Spin Dynamics in Josephson Junctions

Alexander Shnirman

University of Karlsruhe, Germany

In collaboration with

J.-X. Zhu, Z. Nussinov, A.V. Balatsky,
L. Bulaevskii, M. Hruška, D. Smith
Los Alamos National Lab., USA

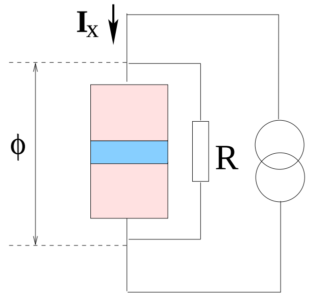
Yu. Makhlin
Uni. Karlsruhe, Germany & Landau Institute, Russia

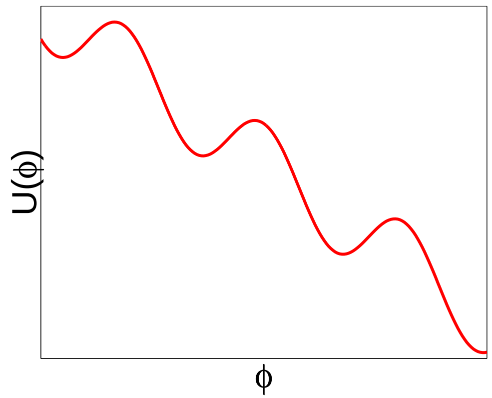
Outline

- Josephson junctions, introduction
- Spins in Josephson junctions, history
- Motivation
- Effective action for phase and spin
- Semiclassical dynamics of big spin
- QND measurements of spin 1/2
- Outlook

Josephson Junctions in the Flux (Phase) Regime

- current-biased Josephson junctions



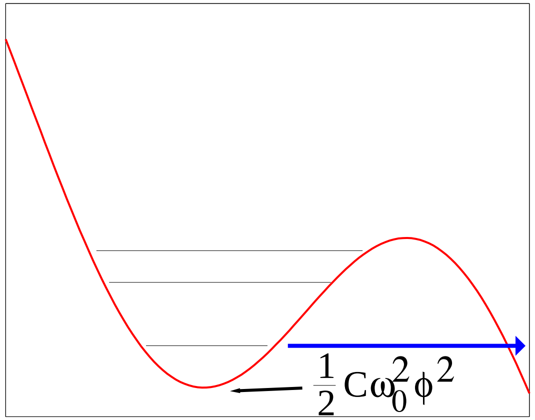


Josephson relations $\quad I_{\mathsf{S}} = I_{\mathsf{C}} \sin\phi \quad ; \quad 2eV = \hbar\dot{\phi} \quad ; \quad I_{\mathsf{C}} \propto \frac{\Delta}{R}$

Balance of current $\quad \frac{\hbar}{2e} C\ddot{\phi} + \frac{1}{R}\frac{\hbar}{2e}\dot{\phi} + I_c \sin\phi = I_x$

Hamiltonian $\quad \mathcal{H} = \frac{Q^2}{2C} - E_{\mathsf{J}} \cos\phi - I_{\mathsf{x}}\phi + H_{\mathsf{diss}}(\phi)$

