Spin and charge response in phase fluctuating d-wave superconductor

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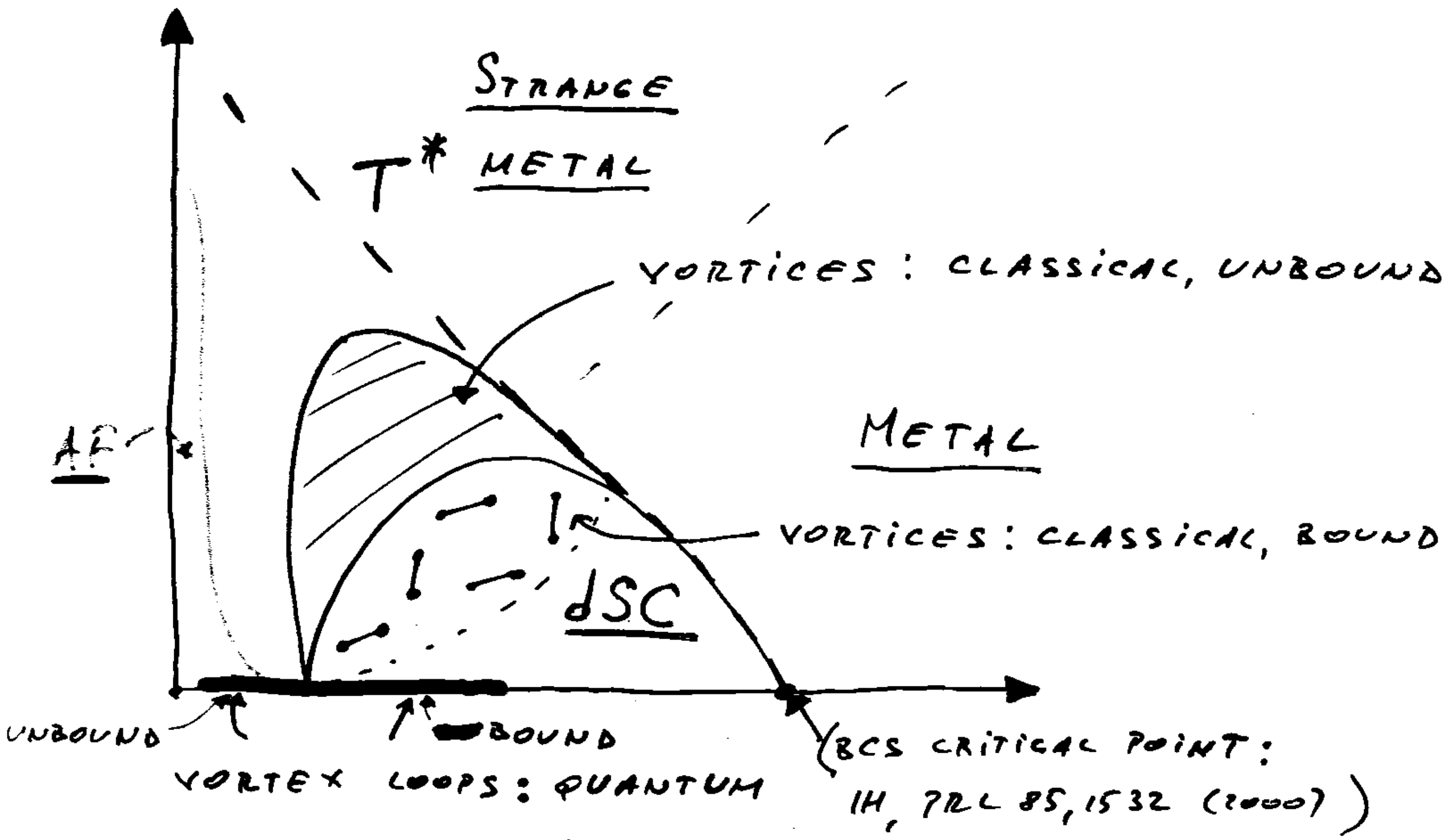
PHASE DIAGRAM: high-T_c SC



