Mesoscopic quantum measurements

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Outline

1. Introduction: Josephson junction qubits:
   - Coulomb blockade of Cooper pair tunneling;
   - coherent oscillations of two coupled charge qubits;
   - variable electrostatic transformer for controlled coupling.
2. Quantum measurement problem.
3. Linear measurements.
4. Quadratic measurements and active suppression of dephasing in Josephson junction qubits.

Collaboration: Ch. Bruder, R. Fazio, A.N. Korotkov, W. Mao, R. Ruskov
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Quantum dynamics of Josephson junctions

- Superconductor can be thought of as a BEC of Cooper pairs: one single-particle state

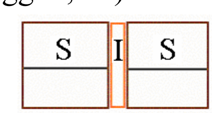
$$\Psi = \sqrt{n} e^{i\varphi}$$

occupied with macroscopic number of particles. The phase φ and the number of particles n are conjugate quantum variables (Anderson, 64; Ivanchenko, Zil'berman, 65):

$$[n, \varphi] = i.$$

This relation describes dynamics of addition or removal of particles to/from the condensate.

- This dynamics manifests itself most directly in Josephson tunnel junctions, and was studied as an example of *macroscopic* quantum dynamics (Leggett, 80).



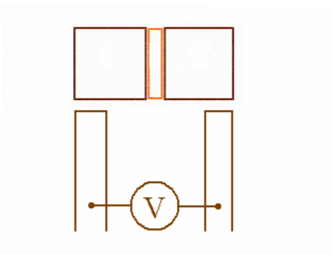
$$H = -E_C \partial^2/\partial\varphi^2 - E_J \cos\varphi + U_{ext}(\varphi), \qquad E_C \equiv (2e)^2/2C.$$

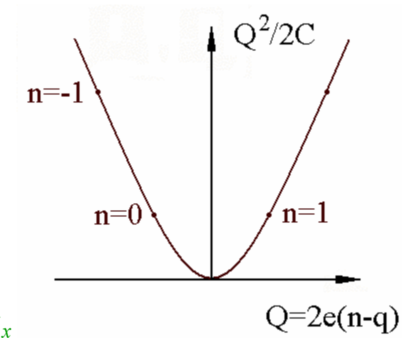
- If quantum fluctuations of phase φ become large, junction behavior can be described as a semiclassical dynamics of charge that leads to controlled transfer of individual Cooper pairs (Averin, Zorin, Likharev, 1985).

$$H = E_C(n-q)^2 - E_J/2(|n\rangle\langle n\pm 1| + |n\pm 1\rangle\langle n|).$$

Charge qubits

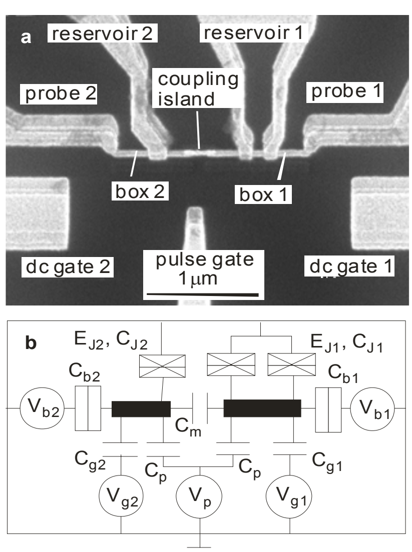
For $E_J << E_C$ and $q \approx 1/2$, the charge tunneling dynamics in an isolated individual junction is directly reduced to the two-state form.





