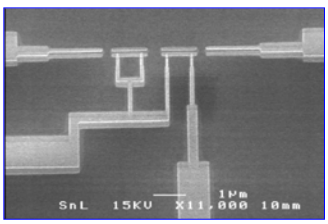
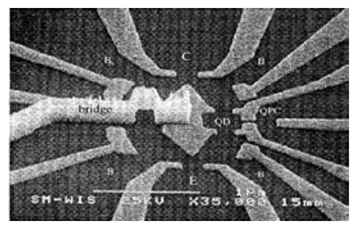
Mesoscopic Detectors and the Quantum Limit

(cond-mat/0211001)

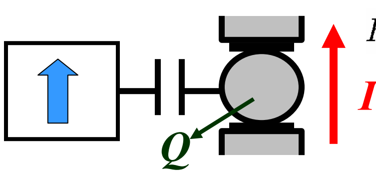
A. A. Clerk, S. M. Girvin, and A. D. Stone
Departments of Applied Physics and Physics,
Yale University
(and many discussions with M. Devoret & R. Schoelkopf)

Q:What characterizes an “ideal” quantum detector?





Generic Weakly-Coupled Detector



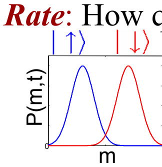
$$H_{qb} = -\frac{\Omega}{2}(\cos\theta\hat{\sigma}_z + \sin\theta\hat{\sigma}_x)$$

$$H_{couple} = A\hat{\sigma}_z\hat{Q}$$

“gain”

$$\langle I\rangle = I_0 + \lambda A\langle\sigma_z\rangle$$

1. ***Measurement Rate***: How quickly can we distinguish the two qubit states?



$$\Gamma_{meas} = (A^2\lambda^2)/S_{II}$$

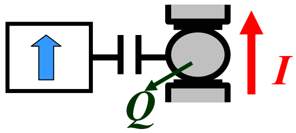
$$S_{II} \equiv 2\int dt\ \langle\delta I(t)\delta I(0)\rangle$$

2. ***Dephasing Rate***: How quickly does the measurement decohere the qubit?

$$\Gamma_\varphi = A^2 S_{QQ}/\hbar^2$$

$$S_{QQ} \equiv 2\int dt\ \langle\delta Q(t)\,\delta Q(0)\rangle$$

The Quantum Limit of Detection



$$\langle I \rangle = I_0 + \lambda A \langle \sigma_z \rangle$$

Quantum limit: the best you can do is measure as fast as you dephase:

$$\chi = \Gamma_{meas} / \Gamma_{\varphi} \leq 1$$

$$(|\uparrow\rangle + |\downarrow\rangle)\, |D\rangle \Rightarrow |\uparrow\rangle\, |D_\uparrow(t)\rangle + |\downarrow\rangle\, |D_\downarrow(t)\rangle$$

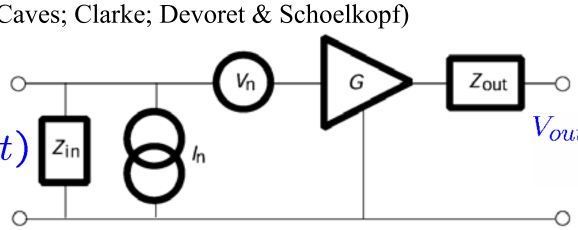
•Measurement? Need $|D_\uparrow(t)\rangle$ ***distinguishable*** from $|D_\downarrow(t)\rangle$

•Dephasing? Need $|D_\uparrow(t)\rangle$ ***orthogonal*** to $|D_\downarrow(t)\rangle$

- What symmetries/properties must an arbitrary detector possess to reach the quantum limit?

Why care about the quantum limit?

- Minimum Noise Energy in Amplifiers:
  (Caves; Clarke; Devoret & Schoelkopf)



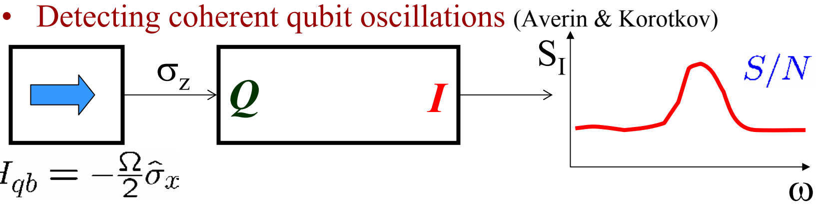
$V_{in}(t)$

$$V_{out}(t) = \lambda V_{in}(t) + V_{noise}(t)$$

- Minimum power associated with V_{noise}?

$$E_N \geq \frac{\hbar\omega}{2}$$

- Detecting coherent qubit oscillations (Averin & Korotkov)



