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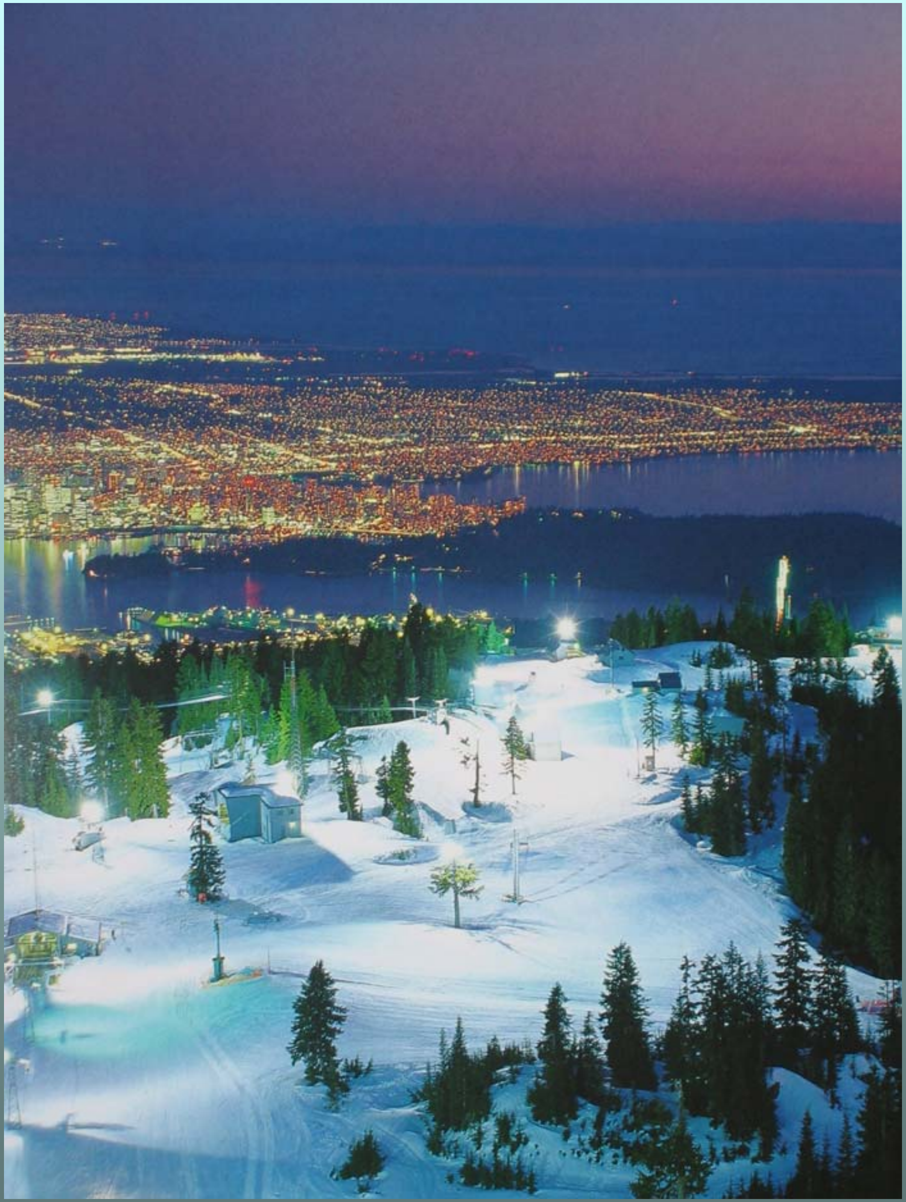


Pacific Institute
for
Theoretical Physics

Limitations of EFFECTIVE HAMILTONIANS- Dissipation and Decoherence

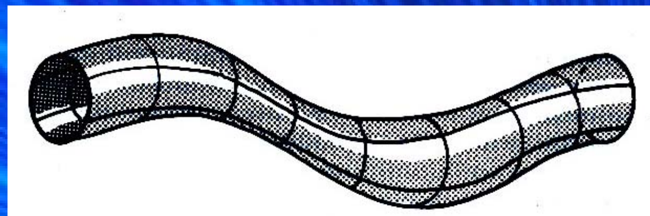
P.C.E. Stamp   Arrows of Time 2004 (Outing Lodge, Dec 2004)

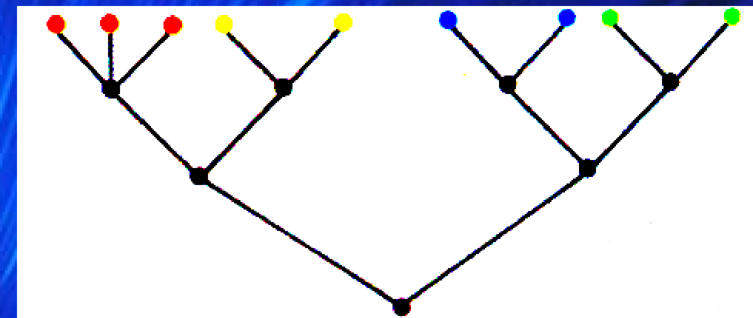
Some key notions for
Thermodynamic Arrow
Coarse/fine-graining
"Exact" solution