- Macroscopic Quantum Tunneling



Caldeira & Leggett '81

Voss & Webb '81

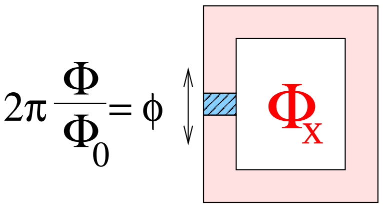
Martinis et al. '87

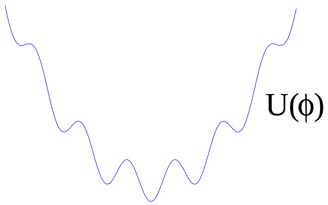
$C \sim 10^{-12}$F

$\Gamma_{\mathrm{th}} \sim e^{-\Delta U/k_{\mathrm{B}}T}$

$\Gamma_0 \sim e^{-7.2\Delta U/\hbar\omega_0}$

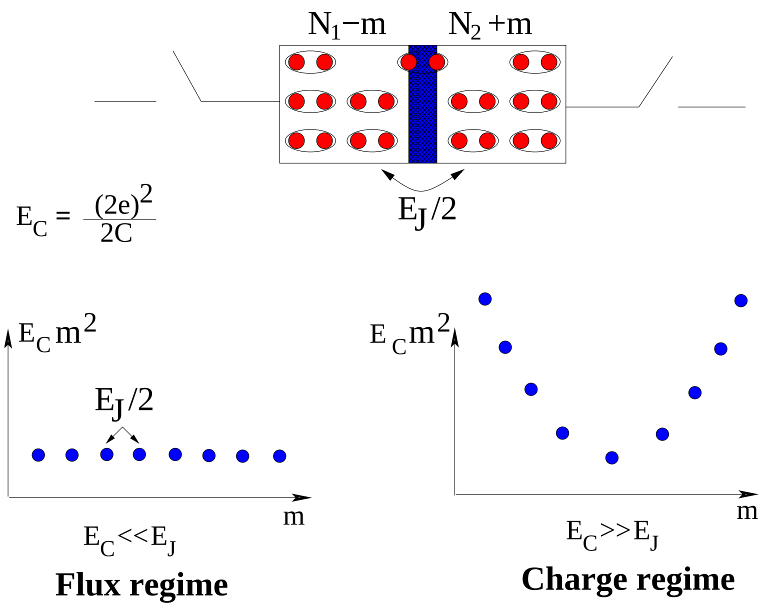
- rf-SQUID





$$\mathcal{H} = \frac{Q^2}{2C} - E_{\mathrm{J}} \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L} + H_{\mathrm{diss}}$$

Charge regime



- The Hamiltonian

$$H = E_C m^2 - \frac{E_J}{2}(|m+1\rangle\langle m| + h.c)$$

$$e^{i\phi} \equiv |m+1\rangle\langle m|$$

- Charge regime

$$|\Psi\rangle \approx |m\rangle$$

$$E \approx E_C m^2$$

- Flux regime

$$|\Psi\rangle \approx \sum_m e^{i\,\phi\,m}|m\rangle$$

$$E \approx -E_J \cos(\phi)$$

Microscopic description

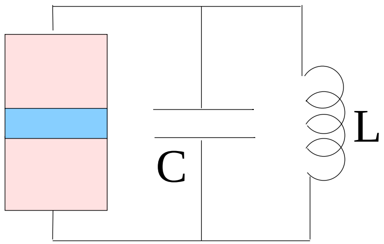
$$H = H^L_{\text{BSC}} + H^R_{\text{BSC}} + H_T$$

$$H_T = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'}(\mathbf{k},\mathbf{p}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} + \text{h.c.}] \quad , \quad T_{\sigma,\sigma'}(\mathbf{k},\mathbf{p}) = T_0\, \delta_{\sigma\sigma'}$$

Josephson, '62 Ambegaokar, Baratoff, '63

Ambegaokar, Eckern, Schön '82,'84
Larkin, Ovchinnikov '83

$$\int D\phi \; \exp\left[i\mathcal{S}_{\text{circuit}}(\phi) + i\mathcal{S}_{\text{eff}}(\phi)\right]$$



$$\mathcal{S}_{\text{circuit}}(\phi) = \int dt \left[\frac{C\dot{\Phi}^2}{2} - \frac{\Phi^2}{2L} + \ldots\right] \qquad \Phi \equiv \frac{\hbar}{2e}\phi$$

Effective action

$$\mathcal{S}_{\text{eff}} = 2i\,T_0^2 \oint_K dt \oint_K dt'\, G(t,t')G(t',t)\ \cos\left[\frac{\phi(t)-\phi(t')}{2}\right]$$

