Interactions and Quantum Decoherence in the Ground State: Persistent Currents and Weak Localization

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1. Introduction

- Quantum decoherence in the ground state of an interacting system: Does it make sense?
- Electrons in disordered conductors: Key features of the model
- What and how to calculate?

2. Persistent currents in mesoscopic rings: Decoherence by interactions at $T \to 0$

3. Weak localization in disordered conductors: Quantum fluctuations and Pauli principle

CALDEIRA-LEGGETT MODEL

$$\hat{H} = \frac{\hat{p}^2}{2m} + \sum_k \left[\frac{\hat{P}_k^2}{2M_k} + \frac{M_k \omega_k^2}{2} \left(\hat{R}_k - \frac{c_k}{M_k \omega_k^2} \hat{x} \right)^2 \right]$$

$$J(\omega) \equiv \frac{\pi}{2} \sum_k \frac{c_k^2}{m_k \omega_k} \delta(\omega - \omega_k) = \eta \omega \theta(\omega_{\mathrm{c}} - \omega)$$

Equilibrium density matrix ($T \to 0$):

$$\hat{\rho}(x_1 - x_2) = \int dR_k \, \Psi_0(x_1, R_k) \Psi_0^*(x_2, R_k)$$

$$= \exp\left[-(x_1 - x_2)^2 / 2L_\varphi^2 \right]$$

Dephasing length: $$L_\varphi = \sqrt{\frac{\pi \hbar}{\eta \ln(m\omega_{\mathrm{c}}/\eta)}}$$

Choice of the basis important!

WHAT to calculate?

- Decay of a non-equilibrium state (e.g. quantum computing)
- Decay of equilibrium correlation functions (e.g. weak localization)
- Equilibrium properties (e.g. persistent currents)

HOW to calculate?

- Perturbation theory
- Non-perturbative techniques

Linear conductance: Kubo formula

$$\sigma = \frac{e^2}{3m} \int_{-\infty}^{t_{\mathrm{f}}} dt' [\nabla_{\mathbf{r}_{1\mathrm{f}}} - \nabla_{\mathbf{r}_{2\mathrm{f}}}] |_{\mathbf{r}_{1\mathrm{f}}=\mathbf{r}_{2\mathrm{f}}} \rho(t_f, t'; \mathbf{r}_{1f}, \mathbf{r}_{2f})$$

$$\rho(t_f, t'; \mathbf{r}_{1f}, \mathbf{r}_{2f}) = \int d\mathbf{r}'_1 d\mathbf{r}'_2 J(t_{\mathrm{f}}, t', \mathbf{r}_{1\mathrm{f}}, \mathbf{r}_{2\mathrm{f}}, \mathbf{r}'_1, \mathbf{r}'_2) \times [\mathbf{r}'_1 - \mathbf{r}'_2]\, \rho(\mathbf{r}'_1, \mathbf{r}'_2) \ ,$$

$$J = \hat{U}_1(t_f, t') \hat{U}_2(t', t_f) \sim \int \mathcal{D}\mathbf{p} \mathcal{D}\mathbf{r} e^{\frac{1}{\hbar}(iS_0 - iS'_0 - iS_R - S_I)}$$

$$\tilde{\rho}_{\mathrm{initial}} = [\mathbf{r}'_1 - \mathbf{r}'_2]\, \rho(\mathbf{r}'_1, \mathbf{r}'_2)$$

Theoretical "arguments":

- General argument: at $T \to 0$ scattering space shrinks to zero (any statistics)
- Perturbation theory + Golden rule

Toy "Cooperon"

- $\tilde{\mathcal{C}}(t) = \theta(t)(1+\alpha t)\mathrm{e}^{-(\alpha+\beta T)t}$,

- $\tilde{\mathcal{C}}(\omega) = \dfrac{1}{-i\omega + \tilde{\Sigma}(\omega)}$,

- $\tilde{\Sigma}(\omega) = \dfrac{(\alpha+\beta T)^2 - i\omega\beta T}{2\alpha + \beta T - i\omega}$