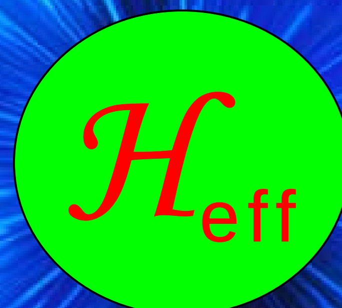
Quantum Arrow
Unitary evolution
"exact" solution
decoherence

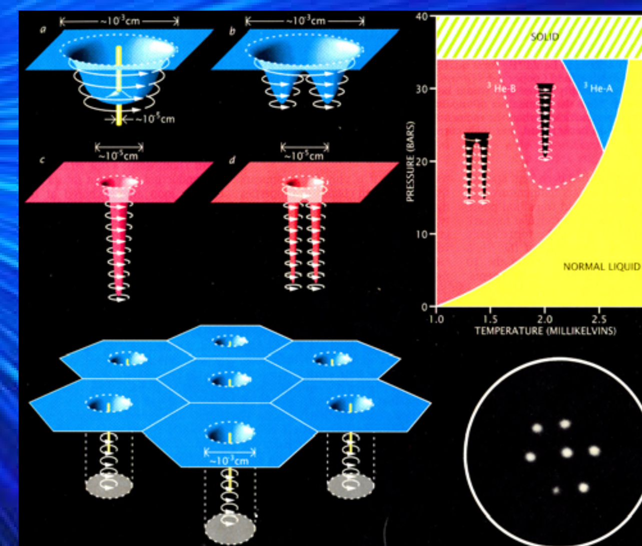
All these notions (& others crucial to discussion of Arrows), rely on notion of
EFFECTIVE HAMILTONIAN

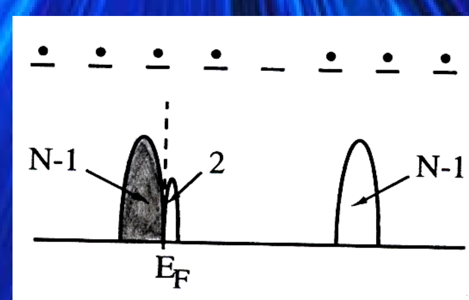
I. Limitations of EFFECTIVE HAMILTONIANS



