First-Principles Study of Integer Quantum Hall Transitions in Mesoscopic Samples

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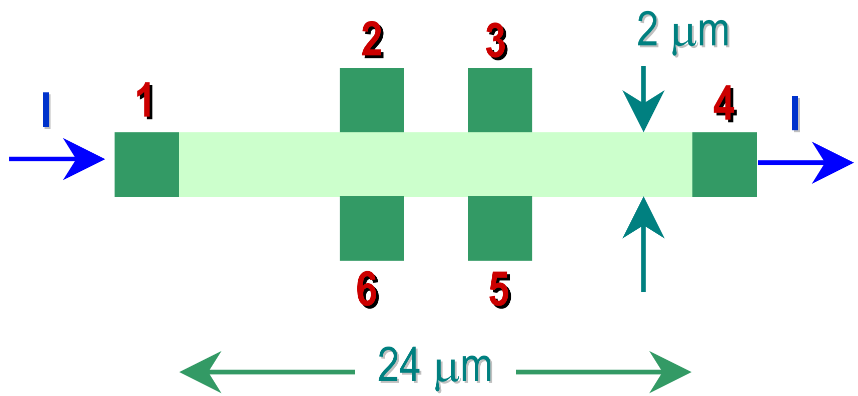
Experimental data:

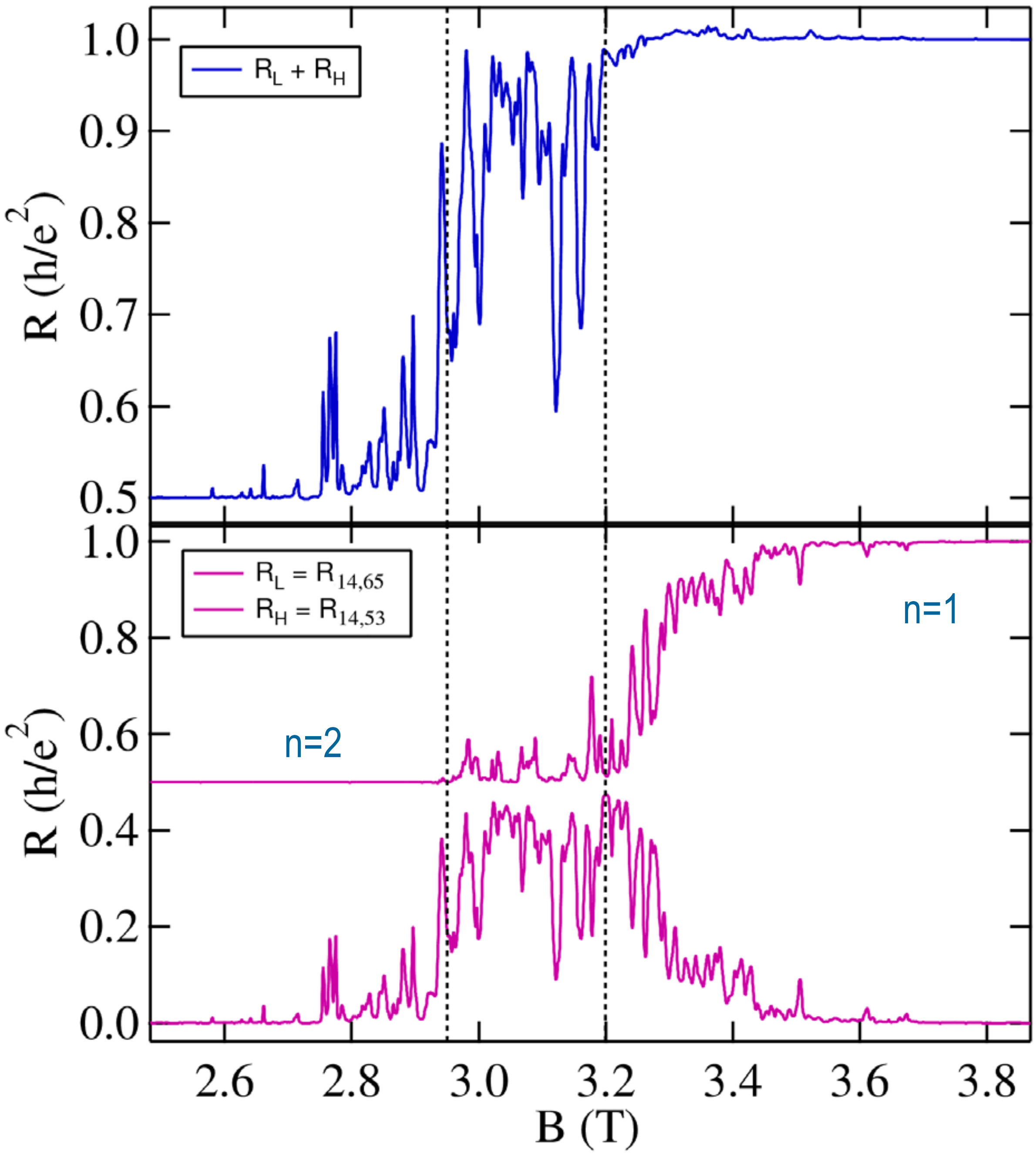
1. "Observation of a Quantized Hall Resistivity in the Presence if Mesoscopic Fluctuations", E. Peled, D. Shahar, Y. Chen, D. L. Sivco and A. Y. Cho, Phys. Rev. Lett. **90**, 246802 (2003);
2. "Near-Perfect Correlation of the Resistance Components of Mesoscopic Fluctuations at the Quantum Hall Regime", E. Peled, D. Shahar, Y. Chen, E. Diez, D. L. Sivco and A. Y. Cho, Phys. Rev. Lett. **91**, 236802 (2003);