$$H = -E_C(q-1/2)\sigma_z - (E_J/2)\sigma_x$$

Two coupled charge qubits

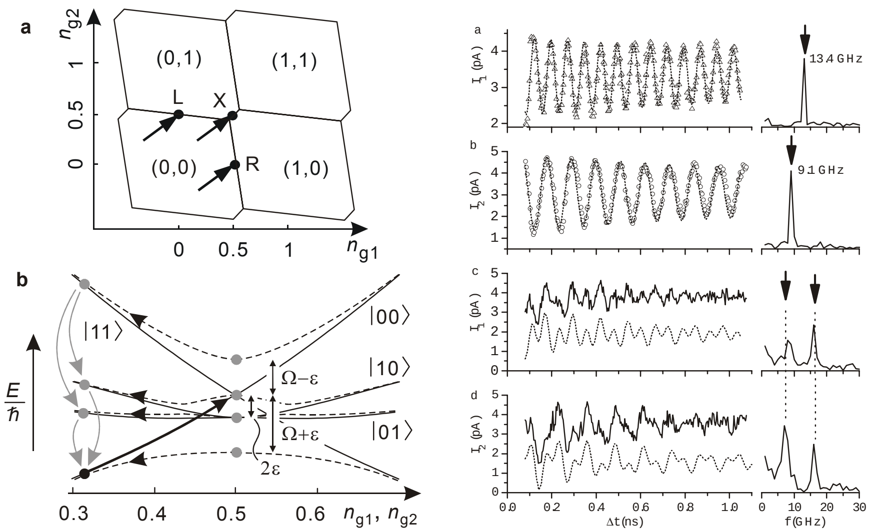


$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

$$E_{n1n2} = E_{c1}(n_{g1}-n_1)^2 + E_{c2}(n_{g2}-n_2)^2 + \\ + E_m(n_{g1}-n_1)(n_{g2}-n_2),$$

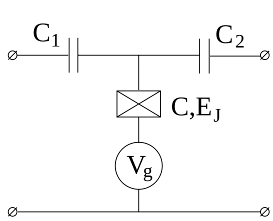
$$E_m = e^2C_m/(C_{\Sigma 1}C_{\Sigma 2} - C_m{}^2)$$

Yu. A. Pashkin *et al.*,
Nature **421**, 823 (2003).

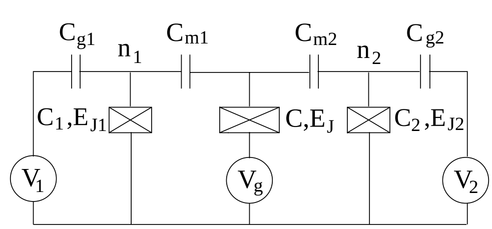


Variable electrostatic transformer: controlled coupling of charge qubits

Equivalent circuit of the variable electrostatic transformer:



Gate-controlled qubit coupling:



coupling capacitance:

$$C \equiv \partial V_{out} / \partial q = \partial^2 \varepsilon_0 (q_g + q) / \partial q^2$$

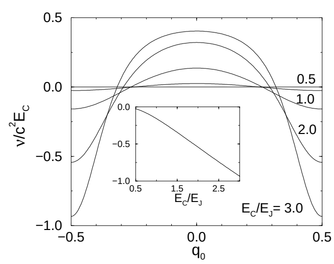
D.V.A. and C. Bruder, cond-mat/0304166.

Coupling strength:

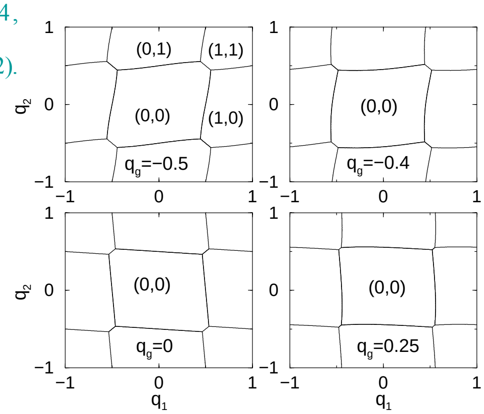
$$H = \nu \sigma_{z1} \sigma_{z2},$$

$$\nu = [\varepsilon_0(q_0 + c) + \varepsilon_0(q_0 - c) - 2\varepsilon_0(q_0 + c)]/4,$$

$$c = C_m / C_\Sigma, \quad q_0 = q_g + c \sum_{i=1,2} (q_i - 1/2).$$



Charging diagram demonstrates transition from positive to negative coupling



Quantum measurement problem

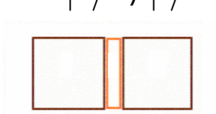
The process of quantum measurement establishes correlations between the states of the measured system and the states of ``macroscopic'' detector.

$$\Psi_S \Psi_D(0) = (\sum_j a_j | j \rangle) \Psi_D(0) \Rightarrow \sum_j a_j | j \rangle \Psi_D^{(j)}(t) \Rightarrow$$

$$\Rightarrow | j_0 \rangle \Psi_D^{(j_0)}(t) \quad \text{with probability} \quad |a_{j_0}|^2$$ - ``wave function collapse''

In the mesoscopic regime, both the detector and the measured systems are of the same ``size''. In addition, there are new simple quantum ``paradoxes''. For instance, measurement of the charge qubit leads to changes in α,β and therefore to transfer of charge (for weak measurements, *gradual*) even if the tunneling is *completely suppressed!*