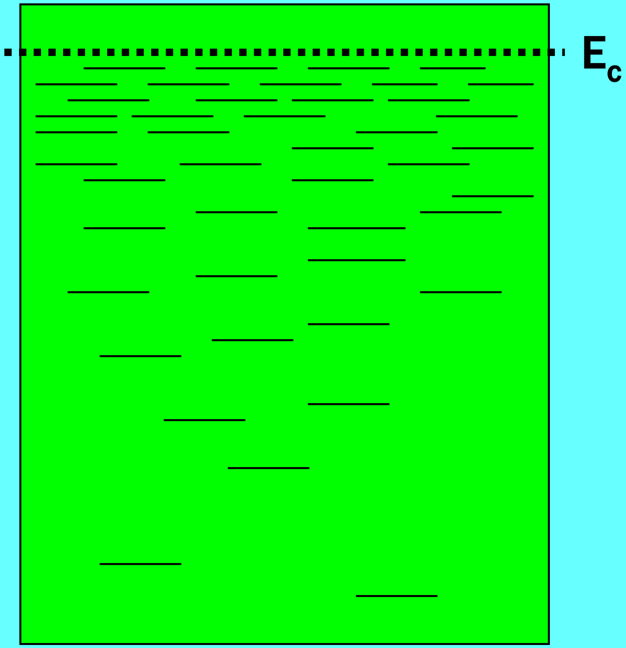






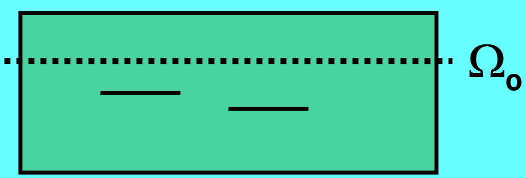


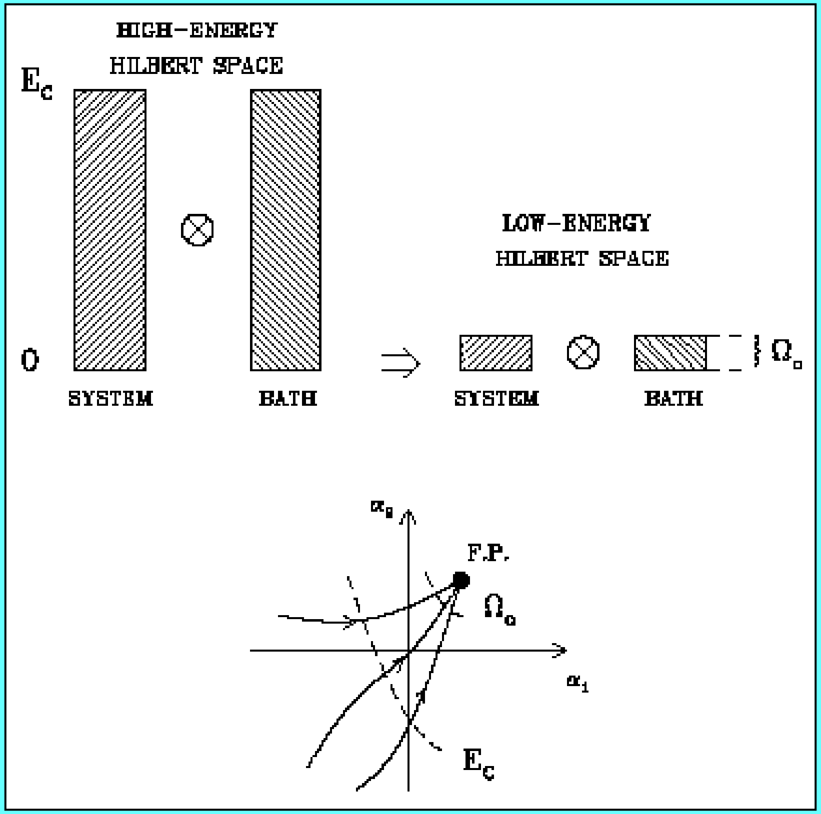
Orthodox view of $\mathcal{H}_{eff}$



Scale out High-E modes

“Renormalisation”



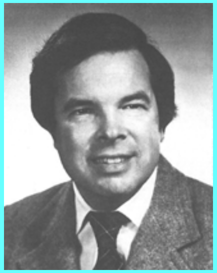


$\mathcal{H}_{eff}\,(E_c) \rightarrow \mathcal{H}_{eff}\,(\Omega_o)$

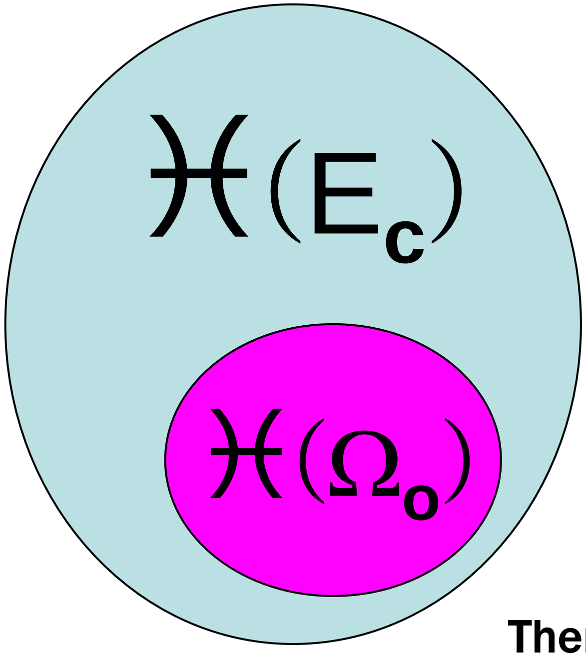
$|\psi_i\rangle\ \ H_{ij}(E_c)\ \ \langle\psi_j| \quad \rightarrow \quad |\phi_\alpha\rangle\ \ \mathcal{H}_{\alpha\beta}(\Omega_o)\ \ \langle\phi_\beta|$

Flow of Hamiltonian & Hilbert space with UV cutoff

The RG mantra is: RG flow
fixed points
low-energy $\mathcal{H}_{eff}$
universality classes



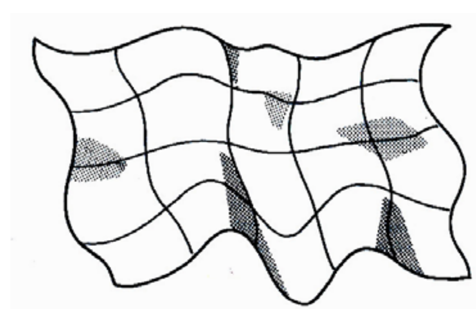
MORE ORTHODOXY



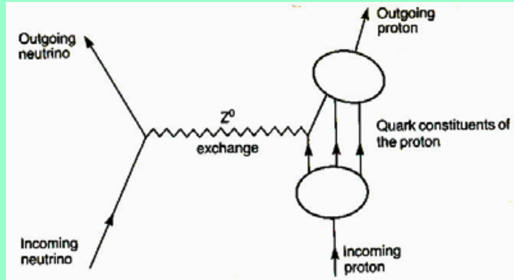
Continuing in the orthodox vein, one supposes that for a given system, there will be a sequence of Hilbert spaces, which the effective Hamiltonian and the other relevant physical operators these are effective operators) are defined.

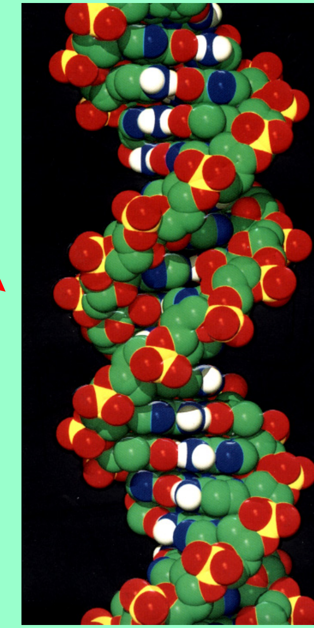
Then, we suppose, as one goes to low energies we approach the '*real vacuum';* the approach to the fixed point tells us about the excitations about this vacuum. This is of course a little simplistic- not only do the effective vacuum and the excitations change with the energy scale (often discontinuously, at phase transitions), but the effective Hamiltonian is in any case almost never one which completely describes the full N-particle states.