Use definition: $\gamma_\varphi = \Sigma(\omega = 0)$

Exact result: $$\gamma_\varphi = \frac{\alpha+\beta T}{1+\frac{\alpha}{\alpha+\beta T}} \quad \overset{T\to 0}{\longrightarrow} \quad \frac{\alpha}{2}$$

Perturbative result: $\gamma_\varphi^{\text{wrong}} = \beta T \quad \overset{T\to 0}{\longrightarrow} \quad 0$

- Electron-electron interactions: nonlinearity is essential (no exact solution possible)

- Long range interactions

- Disorder: causes electron diffusion at long scales

- Disorder: strongly affects: e-e interactions (effective EM environment for electrons)

- Interacting ground state ($T \to 0$)

- Fermi statistics: Pauli principle

Partition function:

$$\mathcal{Z} = \sum_{m=-\infty}^{\infty} \int_0^{2\pi m} \mathcal{D}\theta \exp(i2\pi m\phi_x - S_0[\theta] - S_{\text{int}}[\theta])$$

Free action: $S_0[\theta] = \int_0^\beta d\tau \frac{1}{4E_C}\left(\frac{\partial\theta}{\partial\tau}\right)^2$

Interaction term:

$$S_{\text{int}} = \frac{e^2}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \langle V(\tau,\theta(\tau))V(\tau',\theta(\tau'))\rangle$$

$$\langle VV\rangle = T\sum_{\omega_n}\int \frac{d^3k}{(2\pi)^3}\frac{4\pi}{k^2\epsilon(i|\omega_n|,k)}e^{-i\omega(\tau-\tau')+ikX}$$

Dirty electron gas: $\epsilon(\omega,k) \approx \frac{4\pi\sigma}{-i\omega+Dk^2}$

$$S_{\text{int}}[\theta] = \alpha\int_0^\beta d\tau\int_0^\beta d\tau' \frac{\pi^2T^2K(\theta(\tau)-\theta(\tau'))}{\sin^2[\pi T(\tau-\tau')]}$$

3d: $K(z) = 1 - \frac{1}{\sqrt{4r^2\sin^2(z/2)+1}}$, $\alpha = \frac{3}{8k_F^2 l^2}$

PERTURBATION THEORY

$$K = \sum_n a_n \sin^2 \left[\frac{n(\theta(\tau) - \theta(\tau'))}{2} \right],$$

3d:

$a_n \sim (2/\pi r) \ln(r/n)$ for $1 \leq n \leq r = R/l$

$a_n \approx 0$ otherwise.

Persistent current: $$I = \frac{e}{2\pi} \frac{dE_0}{d\phi_x}$$

$$I = \frac{eE_C}{\pi} \left[\phi_x - \frac{\alpha}{2} \sum_{n=1}^{r} n a_n \ln \left(\frac{n + 2\phi_x}{n - 2\phi_x} \right) \right]$$

Expansion parameter:

$$\alpha \sum_{n=1}^{r} \frac{n a_n}{n} = \alpha \sum_{n=1}^{r} a_n \sim \alpha \ll 1$$

NONPERTURBATIVE EFFECTS

1. High temperatures: semiclassics

$$\theta_{\mathsf{cl}}(\tau) = 2\pi m T \tau$$

$$\mathcal{Z} \sim \sum_{m=-\infty}^{\infty} \exp\left(i2\pi m \phi_x - \frac{\pi^2 m^2 T}{E_C} - 4\pi |m| \alpha r \right).$$

For $T \gg E_C/\pi^2$:

$$I = 2eT \exp\left(-\frac{\pi^2 T}{E_C} - 4\pi\alpha r \right) \sin(2\pi\phi_x)$$

2. Low temperatures: instantons

$$I \sim eE_C^*$$

$$E_C^*/E_C = A(T)\,\exp(-4\pi\alpha r)$$

Parameter:

$$\alpha \sum_{n=1}^{r} n a_n \sim \alpha r$$

Dephasing length:

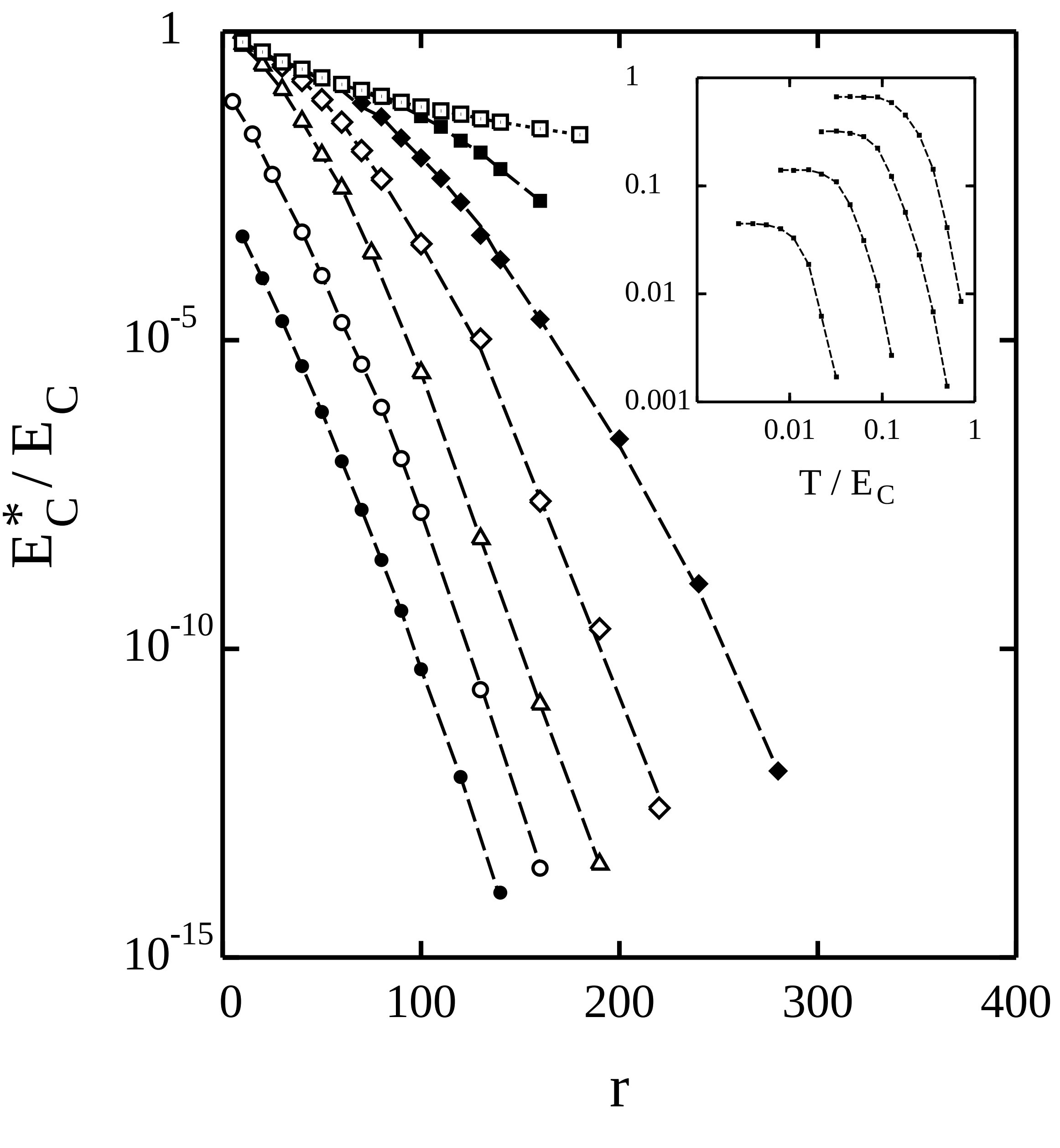
$$L_\varphi \sim l/\alpha \sim l(k_F l)^2$$