$S/N \leq 4$

$H_{qb} = -\frac{\Omega}{2}\hat{\sigma}_x$

How to get to the Quantum Limit

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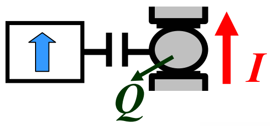


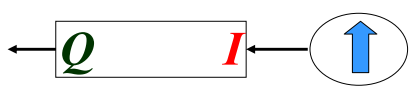
$$\langle I \rangle = I_0 + \lambda A \langle \sigma_z \rangle$$

•Now, we have: $\Gamma_{meas} = (A^2\lambda^2)/S_{II}$ $\quad \Gamma_\varphi = \frac{A^2}{\hbar^2} S_{QQ}$

$$\frac{\Gamma_{meas}}{\Gamma_\phi} = \frac{\hbar^2\lambda^2}{S_{II}S_{QQ}} \leq \frac{\hbar^2\lambda^2}{|S_{IQ}|^2} \leq 1$$

•Quantum limit requires:

- $\langle f|Q|i\rangle \propto \langle f|I|i\rangle$ (i.e. no extra degrees of freedom)
- λ' vanishes (monitoring output does not further dephase)
- λ' is the "reverse gain":

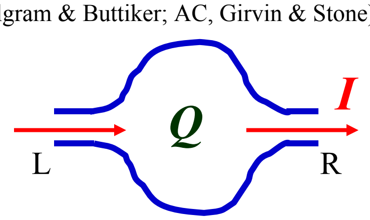


What does it mean?

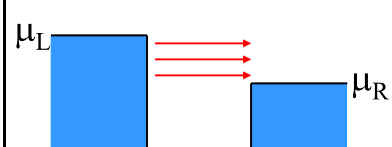
- To reach the quantum limit, there should be no unused information in the detector…

Mesoscopic Scattering Detector:

(Pilgram & Buttiker; AC, Girvin & Stone)



$$s(\varepsilon) = \begin{pmatrix} \sqrt{1-T}e^{i\beta} & \sqrt{T}e^{i\varphi'} \\ \sqrt{T}e^{i\varphi} & -\sqrt{1-T}e^{i\beta'} \end{pmatrix}$$



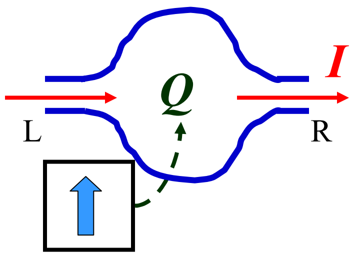
$$I_0 = -\frac{e^2}{h}\int_{\mu_R}^{\mu_L} d\varepsilon T(\varepsilon)$$

What does it mean?

- To reach the quantum limit, there should be no unused information in the detector…

Mesoscopic Scattering Detector:

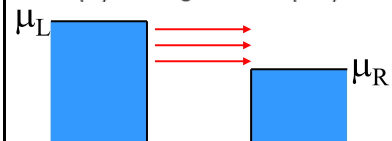
(Pilgram & Buttiker; AC, Girvin & Stone)



Transmission probability depends on qubit: $T_\uparrow - T_\downarrow = A\frac{dT}{d\varepsilon}$

$$s_{\uparrow/\downarrow}(\varepsilon) = s \pm \left(\frac{A}{2}\right)\frac{ds}{d\varepsilon}$$

$$\langle I \rangle = I_0 + \lambda A \langle \sigma_z \rangle$$



$$I_0 = -\frac{e^2}{h}\int_{\mu_R}^{\mu_L} d\varepsilon T(\varepsilon)$$

$$\lambda = -\frac{e^2}{h}\int_{\mu_R}^{\mu_L} d\varepsilon \frac{dT}{d\varepsilon}$$

The Proportionality Condition

- Need: $\langle f|Q|i\rangle \propto \langle f|I|i\rangle$

$$\langle I \rangle = I_0 + \lambda A \langle \sigma_z \rangle$$

$$s(\varepsilon) = \begin{pmatrix} \sqrt{1-T}e^{i\beta} & \sqrt{T}e^{i\varphi'} \\ \sqrt{T}e^{i\varphi} & -\sqrt{1-T}e^{i\beta'} \end{pmatrix}$$

$$\frac{d}{d\varepsilon}(\beta - \varphi) = 0$$

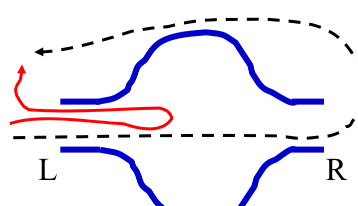
$$\frac{\frac{dT}{d\varepsilon}}{T(1-T)} = \frac{1}{\mathcal{C}}$$

Not usual symmetries!