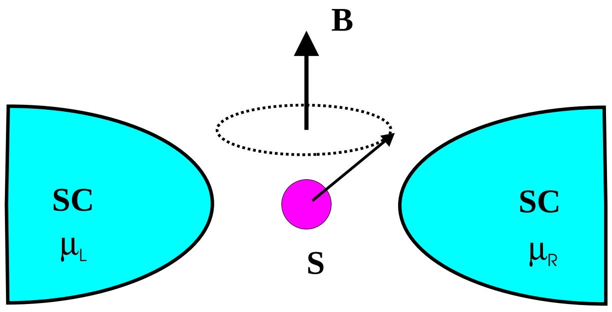
$$+2i\,T_0^2 \oint_K dt \oint_K dt'\, F(t,t')F^\dagger(t,t')\ \cos\left[\frac{\phi(t)+\phi(t')}{2}\right]$$

$$G(t,t') \equiv -i\sum_k \langle T_K c_{\mathbf{k}\sigma}(t)c^\dagger_{\mathbf{k}\sigma}(t')\rangle \qquad F(t,t') \equiv -i\sum_k \langle T_K c_{\mathbf{k}\uparrow}(t)c_{-\mathbf{k}\downarrow}(t')\rangle$$

$$\mathcal{S}_{\text{eff}} \approx \int dt\ E_{\mathsf{J}}\cos\phi(t)$$

$$E_{\mathsf{J}} = \frac{\hbar}{2e}I_c \qquad I_c = \frac{2\pi^2 e}{\hbar}T_0^2\rho^2\Delta$$

The Hamiltonian



$$H = H^L_{\mathsf{BSC}} + H^R_{\mathsf{BSC}} + H_{\mathsf{spin}} + H_T$$

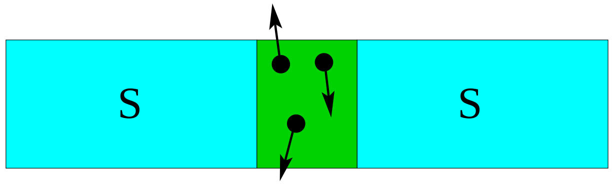
$$H_T = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'}(\mathbf{k},\mathbf{p}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} + \text{h.c.}]$$

$$T_{\sigma,\sigma'}(\mathbf{k},\mathbf{p}) = T_0\, \delta_{\sigma\sigma'} + T_1\, \mathbf{S}(t)\cdot\boldsymbol{\sigma}_{\sigma\sigma'}$$

$$H_{\mathsf{spin}} = -B\, S_z$$

Study: spin dynamics and Josephson current

History



Kulik, Sov. Phys. JETP **22**, 841 (1966)

reduction of Josephson current due to spin flips

$$H_T = \sum_{\mathbf{k},\mathbf{p}} T_0 a^\dagger_{\mathbf{k}\uparrow} b_{\mathbf{p}\uparrow} + T_1 a^\dagger_{\mathbf{k}\uparrow} b_{\mathbf{p}\downarrow} + \ldots$$

$$I_N \propto T_0^2 + T_1^2 \qquad I_S \propto T_0^2 - T_1^2 \qquad I_c \propto \frac{T_0^2 - T_1^2}{T_0^2 + T_1^2} \cdot I_c^{\mathrm{AB}}$$

Shiba & Soda, Prog. Theor. Phys. **41**, 25 (1969)

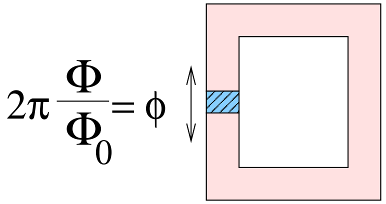
Dynamical spin impurities, Kondo corrections to Josephson current

