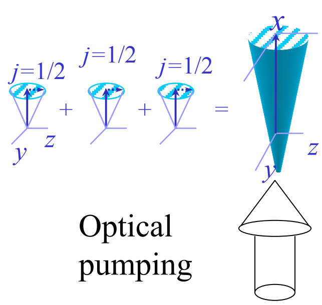
Entangled state:



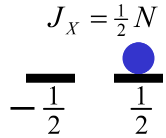
$$a\,|00\rangle + b\,|11\rangle$$

Entangled state, many qubits:

**Ensemble of N atomic spins and more about $\sqrt{N}$**

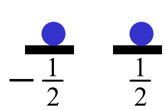



$J_X = \frac{1}{2}N$




Along x: all tails

What is the spin projection along Y or Z?




Zero?

Not really! $\frac{1}{2}\sqrt{N}$

## Ensemble polarized along X, measured along Y or Z

Binomial => Poissonian
N >>1

Zero mean value with random misbalance between heads and tails $\frac{1}{2}\sqrt{N}$

QM: $\left[\hat{J}_y, \hat{J}_z\right] = i\hat{J}_x \Rightarrow \delta J_y \delta J_z \geq \frac{1}{2} J_x = \frac{1}{4} N$

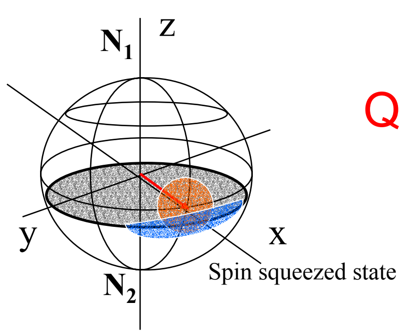
QM: uncorrelated spins => $\delta J_y = \delta J_z = \frac{1}{2}\sqrt{N}$

Alexandrov, Zapassky 1986: "Faraday rotation noise"

Wineland et al 1992: "projection noise"

---

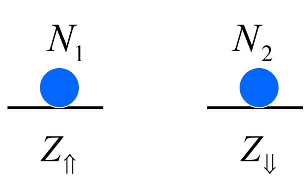
**One atomic ensemble**




QM: $\delta J_y \delta J_z \geq \frac{1}{2} J_x = \frac{1}{4} N$

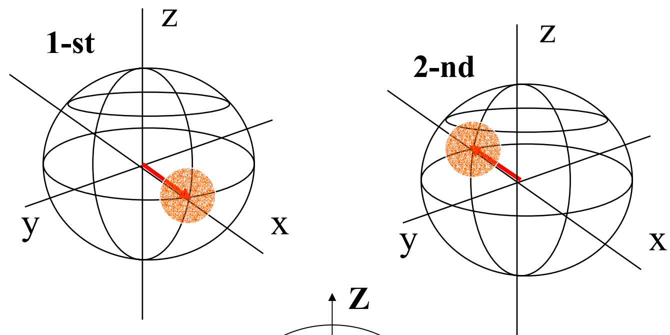
**Like P of a particle**

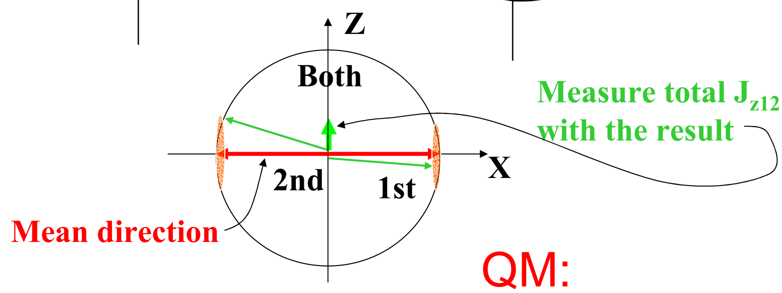
**Like X of a particle**




$Var(N_1 - N_2) < N$

**Two oppositely magnetized atomic ensembles**

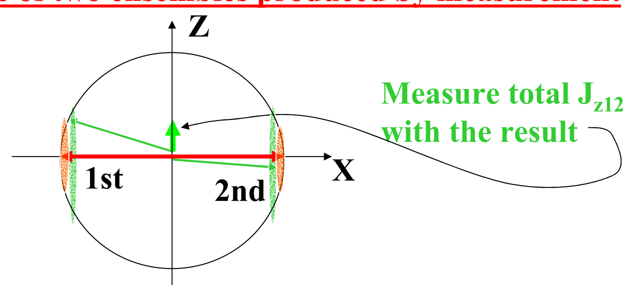


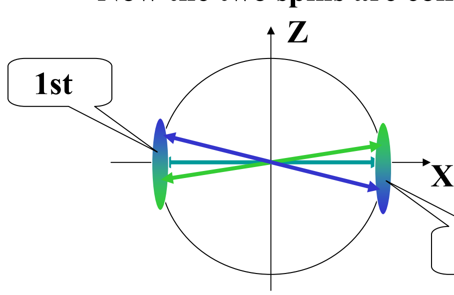

QM:

$$\delta J_{y12}\delta J_{z12} \geq \tfrac{1}{2}(J_{x1}+J_{x2})=0$$

**Entangled state of two ensembles produced by measurement**

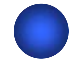



**If $J_{z12}$ is known, it can be made zero by spin rotation:**
**Now the two spins are collinear !**




**Each of the spins has a very uncertain direction –QM rules!**

$\hat{X}_1, \hat{P}_1$



**Compare**

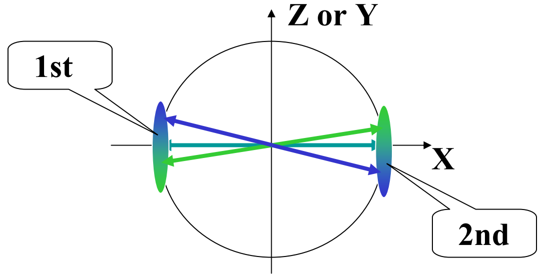
$$\delta J_y \delta J_z \geq \tfrac{1}{2} J_x \qquad\qquad \delta x \times \delta p \geq \tfrac{1}{2}\hbar$$

**If the two macroscopic spins are collinear they must be entangled:**

$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2\hbar$$

**Compare**

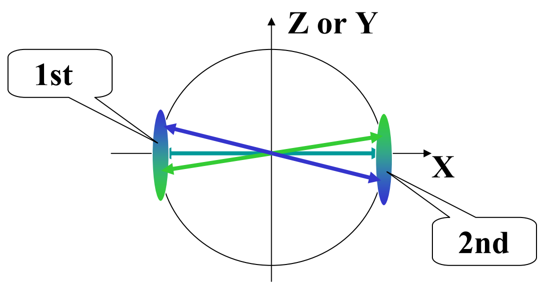
$$\delta(J_{z1} + J_{z2})^2 + \delta(J_{y1} + J_{y2})^2 < 2J_x$$



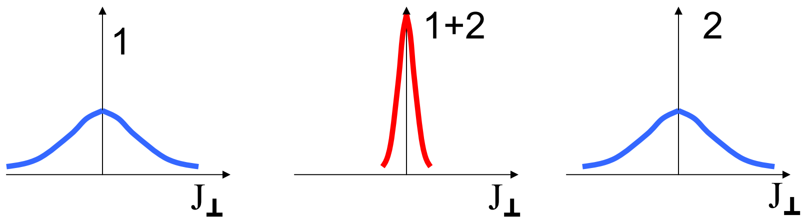

# QM:

$$\delta J_{y12} \delta J_{z12} \geq \tfrac{1}{2}(J_{x1} + J_{x2}) = 0$$

**QM allows such a measurement in principle, now all we need is to find the way to perform it…**



## Stern-Gerlach projection on any axis ⊥ to x:



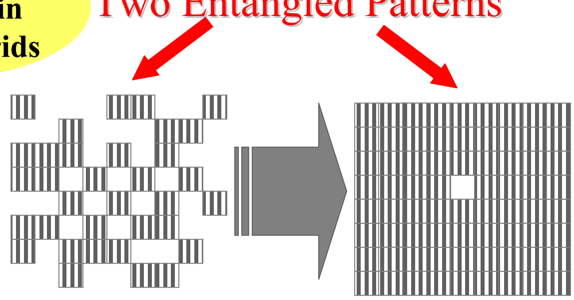

Along y,z: <u>ideally no</u> misbalance between heads and tails of the two ensembles, or, at least, less than random misbalance $\sqrt{N}$

---

**A secret message is encoded in one of the grids**

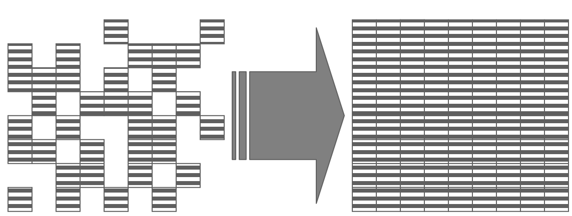
## Two Entangled Patterns

**Projected on vertical grid (Y)**



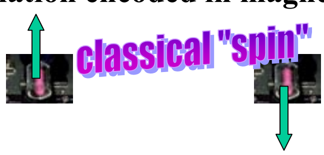
**Projection on one grid erases the other grid!**