Nevertheless, most believe that the basic structure is correct - that the effective Hamiltonian (*& note that ALL Hamiltonians or Actions are effective*) ***captures all the basic physics***



SIDENOTE on ‘EMERGENCE’ vs ‘REDUCTIONISM’





The reductionist view is that all matter can be understood in terms of its ‘basic constituents’. is an atomistic point of view.

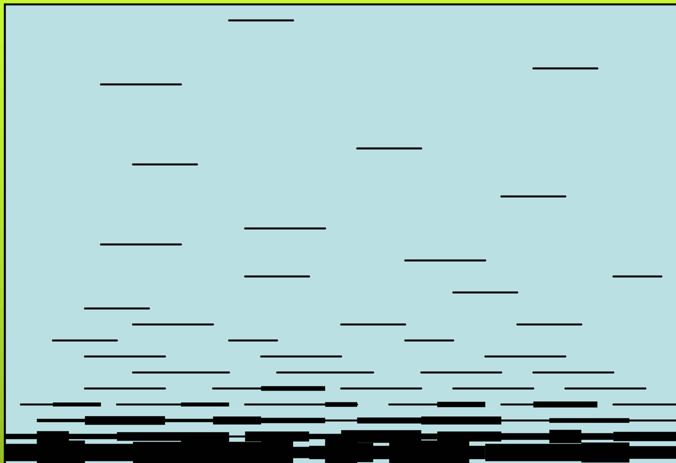
The ‘emergence’ point of view says that structures of matter at higher levels, & in more complex systems, CANNOT be understood in terms of basic constituents- that they have properties that are ineluctably ‘complex’ & which cannot ever be understood in terms of elementary constituents, even in principle.

NB1: *Many if not most ‘emergence’ believers still nevertheless assume that matter is composed of ‘bits’ (the ‘lego’ philosophy, or ‘soft’ emergence)*

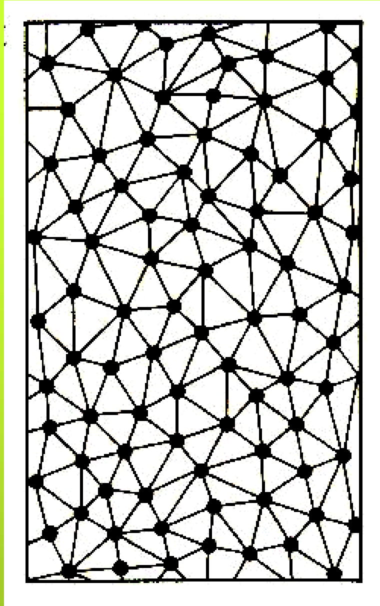
NB2: *In fact there is no obvious end in sight to the long road towards ‘elementary constituents’. Nature may just be ‘wheels within wheels..’.*

1^{ST} CONUNDRUM- the 'GLASS'

States in a glass- piled up at low E



The simple picture of excitations perched above a vacuum gets a shock when we consider Glasses - systems with disorder & 'frustrating interactions'. We are surrounded by these! States pile at low energy, but these can't communicate with each other.

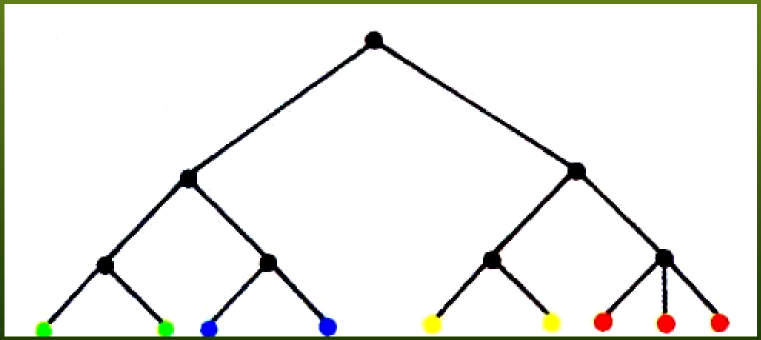


Frustrating interactions

What this means is that no matter what energy or temperature one is working the ground state of the spin glass Hamiltonian is meaningless. **At finite the system can never reach the ground**

