DECOHERENCE IN RESONANTLY DRIVEN BISTABLE SYSTEMS

Sigmund Kohler (Uni Augsburg)

collaboration: Peter Hänggi (Augsburg)
Thomas Dittrich (Bogotá)

- introduction: driven tunneling
- chaotic tunneling
  in resonantly driven systems
- dissipative chaotic tunneling

TUNNELING

wave function

$$\psi(t) = \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar}E_1 t} \varphi_1 + e^{-\frac{i}{\hbar}E_2 t} \varphi_2 \right)$$

time-scale for observables

$$\boxed{\frac{2\pi\hbar}{|E_1 - E_2|}}$$

dominating time-scale, if E_1, E_2 nearly degenerate

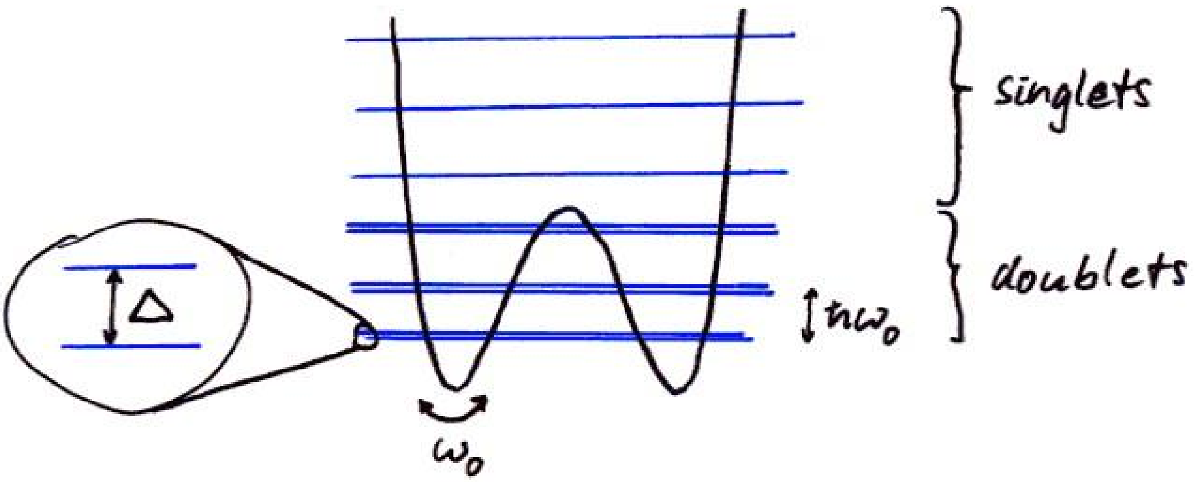
time-dependent field:

correction by non-adiabatic phase

our working model:

quartic double-well potential

$$V(x) = -\frac{1}{4} m\omega_0^2 x^2 + \frac{m^2\omega_0^4}{64 E_B} x^4$$



number of doublets:

$\approx \mathcal{D} = E_B/\hbar\omega_0$ (effective action, class. limit: $\mathcal{D} \to \infty$)

symmetry: parity

$\longrightarrow$ doublets consist of even and odd states which are nearly degenerate

time-scales:

ω_0 intra-well motion

$\Delta/\hbar$ inter-well motion (tunneling)

note: $\frac{\Delta}{\hbar} \lll \omega_0$

DRIVEN TUNNELING

Influence of an external field ?

$$H(t) = H_{DW} + S\, x \cos \Omega t$$
$$= H(t + \tfrac{2\pi}{\Omega})$$

time-periodic !

FLOQUET THEORY

solutions (Floquet states) of type

$$\psi(t) = e^{-i\varepsilon t}\, \varphi(t), \quad \varphi(t) = \varphi(t + \tfrac{2\pi}{\Omega})$$
$$= \sum_n e^{-in\Omega t} \varphi_n$$

quasienergy ε
long-time dynamics

dynamics within driving period

contributions with frequencies

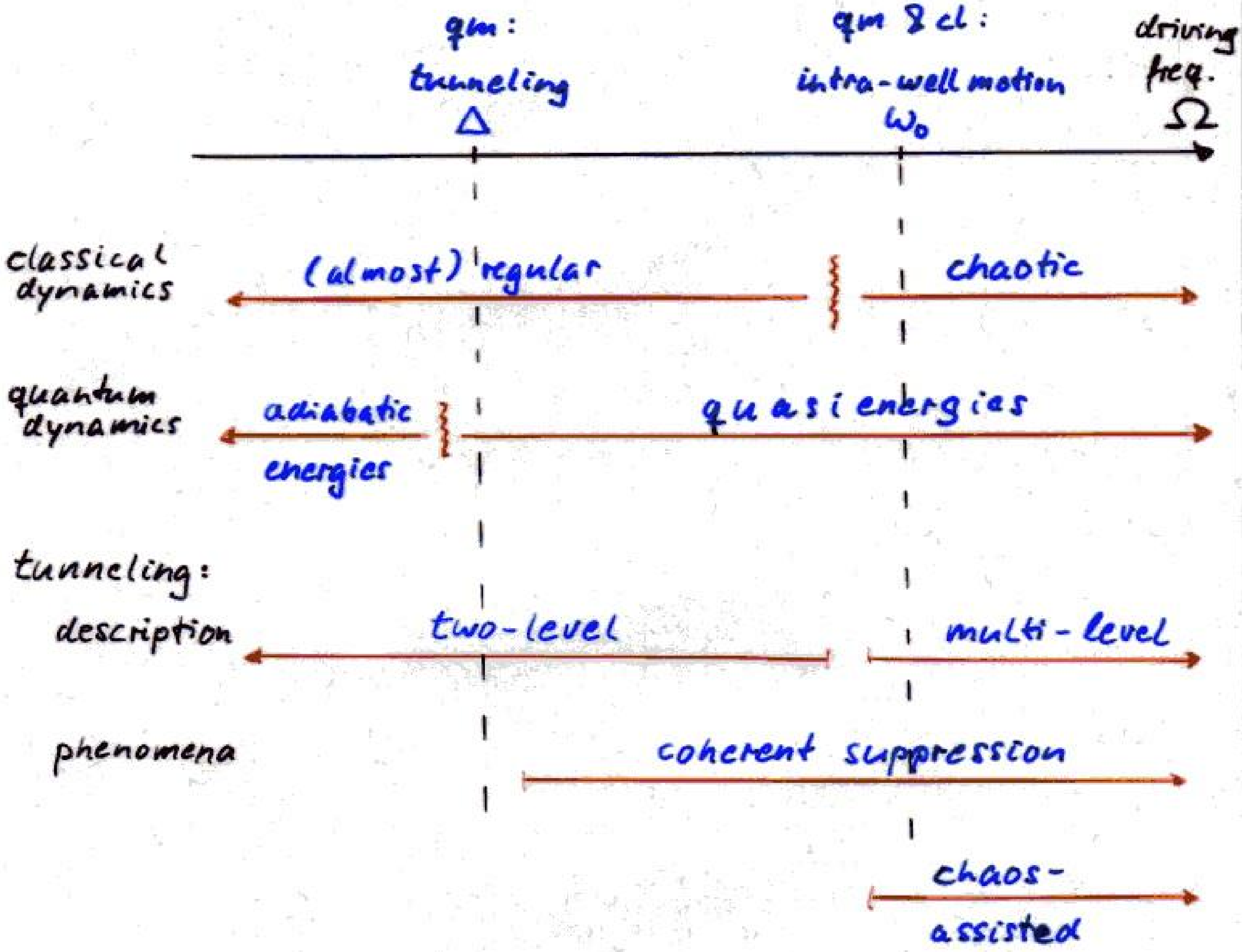
$$\varepsilon_{\alpha,n} = \varepsilon_\alpha + n\Omega$$

