Enhancement of Macroscopic Quantum Tunneling by Landau-Zener Transitions

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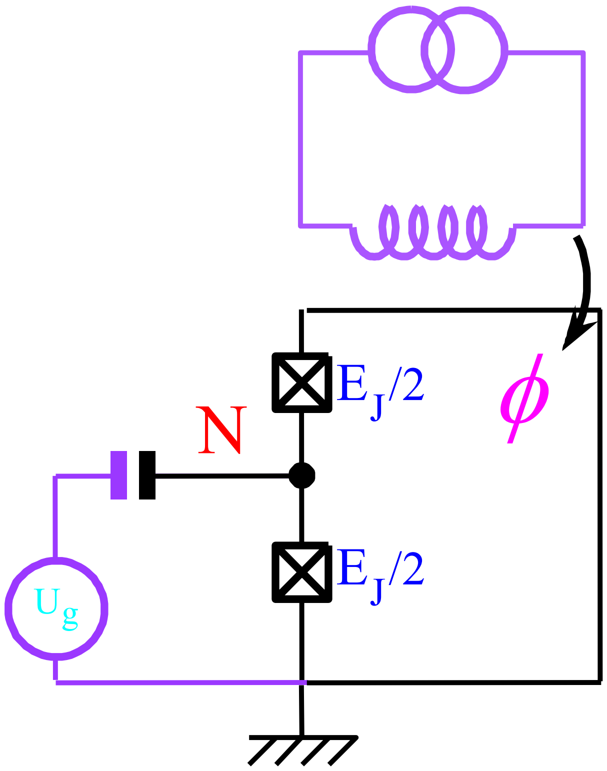
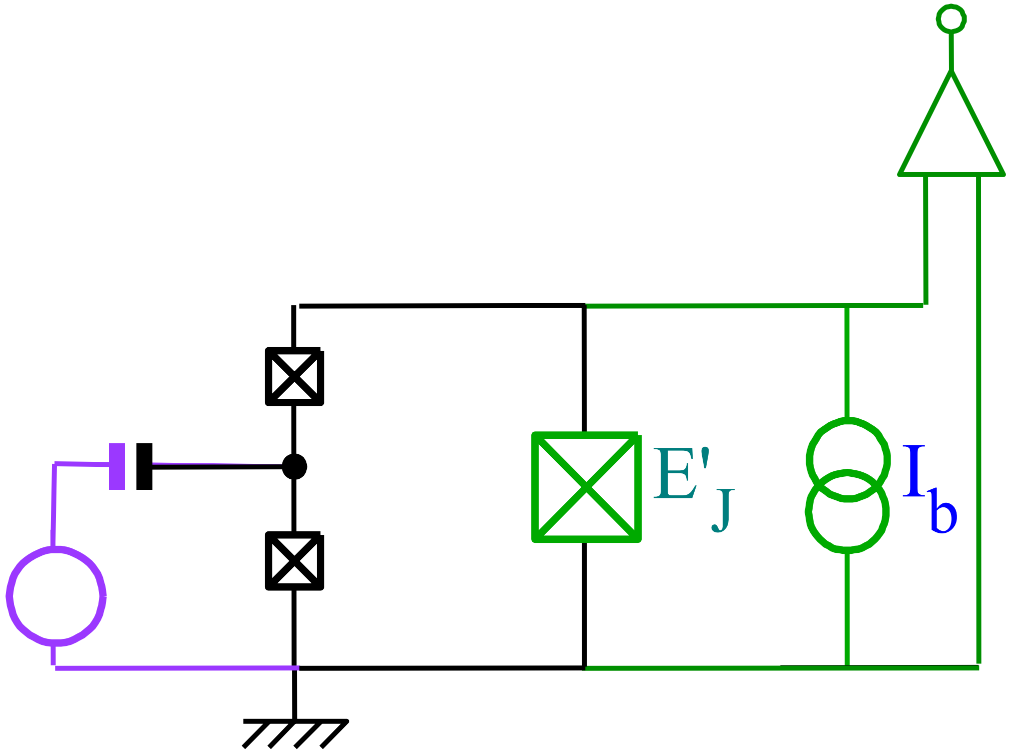
Quantronium Circuit



$$[N, \delta] = -i$$

$$H = E_c (N - N_g)^2 - E_J \cos(\Phi/2) \cos(\delta)$$

Readout Josephson Junction



$$[Q\ ,\ \theta] = 2ie$$

$$H = E_c (N - N_g)^2 - E_J \cos([\Phi + \theta]/2) \cos(\delta)$$

$$+ \frac{Q^2}{2C'} - E'_J \cos(\theta) - \frac{\hbar I_b}{2e}\theta$$

Dimensionless Hamiltonian

$N_g \approx \frac{1}{2}$ only charge states N = 0 & N = 1 relevant

All energies in units of E'_J

$\varepsilon = (E_c / E'_J)(N_g - 1/2)$ asymmetry

$j = E_J / E'_J$ coupling

$i_b = \hbar I_b / 2eE'_J$ bias

$m = C'E'_J / 4e^2$ mass

$$H = \varepsilon\ \sigma_z + j \cos\left(\frac{\Phi + \theta}{2}\right) \sigma_x + \frac{p_\theta^2}{2m} - \cos(\theta) - i_b\theta$$