Current I injected in 1, extracted in 4
→ two pairs of R_H and R_L:

$$R^{H}_{14,62} = \frac{V_6 - V_2}{I}; \qquad R^{H}_{14,53} = \frac{V_5 - V_3}{I};$$

$$R^{L}_{14,23} = \frac{V_2 - V_3}{I}; \qquad R^{L}_{14,65} = \frac{V_6 - V_5}{I};$$



Each QHE transition n→ n+1 has 3 distinct regimes:

i) Low ν (high B):$R_L+R_H=h/ne^2$;
ii) middle – uncorr. fluctuations;
iii) high ν (low B):R_L fluctuates but $R_H=h/e^2(n+1)$;

For 0→1 transition, only (iii) visible, (i) and (ii) replaced by transition to the insulator.

Other interesting features:

$$R^H_{14,62} + R^L_{14,23} =$$

$$R^H_{14,53} + R^L_{14,65} =$$

$$R_{14,63} = R_{63,63}$$

+ interesting symm. B → -B

Theory – numerical simulations:

Response function of the system is the 6x6 conductance matrix g: $I_\alpha = \sum_{\beta=1}^{6} g_{\alpha\beta} V_\beta$

out-going current on lead α

voltage on lead β

If g is known, solve for $V_1,\ldots,V_6$ from

$$\begin{pmatrix} -I \\ 0 \\ 0 \\ I \\ 0 \\ 0 \end{pmatrix} = g \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ 0 \\ V_5 \\ V_6 \end{pmatrix} \rightarrow \text{ compute directly } R^H_{14,63} = \frac{V_6 - V_3}{I}, \text{ etc.}$$