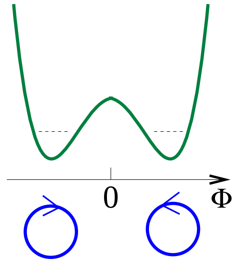
$$H_T = \sum_{\mathbf{k},\mathbf{p}} T_0 a^\dagger_{\mathbf{k}\sigma} b_{\mathbf{p}\sigma} + T_1 a^\dagger_{\mathbf{k}\alpha} b_{\mathbf{p}\beta} \left(\vec{\sigma}_{\alpha,\beta} \cdot \vec{S}\right) + J a^\dagger_{\mathbf{k}\alpha} a_{\mathbf{p}\beta} \left(\vec{\sigma}_{\alpha,\beta} \cdot \vec{S}\right) \ldots$$

$$I_{S/N} \to I_{S/N} \left(1 + J\rho \ln \frac{D}{\Delta}\right)$$

Bulaevskii et al., JETP Lett. **25**, 290 (1977)
possibility of π junctions

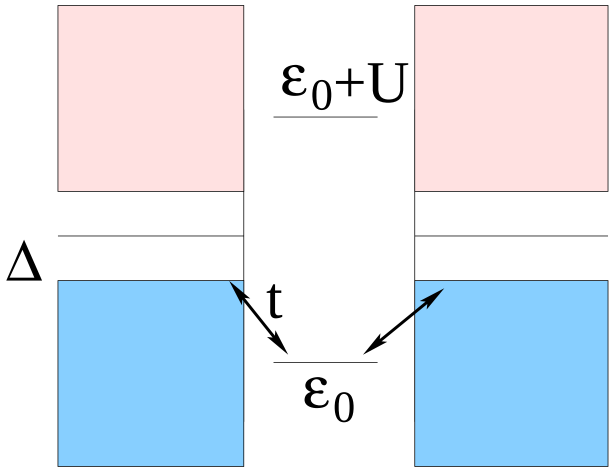
$I_S \propto T_0^2 - T_1^2 \qquad T_1 > T_0 \qquad I_c < 0$





Spontaneous currents

Glazman & Matveev, JETP Lett. **49**, 659 (1989)
$\Gamma \equiv t^2\rho < \Delta$ - perturbative, $\Gamma > T_K > \Delta$ Kondo effect

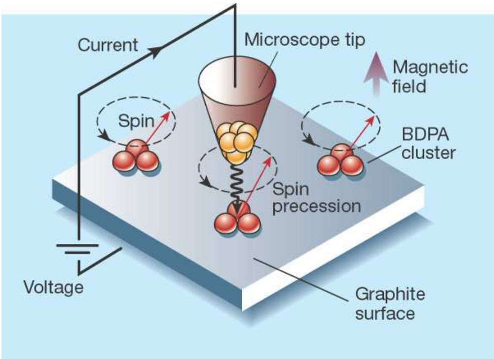


$T_1 \sim \frac{t^2}{|\epsilon_0|}$ or $\frac{t^2}{|\epsilon_0 + U|}$

Kondo enhancement of Josephson current

Our motivations

- Single spin STM detection (ESR-STM)



Manassen, '1989 ...
Durkan & Welland, '2002

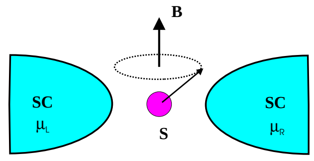
- Spin dynamics and quantum state engineering

  Quantum measurements of spin

  Quantum manipulations of spin

J.-X. Zhu, Z. Nussinov, A.S., A. Balatsky, PRL 92, 107001 (2004).
L. Bulaevskii, M. Hruška, A.S., D. Smith, Yu. Makhlin, cond-mat/0312274
A.S., Z. Nussinov, Jian-Xin Zhu, A. V. Balatsky, Yu. Makhlin, cond-mat/0402548