**Projected on horizontal grid (Z)**

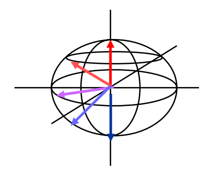


## Communication network: arrays of memory pixels connected with fiber links

**Information encoded in magnetization**



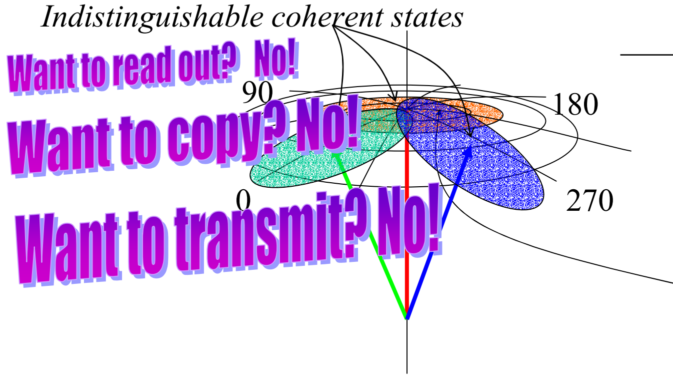

**Quantum spin**

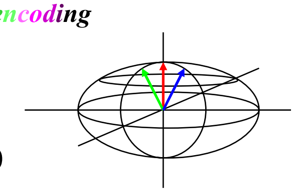


## Spin memory with Coherent Spin States

***Quasi-continuous encoding***

*Indistinguishable coherent states*



$$\delta J_z \delta J_y = 2N$$

•Densely coded states are impossible to read
but possible to transfer via teleportation-like protocols
*Teleportation proposal* – Bennett et al '93
*experiments with light*– Innsbruck'97, Rome'97, Caltech'98

Distributed quantum networking and quantum memory for light can be achieved via atomic teleportation and light-to-atoms state teleportation

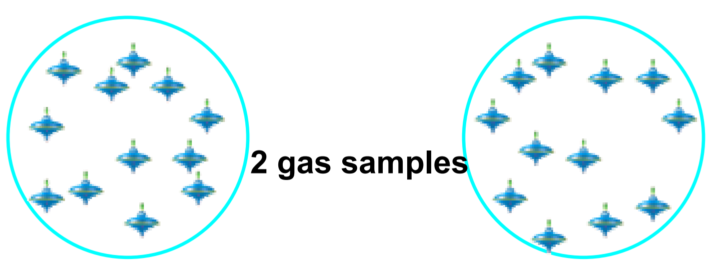
Needed: distant long-lived atomic entangled objects

State-of-the-art experiments:
4 entangled ions in a trap (NIST)
2-3 entangled atoms (ENS, MPQ)
Several atoms in a molecule (NMR)
SQUIDS (?)

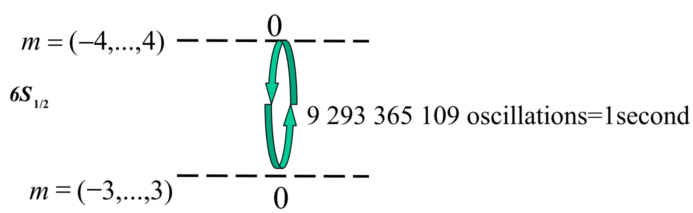
Closely positioned objects – entanglement via localized interactions

---

$10^{12}$ atoms in each ensemble

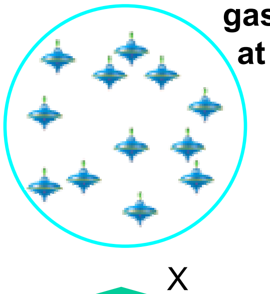



Cesium

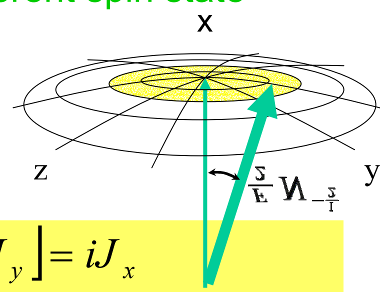


## Macroscopic spin ensemble – coherent spin state

gas sample
at room T

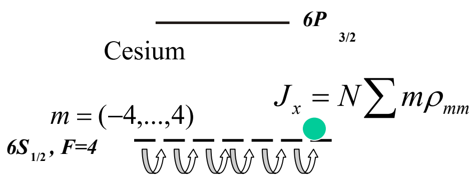


State preparation (optical pumping)