Index n:

- „Brillouin-zone" structure
- semiclassical interpretation: quanta of driving field

ordering possible with respect to mean energy

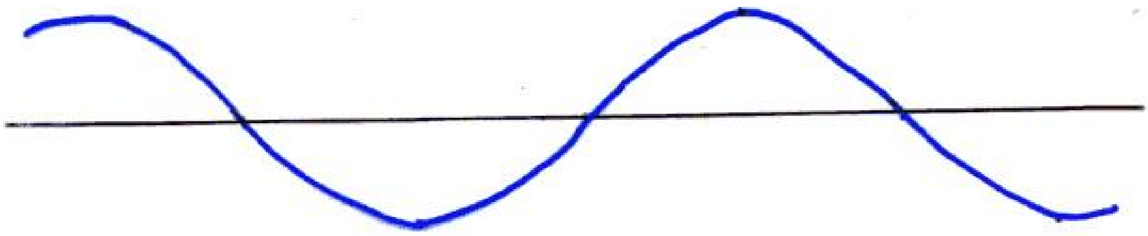
TIME/FREQUENCY SCALES

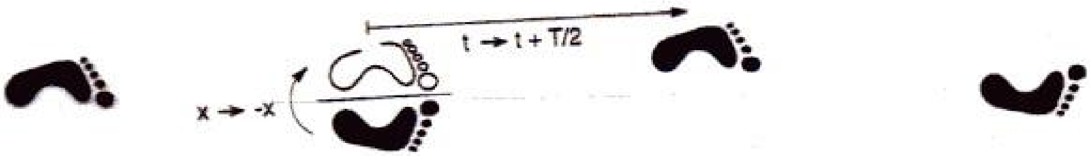


SYMMETRY

$$H(t) = H_{DW} + S\,x \cos \Omega t$$

parity $x \to -x$ parity broken !?





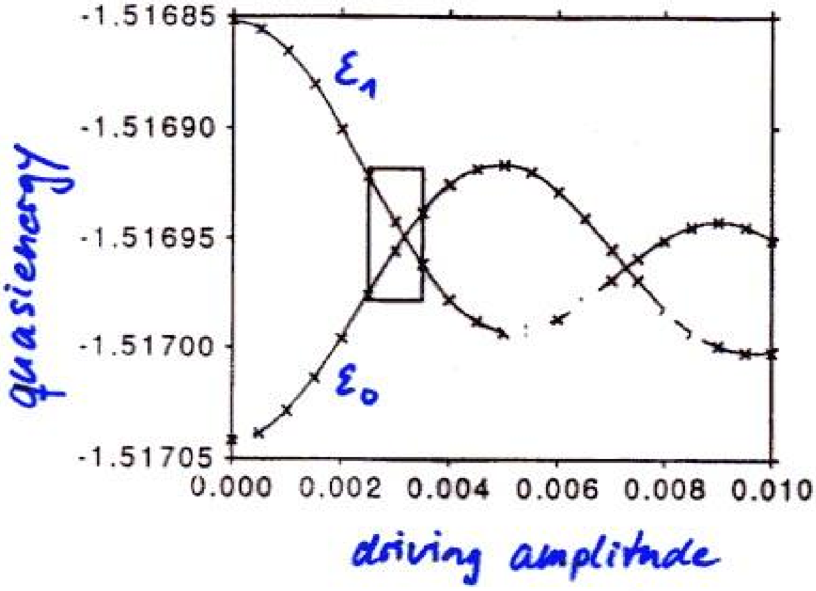
(T. Dittrich, PhiuZ 6/97)

generalized parity :

$$P_{\Omega} : (x, p, t) \longrightarrow \left(-x, -p, t + \frac{\pi}{\Omega}\right)$$