Phase condition?

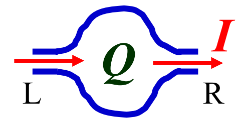
•Qubit cannot alter relative phase between reflection and transmission

•No "lost" information that could have been gained in an ***interference*** experiment….



Transmission Amplitude Condition

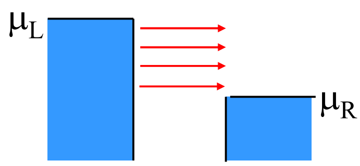
$$\frac{\frac{dT}{d\varepsilon}}{T(1-T)} = \frac{1}{\mathcal{C}}$$



$$\langle I \rangle = I_0 + \lambda A \langle \sigma_z \rangle$$

Ensures that no information is lost when averaging over energy

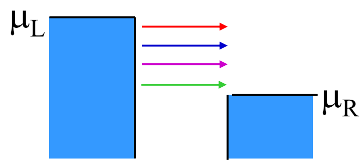
1)



$$\Gamma_{meas} = \frac{A^2\lambda^2}{S_{II}} = \frac{A^2\left(\sum \frac{dT}{d\varepsilon}(\varepsilon_j)\right)^2 \Delta\varepsilon}{\sum T(\varepsilon_j)[1-T(\varepsilon_j)]}$$

versus

2)



$$\sum \Gamma_{meas,j} = \sum \left(\frac{A^2\left(\frac{dT}{d\varepsilon}(\varepsilon_j)\right)^2 \Delta\varepsilon}{T(\varepsilon_j)[1-T(\varepsilon_j)]} \right)$$

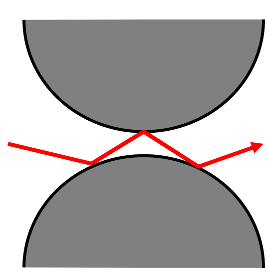
The Ideal Transmission Amplitude

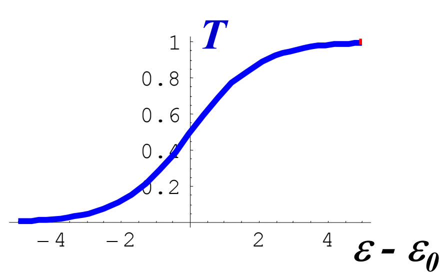
$$\frac{\frac{dT}{d\varepsilon}}{T(1-T)} = \frac{1}{\mathcal{C}} \quad \Rightarrow \quad T(\varepsilon) = \frac{1}{1+e^{(\varepsilon-\varepsilon_0)/\mathcal{C}}}$$

Necessary energy dependence to be at the quantum limit

Corresponds to a real system-- the adiabatic quantum point contact!

(Glazman, Lesovik, Khmelnitskii & Shekhter, 1988)



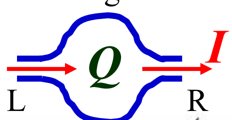


Information and Fluctuations

Reaching quantum limit = no wasted information

- No information lost in phase changes: $\frac{d}{d\varepsilon}(\beta - \varphi) = 0$
- No information lost when energy averaging: $\frac{\frac{dT}{d\varepsilon}}{T(1-T)} = \frac{1}{\mathcal{C}}$

Look at charge fluctuations:



1. $\Gamma_{\varphi} \propto S_{QQ}(0)$
2. $\Gamma_{\varphi} \rightarrow \Gamma_{meas}$

$$S_{QQ} = \frac{e^2\hbar}{2\pi}\int d\varepsilon \left(\frac{\left(\frac{dT}{d\varepsilon}\right)^2}{2T(1-T)} + 2\left[\sqrt{T(1-T)}\frac{d}{d\varepsilon}(\phi - \beta)\right]^2 \right)$$

Γ_{meas} for current experiment

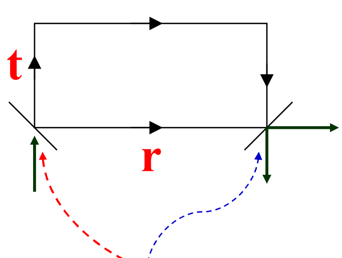
Γ_{meas} for phase experiment

Measurement Rate for Phase Experiment

$$S_{QQ} = \frac{e^2\hbar}{2\pi}\int d\varepsilon \left(\frac{\left(\frac{dT}{d\varepsilon}\right)^2}{2T(1-T)} + 2\left[\sqrt{T(1-T)}\frac{d}{d\varepsilon}(\phi - \beta)\right]^2 \right)$$

