Dephasing of solid-state qubits at optimal points

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- Bloch equations, T_1, T_2
- Nonlinear coupling at optimal points
- Ohmic environment
- 1/f noise

Bloch equations

$$\frac{d}{dt}\vec{M} = \vec{B} \times \vec{M} - \frac{1}{T_1}(M_z\vec{z} - M_0\vec{z}) - \frac{1}{T_2}(M_x\vec{x} + M_y\vec{y})$$

F. Bloch (1946,1957), A.G. Redfield (1957)

For 2-level system (spin 1/2) $\vec{M}$ is determined by density matrix

$$M_z = \langle\sigma_z\rangle = \rho_{00} - \rho_{11}$$

$$M_x = \langle\sigma_x\rangle = \mathrm{Re}\rho_{10},$$

$$M_y = \langle\sigma_y\rangle = \mathrm{Im}\rho_{10}$$

$$\vec{M} = \langle\vec{\sigma}\rangle = \mathrm{tr}\,[\vec{\sigma}\rho]$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

For $\vec{B} = B_z\vec{z}$

$$\dot{\rho}_{00} = -\Gamma_\uparrow\rho_{00} + \Gamma_\downarrow\rho_{11}$$

$$\dot{\rho}_{11} = \Gamma_\uparrow\rho_{00} - \Gamma_\downarrow\rho_{11}$$

$$\dot{\rho}_{01} = -iB_z\rho_{01} - \frac{1}{T_2}\rho_{01}$$

$\frac{1}{T_1} = \Gamma_\downarrow + \Gamma_\uparrow$ and $|M_0| = T_1(\Gamma_\downarrow - \Gamma_\uparrow)$

Thermal equilibrium: $\frac{\Gamma_\downarrow}{\Gamma_\uparrow} = \exp\left(\frac{\Delta E}{k_B T}\right) \Leftrightarrow |M_0| = \tanh\left(\frac{\Delta E}{2k_B T}\right)$; $\Delta E \equiv |\vec{B}|$

Relaxation and dephasing: $\Gamma_{\mathrm{rel}} \equiv T_1^{-1}$, $\Gamma_\varphi \equiv T_2^{-1}$

Golden rule: relaxation rate

$$H = -\tfrac{1}{2}\Delta E\,\sigma_z - \tfrac{1}{2}\,U\,\sigma_z - \tfrac{1}{2}\,V\,\sigma_x + H_{\mathrm{bath}}(U,V)$$

$\uparrow$ $\uparrow$

longitudinal and transverse coupling to bath

Transverse coupling $\Rightarrow$ transitions, relaxation

Golden rule: $|i\rangle, |f\rangle$ - states of bath, E_i, E_f - energies of $|i\rangle, |f\rangle$

$$\Gamma_\downarrow = \tfrac{2\pi}{\hbar}\,\tfrac{1}{4}\;\Sigma_{i,f}\,\rho_i\,|\langle i|\,V\,|f\rangle|^2\,\delta(E_i + \Delta E - E_f)$$

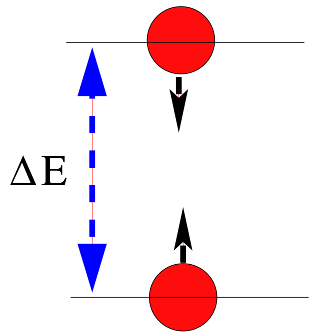
$$= \tfrac{2\pi}{\hbar}\,\tfrac{1}{4}\;\Sigma_{i,f}\,\rho_i\;\langle i|\,V\,|f\rangle\,\langle f|\,V\,|i\rangle\;\tfrac{1}{2\pi\hbar}\int dt\;e^{i\frac{t}{\hbar}\,(E_i+\Delta E-E_f)}$$

$$= \tfrac{1}{4\hbar^2}\;\int dt\;\Sigma_i\,\rho_i\;\langle i|\,V(t)V\,|i\rangle\;\;e^{i\frac{t}{\hbar}\,\Delta E} = \tfrac{1}{4\hbar^2}\,\langle V^2_{\omega=\Delta E/\hbar}\rangle$$

$$\Gamma_\downarrow = \tfrac{1}{4}\,\hbar^{-2}\,\langle V_\omega^2\rangle_{\omega=\Delta E/\hbar}$$

$$\Gamma_\uparrow = \tfrac{1}{4}\,\hbar^{-2}\,\langle V_\omega^2\rangle_{\omega=-\Delta E/\hbar}$$

$$\langle V_\omega^2\rangle \equiv \int\,dt\,e^{i\omega t}\,\langle V(t)V(0)\rangle$$



$$\frac{1}{T_1} = \Gamma_\downarrow + \Gamma_\uparrow = \frac{1}{2}\hbar^{-2}\;S_V(\omega = \Delta E/\hbar)$$

$$S_V(\omega) \equiv \tfrac{1}{2}\left\{\langle V_\omega^2\rangle + \langle V_{-\omega}^2\rangle\right\}$$

Detailed balance (thermal equilibrium bath)

$$\frac{\langle V_\omega^2\rangle_{\omega=\Delta E/\hbar}}{\langle V_\omega^2\rangle_{\omega=-\Delta E/\hbar}} = \frac{\Gamma_\downarrow}{\Gamma_\uparrow} = \exp\left(\frac{\Delta E}{k_{\mathrm{B}}T}\right)$$