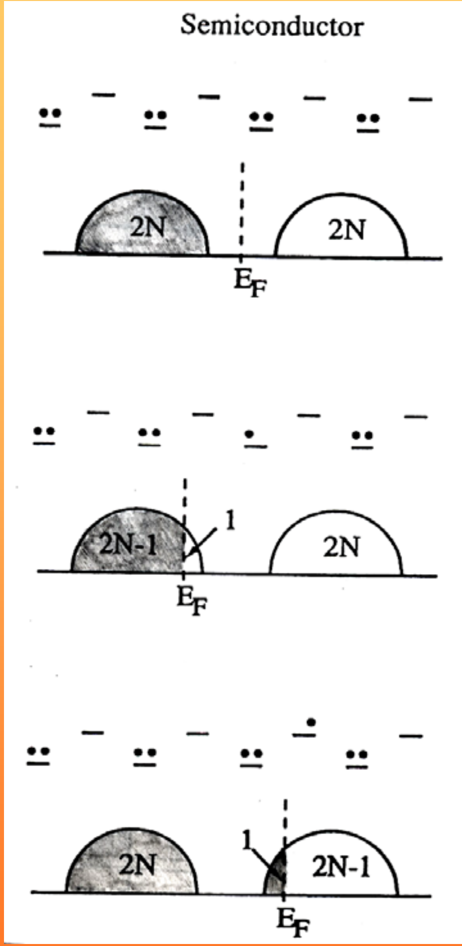
'Frustration' means that at low energy, any local change must re-organize simultaneously a vast number of states. This forces the Hilbert space of the effective Hamiltonian to have an 'ultrametric' geometry.

state, and the finite-T Hilbert space is from any ground state. At zero-T, the system splits subspaces that can never communicate with each other. Thus the effective vacuum & its structure physically meaningless. A glass can only be defined its dynamic (non-equilibrium) properties.



'Ultrametric geometry' of a glass Hilbert space

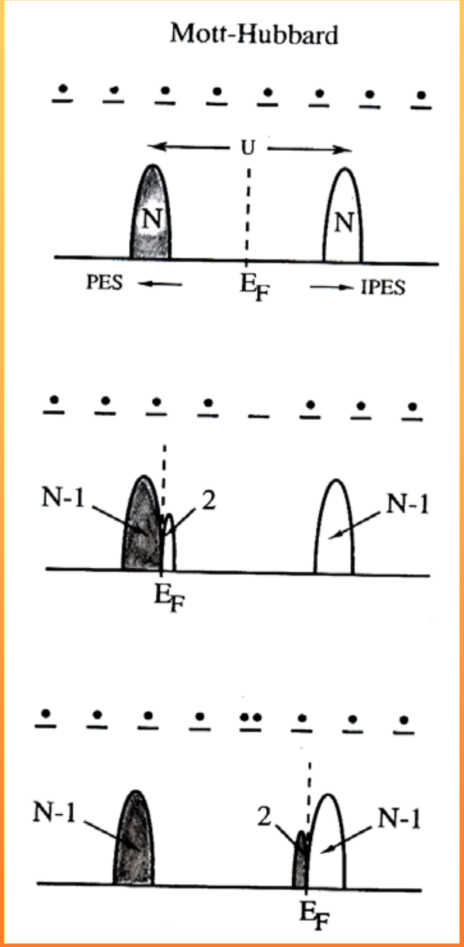
2ND CONUNDRUM- the HUBBARD MODEL



The ‘standard model’ of condensed matter physics for a lattice system is the ‘Hubbard model’, having effective Hamiltonian at electronic energy scales given by

$$H = -t \sum_{\langle i,j \rangle} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

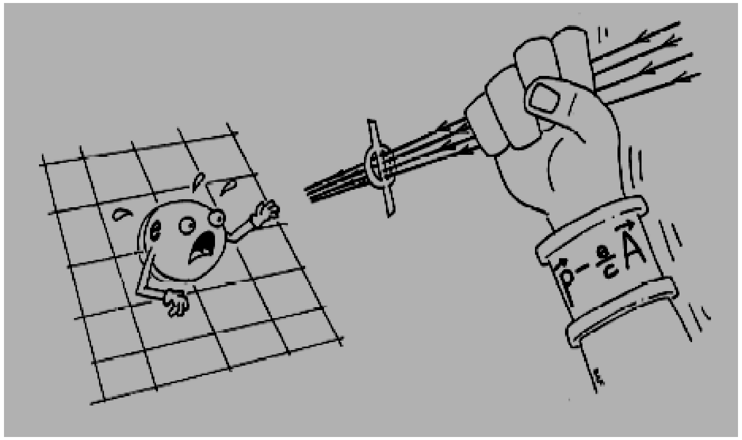
This apparently simple Hamiltonian has very bizarre properties. Suppose we try to a low energy effective Hamiltonian, valid near the Fermi energy- eg., when the is near “half-filling”. We therefore assume UV Cutoff much smaller than the between the Mott-Hubbard sub-bands (we assume that U > t).