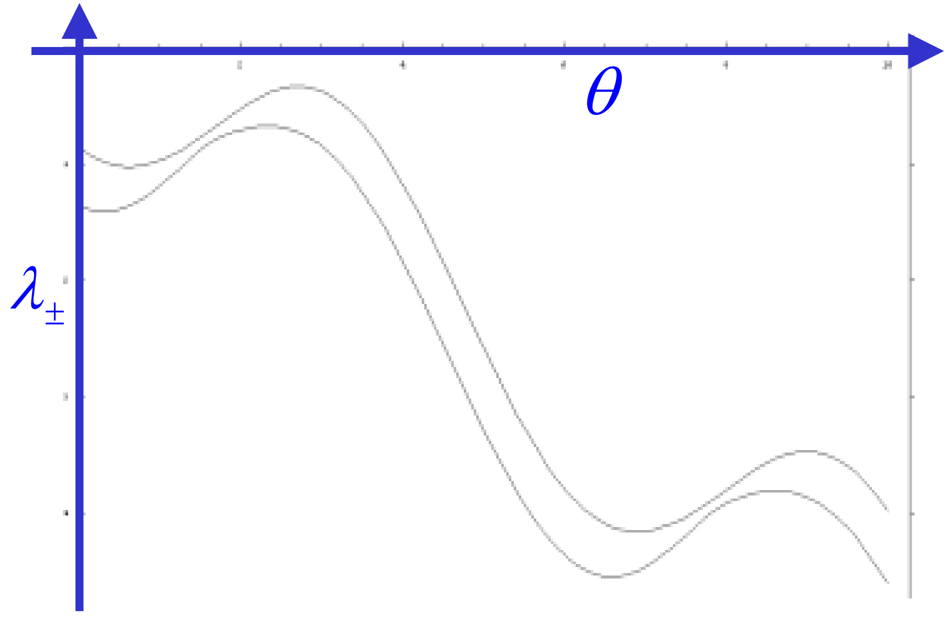
$$[p_\theta\ ,\ \theta] = -i$$

Adiabatic potential surfaces

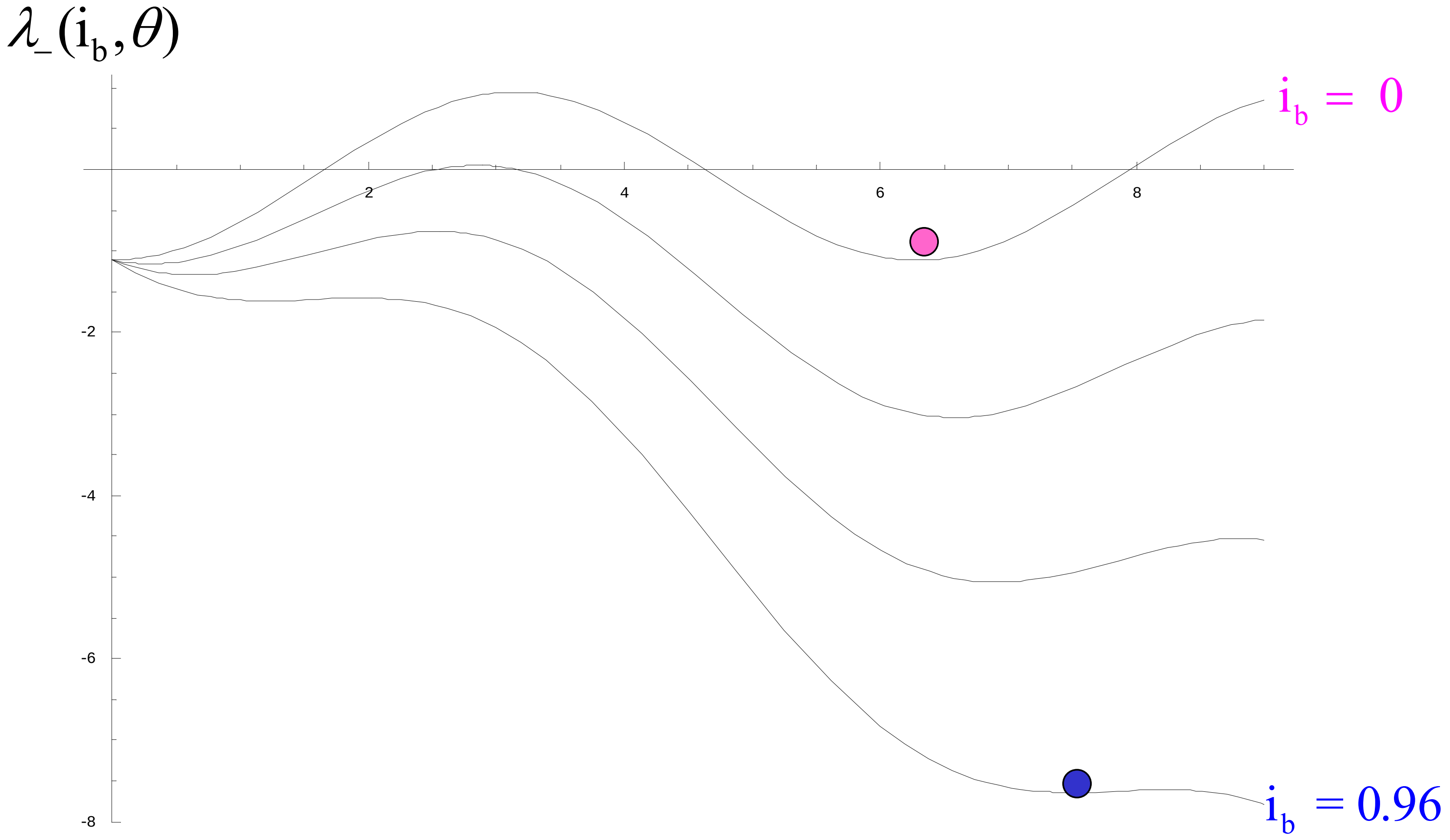
$$H = \varepsilon\,\sigma_z + j\cos\left(\frac{\Phi + \theta}{2}\right)\sigma_x + \frac{p_\theta^2}{2m} - \cos(\theta) - i_b\theta$$