Longitudinal coupling $\Rightarrow$ (pure) dephasing

$$H = -\frac{1}{2}\Delta E\,\sigma_z - \frac{1}{2}U\,\sigma_z + H_{\mathrm{bath}}$$

Simplified calculation: U treated as classical field

$$|\rho_{01}(t)| = |\langle\sigma_+(t)\rangle| \propto \left\langle e^{-\frac{i}{\hbar}\int_0^t dt'\,U(t')}\right\rangle = e^{-\frac{1}{2\hbar^2}\int_0^t dt' \int_0^t dt'' \langle U(t')U(t'')\rangle}$$

$$= e^{-\frac{1}{2\hbar^2}\int \frac{d\omega}{2\pi} S_U(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}} \approx e^{-\frac{1}{2\hbar^2} S_U(\omega\approx 0)\,t}$$

$$\frac{\sin^2(\omega t/2)}{(\omega/2)^2} \approx 2\pi\delta(\omega)\,t$$

$$\frac{1}{T_2} = \Gamma_\varphi = \frac{1}{2\hbar^2}\,S_U(\omega\approx 0)$$

Longitudinal + transverse coupling

$$\frac{1}{T_1} = \Gamma_{\mathrm{rel}} = \frac{1}{2\hbar^2}\,S_V(\omega = \Delta E)$$

$$\frac{1}{T_2} = \Gamma_\varphi = \frac{1}{2}\,\Gamma_{\mathrm{rel}} + \frac{1}{2\hbar^2}\,S_U(\omega\approx 0)$$

Bloch (1957)
Redfield (1957)

$$\Gamma_\varphi = \frac{1}{2}\,\Gamma_{\mathrm{rel}} + \Gamma_\varphi^* \leftarrow \text{“pure dephasing”}$$

Applicability of Bloch-Redfield equations

Bath's correlation time: $\langle\langle U(t)U(0)\rangle\rangle = \langle\langle V(t)V(0)\rangle\rangle = 0$ for $t \gg \tau$

Longitudinal noise: $T_2^{-1} \ll \tau^{-1}$

Transverse noise: $T_1^{-1} \ll \max(\tau^{-1}, \Delta E)$

Example: Spin-Boson model with Ohmic bath

$$\boxed{H = -\frac{1}{2}\Delta E\ \sigma_z - \frac{1}{2}V\ \sigma_x + H_{\rm bath}}$$

$$V = \sum_\alpha c_\alpha x_\alpha$$

$$S_V = \alpha\omega \coth\frac{\omega}{2T} \quad ; \quad \tau \approx T^{-1}$$

$$T_1^{-1} = \alpha\Delta E \coth\frac{\Delta E}{2T}$$

Bloch Redfield applicable if $\alpha \ll 1$

Strong coupling physics for $\alpha \sim 1$ U. Weiss, A. Leggett ...

1/f noise

$$\mathcal{H} = -\frac{1}{2}\,\Delta E\,\hat{\sigma}_z - \frac{1}{2}\,X(t)\,(\cos\eta\,\hat{\sigma}_z - \sin\eta\,\hat{\sigma}_x) + \mathcal{H}_{\text{bath}}$$

- Problem: simple formulas fail for 1/f noise

$$\Gamma_\varphi = \frac{1}{2}\,\Gamma_{\text{rel}} + S_X(\omega = 0)\,\cos^2\eta$$

$$S_X(\omega) = \frac{E_{1/f}}{|\omega|} \qquad \text{diverges at } \omega = 0$$

- To avoid trouble:

  a) $\eta = \frac{\pi}{2}$ (purely transverse coupling)

  b) Echo technique Y. Nakamura 02, D. Vion et al. 02

- Exact classical solution for longitudinal 1/f noise (X being Gaussian)

  Cottet et al. 01

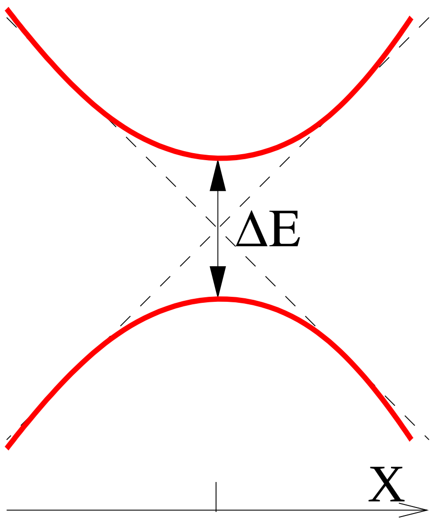
$$|\langle\sigma_+(t)\rangle| \propto \left\langle e^{-\frac{i}{\hbar}\int_0^t dt'\,X(t')}\right\rangle = e^{-\frac{1}{2\hbar^2}\int_0^t dt'\int_0^t dt''\langle X(t')X(t'')\rangle}$$

$$= e^{-\frac{1}{2\hbar^2}\int\frac{d\omega}{2\pi}\,S_X(\omega)\,\frac{\sin^2(\omega t/2)}{(\omega/2)^2}}$$

$$\text{Singular } S_X(\omega) \;\Rightarrow\; \frac{\sin^2(\omega t/2)}{(\omega/2)^2} \neq 2\pi\delta(\omega)t$$

$$|\langle\sigma_+(t)\rangle| \propto \exp\!\left(-\frac{E_{1/f}^2}{2\pi\hbar^2}\,t^2\,|\ln t\omega_{\text{ir}}|\right) \qquad T_2^{-1} \approx \frac{E_{1/f}}{\hbar}\sqrt{\frac{1}{2\pi}\ln\frac{E_{1/f}}{\omega_{\text{ir}}}}$$