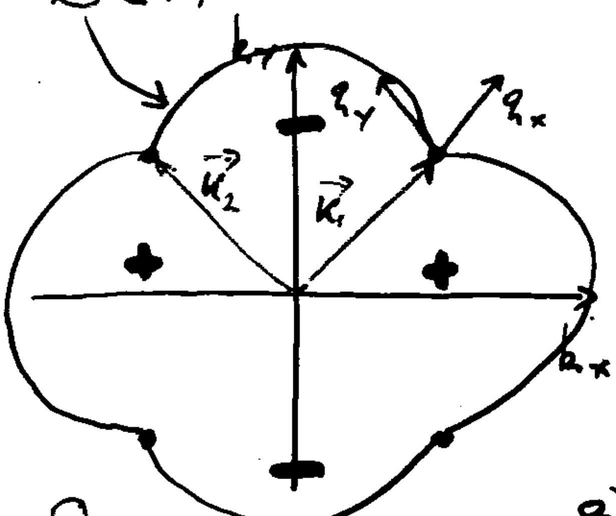
AT $T \ll T^*$: STRONGLY PHASE FLUCTUATING D-WAVE SUPERCONDUCTOR

d-WAVE SUPERCONDUCTOR (IN 2D)

- QUASIPARTICLES ARE WELL DEFINED LOW-ENERGY EXCITATIONS :

$$S_0 = T \sum_{\vec{k}, \omega_n, \sigma} \Big\{ \big(i\omega_n - \xi(\vec{k}) \big) C_\sigma^+(\vec{k},\omega)\, C_\sigma(\vec{k},\omega) - \Delta(\vec{k})\, C_\uparrow^+ C_\downarrow^+ + \text{h.c.} + \mathcal{O}(C^4) \Big\}$$

$\Delta(\vec{k})$ — d-WAVE SYMMETRY



$$\vec{K}_1 \approx \left(\frac{\pi}{2}, \frac{\pi}{2} \right)$$

SYMMETRY:

$|\uparrow\rangle \to \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

a) SPIN ROTATIONS

b) TRANSLATIONS

$$C_\sigma(\vec{k},\omega) \to e^{i\vec{k}\cdot\vec{R}}\, C_\sigma(\vec{k},\omega)$$

EXACT!

c) FOR $|\vec{k}| \simeq \pm \vec{K}_{1,2}$, LINEARIZE: $\vec{k} = \vec{K}_i + \vec{q}$

$$\xi(\vec{k}) = v_F\, q_x + \mathcal{O}(q^2)$$
$$\Delta(\vec{k}) = v_\Delta\, q_y + \mathcal{O}(q^2)$$

THEN:

$$\left[\begin{array}{l}
c_\uparrow^+(\vec{k},\omega) \to i\, c_\downarrow(-\vec{k}+2\vec{K},\omega) \\
c_\downarrow^+(\vec{k},\omega) \to i\, c_\uparrow(-\vec{k}+2\vec{K},\omega) \\
c_\uparrow^+(\vec{k}-2\vec{K},\omega) \to i\, c_\downarrow(-\vec{k},\omega) \\
c_\downarrow^+(-\vec{k},\omega) \to i\, c_\uparrow(\vec{k}-2\vec{K},\omega)
\end{array}\right]$$

ALSO BECOMES A SYMMETRY!

AT LARGE DISTANCES ($|\vec{q}| \ll 1$) SYSTEM HAS A LARGER SYMMETRY! (dSC is "CRITICAL")

"CHIRAL" SYMMETRY:

DIRAC REPRESENTATION (IH, PRL 88, 047006 (2002))

$$\Psi_1(\vec{q}) = \begin{pmatrix} c_\uparrow(\vec{K}_1+\vec{q},\ \omega) \\ c_\downarrow^\dagger(-\vec{K}_1-\vec{q},\ -\omega) \\ \hdashline c_\uparrow(-\vec{K}_1+\vec{q},\ \omega) \\ c_\downarrow^\dagger(\vec{K}_1-\vec{q},\ -\omega) \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{NAMBU SPINOR} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{NAMBU SPINOR} \end{matrix}$$