Effective action



$$H_T = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} + \text{h.c.}] \qquad T_{\sigma,\sigma'} = T_0\, \delta_{\sigma\sigma'} + T_1\, \mathbf{S}(t) \cdot \boldsymbol{\sigma}_{\sigma\sigma'}$$

$$\int D\phi D\mathbf{S}\ \exp\left[\mathbf{i}\mathcal{S}_{\text{circuit}}(\phi) + \mathbf{i}\mathcal{S}_{\text{spin}}(\mathbf{S}) + \mathbf{i}\mathcal{S}_{\text{eff}}(\phi, \mathbf{S})\right]$$

$$\mathcal{S}_{\text{eff}} = 2i \oint_K dt \oint_K dt'\, G(t,t')G(t',t)\ \left[T_0^2 + T_1^2 \mathbf{S}(t) \cdot \mathbf{S}(t')\right] \cos\left[\frac{\phi(t) - \phi(t')}{2}\right]$$

$$+2i \oint_K dt \oint_K dt'\, F(t,t')F^\dagger(t,t')\ \left[T_0^2 - T_1^2 \mathbf{S}(t) \cdot \mathbf{S}(t')\right] \cos\left[\frac{\phi(t) + \phi(t')}{2}\right]$$

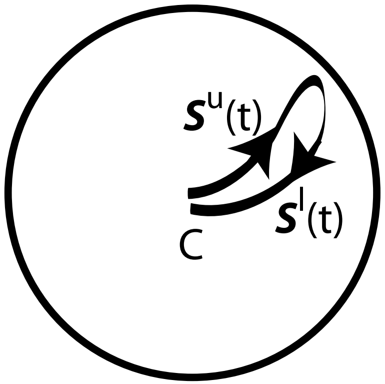
Semiclassics for big spin $S \gg 1$

Wess-Zumino-Novikov-Witten action on Keldysh contour

$$\mathcal{S}_{\text{spin}} = \oint_K dt \mathbf{B} \cdot \mathbf{S} + \mathcal{S}_{WZNW}$$

$$\mathcal{S}_{WZNW} = \frac{1}{S^2} \int_0^1 d\tau \oint_K dt \, [\mathbf{S}(t,\tau) \cdot (\partial_\tau \mathbf{S}(t,\tau) \times \partial_t \mathbf{S}(t,\tau))]$$

$$\mathbf{S(t,0)} = const. \quad , \quad \mathbf{S(t,1)} = \mathbf{S(t)}$$



$$\mathbf{S}_1 \equiv (\mathbf{S}^u + \mathbf{S}^l)/2, \;\; \mathbf{S}_2 \equiv \mathbf{S}^u - \mathbf{S}^l, \;\; \mathbf{S}_1 \cdot \mathbf{S}_2 = 0$$

$$\mathcal{S}_{WZNW} = \frac{1}{S^2} \int dt \, \mathbf{S}_2 \cdot (\mathbf{S}_1 \times \partial_t \mathbf{S}_1)$$

Equations of motion

Voltage bias - imposed phase $\phi(t) = \omega_{\mathsf{J}} t$, $\omega_{\mathsf{J}} = 2eV/\hbar$, $\omega_{\mathsf{J}}, B \ll \Delta$

$$\mathcal{S} = \mathcal{S}_{WZNW} + \int dt \mathbf{B} \cdot \mathbf{S}_2$$

$$+ \int dt \int dt' \left[4D^R(t,t')\, \mathbf{S}_2(t) \cdot \mathbf{S}_1(t') + D^K(t,t')\, \mathbf{S}_2(t) \cdot \mathbf{S}_2(t') \right] \cos\left[\frac{\omega_{\mathsf{J}}(t+t')}{2} \right]$$

$$iD^{>}(t,t') \equiv T_1^2\, F^{>}(t,t') F^{\dagger >}(t,t') \quad , \quad iD^{<}(t,t') \equiv T_1^2\, F^{<}(t,t') F^{\dagger <}(t,t')$$