$1/f$ noise $S_X(\omega) = E_{1/f}^2/|\omega|$, transverse coupling



To protect from leading effect of $1/f$ noise: choose transverse coupling

$$H = -\frac{1}{2}\Delta E\,\sigma_z - \frac{1}{2}X\sigma_x + H_{\rm bath}$$

Golden Rule (1-st order) for $E_{1/f} \ll \Delta E$

$$\Gamma_{\rm rel} = \frac{1}{2}S_X(\Delta E) = \frac{E_{1/f}^2}{2\Delta E} \quad ; \quad \underline{\Gamma_\varphi/\Gamma_{\rm rel} = 1/2}$$

For 1/f noise second order is important !

A.S., Yu. Makhlin, G. Schön, Phys. Script. 02
Yu. Makhlin, A.S., cond-mat/0308297
Yu. Makhlin, A.S., JETP Letters 03

Adiabatic approximation for $\omega \ll \Delta E \;\Rightarrow\; H = -\frac{1}{2}\Delta E(X)\,\sigma_z + H_{\rm bath}$

$$\Delta E(X) = \sqrt{\Delta E^2 + X^2} \approx \Delta E + \frac{X^2}{2\Delta E}$$

linear transverse $\Rightarrow$ quadratic longitudinal coupling

Gaussian approximation: treat X^2 as Gaussian

$$S_{X^2/\Delta E}(\omega) \equiv \frac{1}{2\Delta E^2}\langle\{X^2(t), X^2(0)\}\rangle_\omega \sim \frac{E_{1/f}^4}{\Delta E^2}\frac{1}{|\omega|}\ln\frac{\omega}{\omega_{\rm ir}}$$

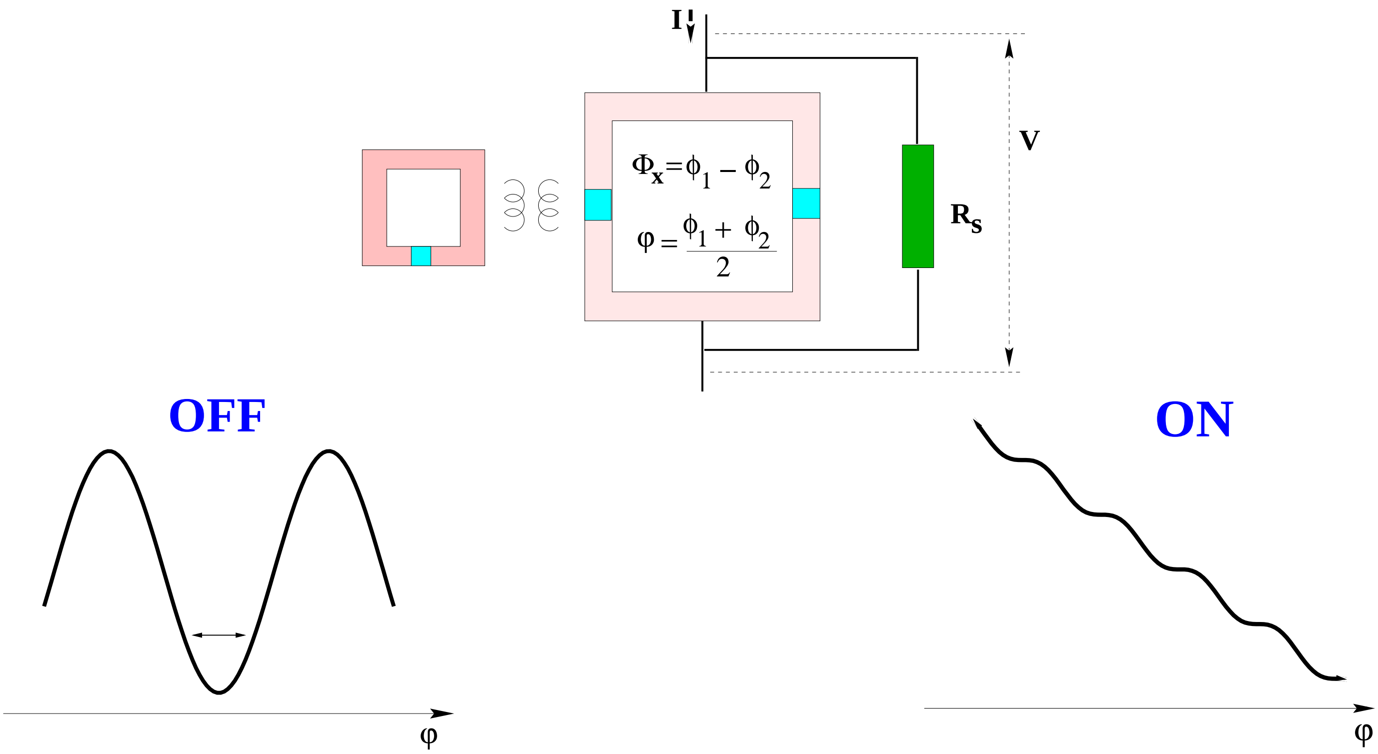
Again 1/f noise with new scale $\tilde{E}_{1/f} \equiv \frac{E_{1/f}^2}{\Delta E}$