- even and odd class of Floquet states
- doublets : one even, one odd partner

Influence of driving on ground-state doublet



$D = 2$, $\Omega = 0.01\, \omega_0$

(F. Großmann, Jung, Dittrich, Hänggi, Z. Phys B 84, 315 (91))

at the crossing: divergent time-scale

$\rightsquigarrow$ COHERENT DESTRUCTION OF TUNNELING

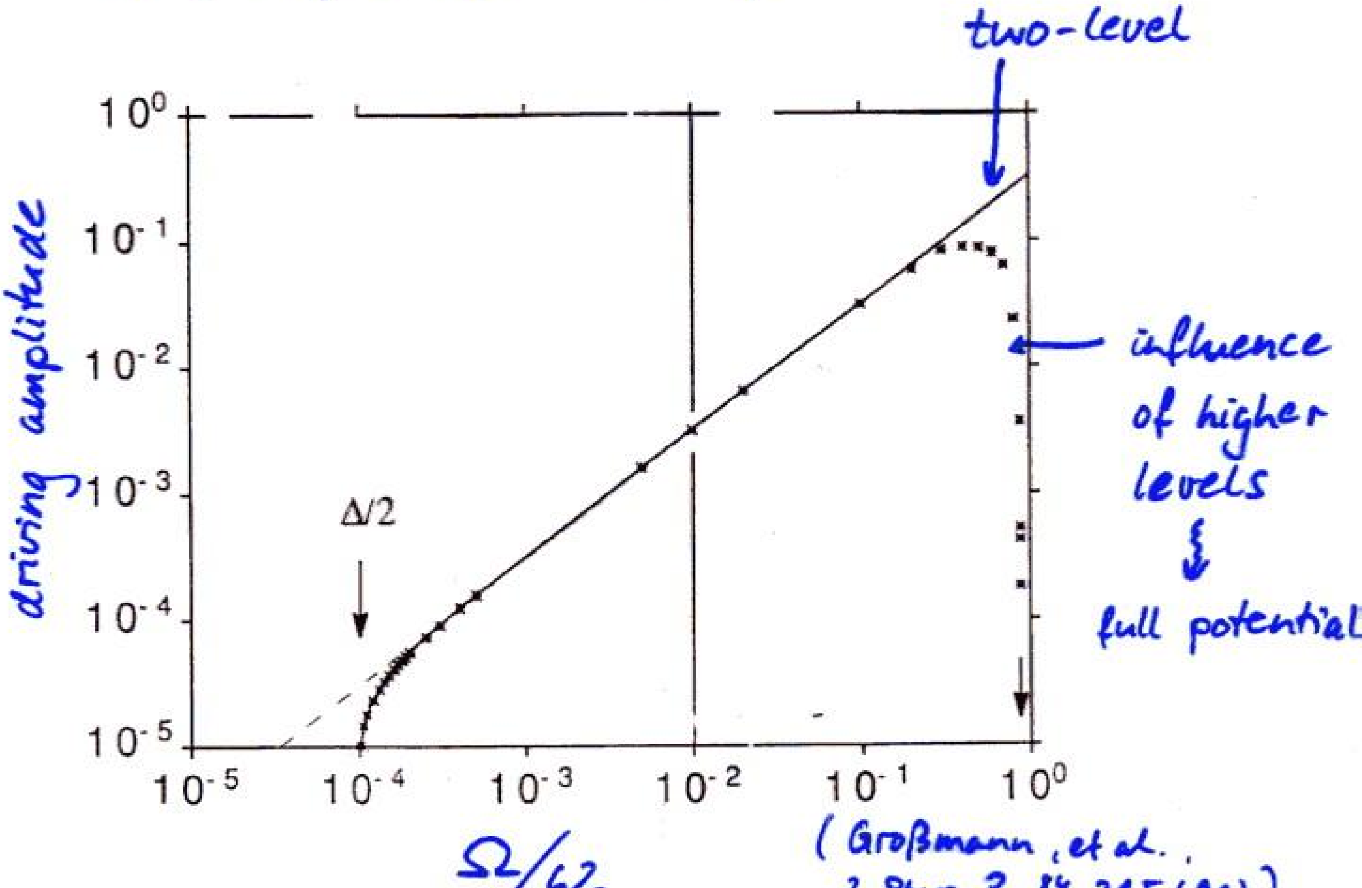
Reduction to a two-state problem

(Rabi problem far from resonance, Shirley 1965)

$$\Delta \rightarrow \Delta \cdot J_0\left(\frac{2b}{\Omega}\right), \quad b = F \langle \varphi_0 | x | \varphi_1 \rangle$$

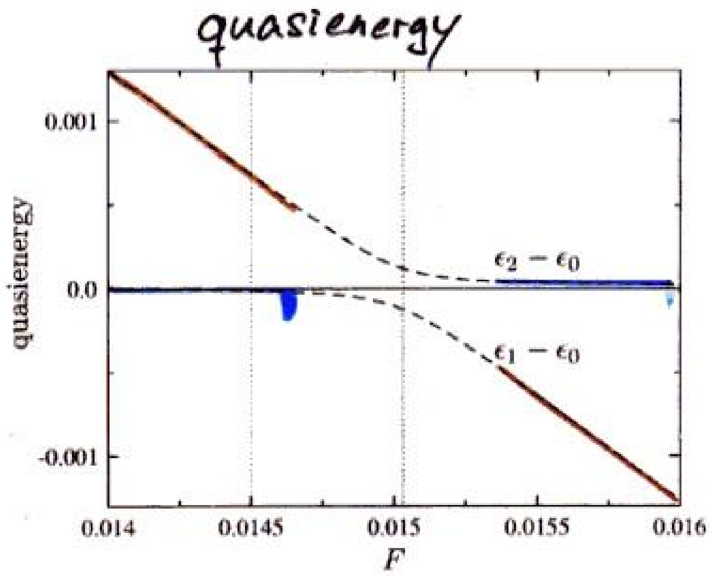
at zeros of Bessel function J_0: localization