THEN:

$$S_0 = \int d^2\vec{r}\, d\tau \Big\{ \bar{\Psi}_1 (\gamma_0 \partial_\tau + v_F \gamma_1 \partial_x + v_\Delta \gamma_2 \partial_y) \Psi_1 + \begin{pmatrix} 1 \leftrightarrow 2 \\ x \leftrightarrow y \end{pmatrix} +$$

$$+ \mathcal{O}\left(\bar{\Psi}\partial^2\Psi,\ (\bar{\Psi}\Psi)^2 \right) \Big\}$$

WHERE: $\bar{\Psi}_1 = \Psi_1^\dagger \gamma_0$, AND

$$\gamma_0 = \begin{array}{c|c} & I \\ \hline I & \end{array}, \quad \gamma_1 = \begin{array}{c|c} & i\sigma_3 \\ \hline -i\sigma_3 & \end{array}, \quad \gamma_2 = \begin{array}{c|c} & -i\sigma_1 \\ \hline i\sigma_1 & \end{array}.$$

OF COURSE: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\mu = 0,1,2$

("CHIRAL" REPRESENTATION)

SYMMETRY:

$$\Psi_1 \longrightarrow e^{i \sum_{i=1}^{4} \theta_i G_i} \Psi_1$$

GENERATORS:

$$G \in \{ \underset{\downarrow}{\underset{U(1)}{I}}, \underbrace{\gamma_5, \gamma_3, i\gamma_3\gamma_5}_{SU_c(2)} \}$$

$U(1) \times SU_c(2)$

PER DIRAC FIELD. IN TOTAL:

$$U(2) \times U(2)$$

(IF $v_F \neq v_\Delta$).

ACTION OF GENERATORS:

1) TRANSLATIONS, $c_\sigma(\vec{k}, \omega) \rightarrow e^{i\vec{k}\cdot\vec{R}} c_\sigma(\vec{k}, \omega)$, IS NOW

$$\Psi_1(\vec{r}) \longrightarrow e^{i(\vec{K}_1 \cdot \vec{R})\gamma_5} \Psi_1(\vec{r} + \vec{R})$$

2) ROTATIONS $\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} \rightarrow e^{i\varphi \sigma_z} \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$, ARE NOW

$$\Psi_1(\vec{v}) \rightarrow e^{i\varphi I} \Psi_1(\vec{v})$$

3) CHIRAL TRANSFORMATION:

$$\gamma_3 = \left(\begin{array}{c|c} & i\tau_2 \\ \hline -i\tau_2 & \end{array}\right), \qquad i\gamma_3\gamma_5 = \left(\begin{array}{c|c} & \tau_2 \\ \hline \tau_2 & \end{array}\right)$$

BROKEN BY SECOND-ORDER DERIVATIVES, AND QUARTIC TERMS! (BUT RESTORED AT LOW ENERGIES!)

BREAKING OF CHIRAL SYMMETRY

"MASS": $m\bar{\Psi}\Psi$

WOULD BREAK γ_3 AND γ_5.

$$m\bar{\Psi}_i\Psi_i = m\sum_{i=1,2}\int d^2\vec{r}\,d\tau\,\cos[2\vec{K}_i\cdot\vec{r}]\left\{C_\uparrow^+ C_\uparrow(\vec{r}) - C_\downarrow^+ C_\downarrow(\vec{r})\right\}$$

SO $m \sim \langle\bar{\Psi}\Psi\rangle$

- IH, PRL 88, 047006 (2002);
- IH, PRB 66, 094504 (2002);
- Z. TEŠANOVIĆ ET AL, PRB 65, 180511 (2002).

IS STAGGERED MAGNETIZATION!

How CAN THIS ARISE?