Decay law in Gaussian approximation

$$|\langle\sigma_+(t)\rangle| \propto \exp\left(-\frac{E_{1/f}^4}{\Delta E^2}\,t^2\,|\ln^2(\omega_{\rm ir}t)|\right) \qquad T_2^{-1} \sim \frac{E_{1/f}^2}{\Delta E}\ln\frac{E_{1/f}^2}{\omega_{\rm ir}\Delta E}$$

$$\underline{T_1/T_2 \approx 1/2 + \ln(..)}$$

Dephasing by symmetric dc-SQUID in the off state
Nonlinear coupling



$$H = -\frac{\epsilon}{2}\,\hat{\sigma}_z - \frac{\Delta}{2}\,\hat{\sigma}_x - \frac{\Phi_0}{2\pi}\,I_c(\Phi_x + \delta\Phi\,\hat{\sigma}_z)\cos\varphi$$

$$H = -\frac{\epsilon}{2}\,\hat{\sigma}_z - \frac{\Delta}{2}\,\hat{\sigma}_x - \frac{\Phi_0}{2\pi}\,I_c(\Phi_x)\cos\varphi - \frac{\Phi_0}{2\pi}\,\delta I_c\,\hat{\sigma}_z\,\cos\varphi$$

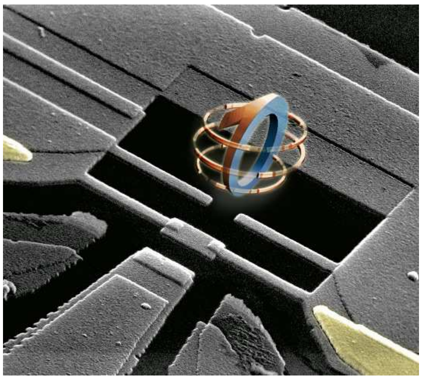
Small fluctuations: $\cos\varphi \approx 1 - \varphi^2/2$

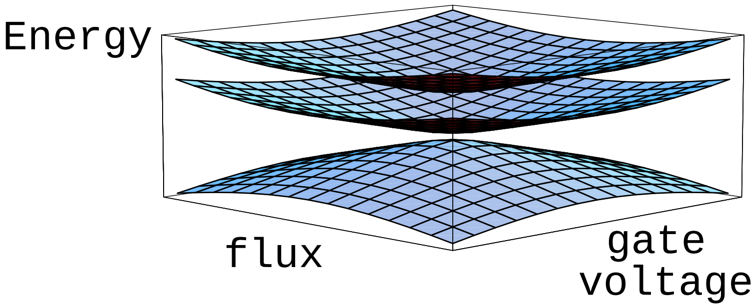
$$\boxed{H_{\rm int} \propto \hat{\sigma}_z \varphi^2}$$

Fluctuations of φ are Gaussian: governed by the shunt resistor

Recent experiments

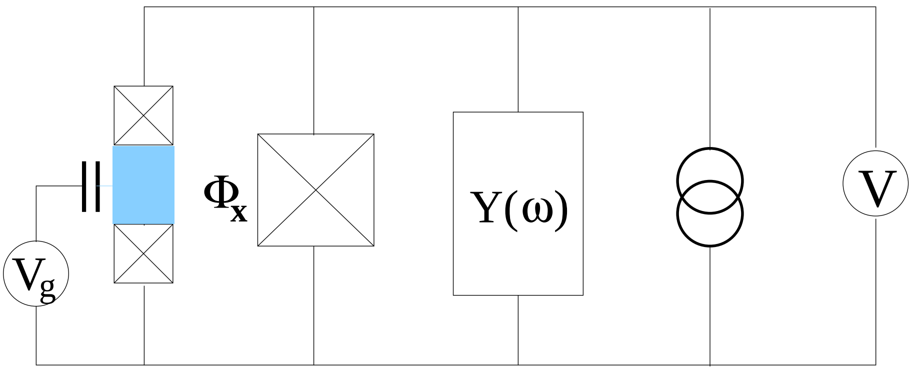
- Charge-flux qubit with phase read-out (Saclay design):





Vion et al. '02

$E_{\rm J} \sim E_{\rm C}$



Symmetry point: $\Delta E_{\rm ch}(V_{\rm g0}) = 0$ and $\partial E_{\rm J}(\Phi_{\rm x0})/\partial\Phi_{\rm x} = 0$:

$$\mathcal{H} = -\frac{1}{2} E_{\rm J}(\Phi_{\rm x0})\,\sigma_x - \frac{1}{2}\frac{\partial \Delta E_{\rm ch}}{\partial V_{\rm g}}\,\delta V_{\rm g}\,\sigma_z - \frac{1}{4}\frac{\partial^2 E_{\rm J}}{\partial \Phi_{\rm x}^2}\,\delta\Phi_{\rm x}^2\,\sigma_x$$

trans. lin. long. quadr.

↓

low ω $\rightarrow$ long. quadr.