$$\left[J_z, J_y\right] = iJ_x$$

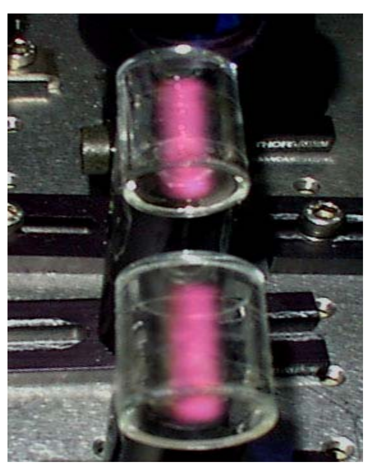
$$\delta J_z^2 = \delta J_y^2 = \tfrac{1}{2} J_x = \tfrac{F}{2} N$$




Coherences determine $J_z, J_y$

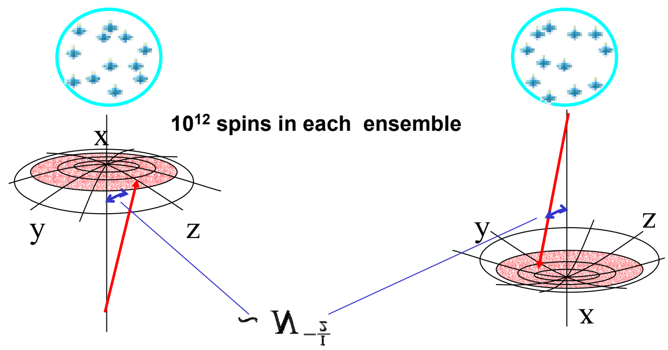
---

***Entangled state of 2 macroscopic objects***



$$\langle (J_{z1} + J_{z2})^2 \rangle + \langle (J_{y1} + J_{y2})^2 \rangle < 2J_x$$

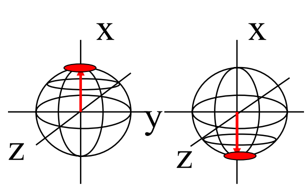
***Experimental long-lived entanglement of two macroscopic objects.***
B. Julsgaard, A. Kozhekin, and E. S. Polzik,
**Nature**, **413**, 400 (2001).



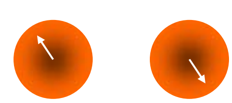

Spins which are "more parallel" than that are entangled

Total $z$ and $y$ components of two ensembles with equal and opposite macroscopic spins can be determined simultaneously with **arbitrary accuracy**

$$\left[\hat{J}_{z1} + \hat{J}_{z2}, \hat{J}_{y1} + \hat{J}_{y2}\right] = i(J_{x1} + J_{x2}) = i(J_x - J_x) = 0$$




Top view:



Parallel spins must be entangled

Therefore entangled state with

$$\delta\left(\hat{J}_{z1} + \hat{J}_{z2}\right)^2 + \delta\left(\hat{J}_{y1} + \hat{J}_{y2}\right)^2 \Rightarrow 0$$

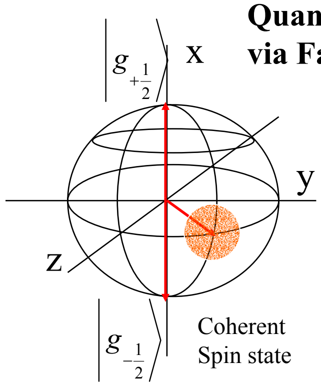
Can be created by a measurement

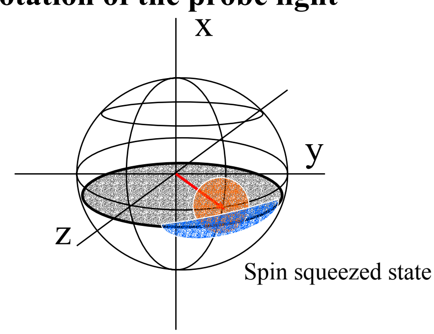
How to measure the total spin projections?