PHASE FLUCTUATIONS IN dSC: VORTICES & SPIN WAVE

$$\propto \quad |\Delta| e^{i\varphi(\vec{r},\tau)} \; c_{\uparrow}(\vec{r},\tau)\, c_{\downarrow}(\vec{r},\tau) + H.\,conj.$$

$$\varphi(\vec{r},\tau) = \underset{\substack{\uparrow \\ \text{SPIN WAVES}}}{\varphi_{reg}} + \underset{\substack{\uparrow \\ \text{VORTICES (LOOPS AT } T=0)}}{\varphi_{sing}}$$

1) "NAIVE" IDEA: (BALENTS ET AL, IJMP B 10, 1033 (98); ...)

$$e^{i\frac{\varphi}{2}} c_{\uparrow} \longrightarrow c_{\uparrow},$$

$$e^{i\frac{\varphi}{2}} c_{\downarrow} \longrightarrow c_{\downarrow},$$

WILL NOT WORK!

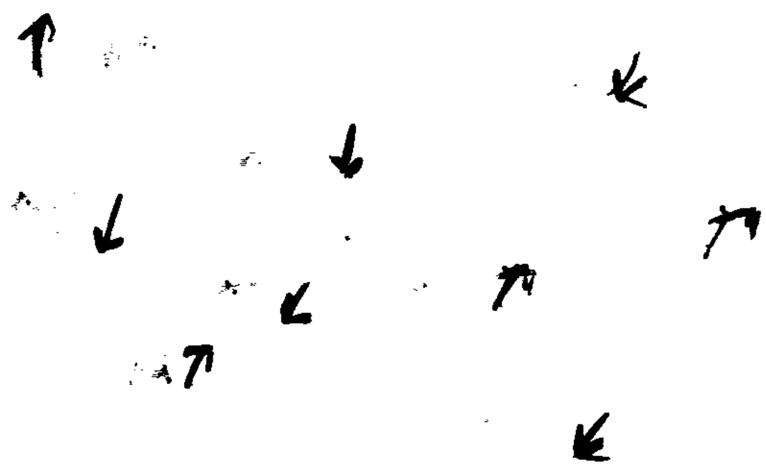
2) BETTER: (ANDERSON, COND-MAT/9812063; YE, PRB 65, 214505 ('02))

$$e^{i\left(\frac{\varphi_{reg}}{2} + \varphi_{sing}\right)} c_{\uparrow} \longrightarrow c_{\uparrow},$$

$$e^{i\left(\frac{\varphi_{reg}}{2}\right)} c_{\downarrow} \longrightarrow c_{\downarrow},$$

WORKS, BUT BY HAND BREAKS $\uparrow \leftrightarrow \downarrow$ SYMMETRY!?

3) GENERAL TRANFORMATION (FRANZ & TEŠANOVIĆ, PRL 87, 257003 (2001))



$$e^{i\left(\frac{\varphi_{reg}}{2}+\varphi_{sing,\uparrow}\right)} c_\uparrow \to C_\uparrow$$

$$e^{i\left(\frac{\varphi_{reg}}{2}+\varphi_{sing,\downarrow}\right)} c_\downarrow \to C_\downarrow$$

THEN:

CHARGE CURRENT

$$\mathcal{L} \to \bar{\Psi}_a \gamma_\mu (\partial_\mu - i a_\mu) \Psi_a + i v_\mu J^\mu_{ch}$$

WITH

(BERRY) $\quad a_\mu = \frac{1}{2} \partial_\mu (\varphi_{sing,\uparrow} - \varphi_{sing,\downarrow})$

(DOPPLER, VOLOVIK) $\quad v_\mu = \frac{1}{2} \partial_\mu (\varphi_{sing,\uparrow} + \varphi_{sing,\downarrow}) + \frac{1}{2} \partial_\mu \varphi_{reg}$

PHASE FLUCTUATIONS & d-WAVE QUASIPARTICLES:

THE GAUGE THEORY (IH, COND-MAT/0302058)

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{GAUGE} + \mathcal{L}_{VORTEX},$$