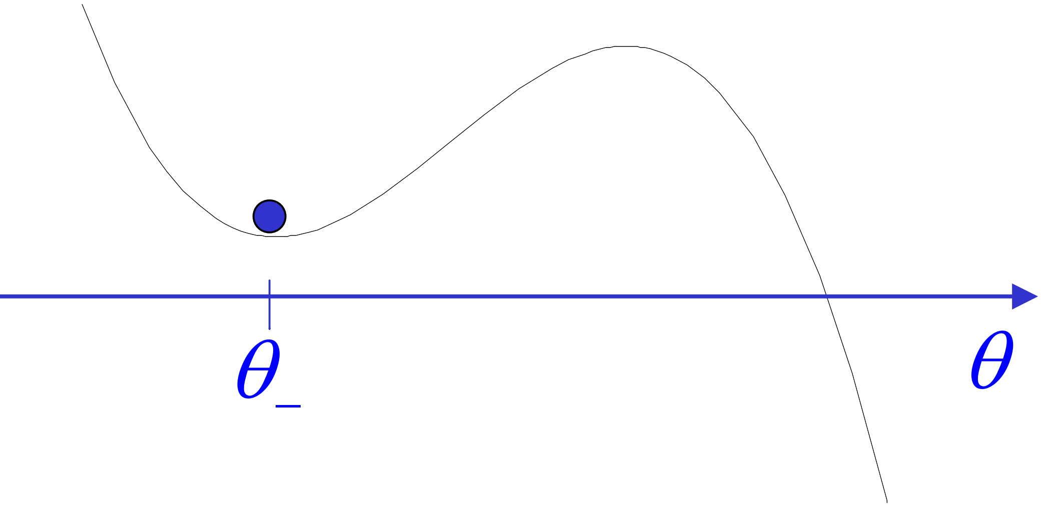
diagonalize adiabatic Hamiltonian: eigenvalues = adiabatic potentials

$$\lambda_\pm(i_b,\theta) = -\cos(\theta) - i_b\theta \pm \sqrt{\varepsilon^2 + j^2\cos^2[(\theta+\Phi)/2]}$$



Initial state before MQT





Initial state

$$|\theta_-, -\rangle$$

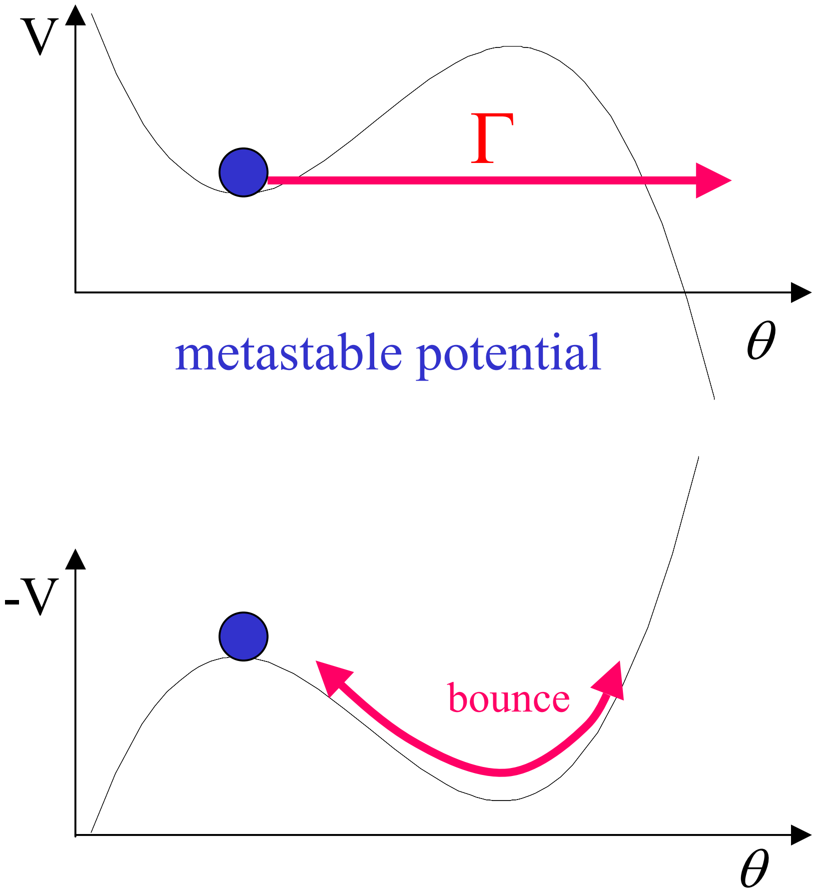
Hamiltonian in basis $|\theta_-, -\rangle, |\theta_-, +\rangle$

$$H = \begin{pmatrix} \frac{p_\theta^2}{2m} + V_+(\theta) & \Delta(\theta) \\ \Delta(\theta) & \frac{p_\theta^2}{2m} + V_-(\theta) \end{pmatrix}$$

