Spin Correlated States in Optical Lattices

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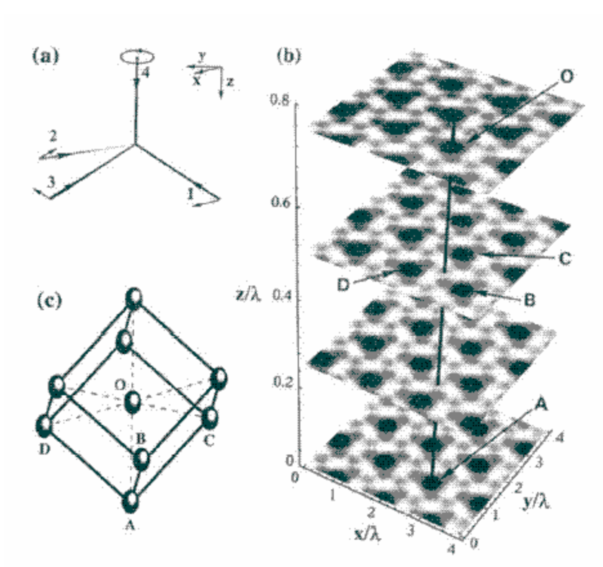
Optical lattices



Information processor and Information storage

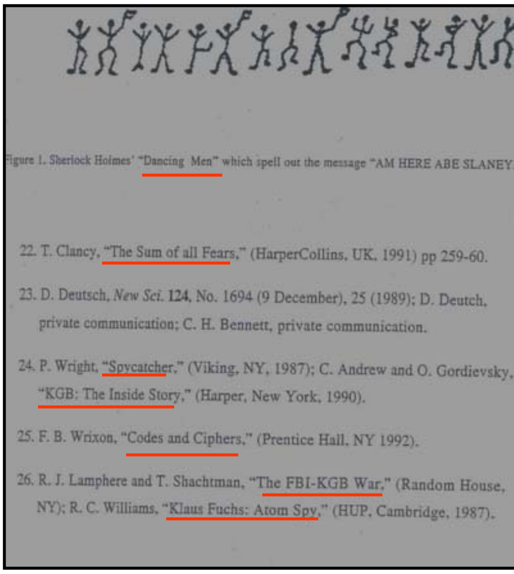


Figure 1. Sherlock Holmes' "Dancing Men" which spell out the message "AM HERE ABE SLANEY."

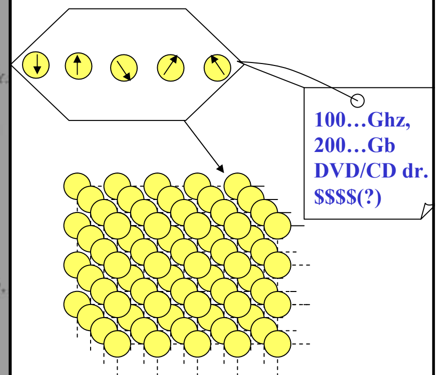
22. T. Clancy, "The Sum of all Fears," (HarperCollins, UK, 1991) pp 259-60.

23. D. Deutsch, *New Sci.* **124**, No. 1694 (9 December), 25 (1989); D. Deutch, private communication; C. H. Bennett, private communication.

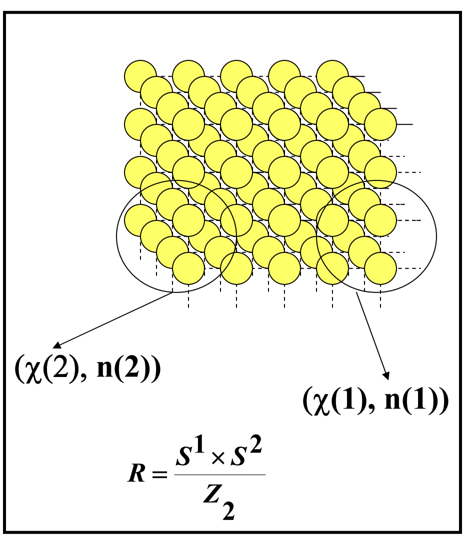
24. P. Wright, "Spycatcher," (Viking, NY, 1987); C. Andrew and O. Gordievsky, "KGB: The Inside Story," (Harper, New York, 1990).