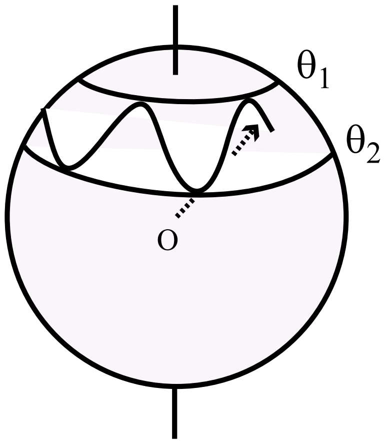
$$\frac{d\mathbf{S}}{dt} = \alpha \mathbf{S} \times \frac{d\mathbf{S}}{dt} \sin \omega_J t + \mathbf{S} \times \mathbf{B} + \mathbf{S} \times \xi$$

$$\alpha \equiv -4 \int_0^\infty d\tau\, \tau D^R(\tau) \sin \frac{\omega_{\mathsf{J}} \tau}{2} \propto g_1 \frac{\omega_{\mathsf{J}}}{\Delta}$$

$g_1 \equiv T_1^2 \rho^2$
spin channel conductance

Spin nutations

$$\frac{d\mathbf{S}}{dt} = \alpha\mathbf{S} \times \frac{d\mathbf{S}}{dt} \sin \omega_J t + \mathbf{S} \times \mathbf{B}$$

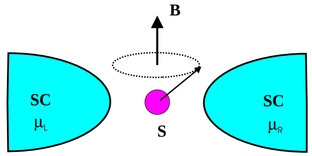


$$\frac{d\phi}{dt} = -\frac{B}{1 + S^2\alpha^2 \sin^2(\omega_J t)} ,$$

$$\frac{d\theta}{dt} = -S\alpha \frac{d\phi}{dt} \sin\theta \sin\omega_J t$$

Amplitude: $\theta_1 - \theta_2 \propto \sin\theta \cdot S \cdot \alpha \cdot \frac{B}{\omega_J} \propto \sin\theta \cdot S \cdot g_1 \cdot \frac{B}{\Delta}$

Quantum spin, $S = 1/2$



$$H_T = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} + \text{h.c.}] \qquad T_{\sigma,\sigma'} = T_0\,\delta_{\sigma\sigma'} + T_1\,\mathbf{S}(t)\cdot\boldsymbol{\sigma}_{\sigma\sigma'}$$

$$\langle T e^{-i\int dt \tilde{\mathcal{H}}_T(t)} \rangle_{\text{el}} = \exp\{i\hat{\mathcal{S}}_{\text{eff}}\} = \exp\left[-\frac{1}{2}\int dt \int dt'\ \langle T\tilde{\mathcal{H}}_T(t)\tilde{\mathcal{H}}_T(t')\rangle_{\text{el}}\right]$$

$$\hat{\mathcal{S}}_{\text{eff}} = i\int dt \int dt'\ F^\dagger(t-t')F(t-t')\left[T_0^2 - T_1^2\left(T\tilde{\mathbf{S}}(t)\cdot\tilde{\mathbf{S}}(t')\right)\right]\times$$

$$\times[Te^{i\tilde{\phi}(t)/2}e^{i\tilde{\phi}(t')/2} + h.c.]$$

$$T\tilde{\mathbf{S}}(t)\cdot\tilde{\mathbf{S}}(0) = \hat{S}_z^2 + (\hat{S}^2 - \hat{S}_z^2)\cos Bt - i\hat{S}_z \sin B|t| = \frac{2\cos Bt + 1}{4} - i\hat{S}_z \sin B|t|$$

Effective Hamiltonian & Josephson Current

$$\hat{S}_{\text{eff}} = -\int dt\, H_{\text{eff}}$$

$$H_{\text{eff}} = -(E_{\text{J}} + \delta E_{\text{J}} \hat{S}_z) \cos\phi$$

$$E_{\text{J}} = (\hbar/2e) I_0 \quad , \quad \delta E_{\text{J}} = (\hbar/2e) I_1$$

$$I_0 = \frac{2\pi^2 e}{\hbar}\left(T_0^2 - \frac{3}{4}T_1^2\right)\rho^2 \Delta \quad , \quad I_1 = \frac{4e}{\hbar} T_1^2 \rho^2 B$$