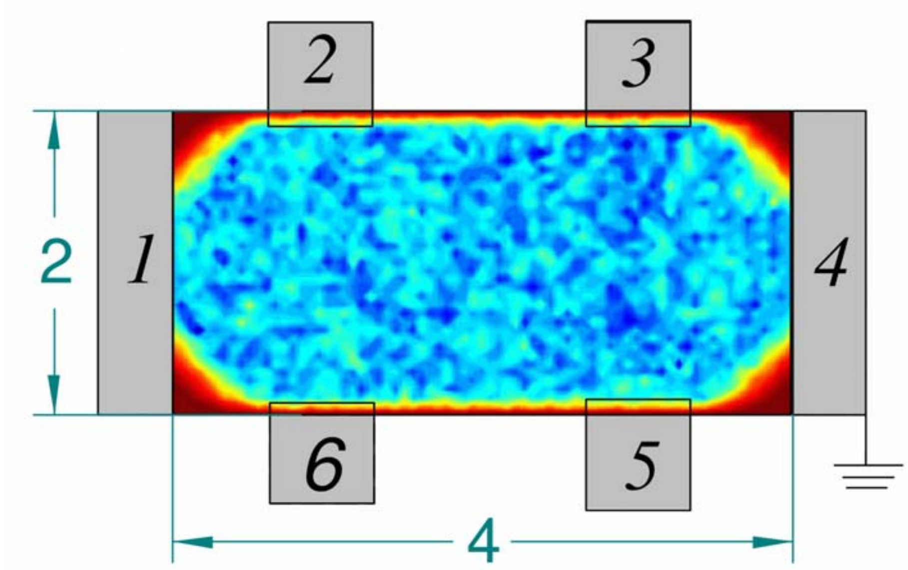
Multi-probe Landauer formula [equiv. to Kubo formula, e.g. H. Baranger and D. Stone, PRB **40** 8169 (1989)]

$$g_{\alpha,\beta\neq\alpha} = \frac{e^2}{h}\sum_{i,j}\left|t_{\alpha i,\beta j}\right|^2 = \frac{e^2}{h} p_{\beta\to\alpha}$$

$$\sum_{\beta=1}^{6} g_{\alpha\beta} = 0 \rightarrow g_{\alpha\alpha} = \sum_{\beta\neq\alpha} g_{\alpha\beta}$$