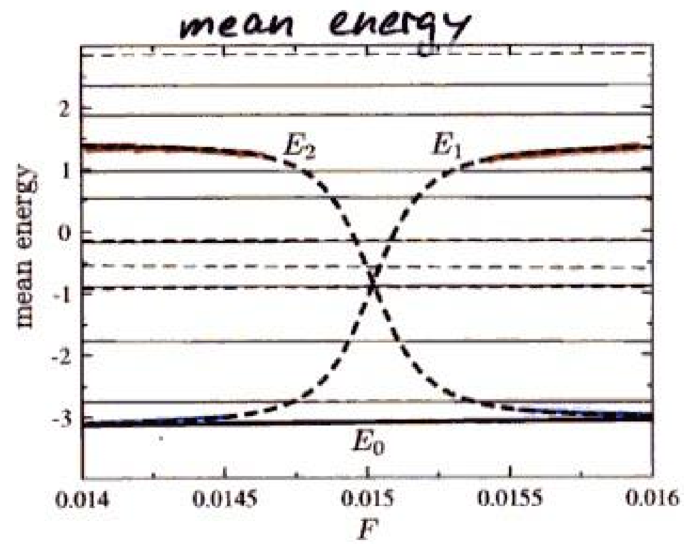
localization manyfold in (frequency, amplitude) - plane



(Großmann, et al., Z. Phys. B $\underline{84}$, 315 (91))

Chaotic tunneling near singlet-doublet crossing

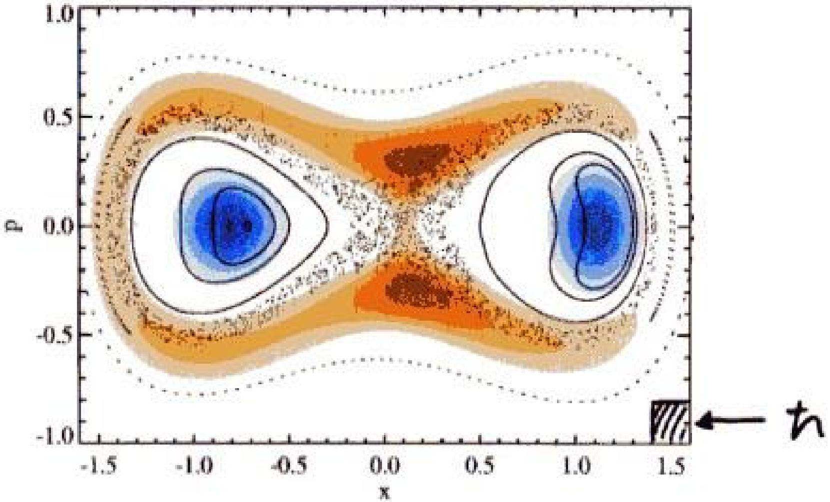




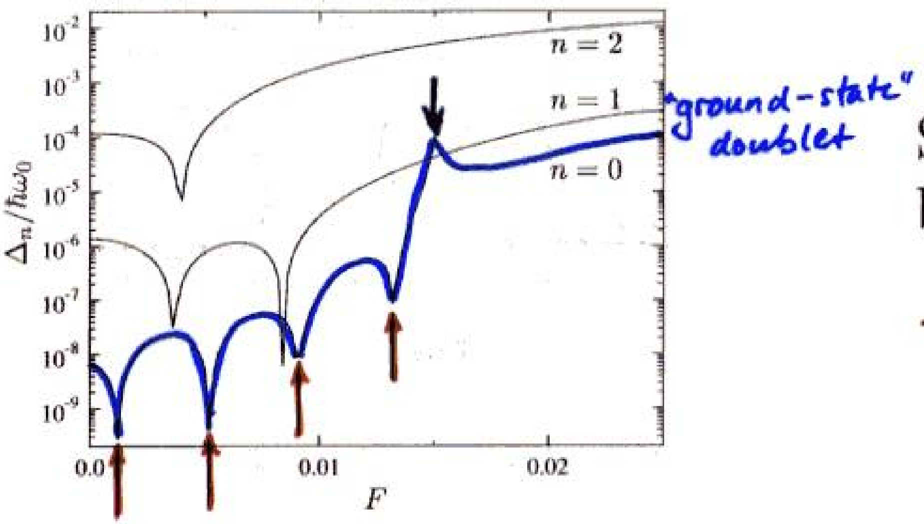
Mixing of the chaotic state $|\phi_c\rangle$ and the odd regular state $|\phi_r^-\rangle$:

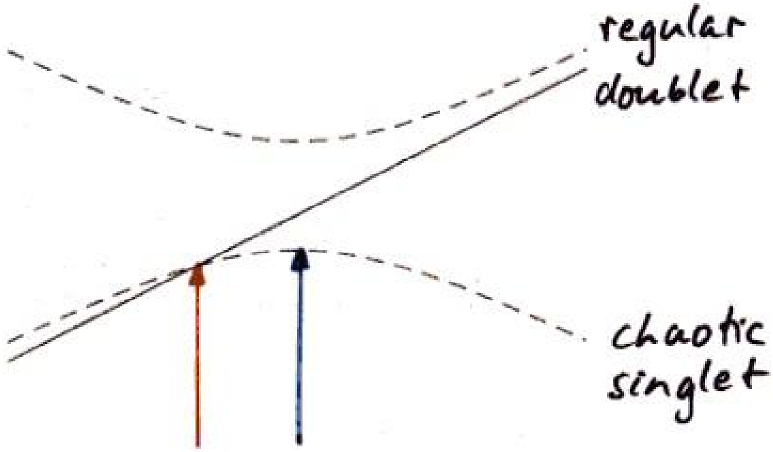
- Quasienergy splittings and tunneling rates are augmented
- Chaotic singlet is temporarily populated during tunneling

Husimi representation of $|\phi_r^-\rangle$, $|\phi_c\rangle$:



Quasienergy splittings near resonance





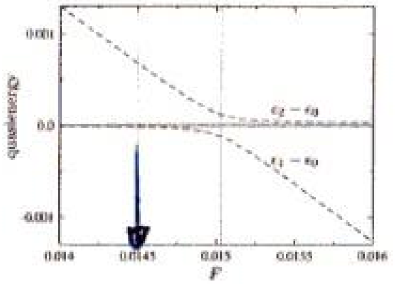
Splitting of the lowest three doublets for $D = 4$, $\Omega = 0.982\,\omega_0$:

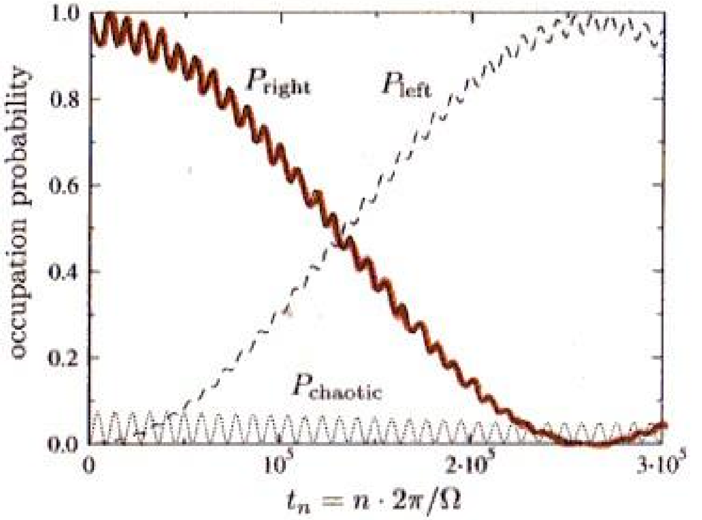
→ intra-doublet crossings:
coherent suppression of tunneling

→ singlet-doublet crossings:
chaos-assisted tunneling
(Bohigas *et al.*, PRL **64**, 1479 (1990))

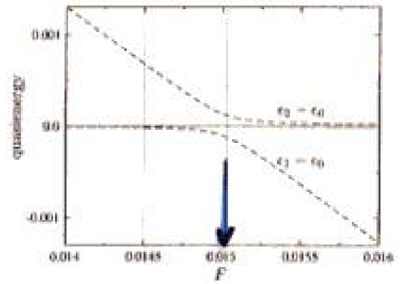
Coherent time-evolution

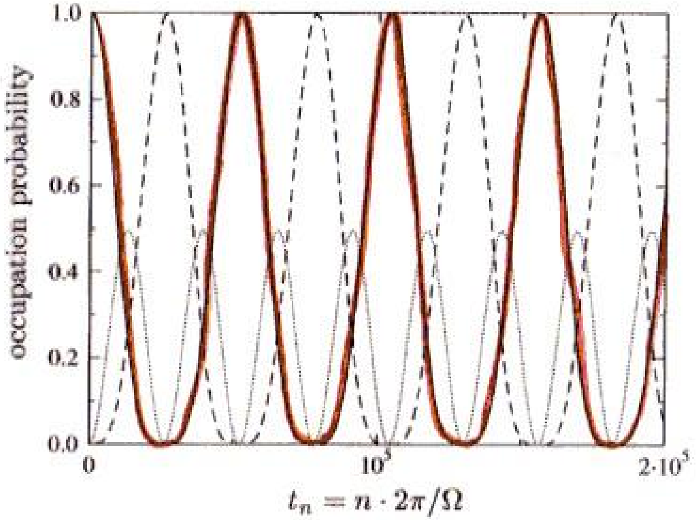
near crossing:





at center of crossing:





so far : coherent

now : DISSIPATION

- pure state evolves into statistical mixture
- decoherence, measured by

  Shannon entropy $\quad \mathrm{tr}(\rho \ln \rho)$

  "linearized entropy" $\quad 1 - \mathrm{tr}\,\rho^2$

- tunneling becomes a transient
- formal description:

  coupling to bath of harmonic oscillators

  ⋮

  Markovian master equation in Floquet basis
  (Blümel et al., PRL 62, 341 (1989))

DISSIPATION IN FLOQUET-MARKOV-DESCRIPTION

(Blümel et al., PRL <u>62</u>, 341 (89))

system-bath model

$$H = H_S(t) + \underset{\substack{\uparrow \\ \text{coupling, linear} \\ \text{in position}}}{H_{SB}} + H_B$$

H_B: ensemble of harmonic oscillators

- projection on system dynamics
  $\rightarrow$ integro-differential equation for ρ_S
- lowest order in system-bath coupling
  (Born-Markov approximation)

$$\dot\rho = -\frac{i}{\hbar}[H_S(t), \rho] + \sum_\nu |c_\nu|^2 \int_0^\infty d\tau \, \Big\{ S_\nu(\tau)\big[x, [x_H(t-\tau, t), \rho]\big] - A_\nu(\tau)\big[x, [x_H(t-\tau,t), \rho]_+\big] \Big\}$$

$\int d\tau \ldots \approx$ Fourier transform of the Heisenberg operator $x_H(t, t')$

$\rightsquigarrow$ Markov approximation depends on the quasispectrum of the system

master equation in Floquet basis

- solve Schrödinger equation for the driven system : Floquet states $|\phi_\alpha\rangle$
- use $\{|\phi_\alpha\rangle\}$ to decompose the master equation
- eq. of motion for density matrix elements

$$\dot{\rho}_{\alpha\beta} = \sum_{\alpha'\beta'} L_{\alpha\beta\alpha'\beta'}(t)\, \rho_{\alpha'\beta'}$$