$V_\pm(\theta)$ diabatic potential surfaces

$\Delta(\theta)$ coupling between spin states $\quad \Delta(\theta_-) = 0$

MQT rate



bounce technique

$$Z = \mathrm{tr}\, e^{-\beta H} = \int D[\theta]\, e^{-S[\theta]}$$

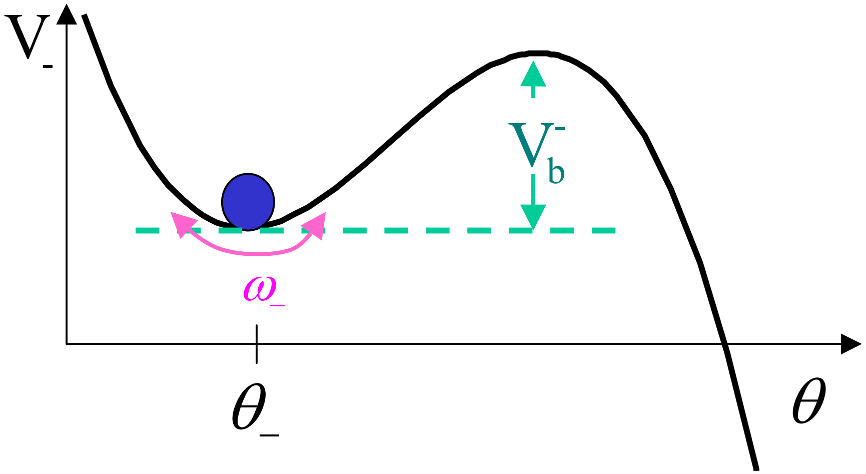
$$Z = Z_{well} + Z_{bounce}$$

$$\Gamma = \lim_{\beta \to \infty} \frac{2}{\beta}\, Im \ln Z$$

$$\Gamma = \lim_{\beta \to \infty} \frac{2}{\beta} \frac{Im\, Z_{bounce}}{Z_{well}} = f\, e^{-S_{bounce}}$$

prefactor f: fluctuations about bounce, zero mode, unstable mode

Decay rate in cubic potential



$V_-(\theta)$: cubic polynominal

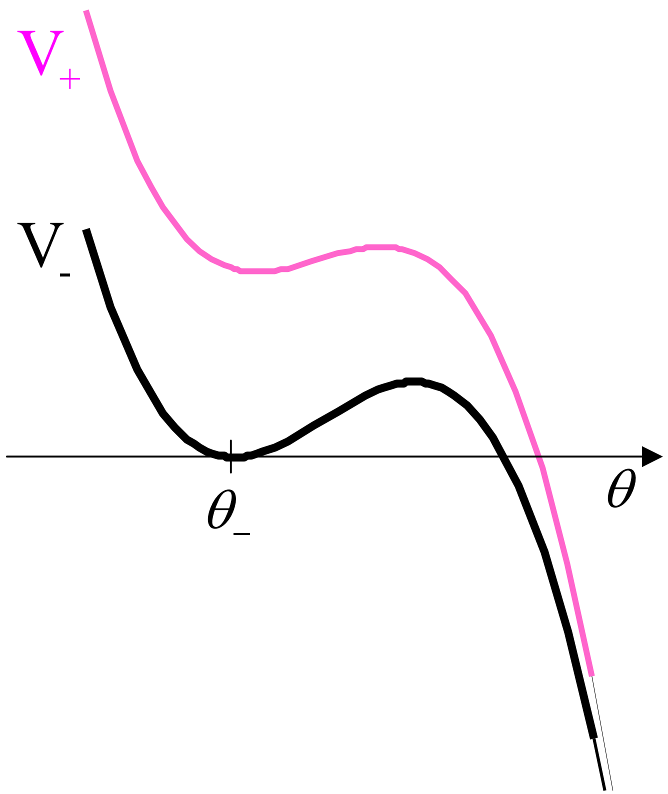
$\omega_-^2 = V_-''(\theta_-)/m$, V_b^-

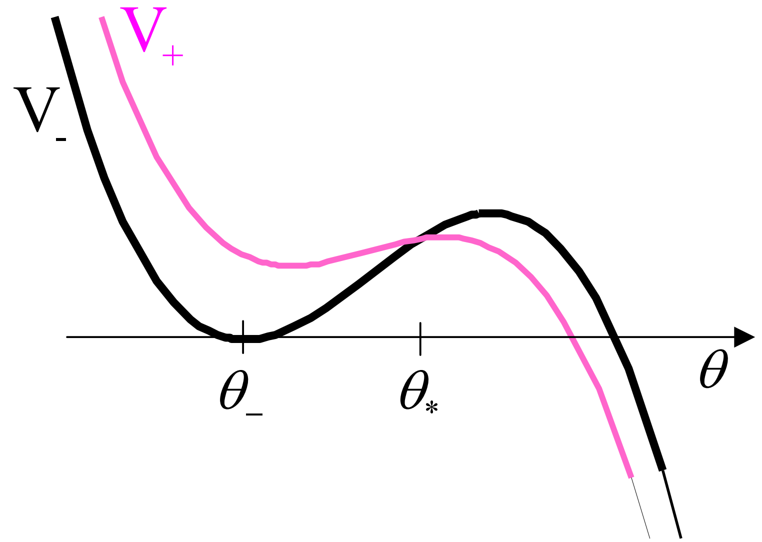
$$\Gamma_0 = 6\sqrt{6\omega_- V_b^- / \pi}\ \exp\left(-\frac{36}{5}\frac{V_b^-}{\omega_-}\right)$$