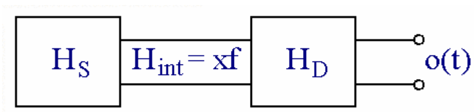
$$\alpha |0\rangle + \beta |1\rangle$$



Linear quantum measurements

Linear-response theory enables one to develop quantitative description of the quantum measurement process with an arbitrary detector provided that it satisfies some general conditions:

- the detector/system coupling is weak so that the detector's response is linear;
- the detector is in the stationary state;
- the response is instantaneous.



$$H = H_S + H_D + xf$$

D.V.A., cond-mat/00044364, cond-mat/0010052, and to be published.
S.Pilgram and M. Büttiker, PRL 89, 200401 (2002).
A.A. Clerk, S.M. Girvin, and A.D.Stone, cond-mat/0211001.

FDT analog for quantum measurements

$$\hbar|\lambda| \le 4\pi[S_f S_q - (\mathrm{Re}\, S_{fq})^2]^{1/2},$$

where λ is the linear response coefficient of the detector, S_f and S_q are the low-frequency spectral densities of the, respectively, back-action and output noise, $\mathrm{Re}S_{fq}$ is the classical part of their cross-correlator.

This inequality shows that finite response coefficient implies that that noise generated by the detector is non-vanishing. Although it was obtained from the linear-response theory, it has broader meaning in that it characterizes the efficiency of the trade-off between the information acquisition by the detector and back-action dephasing of the measured system. The detector that satisfies this inequality as equality is called ``ideal'' or ``quantum-limited''.

Information/back-action trade-off in quantum measurements

Qualitatively, dynamics of the measurement process consists of information acquisition by the detector and back-action dephasing of the measured system. The trade-off between them has the simplest form for measurements of the static system with $H_S=0$. Let $x|j\rangle=x_j|j\rangle$. Then we have for the *back-action dephasing*:

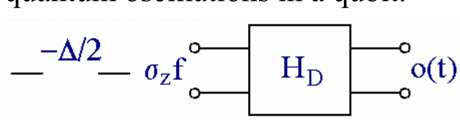
$$\rho_{jj'}(t)=\rho_{jj'}(0)e^{-\Gamma_d t}, \qquad \Gamma_d=\pi(x_j-x_{j'})^2 S_f/\hbar^2 .$$

Information acquisition by the detector is the process of distinguishing different levels of the output signal $\langle o\rangle=\lambda x_j$ in the presence of output noise S_q. The signal level (and the corresponding eigenstates of x) can be distinguished on the time scale given by the by the measurement time τ_m:

$$\tau_m=8\pi S_q/[\lambda(x_j-x_{j'})]^2, \qquad \tau_m\Gamma_d=8(\pi/\hbar\lambda)^2 S_q S_f \geq 1/2.$$

Continuous monitoring of the MQC oscillations

The trade-off between the information acquisition by the detector and back-action dephasing manifests itself in the directly measurable quantity in the case of measurement of coherent quantum oscillations in a qubit.



$$H=-\tfrac{\Delta}{2}\sigma_x+\sigma_z f+H_D$$

Spectral density $S_o(\omega)$ of the detector output reflects coherent quantum oscillations of the measured qubit:

$$S_o(\omega)=S_q+\frac{\Gamma_d\lambda^2}{4\pi}\frac{\Delta^2}{(\omega^2-\Delta^2)^2+\Gamma_d^2\omega^2}.$$

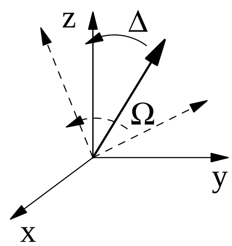
The height of the oscillation peak in the output spectrum is limited by the link between the information and dephasing:

$$S_{max}/S_q\leq 4.$$

Quantum non-demolition measurements of a qubit

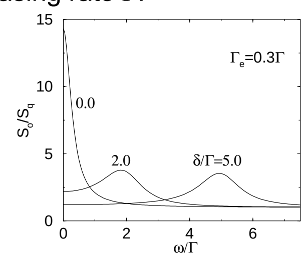
QND measurement avoids the detector backaction by employing specially designed detector-qubit coupling which effectively measures qubit in the rotating frame that follows the qubit oscillations:

$$H=-\frac{\Delta}{2}\sigma_x-\frac{1}{2}(\sigma_z\cos\Omega t+\sigma_y\sin\Omega t)f+H_D$$



D.V.A., PRL **88**, 207901 (2002).