WITH:

$$\frac{K}{2}\left(\partial_\mu \varphi + 2\vec{A}\right)^2$$

$$\mathcal{L}_F = \bar{\Psi}_a \gamma_\mu (\partial_\mu - i a_\mu) \Psi_a .$$

$$\mathcal{L}_{GAUGE} = \frac{i}{\pi}\vec{a}\cdot(\nabla\times\vec{A}_-) + \frac{i}{\pi}\vec{v}\cdot(\nabla\times\vec{A}_+) + \frac{4K}{2}(\vec{v}+\vec{A})^2 + i(\vec{v}+\vec{A})\cdot\vec{J}_{CHARGE}$$

$$\mathcal{L}_{VORTEX} = \left|(\nabla - i(\vec{A}_+ + \vec{A}_-))\phi_1\right|^2 + \left|(\nabla - i(\vec{A}_+ - \vec{A}_-))\phi_2\right|^2 + \mu^2\left(|\phi_1|^2 + |\phi_2|^2\right)$$

$$+ \frac{b_1}{2}\left(|\phi_1|^2 + |\phi_2|^2\right)^2 + \frac{b_2}{2}\left(|\phi_1|^4 + |\phi_2|^4\right).$$

- K = BARE SUPERFLUID DENSITY AT $T=0$
- $\vec{A}$ = TRUE EXTERNAL ELECTROMAGNETIC POTENTIAL
- ϕ_1, ϕ_2 : CREATE VORTICES WITH ↑ AND ↓ LABEL
- $\vec{A}_+$, $\vec{A}_-$: AUXILIARY ("CHERN-SIMONS") FIELDS:

$$\vec{A}_- \longrightarrow \nabla\times\vec{a} = \pi(\vec{J}_{\phi_1} - \vec{J}_{\phi_2}), \quad i\,\vec{a}\cdot\vec{J}_{SPIN}$$

$$\vec{A}_+ \longrightarrow \nabla\times\vec{v} = \pi(\vec{J}_{\phi_1} + \vec{J}_{\phi_2}), \quad i\,\vec{v}\cdot\vec{J}_{CHARGE}$$

• SUPERCONDUCTOR: $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$

$$\mathcal{L}_{VORTEX} \longrightarrow \frac{1}{24\pi m}\left\{ (\nabla \times \vec{A}_+)^2 + (\nabla \times \vec{A}_-)^2 \right\} + \mathcal{O}\left((\nabla \times \vec{A}_\pm)^4 \right)$$

$m^2 = \mu^2 + \mathcal{O}(6)$ "VORTEX SUSCEPTIBILITY"

DIMENSIONAL ANALYSIS $\Rightarrow$ $\boxed{m \propto T_c}$

INTEGRATE $\vec{A}_\pm$:

$$\mathcal{L} \to \underbrace{\left\{ \bar{\Psi}_a (\partial\!\!\!/ - i a\!\!\!/) \Psi + \frac{6m}{\pi} \vec{a}^2 \right\}}_{\text{SPIN}} + \left\{ \frac{24mK}{12m + 4\tilde{\pi}K} \vec{A}^2 + i \frac{12m}{12m + 4\tilde{\pi}K} \vec{A} \cdot \overbrace{\vec{J}_{ch}}^{\text{"CHARGE"}} + \frac{\tilde{\pi}}{24m + 8\tilde{\pi}K} \vec{J}_{ch}^{\,2} \right\}$$

a) $\mathcal{L} = \mathcal{L}_{SPIN} + \mathcal{L}_{CHARGE}$; SPIN-CHARGE SEPARATION

b) $\mathcal{L}_{SPIN}$: 2+1 D THIRRING MODEL

c) $\rho_{SF}(T=0) = \dfrac{48\,mK}{12m + 4\tilde{\pi}K}$ $\xrightarrow{m \to \infty}$ $4K$ (BARE)

$\xrightarrow{m \to 0}$ $\frac{12}{\pi} m \propto T_c$ (UEMURA)

d) $Z = \dfrac{12m}{12m + 4\tilde{\pi}K}$ $\xrightarrow{m \to \infty}$ 1, [illegible] (OVERDOPED)

$\xrightarrow{m \to 0}$ $\frac{3}{\pi}\frac{m}{K} < 1$ (UNDERDOPED)

$$\rho_{sF}(T) = \rho_{sF}(0) - \# \tilde{Z}^2 \frac{T}{v_F}$$

(IOFFE & MILLIS, COND-MAT/0112509, ...)

e) CHARGE INTERACTION: $\propto \dfrac{\pi}{24m + 8\pi K} \rightarrow \dfrac{1}{4K}$, $m \to 0$

STAYS FINITE AND IRRELEVANT!