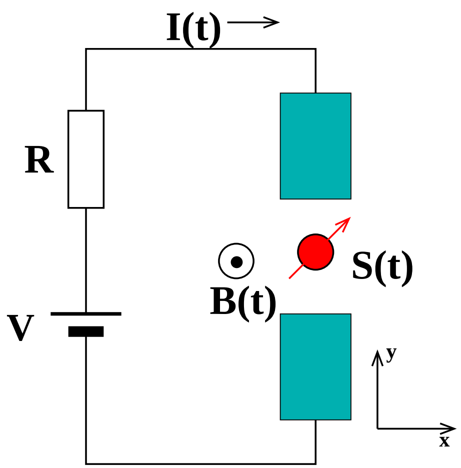
$$\langle \hat{I}(t) \rangle_e = \hat{I}_c \sin\phi(t), \quad \hat{I}_c = I_0 \hat{\mathbf{1}} + I_1 \hat{S}_z$$

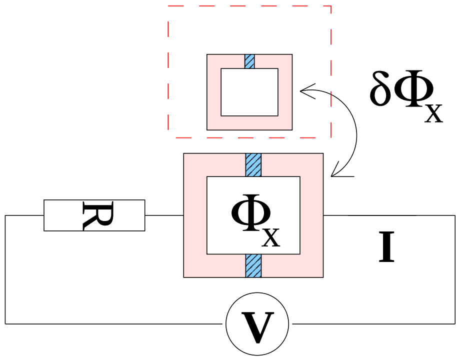
Quantum Non-Demolition (QND) Measurment of S_z

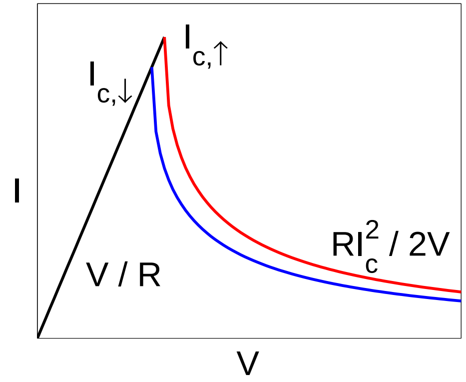
$$\hat{I} = I_0 \sin\varphi + I_1 \sin\phi \cdot \hat{S}_z \quad , \quad H_{\text{spin}} = -B\hat{S}_z - \delta E_{\text{J}} \cos\phi\, \hat{S}_z$$

$$[\hat{S}_z, H_{\text{spin}}] = 0 \qquad \hat{S}_z = const.(\text{QND})$$

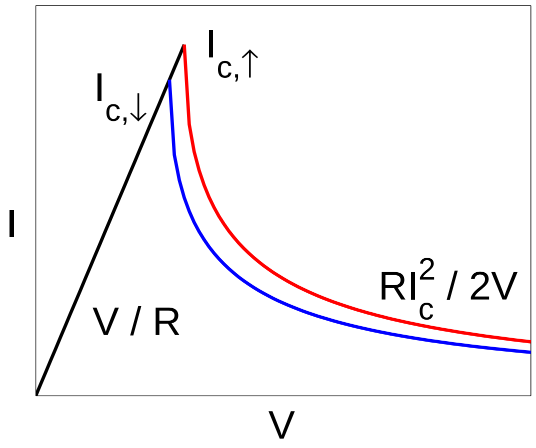
Dissipation needed for real measurements







Measurement and dephasing times



$V \ll I_c R \qquad I = \frac{V}{R}$

$V \gg I_c R \qquad I \approx \frac{R I_c^2}{2V}$

Koch et al., Phys. Rev. Lett. **45**, 2132 (1980)
Averin et al., Physica B **165&166**, 945 (1990)

Incoherent Cooper pair tunneling:

$$I = 2e\Gamma = \frac{2e}{\hbar^2}\frac{E_J^2}{4}\left[P\left(\omega = \frac{2eV}{\hbar}\right) - P\left(\omega = -\frac{2eV}{\hbar}\right)\right], \quad P(t) = \langle e^{i\delta\tilde{\phi}(t)}\, e^{-i\delta\tilde{\phi}(0)}\rangle$$