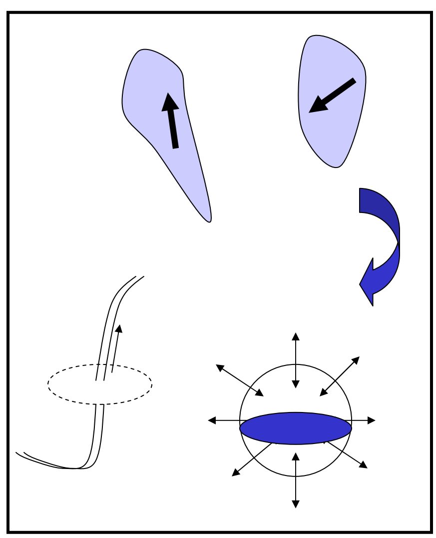
25. F. B. Wrixon, "Codes and Ciphers," (Prentice Hall, NY 1992).

26. R. J. Lamphere and T. Shachtman, "The FBI-KGB War," (Random House, NY); R. C. Williams, "Klaus Fuchs: Atom Spy," (HUP, Cambridge, 1987).



A lab for baby universe ?

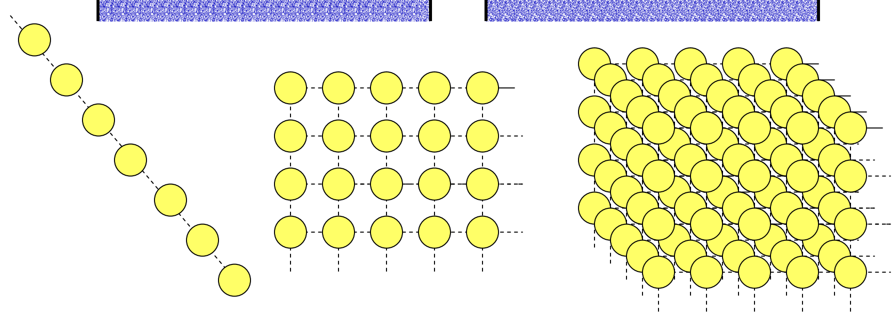




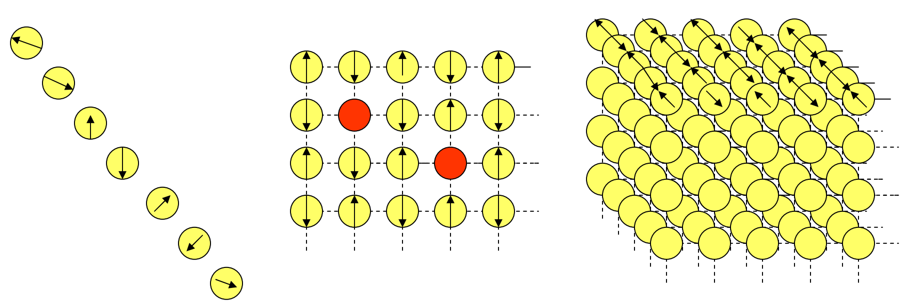
Creation of vortices, monopoles and half vortices.

Atoms in optical lattices
Vs. electrons in Cuprates

a) Free from imperfections	a) Defects or disorder
b) Known interactions	b) Material dependent
c) Tunable coupling constants	c) Barely changeable
d) ld, 2d and 3d lattices	d) layered structures
e) S=0, 1/2, 1, 3/2 atoms	e) S=1/2 electrons



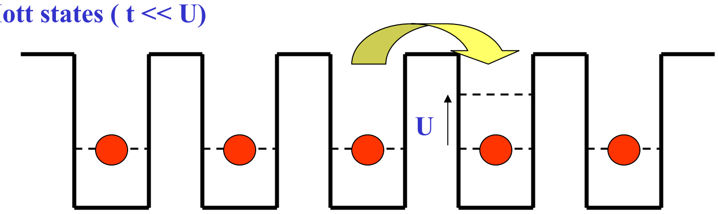
S=1/2 Fermions in optical lattices
(small hopping limit)



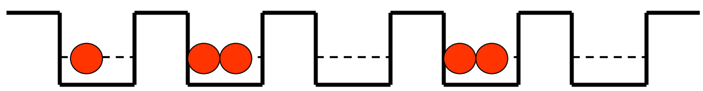
Gapless Spin liquid | *HTcS made of cold atoms?* | *Neel Ordered*