Γ_{meas} for current experiment

Γ_{meas} for phase experiment



$$\Lambda = s' \cdot s$$

$$\Gamma_{meas} = (\Delta I)^2 / S_{II}$$

$$I = |\Lambda_{12'}|^2$$

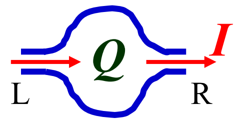
$$\Delta I = \sqrt{T(1-T)}\frac{d}{d\epsilon}(\phi - \beta)$$

$$S_{II} = 2T_{\Lambda}(1 - T_{\Lambda}) = 1/2$$

Information and Fluctuations (2)

Reaching quantum limit = no wasted information

Can connect charge fluctuations to information in more complex cases:



1. Multiple Channels

$$\hat{S}_Q(\varepsilon) = \frac{e^2\hbar}{4\pi}\left(\frac{(\partial_\varepsilon T)^2}{T(1-T)} + 4TR(\phi_U-\phi_V)^2 + 2\left[\phi_U,\sqrt{TR}\right]\left[\sqrt{TR},\phi_V\right] + [\phi_U,T][T,\phi_U] + [\phi_V,T][T,\phi_V]\right)$$

Extra terms due to channel structure

2. Normal-Superconducting Detector

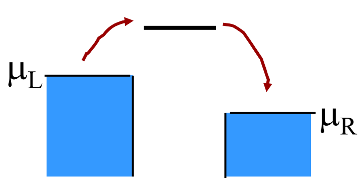
$$S_Q = \frac{e^2\hbar}{2\pi}(eV)\left[\frac{2(\partial_E T)^2}{(2-T)^2(1-T)} + \frac{8T^2(1-T)}{(2-T)^4}\left(\partial_E(\phi-\phi'-\beta)\right)^2\right]$$

Γ_{meas} for current experiment

Γ_{meas} for phase experiment

Partially Coherent Detectors

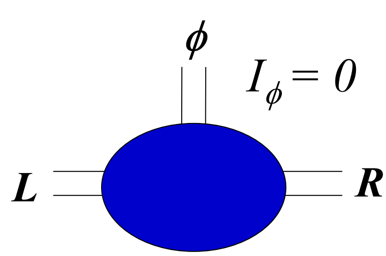
- What is the effect of adding dephasing to the mesoscopic scattering detector? Look at a resonant-level model…



- Symmetric coupling to leads ⇒ no information in relative phase

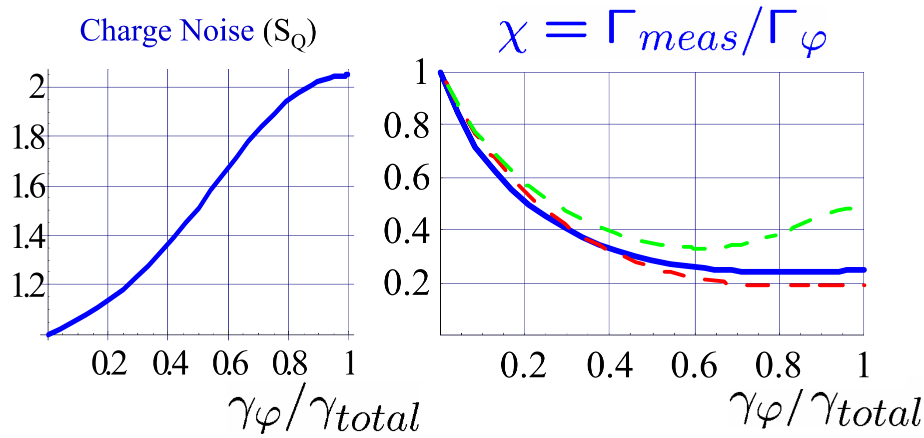
$$t = i\frac{\gamma}{\epsilon+i\gamma} \to i\frac{\gamma}{\epsilon+i\gamma+i\gamma_\varphi}$$

- Assume dephasing due to an additional voltage probe (Buttiker)

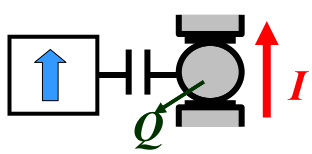


Partially Coherent Detectors

- Reducing the coherence of the detector ***enhances*** charge fluctuations… total accessible information is increased
- A resulting departure from the quantum limit…



Conclusions



$$\frac{\Gamma_{meas}}{\Gamma_\varphi} \leq 1$$

- Reaching the quantum limit requires that there be no wasted information in the detector; can make this condition precise.

- Looking at information provides a new way to look at mesoscopic systems:
  - New symmetry conditions
  - New way to view fluctuations

- Reducing detector coherence enhances charge fluctuations, leads to a departure from the quantum limit