Measurement time: $t_{\text{meas}}^{-1} = \frac{(I_\uparrow - I_\downarrow)^2}{8 S_I} = \frac{R I_1^2}{8eV} = \frac{eR(\delta E_J)^2}{2\hbar^2 V}$ $\qquad S_I = 2eI$

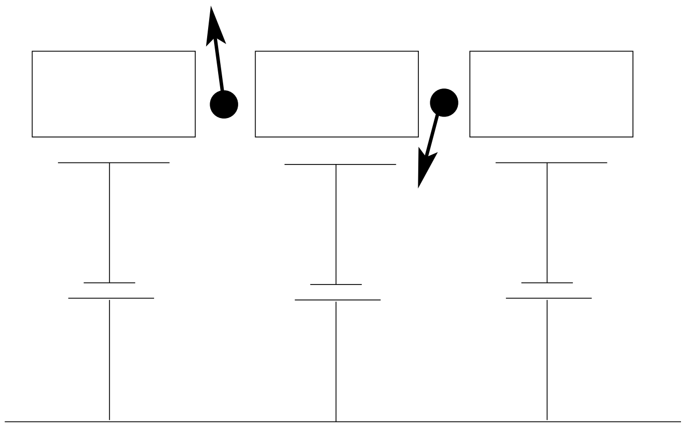
shot noise of C.P.

Dephasing time: $\tau_\varphi^{-1} = \frac{S_{\delta B \equiv \delta E_J \cos\phi}}{2\hbar^2} = \frac{(\delta E_J)^2}{8\hbar^2}\left[P\left(\omega = \frac{2eV}{\hbar}\right) + P\left(\omega = -\frac{2eV}{\hbar}\right)\right]$

At $T = 0$ $\qquad \tau_\varphi = t_{\text{meas}}$ $\qquad$ in general $\tau_\varphi \le t_{\text{meas}}$

Outlook

- Full quantum dynamics with $\mathcal{S}_{\text{eff}}(\phi, \mathbf{S})$
- Manipulation of spins using Josephson currents
- Coupling spins via superconductors (like RKKY)
- Control via gate charges and fluxes in superconducting loops
- Interplay with charge qubits, Coulomb blockade



Physical interpretation

$$H_{\mathsf{spin}} = -BS_z - T_1\,\hat{\mathbf{h}}(t)\cdot\mathbf{S}$$

$$\hat{\mathbf{h}}(t) = -\sum_{\mathbf{k},\mathbf{p},\alpha,\beta} c^{\dagger}_{\mathbf{k}\alpha}(t)(\boldsymbol{\sigma})_{\alpha,\beta}\,c_{\mathbf{p}\beta}(t) + h.c.$$

Oscillating friction $\propto \dot{\mathbf{S}}$: $\mathbf{S}$ changes and acts on $\hat{\mathbf{h}} \rightarrow \hat{\mathbf{h}}$ responds with retardation $\rightarrow \hat{\mathbf{h}}$ acts back on $\mathbf{S}$.

$$\hat{I}(t) = \sum_{\mathbf{k},\mathbf{p},\alpha,\beta} \frac{ie}{\hbar}\hat{c}^{\dagger}_{\mathbf{k}\alpha}(t)[T_0\delta_{\alpha,\beta} + T_1(\hat{\boldsymbol{\sigma}})_{\alpha,\beta}\cdot\hat{\mathbf{S}}(t)]\hat{c}_{\mathbf{p}\beta}(t) = T_0\,\hat{j}(t) + T_1\,\hat{\mathbf{j}}_s(t)\cdot\mathbf{S}(t)$$

Spin dependent Josephson current: $\hat{\mathbf{h}}$ fluctuates and acts on $\mathbf{S} \rightarrow \mathbf{S}$ responds with retardation $\rightarrow$ the response interferes with $\hat{\mathbf{j}}_s(t)$ term in current