transmission amplitude that an electron with E_F injected in channel j of contact β will emerge in channel i of contact α.

Total probability that electron injected in β emerges in α



→ 2μmx4μm sample (~10^4 states in the LLL; we assume no LL mixing)

→ disorder potential = sum of random gaussians scatterers

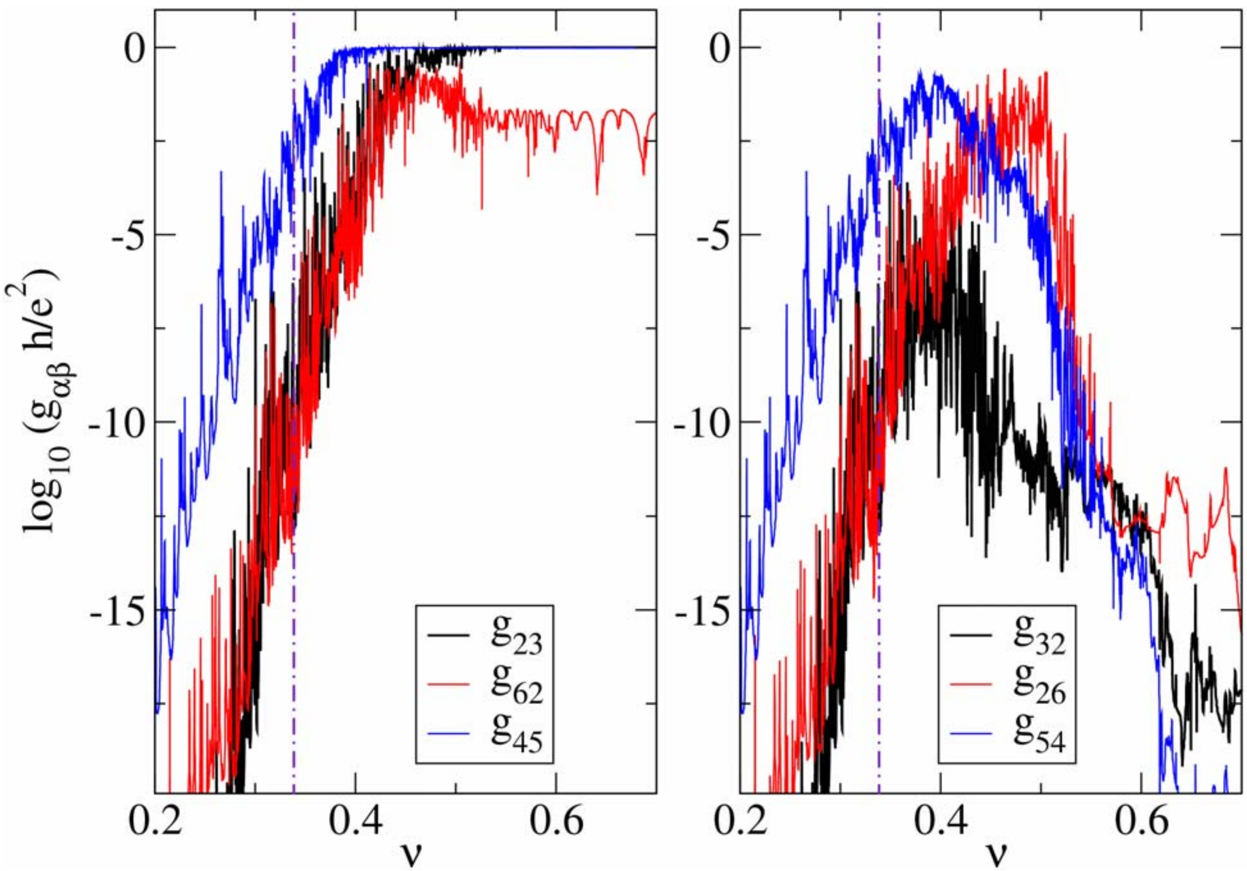
→ add confinement potential (we cut the corners)