MONTE CARLO RESULTS

1. Non-zero temperatures: $\frac{1}{4\pi\alpha r} < \frac{T}{E_C} < 1$

$$E_C^*/E_C = A(T)\,\mathrm{exp}(-4\pi\alpha r), \qquad A(T) \approx \mathrm{exp}(cE_C/T)$$

$$\alpha = 0.019; \qquad \frac{T}{E_C} = 1, 0.5, 0.14, 0.06, 0.03, 0.016, 0$$

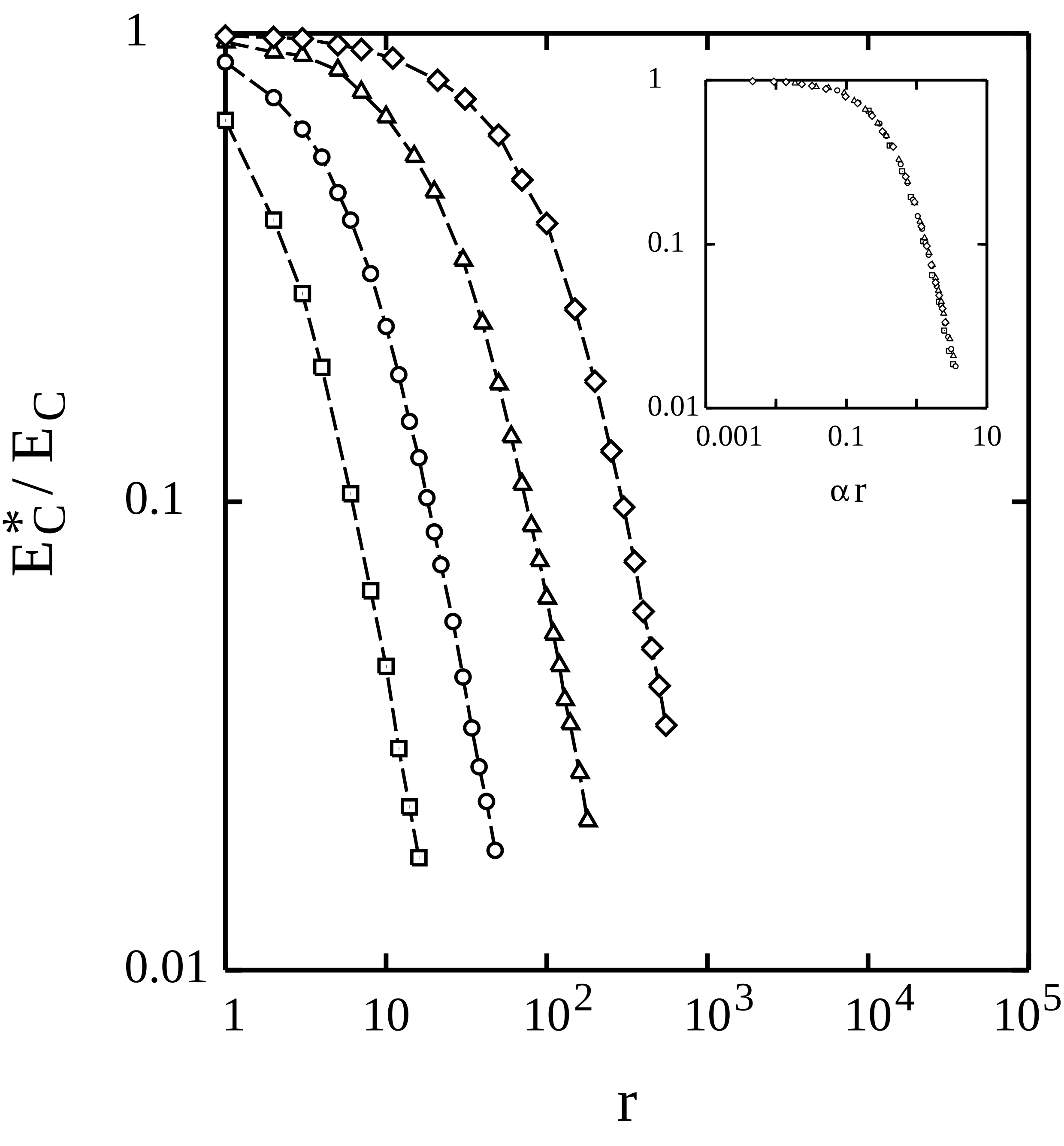


2. Zero temperature limit

$$\alpha r \ll 0.1: \qquad E_C^*/E_C \simeq 1$$

$$\alpha r \gg 0.1: \qquad E_C^*/E_C \propto r^{-\gamma} \quad \gamma \approx 1.8 \div 2$$

$$\alpha = 0.21, 0.07, 0.019, 0.005$$



RETURN PROBABILITY