• NON-SUPERCONDUCTING STATE:

Since $b_2 > 0 \Rightarrow \langle \phi_1 \rangle = \langle \phi_2 \rangle \equiv \langle \phi \rangle$ LABEL SYMMETRY PRESERVED

So:

$$\mathcal{L}_{VORTEX} \rightarrow 2|\langle\phi\rangle|^2 (\vec{A}_+^2 + \vec{A}_-^2) + \sigma ((\nabla \times \vec{A}_\pm)^2)$$

INTEGRATE $\vec{A}_\pm$:

$$\mathcal{L} \rightarrow \underbrace{\left\{ \bar{\Psi}_a (\not{\partial} - i\not{a}) \Psi_a + \frac{(\nabla \times \vec{a})^2}{8\pi^2 |\langle\phi\rangle|^2} \right\}}_{\text{SPIN: } QED_3} + \underbrace{\left\{ \frac{\vec{J}_{eH}^2}{8K} + \frac{\left(\nabla \times (\vec{A} + \frac{i}{4K} \vec{J}_{eH})\right)^2}{8\pi^2 |\langle\phi\rangle|^2} \right\}}_{\text{CHARGE}}$$

(FRANZ ET AL, IH)

a) SPIN-CHARGE STILL SEPARATED $\Leftrightarrow$ LABEL SYMMETRY PRESERVED

b) Spin described by QED_3:

• COUPLING $= 2\pi |\langle\phi\rangle|$

• CHIRAL SYMMETRY BREAKING:

$$\text{STAG. MAGN.} \approx |2\pi \langle\phi\rangle|^2 e^{-\frac{2\pi}{\sqrt{\frac{N_c}{N} - 1}}}$$

• SPINONS CONFINED INTO SPIN WAVES

c) Maxwell term: $\frac{(\nabla \times \vec{A})^2}{8\pi^2 |\langle\phi\rangle|^2} \Rightarrow$ "CHARGE GAP" $= 4\pi^2 |\langle\phi\rangle|^2$

INSULATOR!

SPIN RESPONSE OF THE dSC

$$\mathcal{L}_{SPIN} = \overline{\Psi}_a(\not\partial - i\not a)\Psi_a + \frac{6}{\pi} m \vec{a}^{\,2} \quad ; \quad m \sim T_c$$

$$= \overline{\Psi}_a \not\partial \Psi_a + \frac{\pi}{72 m} \overline{\Psi}\gamma_\mu \Psi \overline{\Psi}\gamma_\mu \Psi \quad ; \text{THIRRING}$$

$$= \overline{\Psi}_a \not\partial \Psi_a + \frac{72 m}{\pi} M_\mu^{ij} M_\mu^{ij} + M_\mu^{ij} \gamma_\mu \Psi_j \overline{\Psi}_i + H.\,conj.$$

ANSATZ : $M_\mu^{ij}(x) = \frac{1}{6} M(x)\, \delta_{ij}\, \gamma_\mu$, so

$$\langle M(x) \rangle = \frac{\pi}{12 N m} \langle S_z(\vec{r}, \tau) \rangle \sum_{i=1}^{2} \cos(2\vec{K}_i \cdot \vec{r}) - \begin{matrix} \text{SDW} \\ \text{OP} \end{matrix}$$

INTEGRATE FERMIONS :

$$S_{SPIN} = N \int \frac{d^3\vec{q}}{(2\pi)^3} \left\{ \frac{24 m \pi - 1}{\pi^2} + \frac{|q|}{8} \right\} M(q) M(-q) \quad + \mathcal{O}(M^4)$$

ALSO :

$$\left\langle M(\vec{q}, \omega) M(-\vec{q}, -\omega) \right\rangle - \frac{\pi}{48 N m} = 2\left(\frac{\pi}{24 N m}\right)^2 \sum_{i=1}^{2} \left\langle S_z(2\vec{K}_i + \vec{q}, \omega) S_z(-2\vec{K}_i - \vec{q}, -\omega) \right\rangle$$