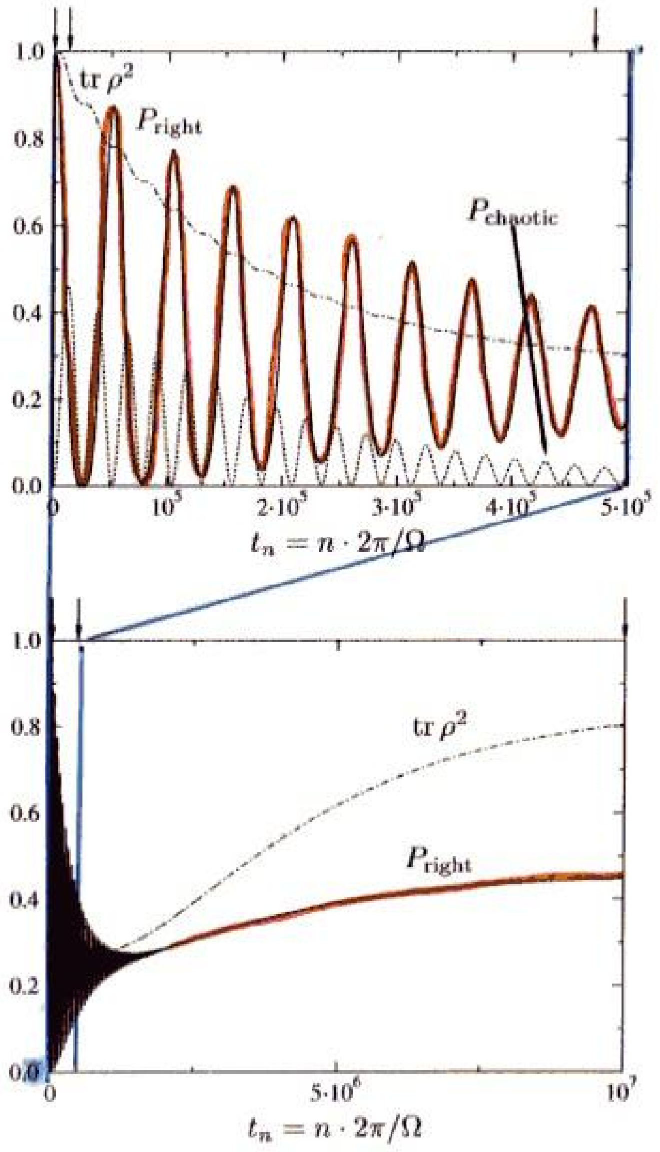
periodically time dependent (with driving period)

  - RWA : average over driving period
  - coherent part : $-i(\varepsilon_\alpha - \varepsilon_\beta)\, \delta_{\alpha\alpha'}\, \delta_{\beta\beta'}$
  - dissipative part : $\langle \phi_\alpha | x | \phi_{\beta'} \rangle$, weighted by $\omega\, n_{th}(\omega)$, $\omega = \varepsilon_\alpha - \varepsilon_{\beta'} + n\Omega$

$\longrightarrow$ computational advantage: keeps basis for ρ small

$\longrightarrow$ quantitatively better (for parametrically driven harmonic oscillator : SK, Dittrich, Hänggi, 1997)

Dissipative time evolution at the center of the crossing



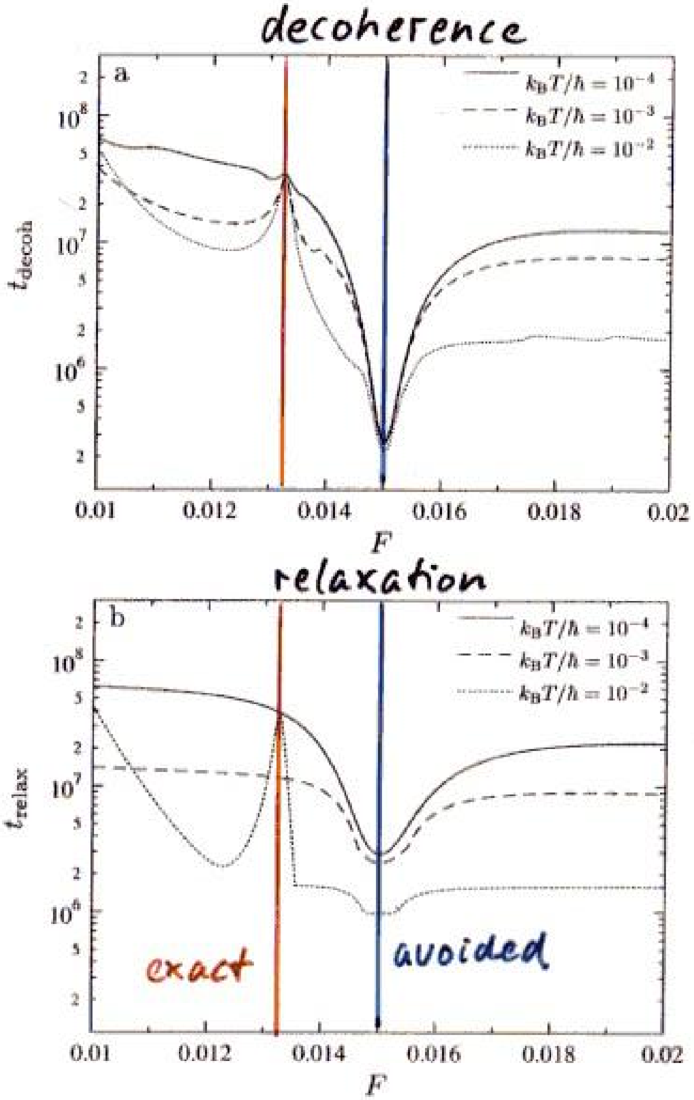
Measure for coherence:

$$\operatorname{tr}\rho^2 \approx \left(\begin{array}{c} \#\text{ of incoherently} \\ \text{populated states} \end{array} \right)^{-1}$$

In the vicinity of the singlet-doublet crossing:

- stepwise decay of coherence during population of the chaotic state
- decay of coherence is much faster than relaxation towards the asymptotic solution