→ lead = collection of semi-infinite 1D hopping chains, each representing a different channel

→ we work at fix B, vary E_F (the electron density, the filling factor ν)

→ we solve the Lippman-Schwinger equation exactly → find all transmission amplitudes

→ we end up with $g(E_F)$ and $\nu(E_F)$ → $g(\nu)$ for $0 < \nu < 1$ (only LLL).

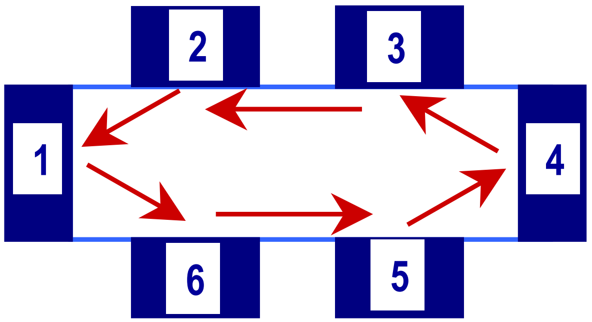


1) For $\nu < 0.35$, g is symmetric : $g_{\alpha\beta} = g_{\beta\alpha}$

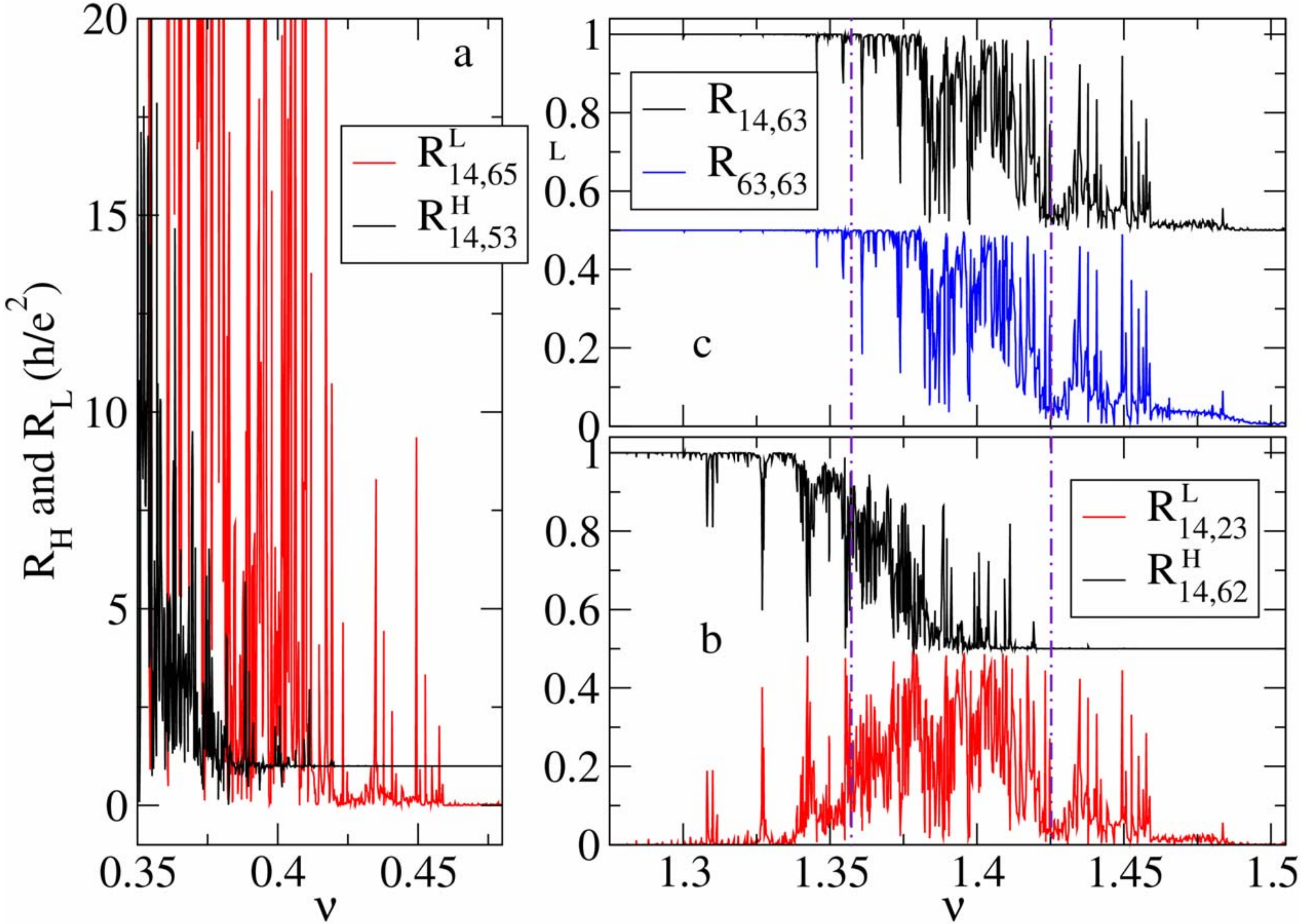
2) For $\nu \rightarrow 1$, $g(\nu) \rightarrow g^0_{\alpha\beta} = e^2/h\,(-\delta_{\alpha\beta}+\delta_{\alpha+1,\beta}+\delta_{\alpha 6}\delta_{\beta 1})$

$$\hat{I} = n\hat{g}^0 \cdot \hat{V} \rightarrow R_H = \frac{h}{ne^2}; R_L = 0 \leftarrow \text{ QH plateau}$$

Variations in $g(\nu)$ from g^0 → fluctuations in R_H, R_L.



Edge currents



$$\hat{I} = \hat{g}(\nu) \cdot \hat{V}$$

$$\hat{I} = \left[\hat{g}(\nu) + \hat{g}^0 \right] \cdot \hat{V}$$

→ our numerical simulations recapture all the experimental results. What is happening?