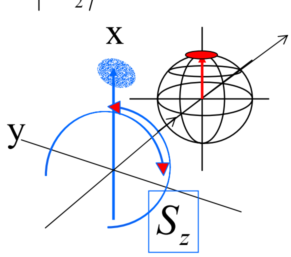
- Send off-resonant light through two atomic samples
- Measure polarization state of light

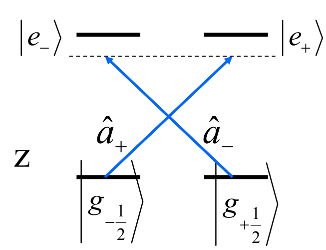
Duan, Cirac, Zoller, EP 2000

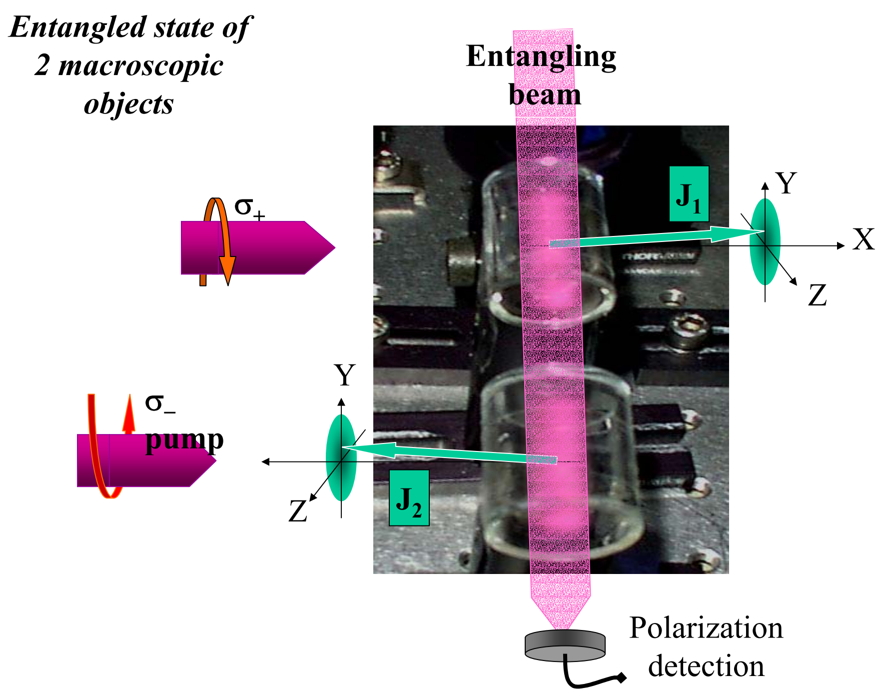
**Quantum non-demolition measurement of spin via Faraday rotation of the probe light**



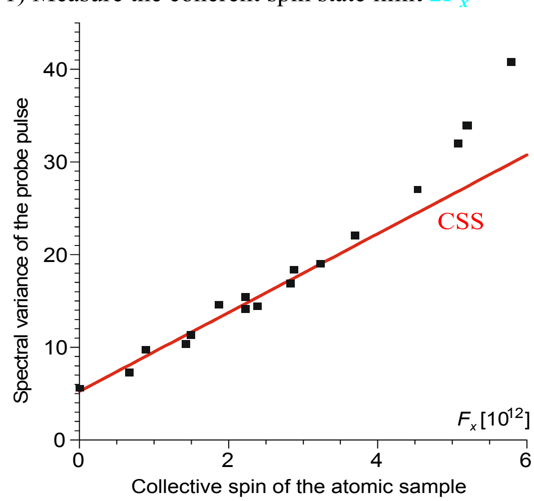



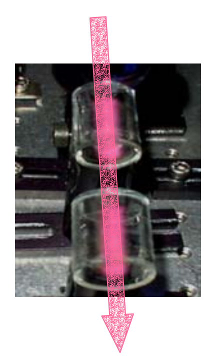



Entanglement criterion:

$$\mathrm{Var}(F_{y,1}+F_{y,2}) + \mathrm{Var}(F_{z,1}+F_{z,2}) < 2F_x$$

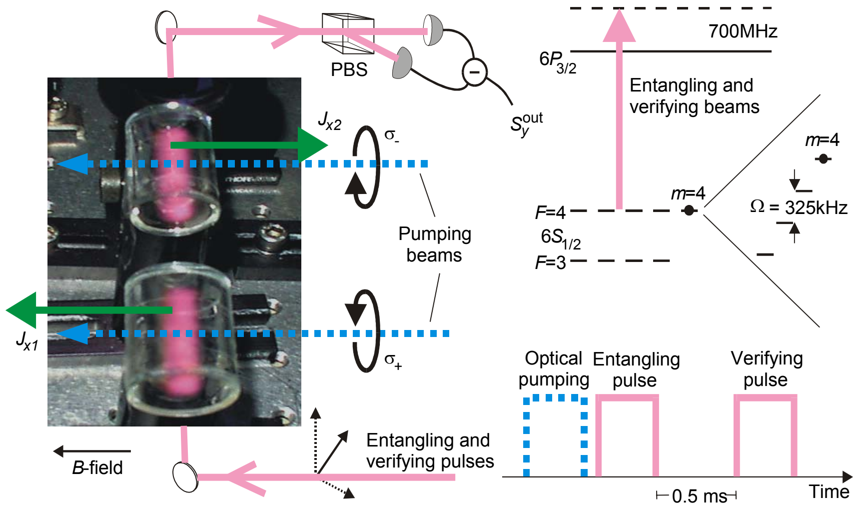
1) Measure the coherent spin state limit $2F_x$