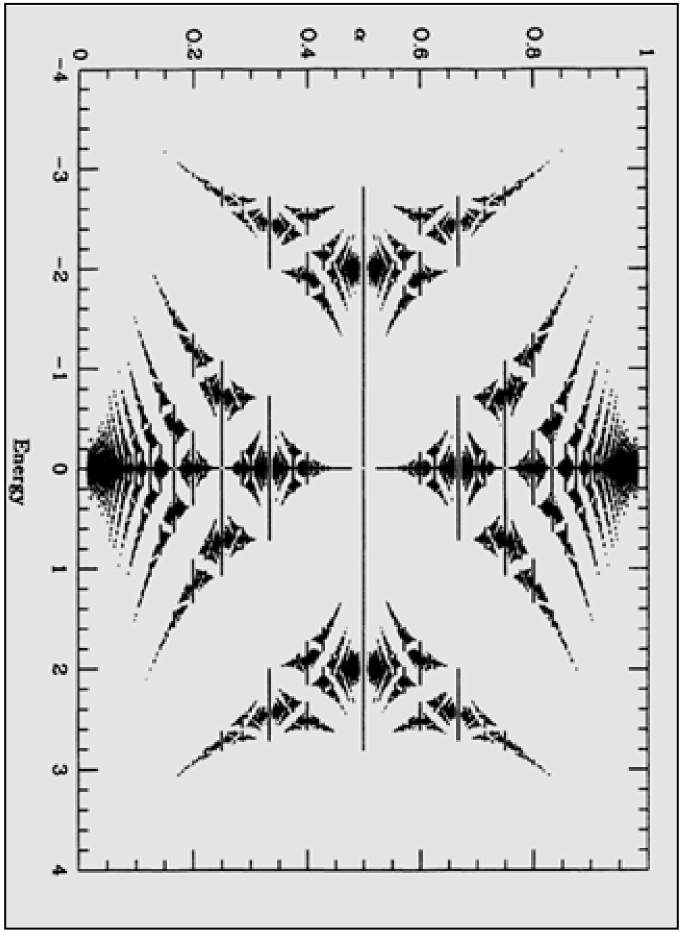
The problem is that this appears to be impossible. Any attempt to write an effective Hamiltonian around the Fermi energy must deal with ‘spectral weight transfer’ from the other Hubbard sub-band- which is very far in energy from the Fermi energy. Thus we cannot disentangle high- and low-energy states. This is sometimes called UV/IR mixing.

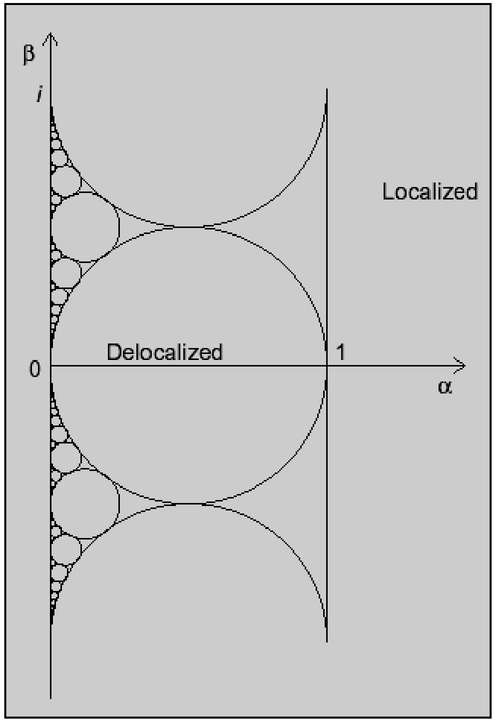
3^{RD} CONUNDRUM- TOPOLOGICAL FIELD THEORIES

RIGHT: A statistical flux attaches itself to an electron to make an 'anyon'- here on a lattice



It is now apparent that the basic theories required in string theory & quantum field theory will be in nature. Theories like Chern-Simons theories have anyonic excitations & topologically different but degenerate vacua. In string theory it is hard to get rid of tachyons, which create the analogue of a lattice potential the strings, leading to the complexity of the famous 'WAH' butterfly (left); once fluctuations & coupling to bosons are added, we get a fractal phase diagram →.





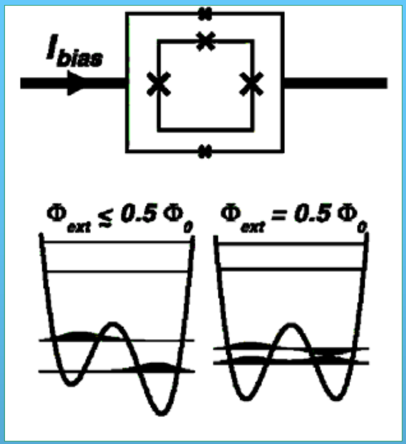
A key feature of all of these theories, and of any non-commutative gauge theory, is the same UV/IR mixing we saw in the Hubbard model- ie., no well-defined effective low-E action or Hamiltonian.

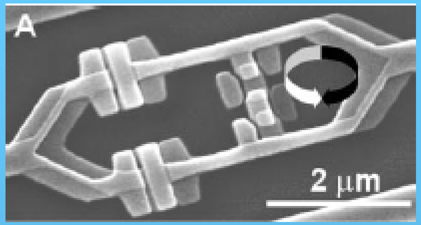
II. ENTANGLEMENT & DECOHERENCE

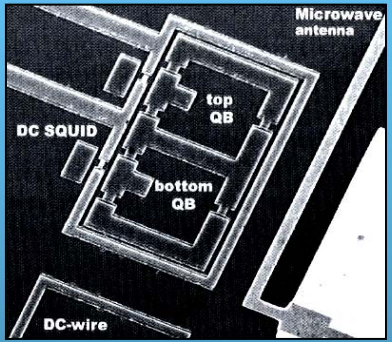
Take 2 systems (A & B) that have once interacted, but are now separated, & whose states are still entangled. It makes no sense to write down 2 separate effective Hamiltonians, one for each- a complete description of A can not be given by a Hamiltonian which operates only in the Hilbert space of the variables of A, no matter what the cut-offs may be.