General structure of g: ($e^2/h=1$)

There are two types of contributions to g: a symmetric part ("resistors", tunneling) and a chiral part.

e.g.: resistor R between 1 and 2 is described by

[each such term involves only two contacts;

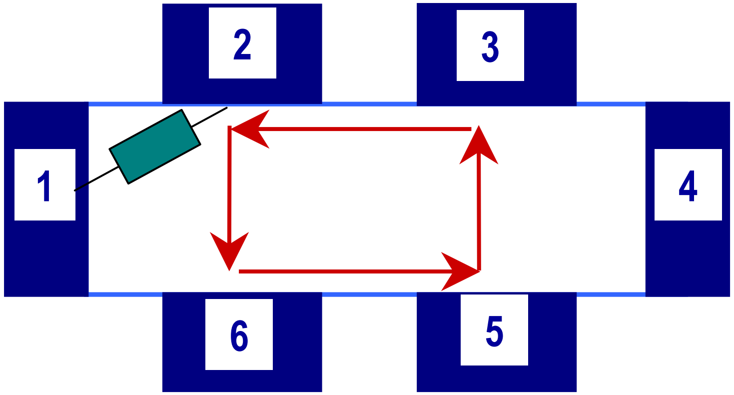
we denote unit resistor by r(α,β)]

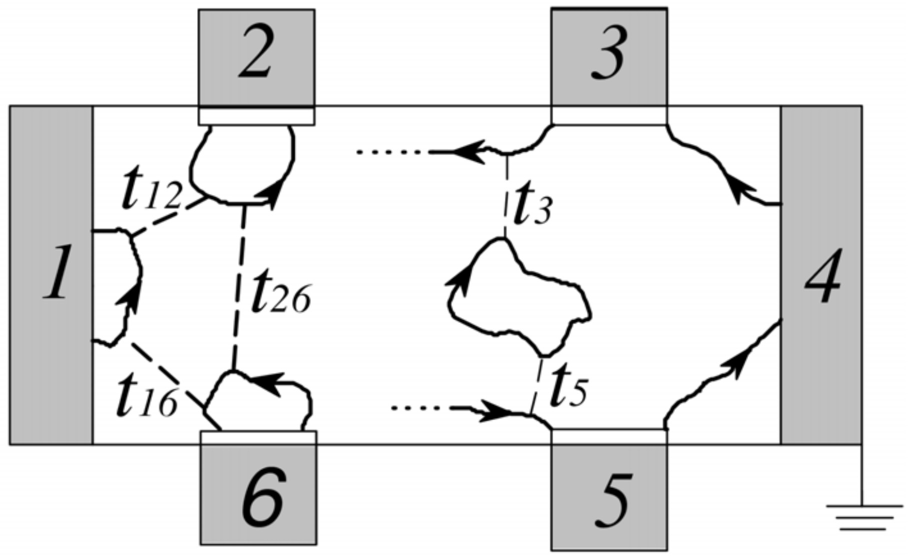
$$\frac{1}{R}\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{cases} I_1 = \dfrac{V_2 - V_1}{R} \\ I_2 = -I_1 \end{cases}$$

A chiral current is a closed loop involving more than 2 contacts, in "chiral" order.

e.g: 2→3→5→6→2 = r(2,3,5,6) ; with this notation, g^0 = r(1,2,3,4,5,6)

$$r(2,3,5,6) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$





Low filling factor ν: all states are localized → electrons can move between different contacts only through direct tunneling→

$p_{1\to 2} \sim p_{2\to 1} = |t_{12}|^2 + O(t^2)$, i.e. $g_{12} \sim g_{21}$.

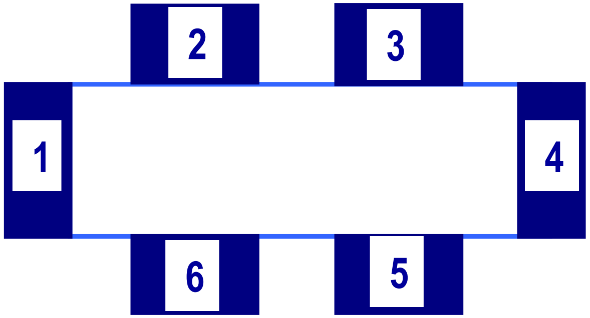
A chiral current $r(1,2,6) = 2|t_{12}t_{26}t_{61}|$ is also established.