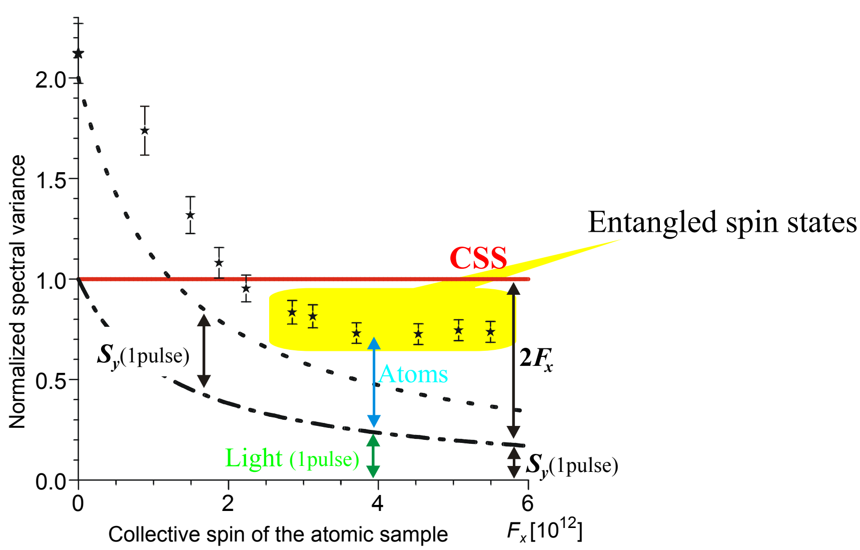



For the coherent spin state:

$$\mathrm{Var}(F_{y,1}+F_{y,2}) + \mathrm{Var}(F_{z,1}+F_{z,2}) = 2F_x$$




2) Create entangled state and measure the state variance




*Julsgaard, Kozhekin, EP*

***Nature* 413**, 400 (2001).

## Decoherence issues

- only collective spin states are entangled
- particles are indistinguishable - high symmetry of the system –
  - robustness against losses. This is not a Schrodinger's cat made of $10^{12}$ atoms!
- no free lunch:
  limited capabilities compared to ideal maximal entanglement

Sources of decoherence:

stray magnetic fields — decoherence time 3 milliseconds

collisions — decoherence time 1-2 milliseconds

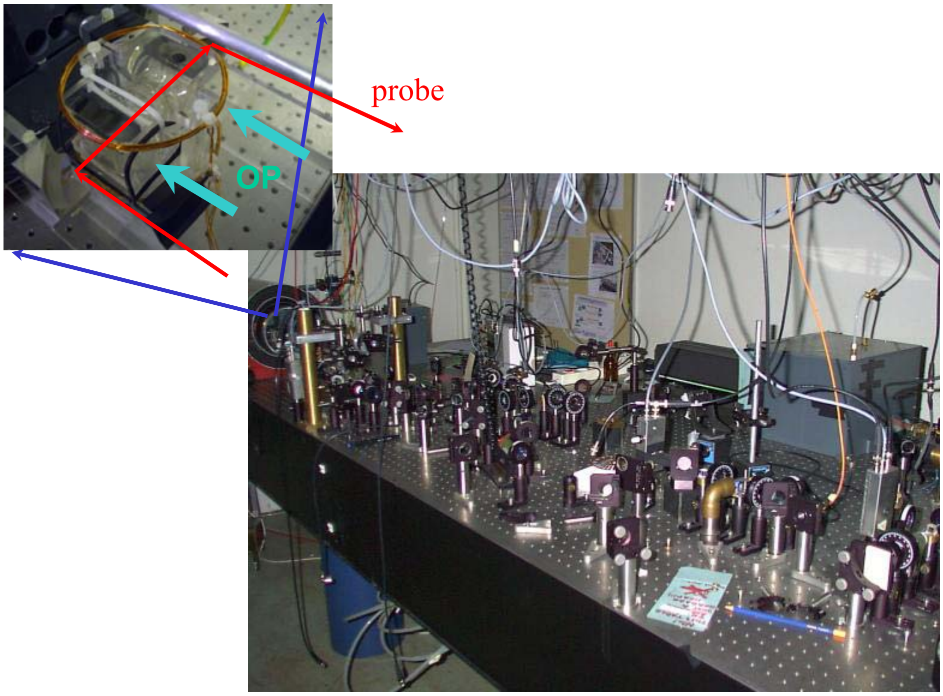
## Phylosophical issues...