Diabatic potential surfaces

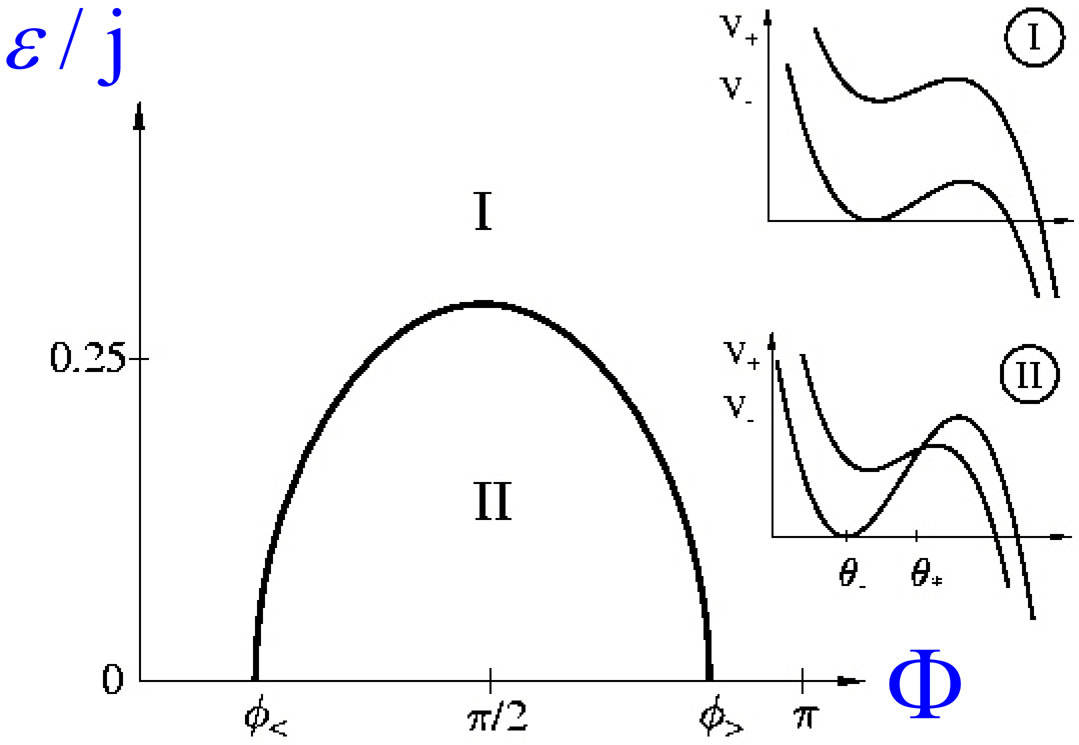
$$V_{\pm}(\theta) = -\cos(\theta) - i_b\theta \;\pm\left(\sqrt{\varepsilon^2 + V_0^2} + \kappa_0\left[V(\theta) - V_0\right]\right)$$

$$V(\theta) = j\cos\left[(\Phi+\theta)/2\right]\;;\qquad V_0 = V(\theta_-)\;;\qquad \kappa_0 = 2\,\frac{2V_0\left(\varepsilon+\sqrt{\varepsilon^2+V_0^2}\right)}{\left(\varepsilon+\sqrt{\varepsilon^2+V_0^2}\right)^2+V_0^2}$$





MQT Phase Diagram



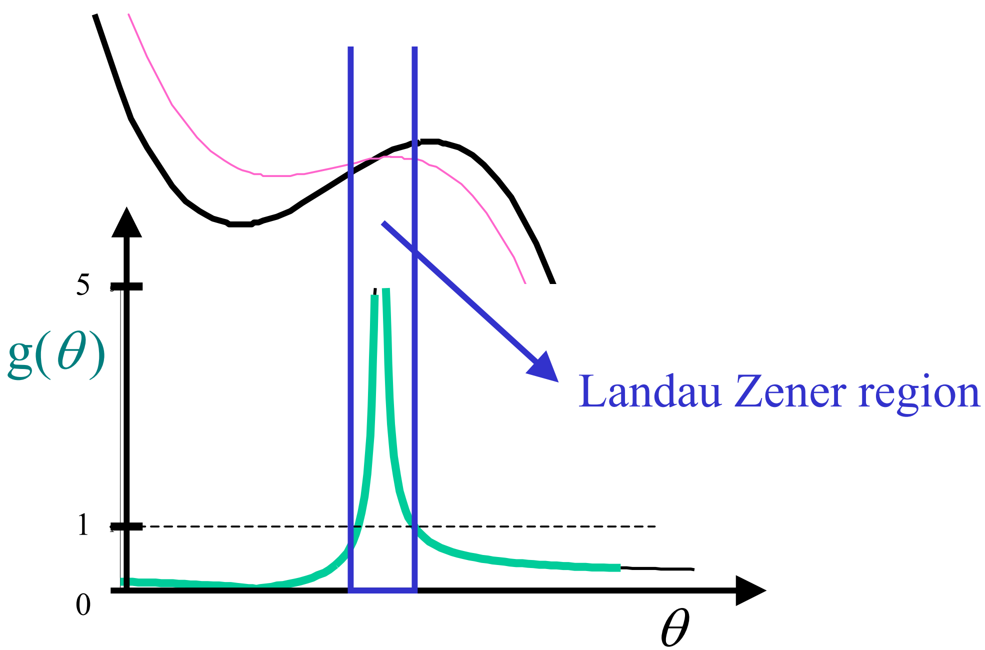
Landau-Zener transitions

(in imaginary time)

$$H = \begin{pmatrix} \frac{p_\theta^2}{2m} + V_+(\theta) & \Delta(\theta) \\ \Delta(\theta) & \frac{p_\theta^2}{2m} + V_-(\theta) \end{pmatrix}$$

initial state $|\theta_-, -\rangle$

coupling parameter $g(\theta) = \left| \frac{\Delta(\theta)}{V_+(\theta) - V_-(\theta)} \right|$