In experiment

$T_1/T_2 \approx 3$

Quadratic longitudinal dephasing

$$\mathcal{H} = -\frac{1}{2}\,\Delta E\;\hat{\sigma}_z - \frac{1}{2}\,\frac{X^2}{E_0}\;\hat{\sigma}_z + H_{\rm bath}$$

$$\langle \hat{\sigma}_+(t)\rangle = e^{-\frac{i\Delta E t}{\hbar}}\;{\rm Tr}\,\left[S^\dagger(t,0)\,\hat{\sigma}_+\,S(t,0)\hat{\rho}_0\right]$$

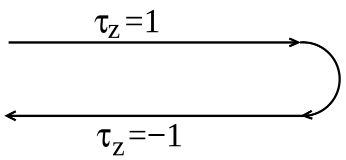
$$S(t,0) \equiv T\exp\left[\frac{i}{\hbar}\int\limits_0^t \frac{X^2(t')}{2E_0}\,\hat{\sigma}_z\,dt'\right]$$

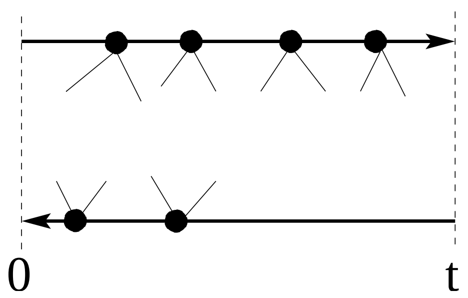
If initial conditions factorized, $\hat{\rho}_0 = \hat{\rho}_{\rm s}\otimes\hat{\rho}_{\rm bath}$

$$\langle \hat{\sigma}_+(t)\rangle = P(t)\;e^{-i\frac{\Delta E}{\hbar}t}\;\langle \hat{\sigma}_+(0)\rangle$$

$$P(t) = \langle \tilde{T}\exp\left(\frac{i}{\hbar}\int\limits_0^t \frac{X^2}{2E_0}\,dt'\right)\;T\exp\left(\frac{i}{\hbar}\int\limits_0^t \frac{X^2}{2E_0}\,dt'\right)\rangle$$

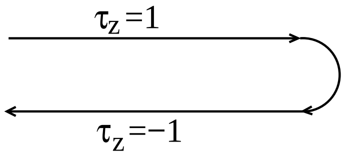
$$P(t) = \langle T_{\rm K}\exp\left(\frac{i}{\hbar}\oint \frac{X^2(t')\,\tau_z(t')}{2E_0}\,dt'\right)\rangle$$

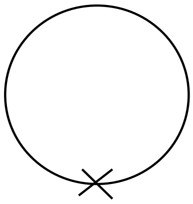
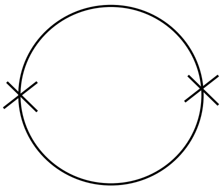
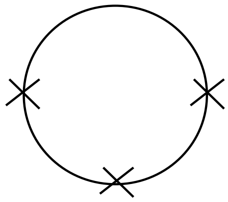


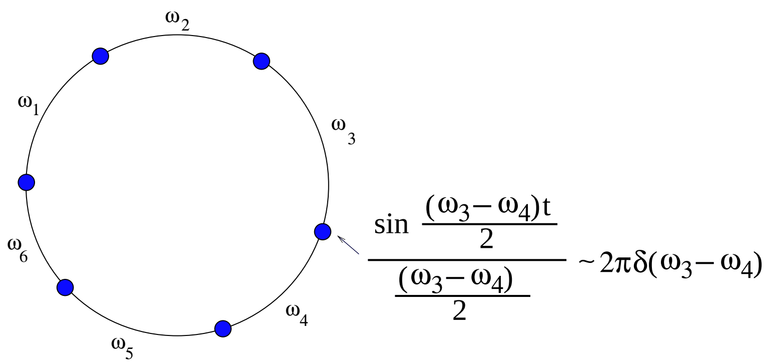


Quadratic longitudinal dephasing

$$P(t) = \langle T_{\rm K} \exp\left(\frac{i}{\hbar}\oint \frac{X^2(t')\,\tau_z(t')}{2E_0}\,dt'\right)\rangle$$



$$\ln P(t) = \sum_{n=1}^{\infty}\frac{1}{n}F_n(t) =$$  $+\frac{1}{2}$  $+\frac{1}{3}$ 



$$F_n(t) = \frac{1}{2}\left(\frac{-1}{E_0}\right)^n \mathrm{Tr}\int_0^t\int_0^t\int_0^t dt_1dt_2dt_3 ... \hat{D}(t_1-t_2)\,\hat{1}\,\hat{D}(t_2-t_3)\,\hat{1}\ldots =$$

$$= \frac{1}{2}\left(\frac{-1}{E_0}\right)^n \mathrm{Tr}\int \frac{d\omega_1 d\omega_2..}{(2\pi)^n}\hat{D}(\omega_1)\hat{D}(\omega_2)...\frac{\sin\left[(\omega_1-\omega_2)t/2\right]}{(\omega_1-\omega_2)/2}\,\frac{\sin\left[(\omega_2-\omega_3)t/2\right]}{(\omega_2-\omega_3)/2}...$$

The Green's functions

$$\begin{pmatrix} D^c & D^< \\ D^> & D^{ac}\end{pmatrix} = \begin{pmatrix} -i(S_X - i\chi') & -i(S_X-\chi'') \\ -i(S_X+\chi'') & -i(S_X+i\chi')\end{pmatrix}$$

$$S_X(t) \equiv \frac{1}{2}\{X(t),X(0)\}_+ \quad , \quad \chi(t)\equiv i\,\theta(t)[X(t),X(0)]_-$$