ROTATIONAL INVARIANCE:

$$\langle S_\alpha(\vec{k},\omega)\, S_\beta(-\vec{k},-\omega)\rangle = \chi(\vec{k},\omega)\,\delta_{\alpha\beta}$$

So:

$$\chi''(2\vec{K}_i \pm \vec{q},\omega) = \left(\frac{12Nm}{\pi}\right)^2 \operatorname{Im}\langle M(\vec{q},\omega)\, M(-\vec{q},\omega)\rangle$$

FINALLY: $i\omega \to \omega$

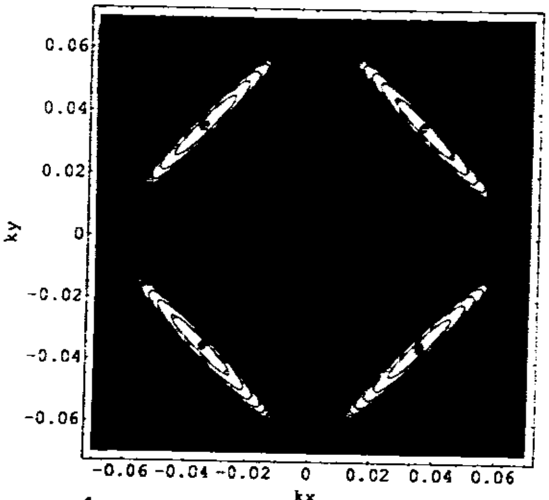
$$\boxed{\chi''(2\vec{K}_i + \vec{q},\omega) = \left(\frac{24m}{\pi}\right)^2 \frac{N\,\Theta(\omega^2 - \vec{q}^{\,2})\sqrt{\omega^2 - \vec{q}^{\,2}}}{\left(\frac{192m}{\pi}\right)^2 + \omega^2 - \vec{q}^{\,2}}}$$

WHERE:

- $\vec{q}^{\,2} = v_F^2 q_x^2 + v_\Delta^2 q_y^2 \quad ; \quad v_F \gg v_\Delta$
- $m \sim T_c \quad \left(\frac{64m}{\pi} = 5.6\,T_c \text{, FROM UEMURA SCALING} \right)$

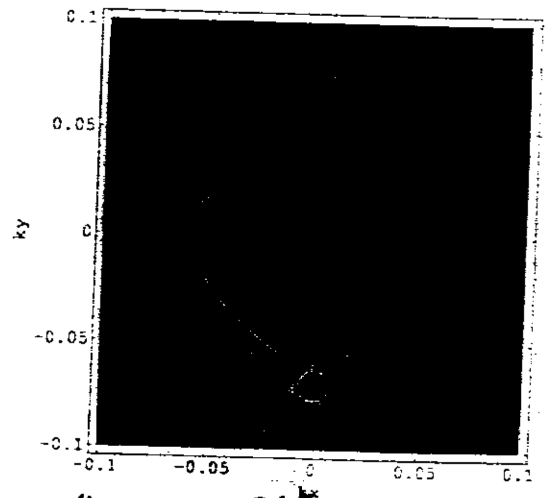
FOUR REGIMES: (D. LEE & IK, COND-MAT/0305512)

I) LOW ω: $\omega = 6\,meV$



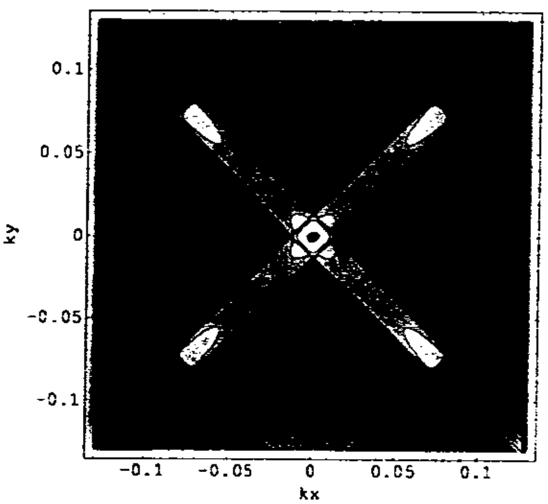
"DIAGONAL" INCOMM.