S=0 bosons in lattices

Mott states (t << U)

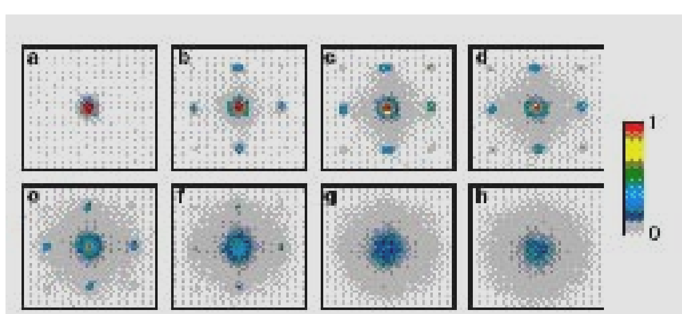


Condensates (t >>U)



In (a) and (b), one boson per site. t is the hopping and can be varied by tuning laser intensities of optical lattices; U is an intra-site interaction energy. In a Mott state, all bosons are localized.

M. P. A. Fisher et al., PRB 40, 546 (1989);
On Mott states in a finite trap, see
Jaksch et al., PRL. 81, 3108-3111(1998).



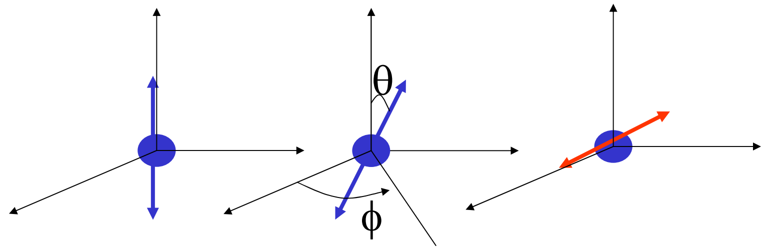
Absorption images of interference patterns

as the laser intensity is increased (from a to h).

(a-d) BECs and (g-h) Mott insulating states

Greiner et al., Nature 415, 39(02)

S=1 bosons with Anti-ferromagnetic interactions



$$|\vec{n}(\theta,\phi)>= \frac{\sin\theta e^{-i\phi}}{\sqrt{2}}|1> + \cos\theta\,|0> - \frac{\sin\theta e^{i\phi}}{\sqrt{2}}|-1>$$

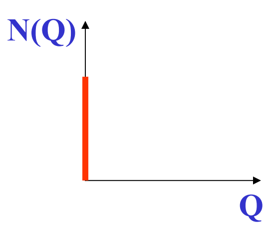
$$:|-\vec{n}>= (-1)\quad|\vec{n}>, \qquad <\vec{n}|S_{\alpha}|\vec{n}>= 0.$$

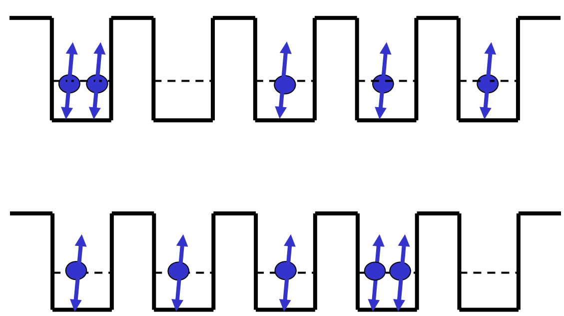
$$B:(0,\phi) \Leftrightarrow \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad R:(\frac{\pi}{2},0) \Leftrightarrow \begin{pmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\end{pmatrix}.$$

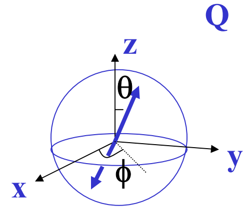
$$U_F(r_1-r_2)=\delta(r_1-r_2)g_F,$$
$$g_F=\frac{4\pi\hbar a_F}{M}, g_2>g_0,$$
$$F=0,2.$$

Condensates of spin one bosons

(d>1)