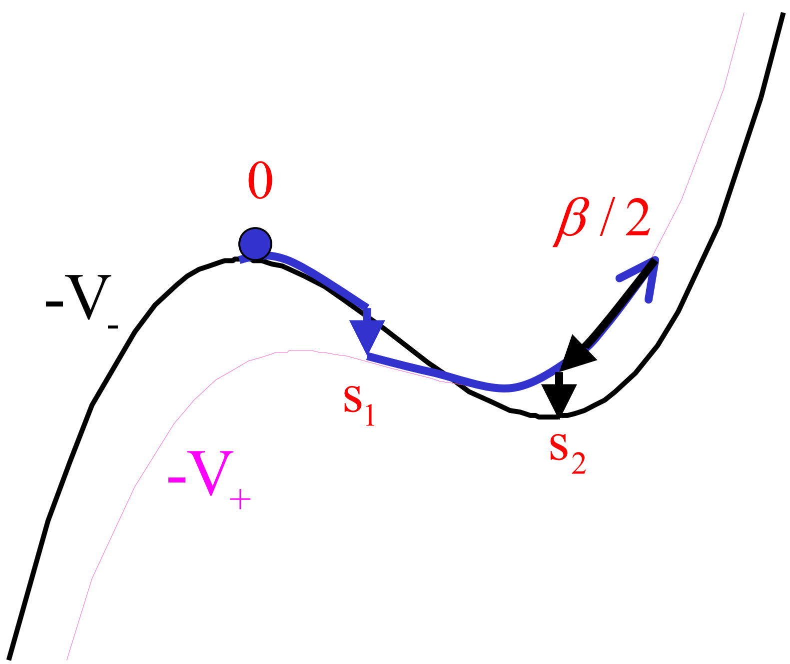


Partition function in semiclassical limit

$$Z = \mathrm{tr}\left\{\left|\theta_-,-\right\rangle\left\langle\theta_-,-\right| e^{-\beta H}\right\} = Z_0 + Z_2 + \ldots$$

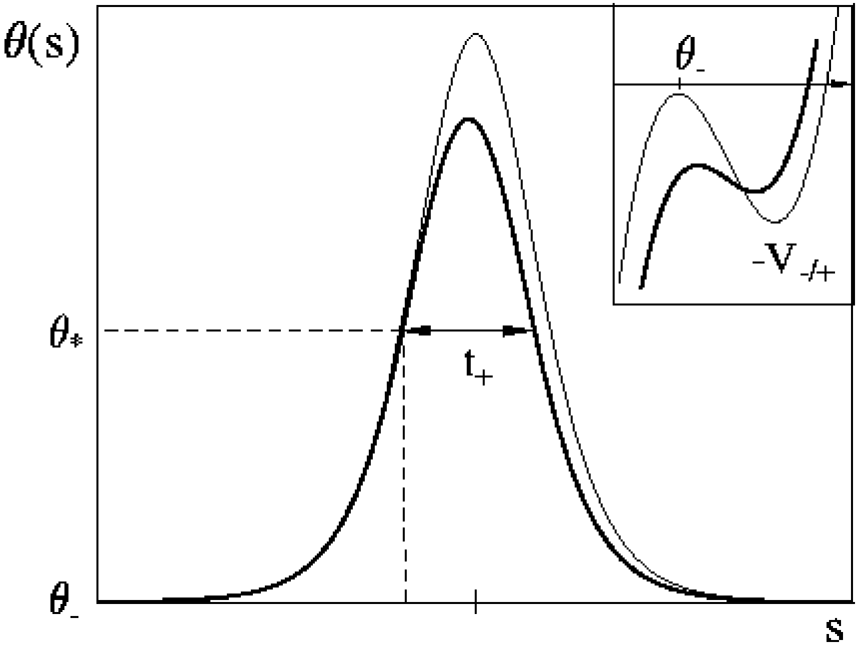
$$Z_0 = \int D[\theta]\, e^{-S_-[\theta]}$$

$$Z_2 = \int D[\theta]\, e^{-S_-[\theta]} \int_0^\beta ds_2 \int_0^{s_2} ds_1\, \Delta\big(\theta(s_2)\big)\Delta\big(\theta(s_1)\big)\exp\left[\int_{s_1}^{s_2} d\tau\left[V_-(\theta) - V_+(\theta)\right]\right]$$



Flip bounce

optimal flips occur at crossing point $\theta = \theta_*$



Action of flip bounce

$$S_* = \frac{18V_b^-}{5\omega_-}\left[\sqrt{\left(1-x_*^-\right)^3}\left(3\sqrt{1-x_*^-}-5\right)+2\right]$$
$$+\frac{18V_b^+}{5\omega_+}\left[\frac{2x_0-2\lambda^2+9\rho}{2\lambda}F(\varphi|\overline{m})+2\lambda E(\varphi|\overline{m})\right]$$
$$-\frac{1}{2}\sqrt{-P(x_*^+)}\left(\frac{4}{\lambda^2+x_0-x_*^+}+P''(x_*^+)\right)$$

$$P(x)=x^3-x^2-\frac{4}{27}\rho\ ;\ \ \rho=\frac{V_+(\theta_+)-V_-(\theta_-)}{V_b^+}\ ;\ \ \lambda^2=\sqrt{P'(x_0)}\ ;\ \ P(x_0)=0$$

$$\overline{m}=\frac{1}{2}+\frac{P''(x_0)}{8\lambda^2}\ ;\ \ \cos(\varphi)=\frac{\lambda^2-\left(x_0-x_*^+\right)}{\lambda^2+\left(x_0-x_*^+\right)}\ ;\ \ x_*^\pm=\frac{\theta_*-\theta_\pm}{\theta_\pm^0-\theta_\pm}\ ;\ \ V_\pm(\theta_\pm^0)=V_\pm(\theta_\pm)$$