II): $\omega \lesssim \omega_{RES}$; $\omega = 15\,meV$



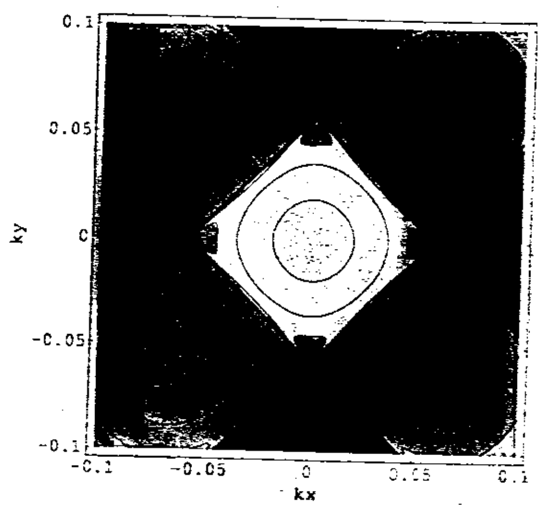
"PARALLEL" INCOMM.

III "RESONANCE"



$\omega = 72\,meV$

IV HIGH ω : $\omega = 110\,meV$



"PARALLEL" INCOMM.
(WEAK)

PARAMETERS: $v_F = 1.2\,eV\,Å$

$T_c \approx 67\,K$

$v_F / v_\Delta = 7$

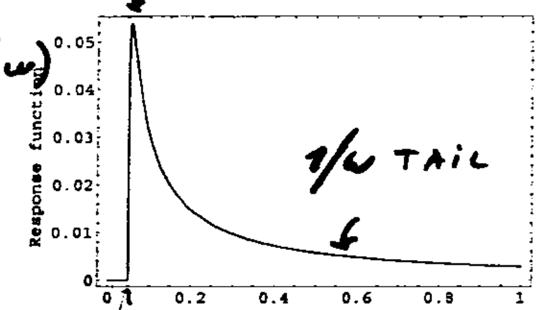
$2\vec{K}_1 = 0.465\,(2\pi, 2\pi)$

"RESONANCE" : $\vec{Q} = (\pi, \pi)$

P. DAI ET. AL.
PRB 63, 054525 (2001)

"RES. ENERGY"

$\chi''(\vec{Q}, \omega)$

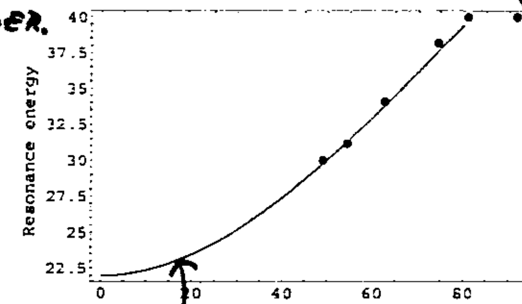


1/ω TAIL

ω

"SPIN GAP"

RES. ENER.



T_c

$$\omega_{res}^2 = (cT_c)^2 + \omega_{SG}^2$$

$c \approx 5$

$\omega_{SG} = 22 \text{ meV}$

$\boxed{\text{CONCLUSION:}}$

- "CHIRAL" SYMMETRY OF dSC
- QUANTUM FLUCTUATIONS $\Rightarrow U(1)$ GAUGE TH.

$\boxed{\text{POSTDICTIONS \& PREDICTIONS}}$:

- SDW INSULATOR
- UEMURA SCALING
- "CHARGE" RENORMALIZATION $(Z \to 0)$
- INCOMMENS. - COMMENS. - INCOMM. REGIMES IN SPIN RESPONSE OF dSC
- "SPIN GAP" AT $\vec{Q} = (\pi, \pi)$
- "RESONANCE ENERGY" $\sim T_c$ (BUT SATURATES)