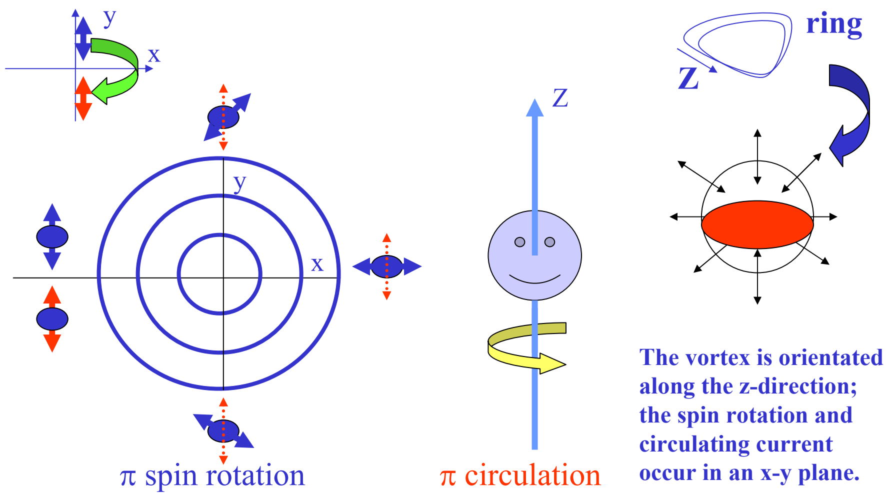


Snap shots

$$\psi_{pBEC} \sim \frac{(C^{+}_{Q=0,\alpha}n_{\alpha})^{N\times V_T}}{\sqrt{(N\times V_T)!}};C^{+}_{0,\alpha}=\frac{1}{\sqrt{V_T}}\sum_k C^{+}_{k\alpha}, \alpha=x,y,z.$$

Half vortices

In a half vortex, each atom makes a π spin rotation; a half vortex carries **one half circulation** of an integer vortex. A half vortex ring is also a hedgehog.



The vortex is orientated along the z-direction; the spin rotation and circulating current occur in an x-y plane.

S=1 bosons with anti-ferromagnetic interactions in optical lattices (3D and 2D, N=2k)

Spin Singlet MI	Nematic MI	Polar BEC
(c)	(b)	(a)

→ t: Hopping

The critical value of η is determined numerically.

$$a)\ zt(V_{opt}) > \eta_1 E_C;$$

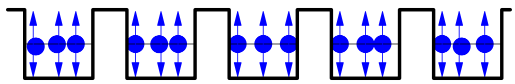
$$b)\ \eta_1 E_C > zt(V_{opt}) > \eta_C \sqrt{E_C E_S};$$

$$c)\ \eta_C \sqrt{E_C E_S} > zt(V_{opt}).$$

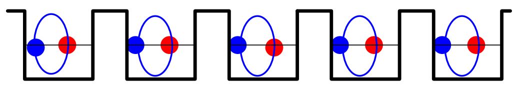
$$E_C = \frac{4\pi\rho(2a_2 + a_0)}{3MN_0} \gg E_S = \frac{4\pi\rho(a_2 - a_0)}{3MN_0}.$$

Schematic of microscopic wave functions

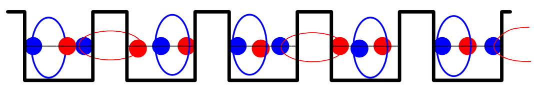
a) NMI; b) SSMI (N=2k); c) SSMI (N=2k+1 in 1d).
Each pair of blue and red dots with a ring is a spin singlet.



(a)



(b)



(c)

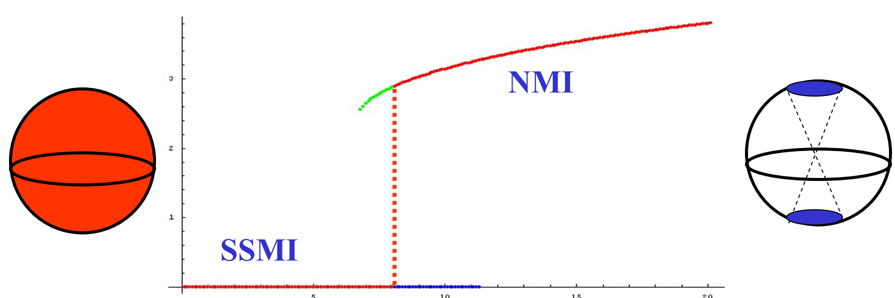
$$O^2_{\alpha\beta} = \langle C^+_\alpha C_\beta \rangle - \frac{1}{3}\delta_{\alpha\beta} \langle C^+_\gamma C_\gamma \rangle .$$