Dissipative time scales



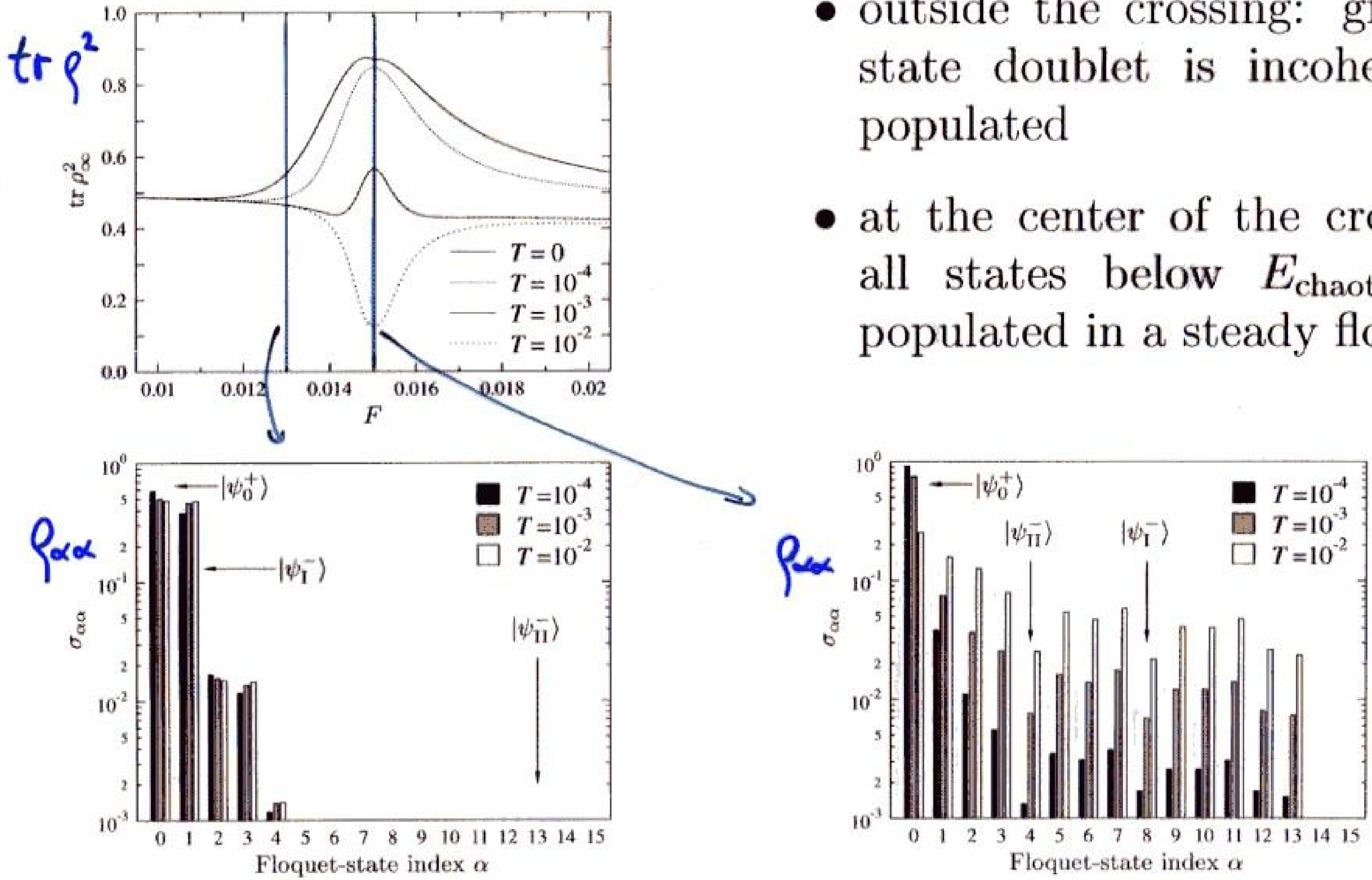
$D = 4$, $\Omega = 0.982$, $\gamma = 10^{-6}$, $T = 10^{-4}$, dotted line: center of singlet-doublet crossing.

Decay of coherence:

$$\frac{1}{\tau} \equiv - \left\langle \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{tr}\, \rho^2 \right\rangle_{\text{tunnel cycle}}$$

Time scale of relaxation towards asymptotic solution: eigenvalue of L with smallest real part.

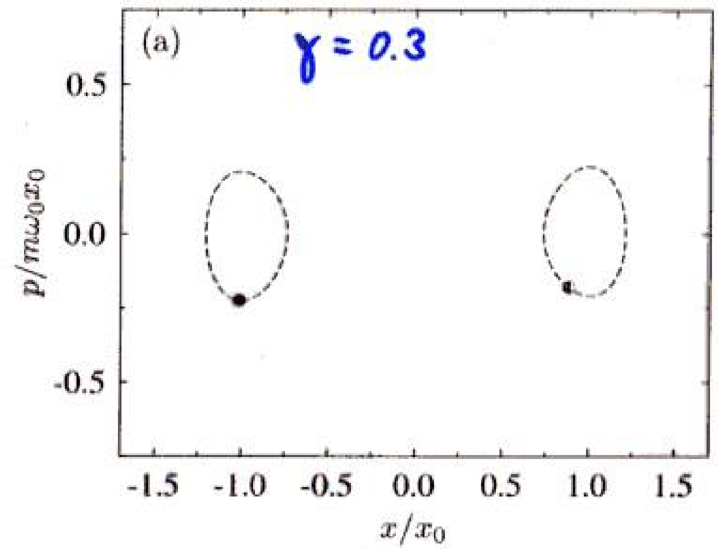
Asymptotic state: coherence and population