SOLID-STATE QUBITS: Theoretical Designs & Experiments

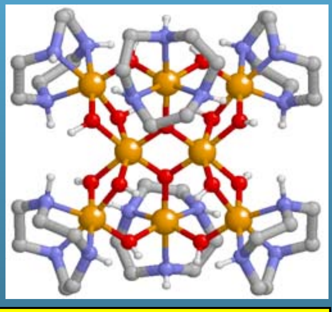
Here are a few: **(1) Superconducting SQUID qubits (where qubit states are flux states); all parameters can be controlled.**

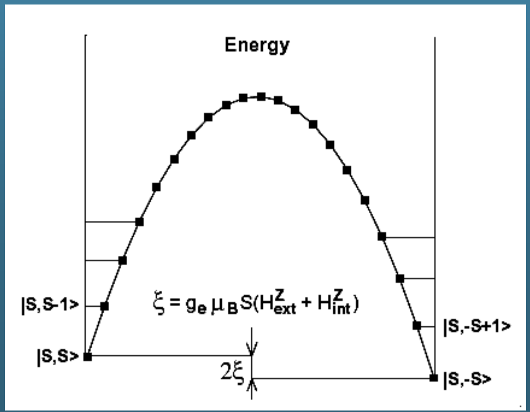


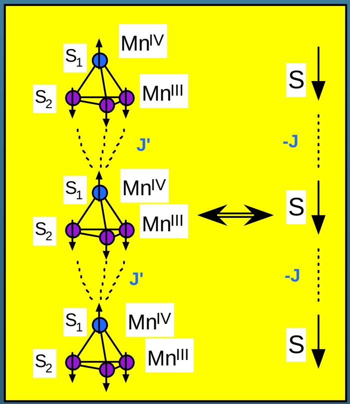




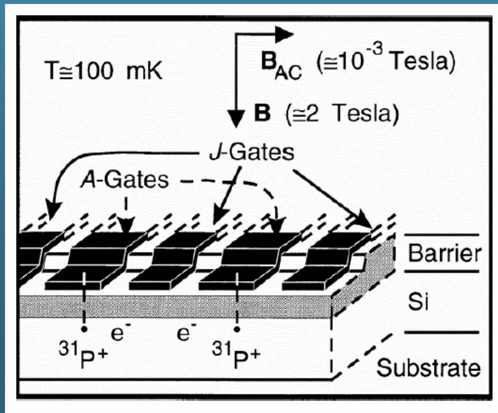
(2) Magnetic molecule qubits (where an easy axis anisotropy gives 2 low energy spin states, which communicate via tunneling, and couple via exchange or dipolar interactions. Control of individual qubit fields is easy in principle- interspin couplings less so…







(3) Spins in semiconductors (or in Q Dots). Local fields can be partially controlled, & the exchange coupling is also controllable.



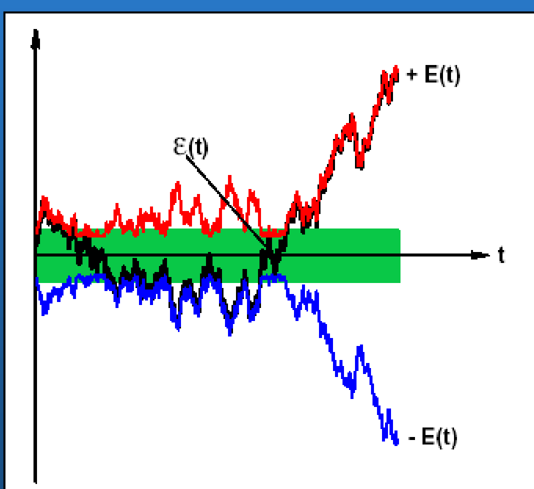
DECOHERENCE DYNAMICS from an EFFECTIVE $\mathcal{H}$

Consider the following $\mathcal{H}_{\mathbf{eff}}$:

$$H(\Omega_o) = \{ [\Delta\tau_+ \exp(-i \Sigma_k\, \alpha_k.\sigma_k) + \text{H.c.}] + \varepsilon_o \tau_z \qquad \textbf{(qubit)}$$
$$+ \tau_z\, \omega_k.\sigma_k + h_k.\sigma_k \qquad \textbf{(bath spins)}$$
$$+ \text{ inter-spin interactions}$$

At first glance a solution of this seems very forbidding. However it turns out one can solve for the reduced density matrix of the central spin in all interesting parameter regimes- & the decoherence mechanisms are easy to identify:

(i) Noise decoherence: Random phases added to different Feynman paths by the noise field.