This is why for small ν, g is a symmetric matrix.

Medium filling factor ν: edge states connecting neighboring contacts become established → chiral loops start to form. One expects direct competition between both types of transport.

Large filling factor ν: most transport due to chiral currents. Tunneling contributes to back-scattering (Jain-Kivelson model).

For each pair α,β: $\min(g_{\alpha\beta},g_{\beta\alpha})$ is symmetric term ($\alpha\beta$ resistor); remaining term contributes to one or more chiral currents.



Consider the general form:

$$\hat{g} = n\hat{g}^0 + r_{12}\hat{r}(1,2) + r_{26}\hat{r}(2,6) + r_{16}\hat{r}(1,6) + \\ + r_{34}\hat{r}(3,4) + r_{45}\hat{r}(4,5) + r_{35}\hat{r}(3,5) + \\ + c_1\hat{r}(1,2,6) + c_2\hat{r}(2,3,5,6) + c_3\hat{r}(3,4,5) + \\ + c_4\hat{r}(1,2,3,5,6) + c_5\hat{r}(2,3,4,5,6) + c_0\hat{g}^0$$

$$\downarrow$$

$$R^H + R^L = R_{14,63} = R_{63,63} = \frac{h}{e^2}\cdot\frac{1}{n + c_0 + c_2 + c_4 + c_5}$$

Largest neglected terms: r_{23} and r_{56} ~10^{-4}

Conclusions:

a) $R_{14,63} = R_{63,63}$ for all filling factors ν.

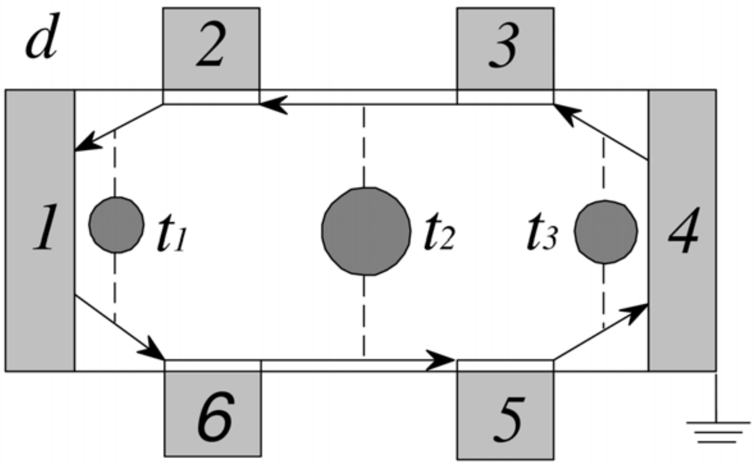
b) At small ν, all c's $=O(t^2)$ → $R^H + R^L = h/ne^2$

Large filling factor ν:

$$\hat{g} = n\hat{g}^0 + (1-t_1-t_2-t_3)\hat{g}^0 + t_2[\hat{r}(1,2,6)+\hat{r}(3,4,5)] + t_3\hat{r}(1,2,3,5,6)+t_1\hat{r}(2,3,4,5,6)$$

$$\downarrow$$

$$\begin{cases} R^H_{14,62} = R^H_{14,53} = \dfrac{h}{e^2(n+1)} \\ R^L_{14,23} = R^L_{14,65} = \dfrac{h}{e^2(n+1)} \cdot \dfrac{t_2}{n+1-t_2} \end{cases}$$



Summary:

: low ν, $R^H + R^L = h/ne^2$ holds until edge state is established between 2-3 or 5-6, i.e. the localization length becomes comparable to distance between 2-3 contacts.

: high ν, $R^H = h/(n+1)e^2$ while R^L fluctuates: only if localization length comparable to 3-5 distance.

→ The central regime corresponds to the "critical region"