MQT rate in region II

$$\Gamma = \lim_{\beta \to \infty} \frac{2}{\beta} \, Im \ln Z \qquad Z \approx Z_{well} + Z_{bounce} + Z_{flip}$$

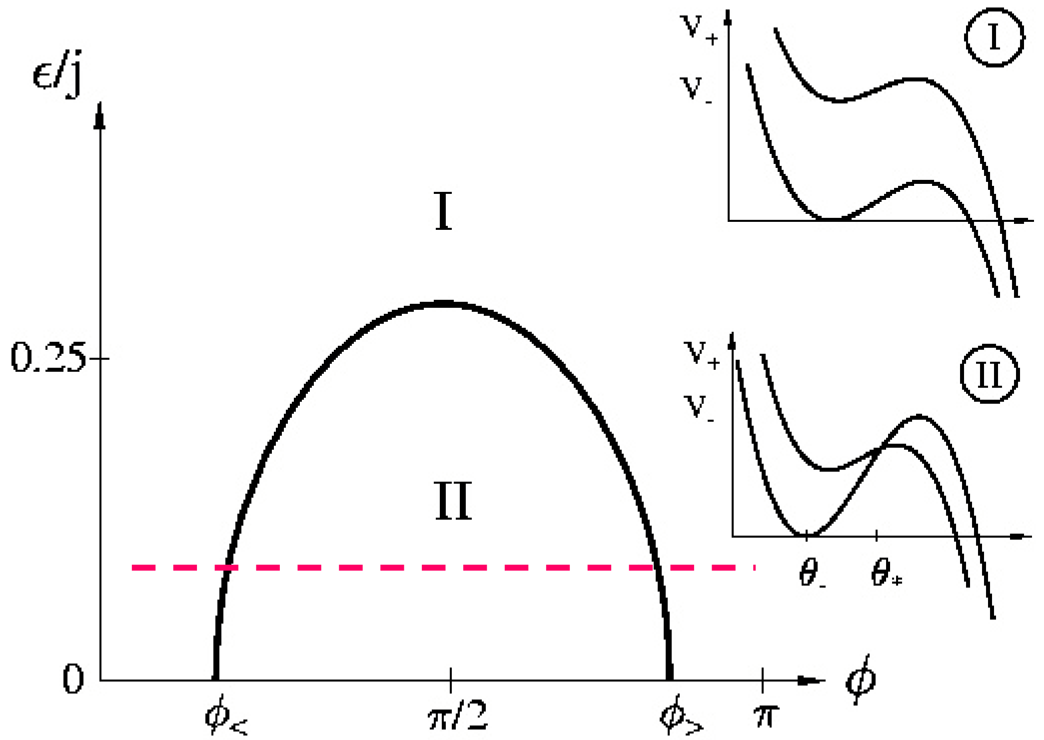
$$\Gamma = \lim_{\beta \to \infty} \frac{2}{\beta} \frac{Im\,(Z_{bounce} + Z_{flip})}{Z_{well}} = \Gamma_0 + \Gamma_2$$

fluctuations about flip bounce include variations of flip times s_1, s_2

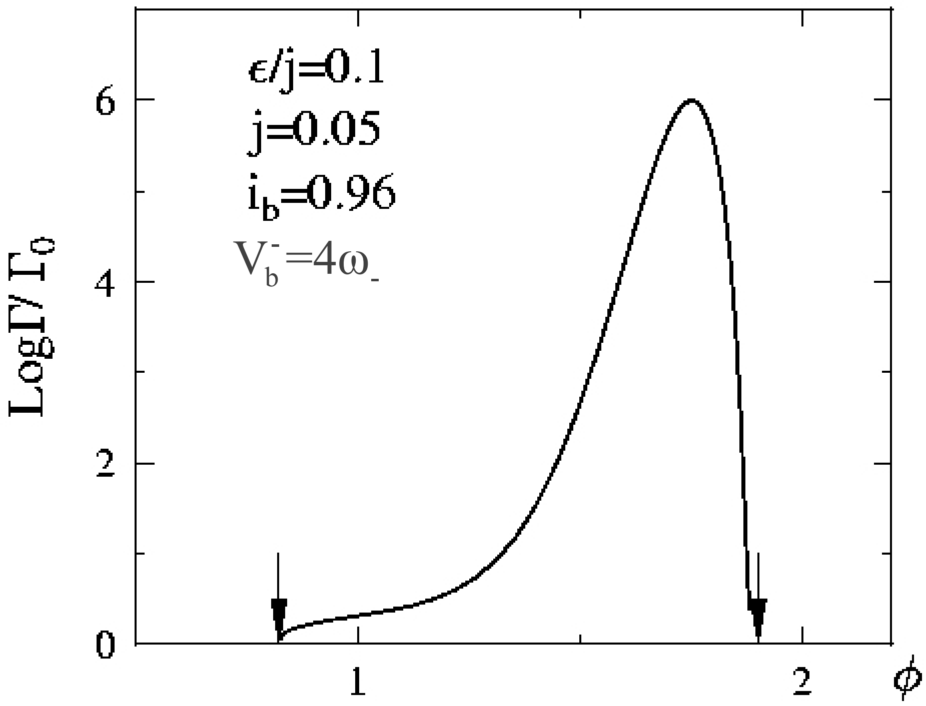
$u = s_1 + s_2$ zero mode, $v = s_1 - s_2$ breathing mode