Ohmic case

Regular spectrum

$$\frac{\sin(\omega t/2)}{\omega/2} \to 2\pi\delta(\omega)$$

$$F_n(t) = t\,\frac{(-1)^n}{2E_0^n}\,\mathrm{Tr}\int\frac{d\omega}{2\pi}\,\hat{D}^n(\omega)$$

Sum the series

$$\ln P(t) = \sum_{n=1}^{\infty}\frac{1}{n}\,F_n(t) = -\frac{t}{2}\int\frac{d\omega}{2\pi}\,\ln\mathrm{Det}\left(1+\frac{\hat{D}(\omega)}{E_0}\right)$$

Second order (Gaussian approximation)

$$\mathrm{Re}\,\ln P(t) \approx \frac{1}{2}\,F_2(t) = t\int\frac{d\omega}{2\pi}\left[\frac{4D^>D^< + D_\mathrm{R}^2 + D_\mathrm{A}^2}{4E_0^2}\right]$$

$$\langle\hat{\sigma}_+(t)\rangle \propto e^{-\Gamma_\varphi t} \qquad \Gamma_\varphi = \frac{S_{X^2}(\omega=0)}{2E_0^2}$$

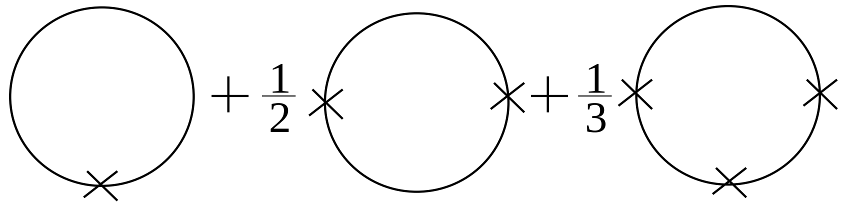
$$S_X \equiv \frac{1}{2}\{X(t),X(0)\}_\omega = \alpha\,\omega\,\coth\frac{\omega}{2T}$$

$$S_{X^2} \equiv \frac{1}{2}\{X^2(t),X^2(0)\}_\omega = \frac{\alpha^2}{3\pi}(\omega^2+4\pi^2T^2)\,\omega\,\coth\frac{\omega}{2T}$$

$$\Gamma_\varphi = \frac{4\pi}{3}\,\alpha^2\,\frac{T^3}{E_0^2}$$

Higher orders small if $|D_R(\omega\approx 0)| \approx \chi(\omega=0) \approx \alpha\omega_c \ll E_0$

1/f noise, quadratic longitudinal coupling

$$\ln P(t) = \sum_{n=1}^{\infty} \frac{1}{n} F_n =$$ 

$$F_n(t) = \frac{1}{2}\left(\frac{-1}{E_0}\right)^n \mathrm{Tr} \int_0^t \int_0^t \int_0^t dt_1 dt_2 dt_3 ... \hat{D}(t_1 - t_2)\hat{D}(t_2 - t_3)... =$$

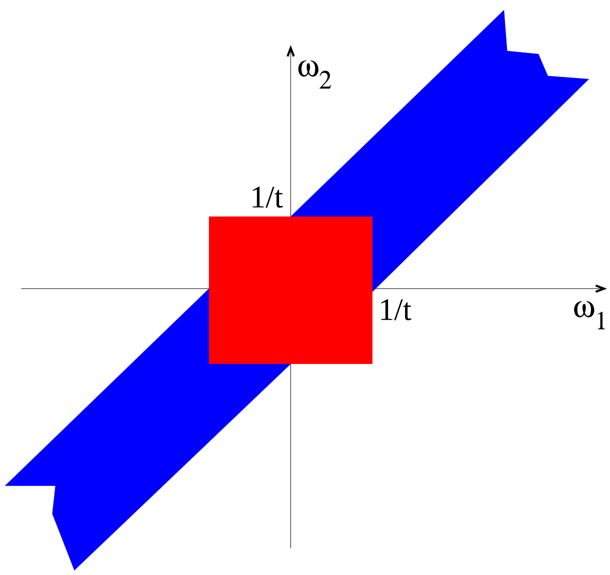
$$= \frac{1}{2}\left(\frac{-1}{E_0}\right)^n \mathrm{Tr} \int \frac{d\omega_1 d\omega_2 ..}{(2\pi)^n} \hat{D}(\omega_1)\hat{D}(\omega_2) ... \frac{\sin\left[(\omega_1 - \omega_2)t/2\right]}{(\omega_1 - \omega_2)/2} \frac{\sin\left[(\omega_2 - \omega_3)t/2\right]}{(\omega_2 - \omega_3)/2} ...$$

The Green's functions

$$\begin{pmatrix} D^c & D^< \\ D^> & D^{ac} \end{pmatrix} = \begin{pmatrix} -i(S - i\chi') & -i(S - \chi'') \\ -i(S + \chi'') & -i(S + i\chi') \end{pmatrix} \approx \begin{pmatrix} -iS & -iS \\ -iS & -iS \end{pmatrix}$$