$$W = W^{(0)}\mathcal{F}, \qquad \mathcal{F} = \exp(iS_R - S_I)$$

Identical paths: $S_R = S_I = 0 \quad \rightarrow \quad W_1 = W^{(0)}$

Time-reversed paths:

$$S_R = 0 \;\; S_I > 0 \;\; \rightarrow W_2 = W^{(0)} \exp(-S_I)$$

$$S_I = \frac{e^2 t}{2\pi v} \int_0^{1/l} dk \int_{-kv}^{kv} d\omega \, \frac{\omega \coth \frac{\omega}{2T}}{4\pi\sigma k}$$

$$S_I(T \to 0) = \frac{e^2 L}{16\pi^2 \sigma l^2} \sim \frac{L}{l\alpha}$$

$$L \equiv 2\pi R = vt, \quad L_\varphi \sim l(k_F l)^2$$

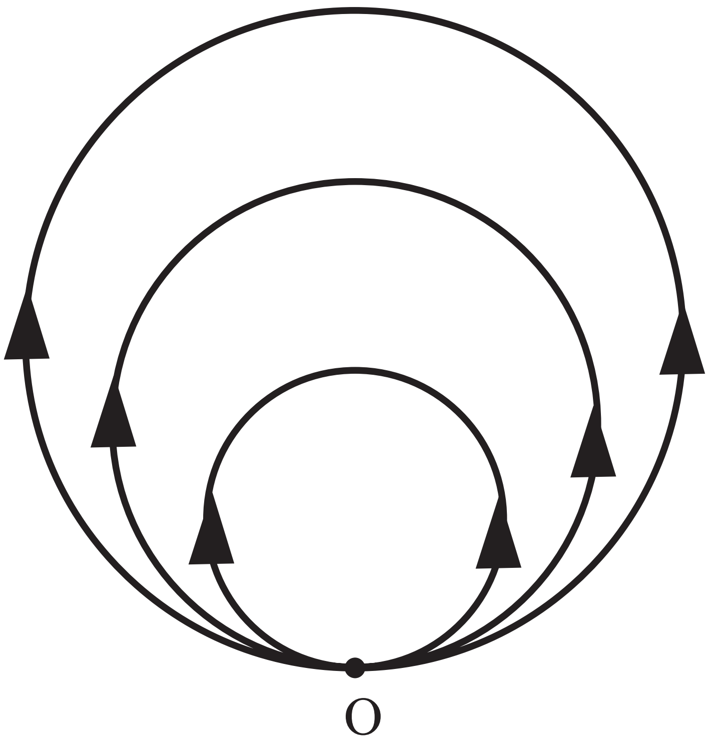
DEPHASING, not renormalization!