Phylosophical issues...

Realism – two noncommuting spin components cannot be measured – therefore do not exist? But can be entangled

Non-locality – two entangled macroscopic objects can be used to violate Bell-type inequalities via distillation

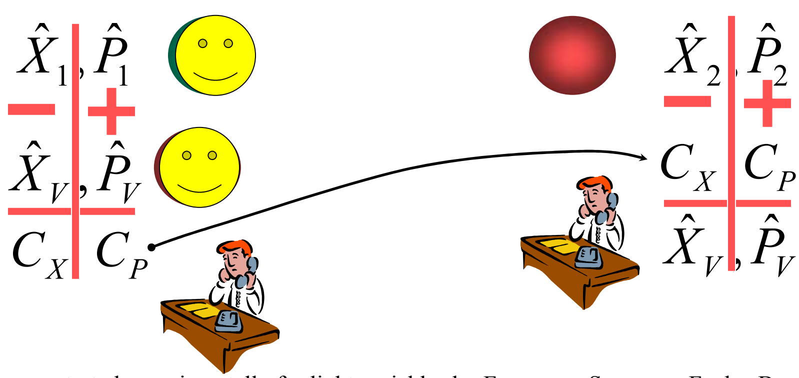
All entangled Gaussian two-mode states are distillable (Giedke et al)




**Teleportation principle (continuous variables)**

$$[X,P]=i,[X_1-X_2,P_1+P_2]=0$$

L.Vaidman

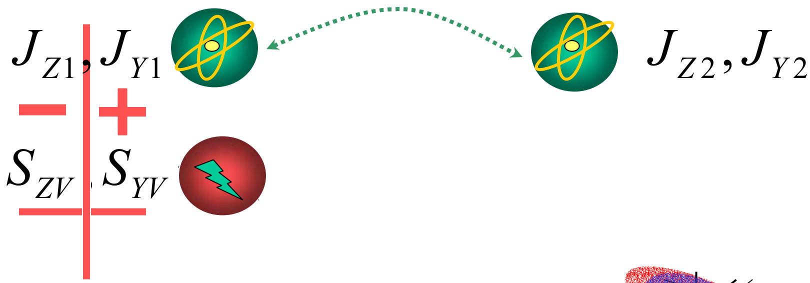



Demonstrated experimentally for light variables by Furusawa, Sørensen, Fuchs, Braunstein Kimble, Polzik. *Science 1998*

Einstein-Podolsky-Rosen entangled state

$$X_1 - X_2 = 0, P_1 + P_2 = 0$$

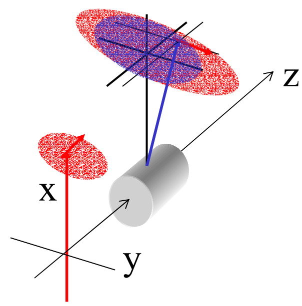
## Light-to-Atoms Teleportation




$$J_y^{out} = J_y^{in} + kS_z$$
$$S_y^{out} = S_y^{in} + kJ_z$$

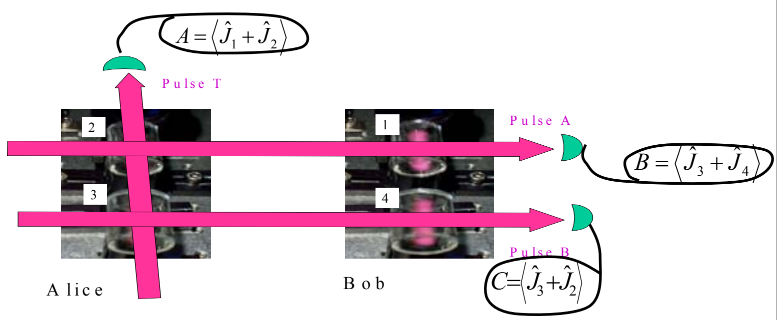
k=1

$$\hat{X}^{out}_{light} = \hat{X}^{in}_{light} + \hat{P}^{in}_{atoms}$$




Kuzmich, EP 2000

## Teleportation of an entangled atomic state




- Every measurement changes the single cell spin, BUT does not change the measured sum
- Every pulse measures both y and z components of the sum – entanglement is created

