Quasiparticle interference in the *pseudogap* phase of cuprate superconductors

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Pseudogap: the key mystery

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Phase diagram of cuprates. [For exp. review see Timusk and Statt, Rep. Prog. Phys. 62, 61 (1999).]
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Gap in the single-particle DOS above \( T_c \) [tunneling data from Renner et al., PRL 80, 149 (1998)].
Two schools of thought on the origin of pseudogap

Ascribe the pseudogap phenomenon to:

- Remnants of superconducting order
  - Franz and Millis, PRB 58, 14572 (1998)
  - Balents, Fisher and Nayak, PRB 60, 1654 (1999)
  - Laughlin, cond-mat/0209269
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- **Static or fluctuating competing order in p-h channel (SDW, CDW, DDW, . . .)**
  - Chakravarty, Laughlin, Morr, and Nayak, PRB **63**, 094503 (2001)
Who is right?

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Need a decisive “smoking gun” experiment

Our proposal: use the recently developed technique of Fourier Transform scanning tunneling spectroscopy (FT-STS).

- Pereg-Barnea and Franz, cond-mat/0401594
STM Basics

STM measures differential conductance

\[ n(r, \omega) \sim \left( \frac{dI(r, eV)}{dV} \right)_{eV=\omega} \]

with potentially atomic resolution.

To reasonable approximation \( n(r, \omega) \) is proportional to the Local Density of States (LDOS) of the sample at point \( r \) directly under the STM tip.
Tunneling spectroscopy in cuprates

Topography of BiSCCO:
Tunneling spectroscopy in cuprates

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Spectroscopy of Ni impurities:
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Topography of BiSCCO:

LDOS inhomogeneity:

Spectroscopy of Ni impurities:
FT-STS: “Fourier Transform Scanning Tunneling Spectroscopy”

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FT
FT-STS peaks disperse as a function of applied bias

The “Octet Model”

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Theory vs. Experiment

T-matrix calculation [Wang and Lee, PRB 67, 020511(R) (2003)]
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- The interference patterns depend crucially on the electron \textit{wavefunctions}, i.e. they are sensitive to BCS \textit{coherence factors}.
- System with \textit{identical DOS} but different type of electron order will exhibit qualitatively different FT-STS patterns.
STM measures the quantity

\[ n(r, \omega) = -\frac{1}{\pi} \text{Im}[G_{11}(r, r, \omega) + G_{22}(r, r, -\omega)], \]

where \( G(r, r', \omega) \) is a full electron propagator. In the presence of disorder potential \( V \) we can write

\[ G(k, k', \omega) = G^0(k, \omega)\delta_{k,k'} + G^0(k, \omega)\hat{T}_{kk'}(\omega)G^0(k', \omega), \]

with \( G^0(k, \omega) = [\omega - \sigma_3 \epsilon_k - \sigma_1 \Delta_k]^{-1} \) the bare Green's function and \( \hat{T}_{kk'}(\omega) \) the T-matrix that satisfies the Lippman-Schwinger equation

\[ \hat{T}_{kk'}(\omega) = \hat{V}_{kk'} + \sum_q \hat{V}_{kq} G^0(q, \omega)\hat{T}_{qq'}(\omega). \]
FT-STS measures $n(q, \omega)$, a spatial Fourier transform of $n(r, \omega)$.

It is useful to consider a limit of weak disorder (i.e. Born limit) in which one can express the non-uniform part $\delta n(q, \omega)$

$$\delta n(q, \omega) = -\frac{1}{\pi}|V_q|\text{Im} [\Lambda_{11}(q, \omega) + \Lambda_{22}(q, -\omega)],$$

where, for scattering in the charge channel,

$$\Lambda(q, \omega) = \sum_k G^0(k, \omega)\sigma_3 G^0(k - q, \omega).$$

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For weak disorder FT-STS provides information about the underlying electron order.
Nodal approximation: importance of coherence factors

One finds

$$\Lambda(q, i\omega) = \frac{1}{L^2} \sum_{k} \frac{(i\omega + \epsilon_+)(i\omega + \epsilon_-) - \Delta_+\Delta_-}{(\omega^2 + E_+^2)(\omega^2 + E_-^2)},$$

with $\epsilon_\pm = \epsilon_{k\pm q/2}$, $\Delta_\pm = \Delta_{k\pm q/2}$ and $E_\pm = \sqrt{\epsilon_\pm^2 + \Delta_\pm^2}$. Linearize near the nodes to obtain

$$\Lambda_{\text{lin}} = \frac{1}{v_F v_\Delta} \int \frac{d^2k}{(2\pi)^2} \left[ -\omega^2 + (k_1^2 - k_2^2) - (\tilde{q}_1^2 - \tilde{q}_2^2) \right] \left[ \omega^2 + (k + \tilde{q})^2 \right] \frac{\omega^2 + (k - \tilde{q})^2}{[\omega^2 + (k + \tilde{q})^2][\omega^2 + (k - \tilde{q})^2]}.$$
For intranodal scattering we thus get

Magnetic and non-magnetic scattering differ only in the coherence factors, DOS is exactly the same. Yet, the FT-STS patterns are qualitatively different!
The full picture

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Alternately, one can evaluate $\Lambda(q, \omega)$ exactly using numerical techniques:
The pseudogap state

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- In the following we illustrate this general thesis on the comparison between QED$_3$ theory of phase disordered $d$SC and $d$-density wave (DDW) scenario for pseudogap.
This theory describes fermionic excitations in a phase-disordered \textit{d}-wave superconductor. The electron propagator reads

\[ G^0(k, i\omega) = \lambda^{-\eta} \frac{i\omega + \epsilon_k \sigma_3}{\left[\omega^2 + \epsilon_k^2 + \Delta_k^2\right]^{1-\eta/2}}, \]

where \( \lambda \) is a high energy cutoff and \( \eta \) is the anomalous dimension exponent which encodes the physics of phase fluctuations. \( \eta \) is a small positive number, whose precise value is still under debate.
DDW

[Chakravarty, Laughlin, Morr, and Nayak PRB 63, 094503 (2001)]

Also known as the “flux phase”, this theory describes the pseudogap as a mean-field state with staggered pattern of currents, breaking the translational symmetry of the square lattice. We have

\[ G^0(k, i\omega) = \left( (i\omega - \epsilon'_k) - \epsilon''_k \sigma_3 - D_k \sigma_2 \right)^{-1}, \]

with \( \epsilon'_k = \frac{1}{2}(\epsilon_k + \epsilon_{k+Q}) \), \( \epsilon''_k = \frac{1}{2}(\epsilon_k - \epsilon_{k+Q}) \), and the DDW gap \( D_k = \frac{1}{2}D_0(\cos k_x - \cos k_y) \).

At half filling \( (\mu = 0) \) and with nn dispersion \( (t' = 0) \) DDW has the same DOS as the \( d\text{SC} \).
Conclusions

- By analyzing the quasiparticle interference patterns in the nodal approximation we gained some crucial insights into FT-STS in the superconducting state.

- FT-STS is sensitive to both the quasiparticle DOS and the coherence factors.

- This sensitivity can be used to determine the nature of the condensate responsible for the pseudogap phenomenon in the cuprates.

- Several experimental groups are now actively pursuing this goal.