$$\Gamma_2 = 6\sqrt{\frac{3V_b^-}{\omega_-}\frac{\Delta(\theta_*)^2}{\Omega_1}} \; \mathrm{erfc}\left[-\frac{\Omega_1 t_+}{\sqrt{2}}\right] \exp(-S_*)$$

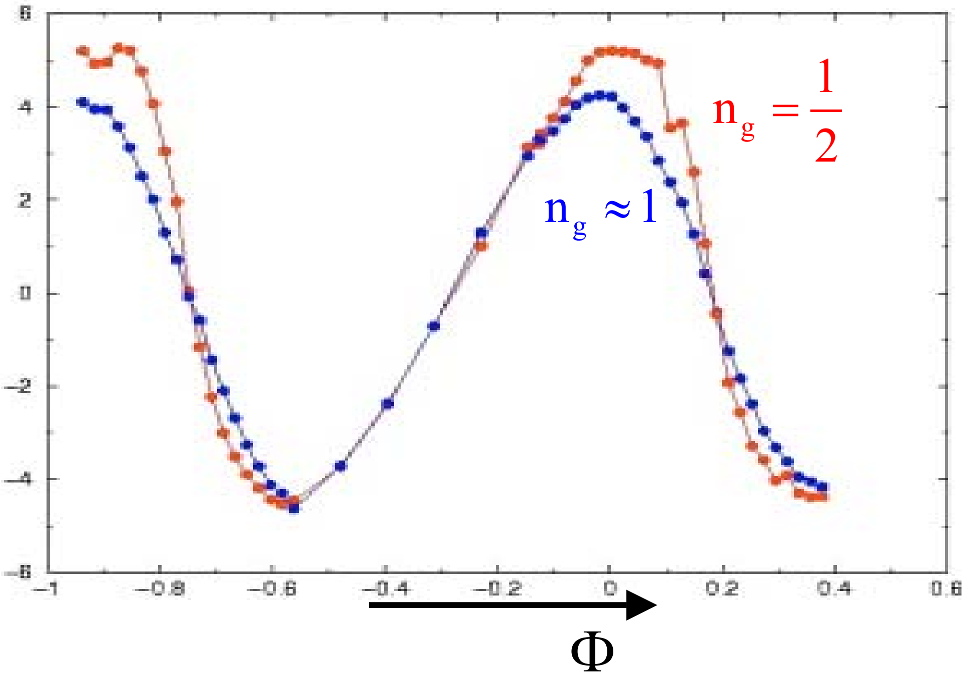
Effect of Landau-Zener transitions



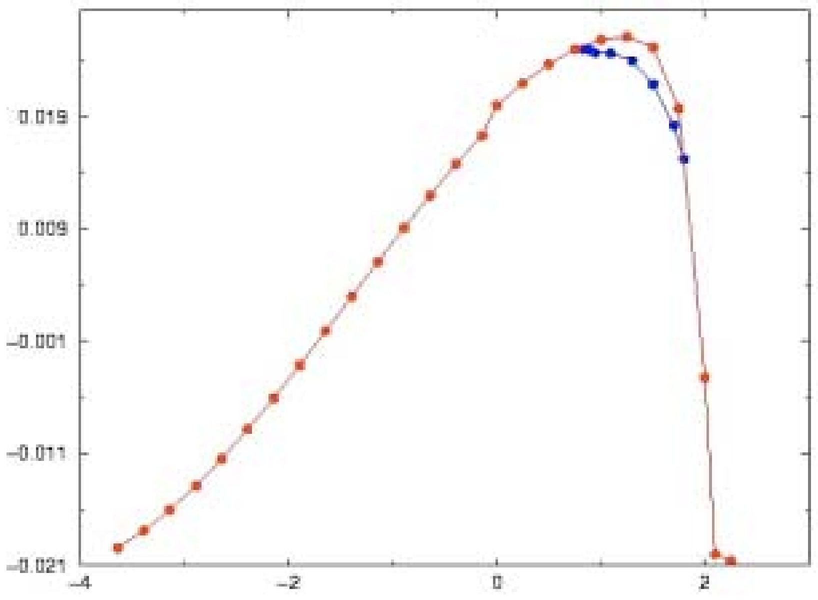
Rate enhancement



Peak current vs. magnetic field



Saclay



Theory

Conclusions

MQT in presence of Landau-Zener transitions

2 regions in Φ, ε/j plane

rate enhancement by spin flips in region II

nonadiabatic theory $\Gamma = \Gamma_0 + \Gamma_2$
at most 1 spin flip during crossing of LZ region

boundary region: multiple spin flips
prefactor correction smoothing rate

At least qualitative agreement with experimental observation SACLAY

Future work

tunneling from upper adiabatic potential surface
decay of an excited state

effects of dissipation
dephasing of spin, influence of lead impedance

tunneling in presence of microwave excitation

Reference: cond-mat/0304232