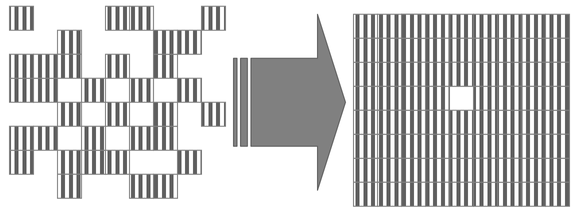
To complete teleportation of entanglement onto cell 1 and cell 4:
rotate spin 4 by A+B+C:

$$\hat{J}_4^{Tel} = \hat{J}_4 - A - B + C = \hat{J}_4 - \hat{J}_1 - \hat{J}_2 - \hat{J}_3 - \hat{J}_4 + \hat{J}_2 + \hat{J}_3 = -\hat{J}^{Tel}{}_1$$

A secret message is encoded in one of the grids

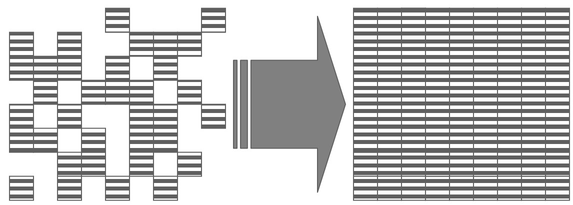
Two Entangled Spin Samples

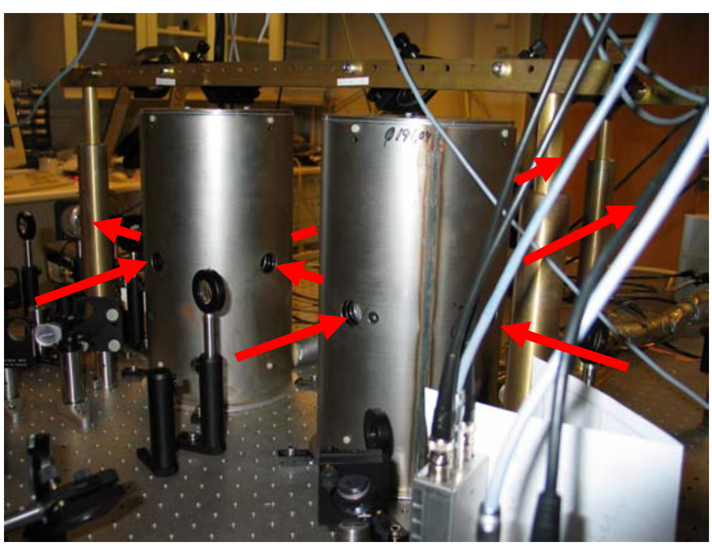
Projected on vertical grid (Y)



Projection on one grid erases the other grid!

Projected on horizontal grid (Z)

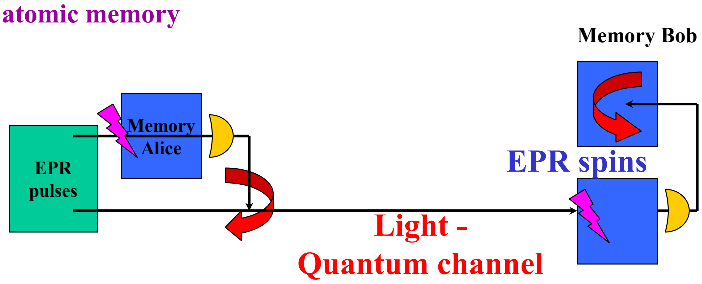




## Communication networks based on continuous spin variables

**Operation:**
**Storage of light and read-out from atomic memory**

**Resources: local entanglement**




**Symbols :**
- polarization rotation
- detection of light

**Input-Output interaction:**
<u>free space</u> off-resonant dipole interaction

**Continuous variables:**
- polarization state of light
- spin state of atoms

## Scalability – an array of dipole traps or solid state implementation – quantum holograms

Requirements:
- Long-lived 2 or more level state of a single particle
- The ability to optically pump the system into one of these states
- Dipole coupling to light
- Optically dense medium

Example: pencil-shaped Rubidium BEC
4 microns by 100 microns
$10^5$ atoms, $10^7$ photons per pulse

Quantum state engineering in Atoms clocks

Cesium atoms