DEPHASING TIME

$$\tau_\varphi \sim L_\varphi / v$$

Metals: $v \approx v_F \quad \rightarrow \quad \tau_\varphi \sim \tau_e (k_F l)^2$

Diffusion: $L_\varphi \sim \sqrt{D\tau_\varphi} \sim l(k_F l)$



Particle + electron gas

$$S_R = \frac{e^2}{2}\int_0^t \Big[R(t'-t'',\mathbf{r}'_1-\mathbf{r}''_1) - R(t'-t'',\mathbf{r}'_2-\mathbf{r}''_2) + R(t'-t'',\mathbf{r}'_1-\mathbf{r}''_2) - R(t'-t'',\mathbf{r}'_2-\mathbf{r}''_1)\Big]\, dt'dt'',$$

$$S_I = \frac{e^2}{2}\int_0^t \Big[I(t'-t'',\mathbf{r}'_1-\mathbf{r}''_1) + I(t'-t'',\mathbf{r}'_2-\mathbf{r}''_2) - I(t'-t'',\mathbf{r}'_1-\mathbf{r}''_2) - I(t'-t'',\mathbf{r}'_2-\mathbf{r}''_1)\Big]\, dt'dt''$$

$$I(t,\mathbf{r}) = \int \frac{d\omega d^3k}{(2\pi)^4}\,\mathrm{Im}\left(\frac{-4\pi}{k^2\epsilon(\omega,k)}\right)\coth\frac{\hbar\omega}{2T}\, e^{-i\omega t + i\mathbf{k}\mathbf{r}},$$

$$R(t,\mathbf{r}) = \int \frac{d\omega d^3k}{(2\pi)^4}\,\frac{4\pi}{k^2\epsilon(\omega,k)}\, e^{-i\omega t + i\mathbf{k}\mathbf{r}}$$

Indistinguishable electrons

Golubev, A.D.Z.'98:

$$R(t, \mathbf{r}) \to R(t, \mathbf{r})(1 - 2f(H_0(\mathbf{p}, \mathbf{r}))$$

$$T \to 0: \qquad 1 - 2f(\xi) \to \mathrm{sign}(\xi - \mu)$$

Energy-dependent dissipation

Linear conductance

$$\sigma = \frac{e}{3i\hbar}\int_{-\infty}^{t} dt' \left\langle \mathrm{tr}\left(\hat{j}(x)\hat{U}_1(t,t')[\hat{x},\hat{\rho}(t')]\hat{U}_2(t',t)\right)\right\rangle_V$$

Exact representation:

$$\hat{U}_1(t,t') = [(1-\hat{\rho}(t'))\hat{u}_1(t',t) + \hat{\rho}(t')\hat{u}_2(t',t)]^{-1},$$
$$\hat{U}_2(t',t) = [\hat{u}_2(t,t')(1-\hat{\rho}(t')) + \hat{u}_1(t,t')\hat{\rho}(t')]^{-1}.$$

$$\hat{u}_{1,2}(t,t') = \hat{T}\exp\left[-\frac{i}{\hbar}\int_{t'}^{t} d\tau\,(\hat{H}_0 - eV_{1,2}(\tau))\right]$$

- $\rho \to 0$: $\hat{U}_{1,2}(t,t') = \hat{u}_{1,2}(t,t')$
  $\langle \hat{u}_i(t,0)\hat{u}_j(0,t)\rangle_{\mathrm{t.r.p.}} \propto \exp(-\frac{at}{\hbar})$

- Include ρ:
  $\langle \hat{U}_1(t,0)\hat{U}_2(0,t)\rangle_{\mathrm{t.r.p.}} \propto \exp(-\frac{at}{\hbar}+\frac{at}{\hbar})$???

Theorem. For small $\hbar$ or at long times:

$$\langle \widehat{U}_1(t,0)\widehat{U}_2(0,t)\rangle_{\text{t.r.p.}} \propto \exp(-\frac{at}{\hbar})$$

Proof:

$$\widehat{U}_1 = \widehat{U}_1^{-1}[(1+(\widehat{\rho}-\widehat{\rho}^2)(\widehat{u}_1\widehat{u}_2^{-1}+\widehat{u}_2\widehat{u}_1^{-1}-\widehat{2})]^{-1}$$

Dependence on $\hbar$:

- $\widehat{u}_i\widehat{u}_j^{-1} = \widehat{1} + O(\hbar)$
- $\widehat{U}_1 = \widehat{U}_1^{-1}[1+(\widehat{\rho}-\widehat{\rho}^2)O(\hbar)] \simeq \widehat{U}_1^{-1}$