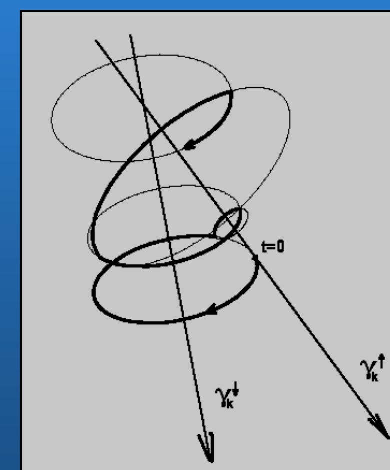


Noise decoherence source

(ii) Precessional decoherence: the phase accumulated by bath spins between qubit flips.

(iii) Topological Decoherence: The phase induced in the bath spin dynamics by the qubit flip itself

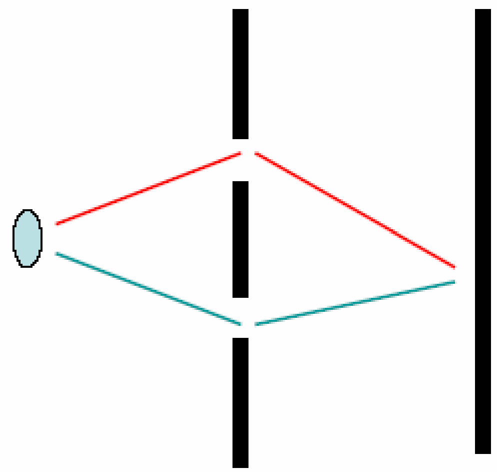
USUALLY PRECESSIONAL DECOHERENCE DOMINATES



Precessional decoherence

This leads to the very interesting result that one can have decoherence dominated by processes which cause little or no dissipation

3rd PARTY DECOHERENCE

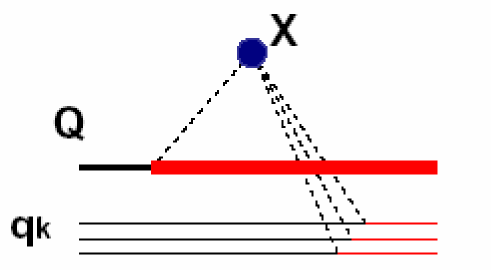


This is fairly simple- it is decoherence in the dynamics of a system A (coordinate Q) caused by *indirect* entanglement with an environment E- the entanglement is achieved via a 3rd party B (coordinate X).

Ex: Buckyball decoherence

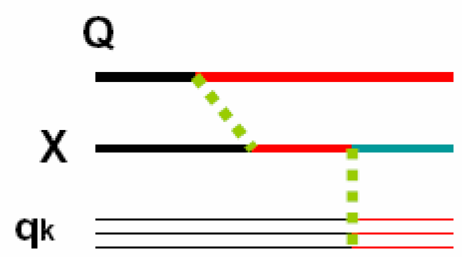
Consider the 2-slit buckyballs. The coordinate Q of the buckyball does not couple directly to the vibrational {q_k} of the buckyball- by definition. However BOTH couple to the slits in system, in a distinguishable way.

Note: the state of the 2 slits, described by a coordinate X, is irrelevant- not need to change at all. We can think of it as a scattering potential, by a system with infinite mass (although recall Bohr's response to which includes the recoil of the 2 slit system). It is a PASSIVE 3rd party.



ACTIVE 3rd PARTY: **Here the system state correlates with the 3rd party, which then goes on to environment to correlate with Q. We can also think of the 3rd party X as PREPARING the states of both and environment. Alternatively we can think of the system and the environment as independently state of X. In either case we see that system and environment end up being correlated/entangled.**

Note the final state of X is not necessarily relevant- it can be changed in an arbitrary way after the 2nd interaction of X. Thus X need not be part of the environment. Note we could also have more than one intermediary- ie., X, Y, etc.- with correlations/entanglement are transmitted along a chain (& they can wiped out before the process is finished).



REMARKS

R1: **One could argue that despite all this, the idea that we can still think of matter as made of 'elementary constituents' (the lego philosophy) is nevertheless intact.**

If so, one would like to know how to formulate this in physical theory- at the present time the fundamental formulation of the properties of any physical system is in terms of an effective Hamiltonian or effective action- and this poses the problems discussed herein.

R2: **Despite the literature and the beliefs of reductionists in the particle physics community, these are not just problems of condensed matter physics- they arise in high energy physics as well.**

Notice that whereas the IR / UV mixing comes in in condensed matter systems typically in the presence of a lattice, this is not necessary- in non-commutative gauge theory or open string theory there is no lattice. In any case- since all Hamiltonians are effective, the problems we address are generic to all 'many-body' quantum theories, in condensed matter, particle theory, or quantum gravity.

R3: **Some of the problems discussed so far exist in a classical theory. However features like IR / UV mixing seem to be quantum mechanical. And of course, the ineluctable role of entanglement is entirely a QM feature.**

Note that some formulations of QM make the description of any quantum system dependent on macroscopic objects, and their entanglement with them (eg., Copenhagen/Bohr).

R4: **The idea of 'Emergence' results, in what we discuss above, from the impossibility mapping from one effective Hamiltonian to another, even for the same system. 'Soft' is basically the logo philosophy. Hard emergence denies even this, and says that the properties of the microsystem may even be dependent on those of the macrosystem.**

TALK: see http://physics.ubc.ca/~stamp