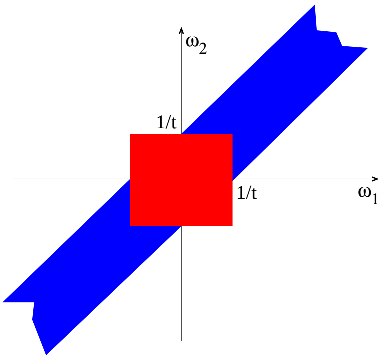
$$S(\omega) = \frac{E_{1/f}}{|\omega|} \qquad \chi''(\omega) = ? \ (\text{non-singular}) \qquad \chi'(\omega) = \int \frac{d\nu}{\pi} \frac{\chi''(\nu)}{\nu - \omega}$$

$$F_n(t) = \frac{1}{2}\left(\frac{2i}{E_0}\right)^n \int \frac{d\omega_1 d\omega_2 ..}{(2\pi)^n} S(\omega_1)S(\omega_2) ... \frac{\sin\left[(\omega_1 - \omega_2)t/2\right]}{(\omega_1 - \omega_2)/2} \frac{\sin\left[(\omega_2 - \omega_3)t/2\right]}{(\omega_2 - \omega_3)/2} ...$$



1/f noise, quadratic longitudinal coupling

$$\Gamma_\varphi \equiv \frac{E_{1/f}^2}{E_0}$$



$$F_n = \frac{1}{2}\left[\frac{2it}{\pi}\int_{\omega_{\rm ir}}^{1/t}\frac{\Gamma_\varphi d\omega}{|\omega|}\right]^n = \frac{1}{2}\left[\frac{2}{\pi}it\Gamma_\varphi \ln\frac{1}{\omega_{\rm ir}t}\right]^n$$

$$F_n = t\,(2i\Gamma_\varphi)^n \int_{1/t}^{\infty}\frac{d\omega}{2\pi}\,\omega^{-n}$$

- Decay law for $t < \Gamma_\varphi^{-1}$

$$|\langle\sigma_+(t)\rangle| \sim \left[1+\left(\frac{2}{\pi}\Gamma_\varphi\, t\ln(\omega_{\rm ir}t)\right)^2\right]^{-\frac{1}{4}}$$

Gaussian approximation $|\langle\sigma_+(t)\rangle| \sim \exp\left(-\frac{\Gamma_\varphi^2}{\pi^2}\,t^2\,\ln^2(\omega_{\rm ir}t)\right)$

works only at short times $t < \left(\Gamma_\varphi \ln\left(\frac{\Gamma_\varphi}{\omega_{\rm ir}}\right)\right)^{-1}$ when

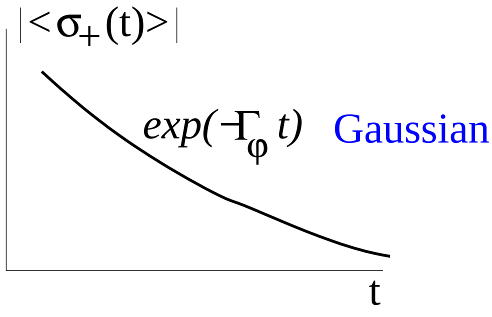
$|\langle\sigma_+(t)\rangle| \sim 1 - \frac{\Gamma_\varphi^2}{\pi^2}\,t^2\,\ln^2(\omega_{\rm ir}t)$

- Decay law for $t > \Gamma_\varphi^{-1}$

$$|\langle\sigma_+(t)\rangle| \sim \exp\left(-\frac{1}{2}\Gamma_\varphi t\right)$$

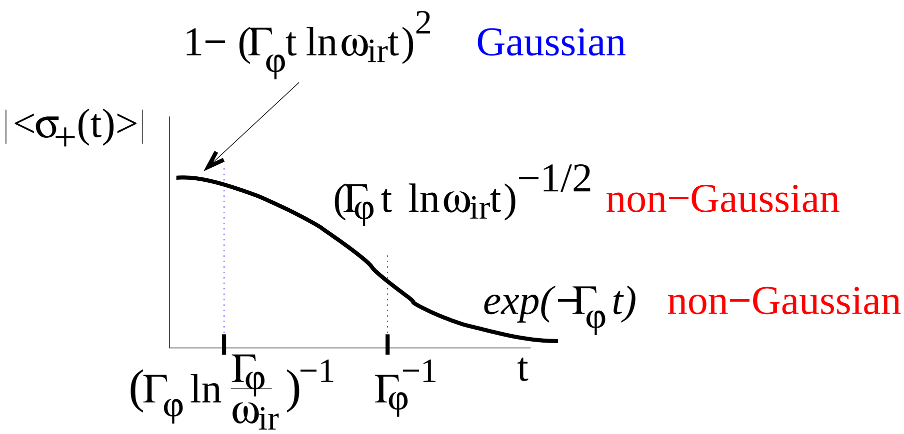
Results

Ohmic noise:



$$\Gamma_\varphi = \frac{\alpha^2 T^3}{E_0^2}$$

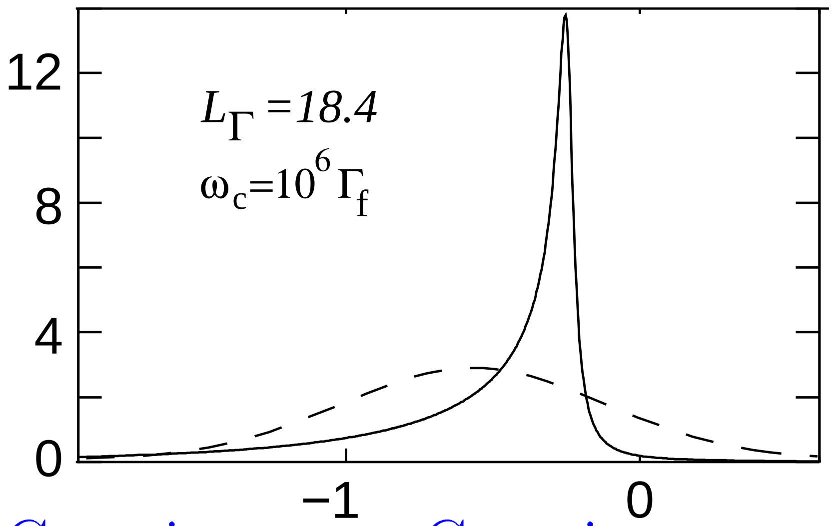
$1/f$ noise:



$$\Gamma_\varphi = \frac{E_{1/f}^2}{E_0}$$

Transverse susceptibility of the qubit ($1/f$ noise)

$$\chi_{\sigma_x} \equiv i\,\theta(t)\,\langle[\sigma_x(t), \sigma_x(0)]\rangle$$



$$\chi''(\omega) \propto \sqrt{|\omega - \Delta E|}$$

height $\propto 1/\sqrt{\Gamma_\varphi}$

Gaussian - non-Gaussian

We assume X Gaussian - big bath, many fluctuators

Few fluctuators, small bath - X - non-Gaussian

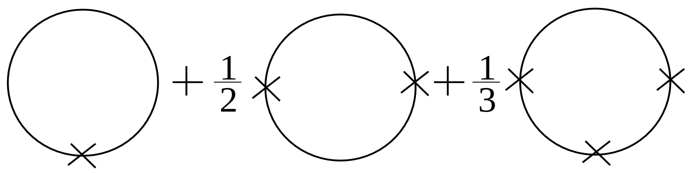
Paladino, Faoro, Falci, Fazio, PRL '02

Galperin, Altshuler, Shantsev, cond-mat/0312490

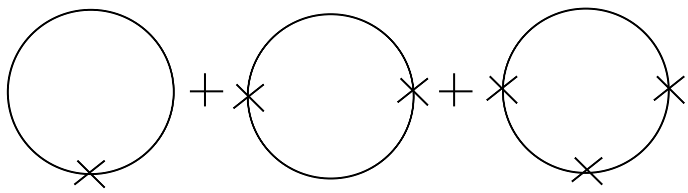
Both cases realize experimentally

"Screening" of $1/f$ noise

$$\ln P(t) = \sum_{n=1}^{\infty} \frac{1}{n} F_n =$$



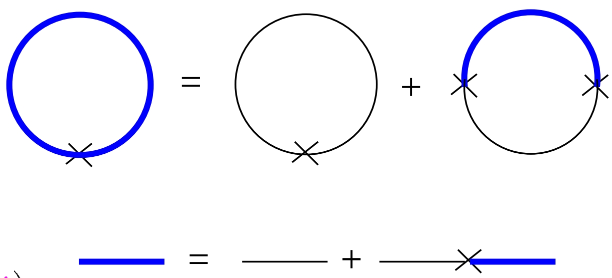
$$\frac{\partial \ln P(t)}{\partial \ln(1/E_0)} = \sum_{n=1}^{\infty} F_n =$$



$$\frac{\partial \ln P(t)}{\partial \ln(1/E_0)} = \tilde{F}_1(t) = F_1 + \tilde{F}_2$$

$$\tilde{F}_1 = -\frac{1}{2E_0} \,\mathrm{Tr} \int_0^t dt' \, \tilde{D}(t', t'; t)$$

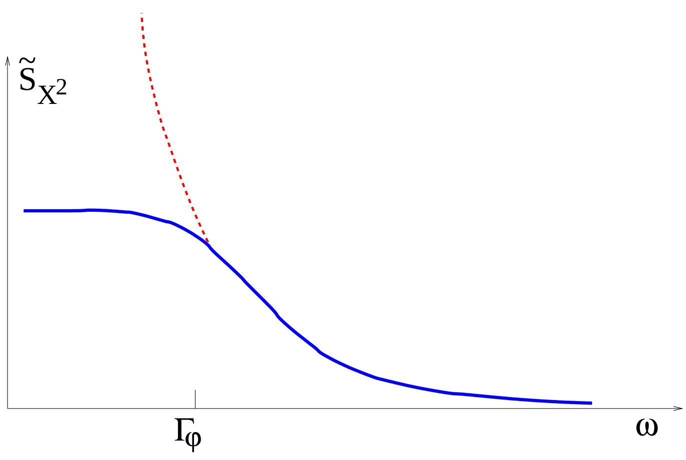
$$\tilde{F}_2 = \frac{1}{2E_0^2} \,\mathrm{Tr} \int_0^t \int_0^t dt' dt'' \, D(t' - t'') \, \tilde{D}(t', t'; t)$$



Dyson equiation

$$\tilde{D}(t', t''; t) = D(t' - t'') - \frac{1}{E_0} \int_0^t d\tau D(t' - \tau) \, \tilde{D}(\tau, t''; t)$$

Screening at $t > \Gamma_\varphi^{-1}$



Summary

1. Nonlinear longitudinal coupling is essential at optimal points

2. Gaussian approximation works for Ohmic noise: $\Gamma_{\rm rel} \propto T^3$

3. Strong non-Gaussian effects for $1/f$ noise: $\langle \sigma_+(t) \rangle \propto 1/\sqrt{t}$

4. At long time: screening of the low frequency correlations