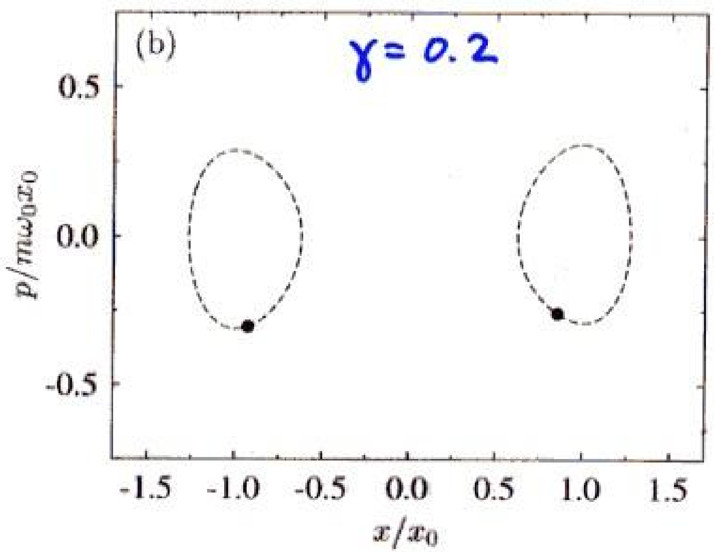


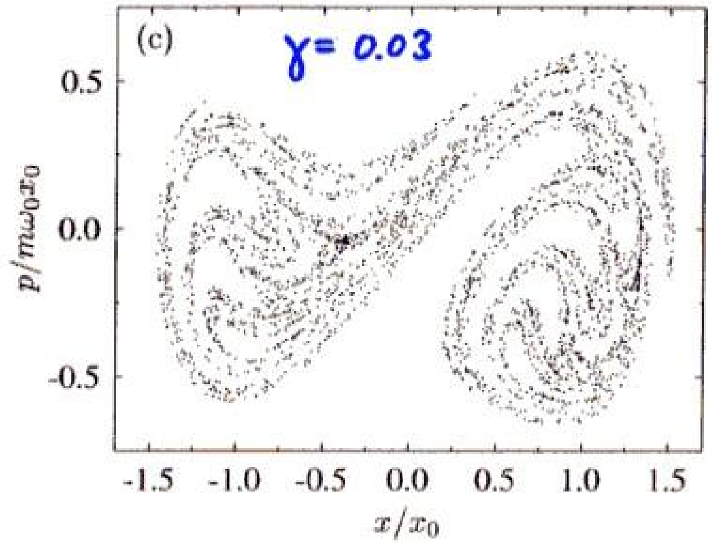
- outside the crossing: ground-state doublet is incoherently populated
- at the center of the crossing: all states below E_{chaotic} are populated in a steady flow

Strange attractor vs. isolated fixed point

$F = 0.09$
$\Omega = 0.9$



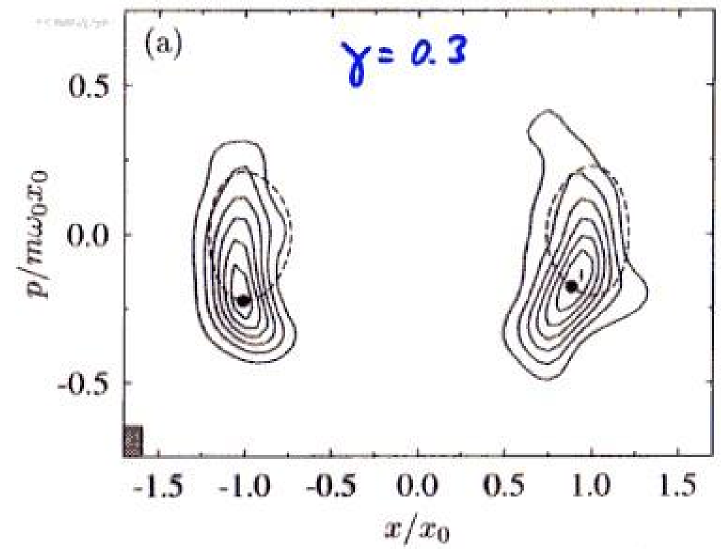


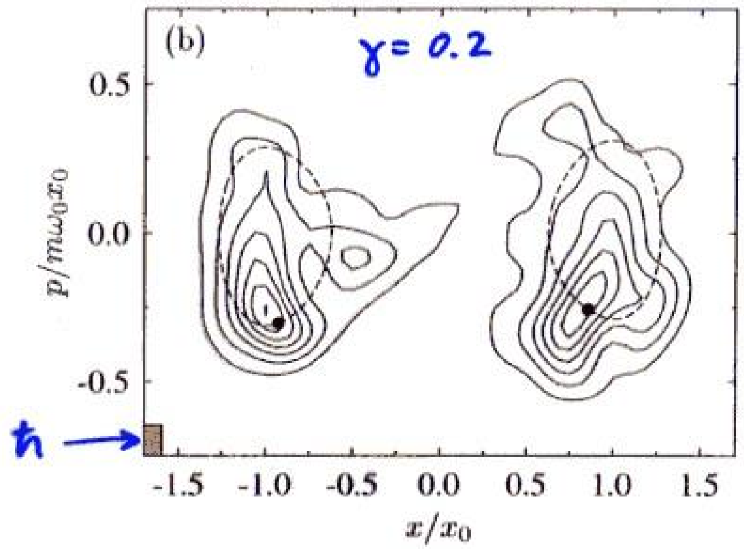


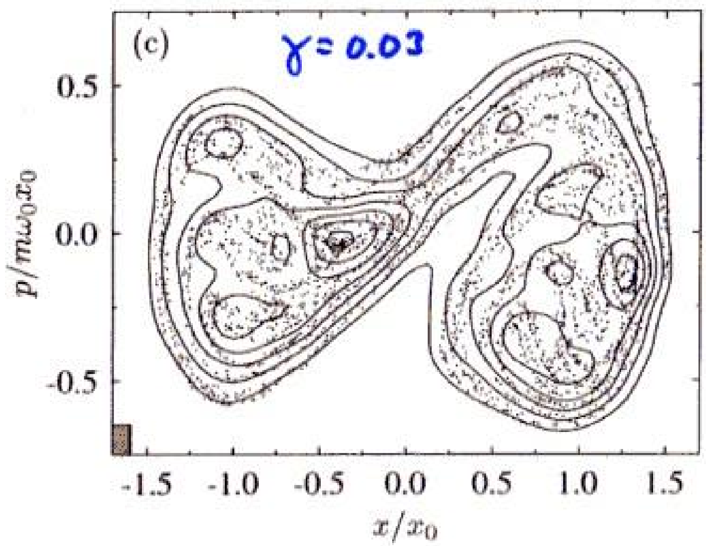
- larger driving amplitude, regular islands resolved in the chaotic sea
- time-reversal symmetry $(x, p, t) \to (x, -p, -t)$ obviously broken

... corresponding quantum attractor

$F = 0.09$
$\Omega = 0.9$
$D = 12$







- off-diagonal matrix elements relevant
- quantum attractor also undergoes qualitative change
- transition is in the quantum case smoother

SUMMARY & OUTLOOK

* slow driving:
  coherent suppression of tunneling

* near resonance:
  chaos-assisted tunneling

* dissipative effects:
  - stepwise decay of coherence
  - attractor: chaos-induced coherence / decoherence
  - three-level approximation clearly fails

* driving and dissipation stronger: full chaos
  quantum „strange attractor"

* stochastic resonance