$$NMI: O^2_{\alpha\beta} = \varepsilon N(\vec{n}_\alpha \vec{n}_\beta - \frac{1}{3}\delta_{\alpha\beta});$$

$$SSMI: O^2_{\alpha\beta} = 0.$$

$$\eta = \frac{tz}{\sqrt{E_s E_c}}; \eta >> 1, \psi_{NMI} \sim \prod_k \frac{(C^+_{k\alpha}\vec{n}_\alpha)^N}{\sqrt{N!}} | vac >; \eta = 0, \psi_{SSMI} \sim \prod_k \frac{(C^+_{k\alpha} C^+_{k\alpha})^{N/2}}{\sqrt{N!}} | vac > .$$

Numerics I: Large **N=2k** *limit*

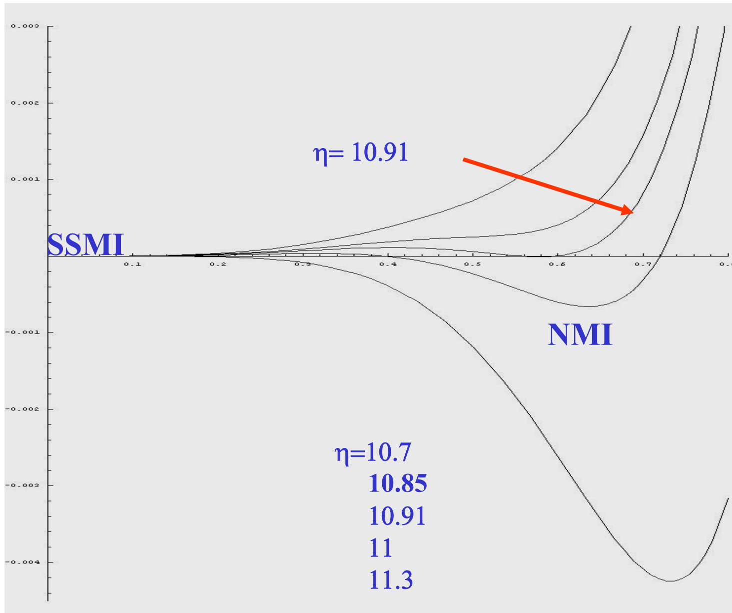


η=10.91

σ vs. η (proportional to hopping) is plotted here. Blue and Green lines represent metal stable states close to the critical point.

$$\Psi(\vec{n}) \sim \exp[-\sigma \vec{n}_\alpha \vec{n}_\beta Q^0_{\alpha\beta}]$$

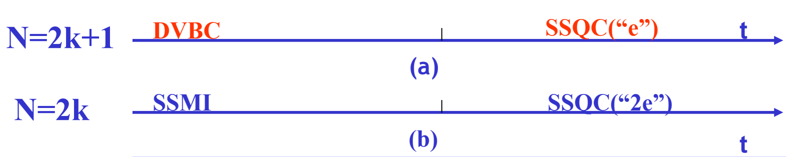
The energy Vs. σ



Spin singlet quantum "condensates" in 1D optical lattices (SSQC)

S=1, "Q=e" bosons with AF interactions ===>

S=0 , "Q=e" bosons interacting via Ising gauge fields



$$H_{fqc} = H_{m.} + H_{Z_2}$$

$$H_{m.} = -t_0 \sum_{<kl>} \sigma^z_{kl} (b^+_k b_l + h.c.) + E_C \sum_k (\hat{N}_{kb} - N)^2;$$

$$H_{Z_2} = \Gamma_a \sum_{<kl>} \sigma^x_{kl}$$

$$\hat{C}_k \Psi = \Psi, \hat{C}_k = \exp(i\pi [N_{kb} + \sum_{+} \sigma^x_{kl}]).$$

Spin one bosons in optical lattices

We have found

1) Polar Condensates
2) Nematic Mott insulators
3) Spin singlet Mott insulators
4) Valence bond crystals (N=2k+1,1D)
5) Spin singlet condensates (1D)

Work in progress

- Towards topological fault tolerant quantum information storage