Suppression of backaction should manifest itself as more pronounced oscillation line in the output spectrum of detector S_0 when the detuning $\delta=\Delta-\Omega$ is small in comparison to the backaction dephasing rate Γ:

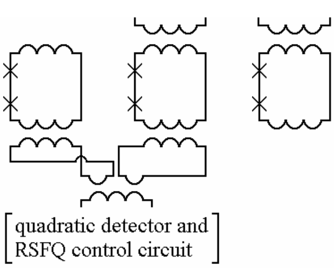


For flux qubits, the QND coupling can be implemented with SFQ circuits, either directly or as a periodic sequence of the ``single-shot'' measurements.

Quadratic measurements

Quadratic detectors enable the measurements of product operators for pairs of qubits:

$$t(\sigma_{z1}+\sigma_{z2})=t'+(\delta/2)(\sigma_{z1}+\sigma_{z2})^2=t+\delta\sigma_{z1}\sigma_{z2}$$



Back-action dephasing rate:

$$\Gamma_d=(1/2)(\sqrt{\gamma_1}-\sqrt{\gamma_0})^2.$$

Spectrum of continuously measured oscillations in two qubits:

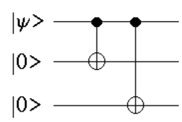
$$S_o(\omega)=S_q+\frac{\Gamma_d}{\pi}\frac{(\gamma_1-\gamma_0)^2\Delta^2}{(\omega^2-4\Delta^2)^2+\Gamma_d^2\omega^2}.$$

Basic error correction

Bit-flip errors

Errors: $|\Psi\rangle \rightarrow \sigma_X|\Psi\rangle$ (i.e. $0 \leftrightarrow 1$)

Encoding: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$



Phase-flip errors

Errors: $|\Psi\rangle \rightarrow \sigma_Z|\Psi\rangle$

Encoding: the same as for bit-flip errors but includes Hadamard transform that effectively turns phase error into bit errors: $\sigma_X = H^+ \sigma_Z H$

Majority code for dephasing errors – (I)

One can correct k dephasing errors by encoding a qubit of information into the 2k+1 physical qubits:

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|00\ldots0\rangle+\beta|11\ldots1\rangle .$$

The main element of the error-correction procedure is the set of the 2k projective measurement of operators $\sigma_x^{(j)}\sigma_x^{(j+1)}$, $j=1,\ldots,2k$. These measurements reduce the state space of the qubit system to the 2k 2×2 subspaces spanned by the states $\{|\Psi\rangle, R|\Psi\rangle\}$, where R is an inversion of all qubit states. Subsequent application of the error-correcting pulses returns all the states into the initial subspace $\{|00\ldots0\rangle, |11\ldots1\rangle\}$.

This procedure correctly reverses all error up to an order k, but the errors of order k+1 exchange the basis states of the initial subspace.

Code for dephasing errors –(II)

Since the period T of the error-correction is necessarily short, *quasi-continuous evolution of the density matrix* in this subspace under the error-correction transformation is governed by the equations:

$$d\rho_{11}/dt = \Gamma^{(k)}(\rho_{22}-\rho_{11}) , \quad d\rho_{12}/dt = \Gamma^{(k)}(\rho_{21}-\rho_{12}).$$

``Rotating'' these equations back to the σ_z basis we see that they describe the usual suppression of the off-diagonal elements of the density matrix with the reduced dephasing rate $\Gamma^{(k)}$:

$$\Gamma^{(k)} = 1/T \sum_{j1> \ldots >jk+1} P_{j1} \ldots P_{jk+1} .$$

In the classical regime, and when initial dephasing rates for all qubits are the same,

$$\Gamma^{(k)} = \Gamma\, C^{k}_{2k+1}\, (\Gamma T)^k \approx \Gamma\, (4\Gamma T)^k ,$$

where the last equation assumes $k>>1$.

In the case of correlated noise, the dephasing rate of the encoded quantum information is increased by renormalized qubit-qubit interaction and directly by the correlations, e.g., for $k=1$:

$$\Gamma^{(1)} = 1/T \sum_{j>j'}[(V_{jj'}T)^2 + P_j P_{j'} +2\, P_{jj'}] .$$

The exponential decrease of the dephasing rate of the encoded quantum information with k is limited by the inaccuracies in the measurement/correction procedure. The most important are inaccuracies in the measurement, which can introduce direct dephasing with rate $\gamma_i k$ of the encoded state:

$$d\rho_{12}/dt = \Gamma^{(k)}(\rho_{21}-\rho_{12}) - \gamma_i k \rho_{12} .$$

The main, but probably obvious, conclusion is that for the error-correction to make sense, the introduced dephasing should be at least smaller than the original qubit dephasing.